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Sierra/SD: A Massively Parallel Finite Element Code for Structural Dynamics and Acoustics Analysis

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Introduction / Outline

- History
- Domain Decomposition
- Solution Methods
- Element Types
- Structural Acoustics Formulation
- Quadratic Eigenvalue Problem
- Structural Acoustic Tying/Mortars
- Infinite Elements
- Inverse Methods
- Conclusions

History and Intent

- **Sierra/SD was created in 1990's as part of the Accelerated Strategic Computing Initiative (ASCI) of the US Dept. of Energy**
- **Intended for *extremely* complex finite element analysis**
  - Models with 10s or 100s of millions of DOF
- **Scalability**
  - Ability to solve *n*-times larger problem using *n*-times more compute processors in nearly constant CPU time
- **Code portability**

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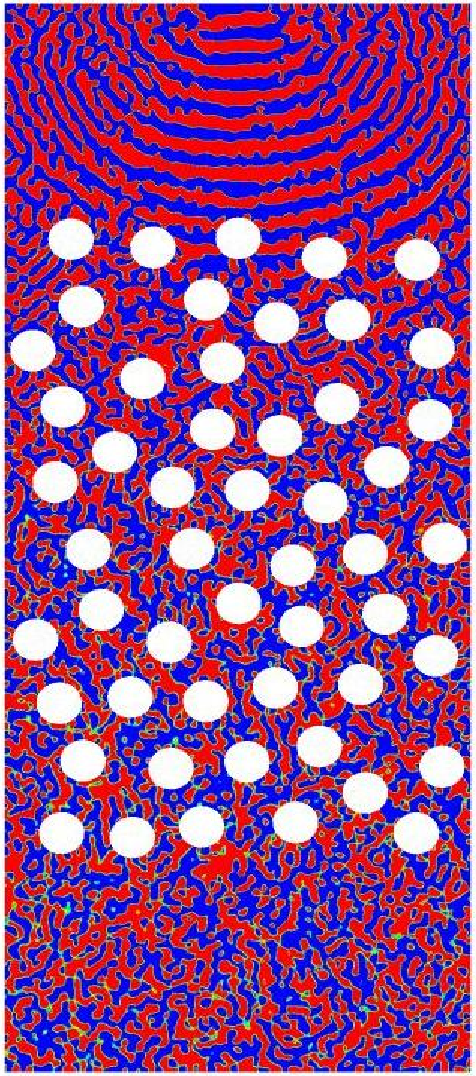
An Illustration of Intent

- **Ultrasonic wave propagation in elastic plate**
  - 4x10x1 in. Aluminum
  - 1 MHz FRF shown
- **Examine hole size/shape effects on scattering**
  - Visualize diffuse field development in elastic solids
- **For results shown:**
  - 32 elements/λ
  - 57,255,317 nodes
  - 343,531,902 degrees of freedom



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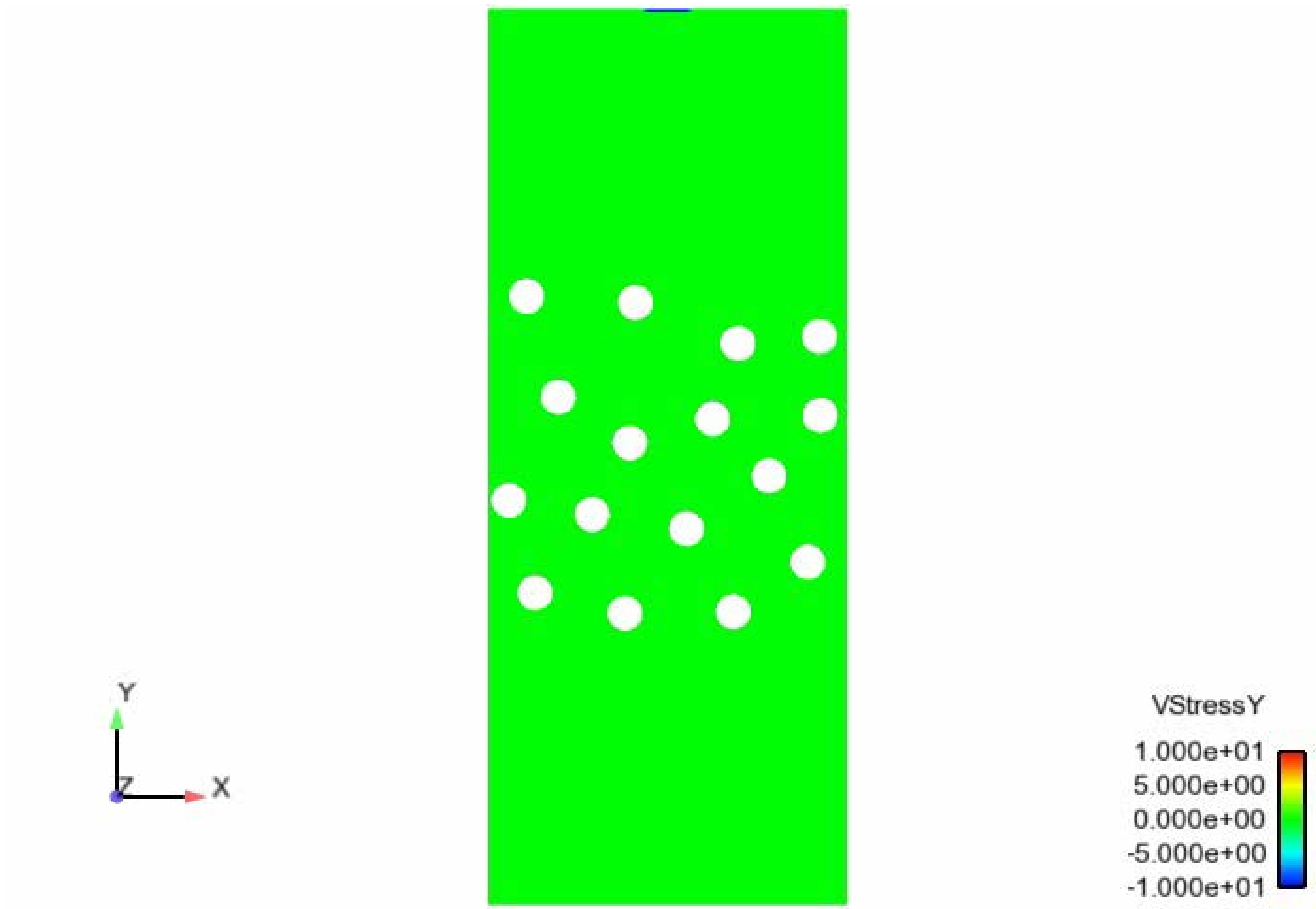
An Illustration of Intent: 1μs Pulse



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To Meet ASCI Requirements

- **Employ Domain Decomposition Methods**
  - First performed by Schwarz in the 1870s
- **Massively Parallel**
  - Distribution of processors (nodes), each with own memory, linked together by a specialized network communication system
- **Began First Using FETI-DP solver**
  - "Finite Element Tearing and Interconnecting" *(C. Farhat, et al., 2000)*
  - Versatile iterative solver
- **Current Solvers:**
  - FETI-DP and FETI-DPH
  - GDSW *(C. Dohrmann, et al., 2007)*
  - Others

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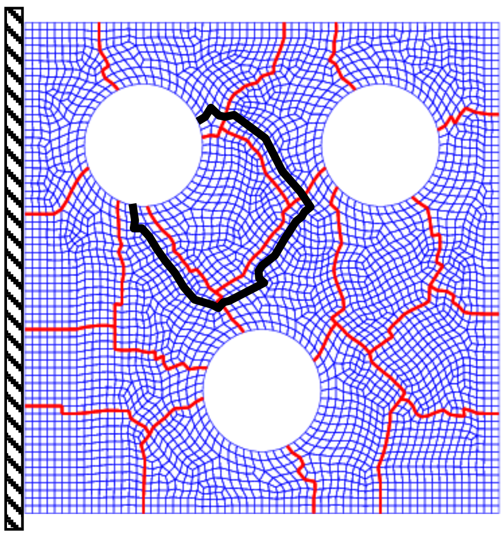
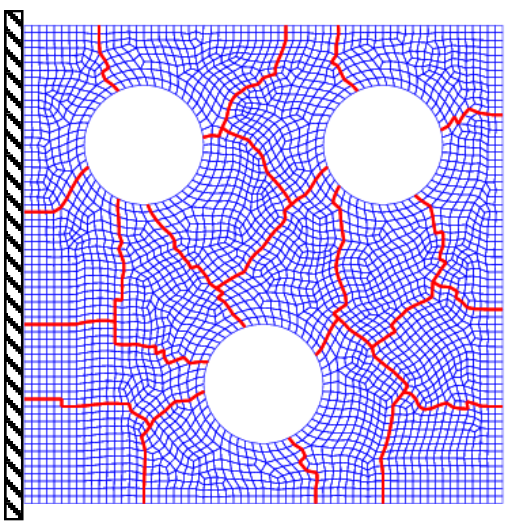
Domain Decomposition



Schwarz Methods
(Overlapping)



Schur Complement
Methods
(Iterative
Substructuring)

- Decompose model into smaller subdomains
- Each subdomain is often assigned to one processor
- Two-level methods have "local" subdomain solves and "global" coarse solve
- Solve using preconditioned conjugate gradients or GMRES

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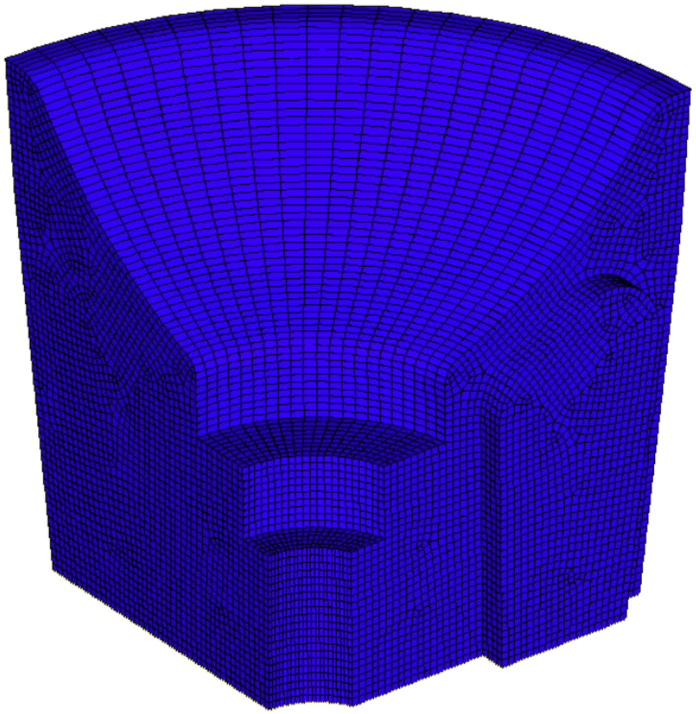
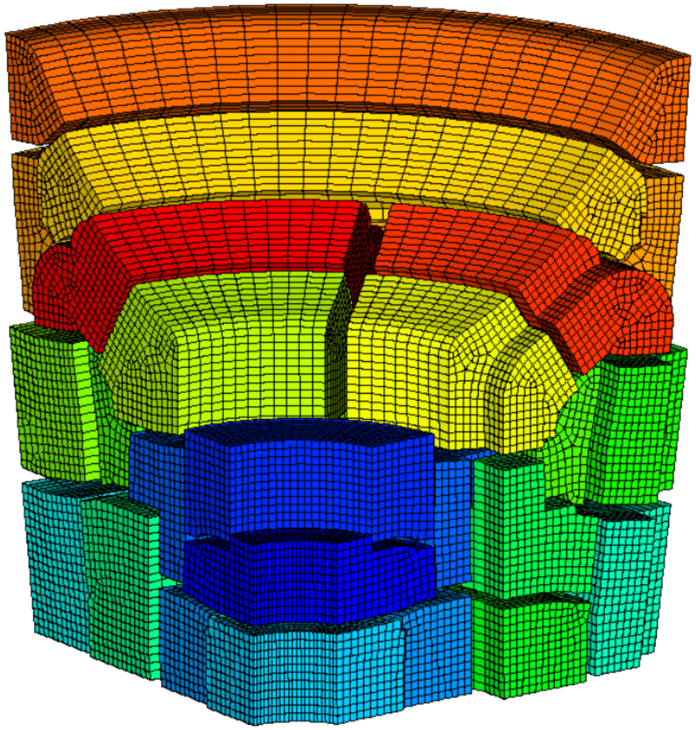
Domain Decomposition Example

Single Mesh Decomposed Into 20 Meshed Subdomains





Current State of High Performance Computing

- **1.37 petaFLOPS capability system, built by Cray, Inc**
- **Installed 2010-2011 at Los Alamos National Laboratory**
- **Compute nodes: 8,944**
  - Each compute node: 2 AMD G34 Opteron Magny-Cours 2.4 GHz 8 core processors for a total of 143,104 cores

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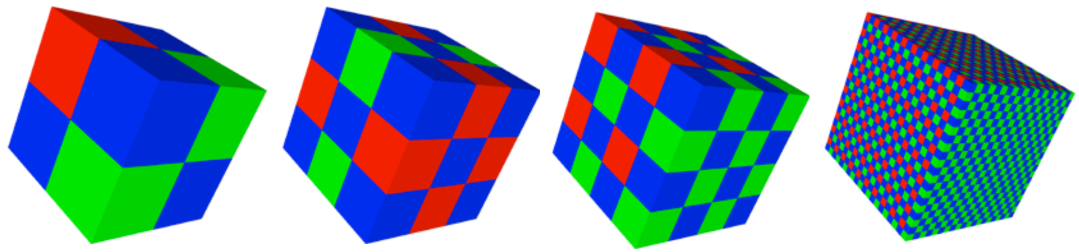
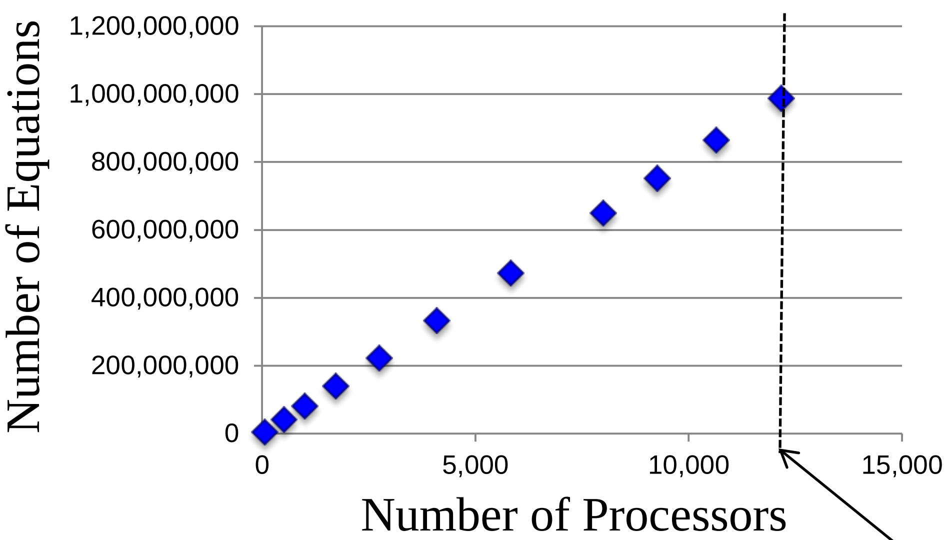
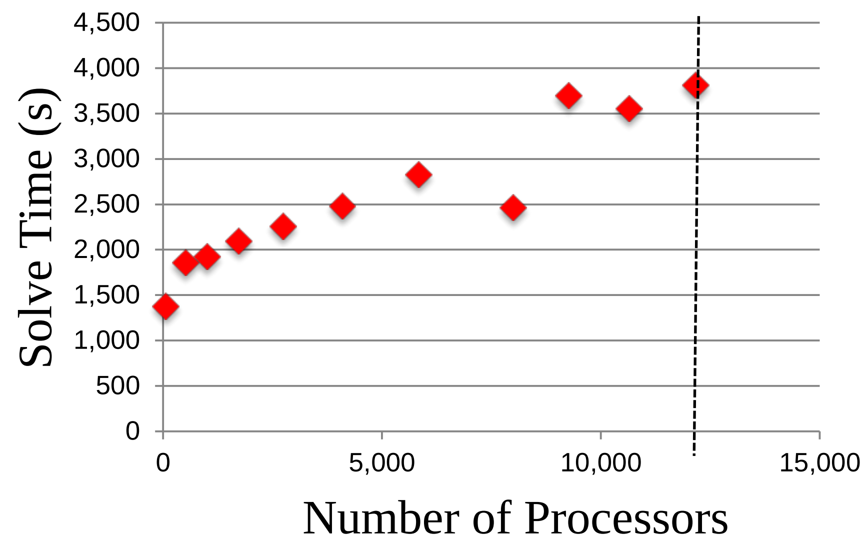
Eigenvalue Scaling Studies

Scaling studies were performed to characterize solver performance to 1 billion equations, well beyond previous work







Hit 32-bit integer limitation in Sierra

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Sierra/SD Solution Methods

- **Linear and Nonlinear Statics and Transient Dynamics**
- **Eigenanalysis**
  - Real and complex (quadratic)
- **Direct Frequency Response**
- **Random Vibration Analysis**
- **Modal Based Solutions for Transient Dynamics, SRS, Frequency Response**
- **Coupled Nonlinear-Linear Analysis**
  - With Adagio/Presto *(Sandia in-house codes)*

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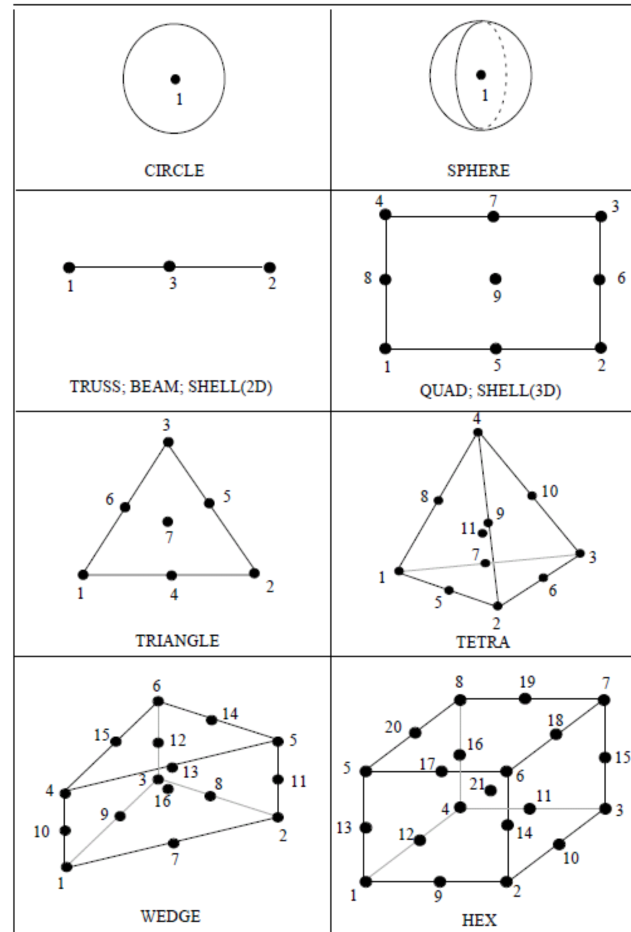
Large Element Library

- **Solid Elements**
  - Hexahedral, Tetrahedral, Wedge
- **Shell Elements**
  - Triangle, Quadrilateral, HexShell (hybrid)
- **Bar/Beam Elements**
  - Beam, Truss, Spring, Dashpot
- **Point Elements**
  - Conmass (concentrated mass)
- **Specialty Elements**
  - Iwan, Hys, Shys, Joint2G, Gap



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Structural Acoustics

- **Formulations for Structural Acoustics:**

Scalar Based
  - Velocity potential formulation *(Everstine, 1981, 1997)*
  - Mixed pressure-potential symmetric formulation *(Felippa & Ohayon, 1990; Pinsky, 1991; Ohayon 1996)*

Vector Based
  - Displacement-based formulation *(Hamdi & Ousset 1978; Belytschko, 1980; Wilson, 1983; Chen 1990; Bermudez 1994)*
  - Space-time formulation *(Harari et al., 1996; Thompson and Pinsky, 1996)*

  - Others …

- **All fully-coupled formulations (monolithic)**

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Structural Acoustics Formulation

- **Applied two-field formulation of Everstine[1]**
  - Structural displacement
  - Fluid velocity potential
- **Exterior problems straightforward**
  - Compared to other formulations
- **Symmetric, indefinite matrices**
  - Best suited for domain decomposition-based solvers
- **Results in 2nd order equations**
  - Compatible with Newmark beta and alpha time integration
- **Added by Tim Walsh beginning in 2003**

[1] G. C. Everstine, "Finite Element Formulations For Structural Acoustics Problems," *Computers & Structures* **65**: 307-321, (1997).

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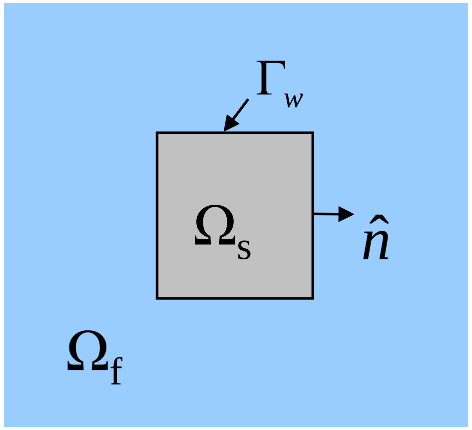
Structural Acoustics Formulation



Structure: $\rho_s \frac{\partial^2 \vec{u}}{\partial t^2} - \vec{\nabla} \cdot \tau = \vec{f}(\vec{x}, t) \quad \Omega_s \times [0, T]$

Fluid: $\nabla^2 \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = 0 \quad \Omega_f \times [0, T]$

Fluid-Structure B.C.' s:

$$\tau \cdot \hat{n} = -\frac{\partial \varphi}{\partial t}$$

$$\rho_f \frac{\partial \vec{u}}{\partial t} \cdot \hat{n} = -\vec{\nabla} \varphi \cdot \hat{n}$$

- **Resulting time domain finite element form:**

$$\begin{bmatrix} M_s & 0 \\ 0 & \tilde{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} C_s & L \\ L^T & \tilde{C}_f \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\varphi} \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & \tilde{K}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} = \begin{Bmatrix} f_s \\ \tilde{f}_f \end{Bmatrix}$$

Coupling occurs
in damping matrix

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Structural Acoustics Solvers/Capabilities

- **Full massively parallel functionality**
- **Hex, wedge, and tetra acoustic elements**
- **Acoustic coupling with both 3D and shell (2D) structural elements**
- **Allows for mismatched acoustic/solid meshes**
  - Inconsistent Tying
  - Standard Mortars
- **Solvers: FETI-DP, GDSW**
- **Solution Procedures:**
  - Frequency Response (frequency-domain)
  - Transient (time-domain)
  - Eigenvalue Analysis (real and quadratic)
- **Nonlinear Acoustics – Kuznetsov Equation**

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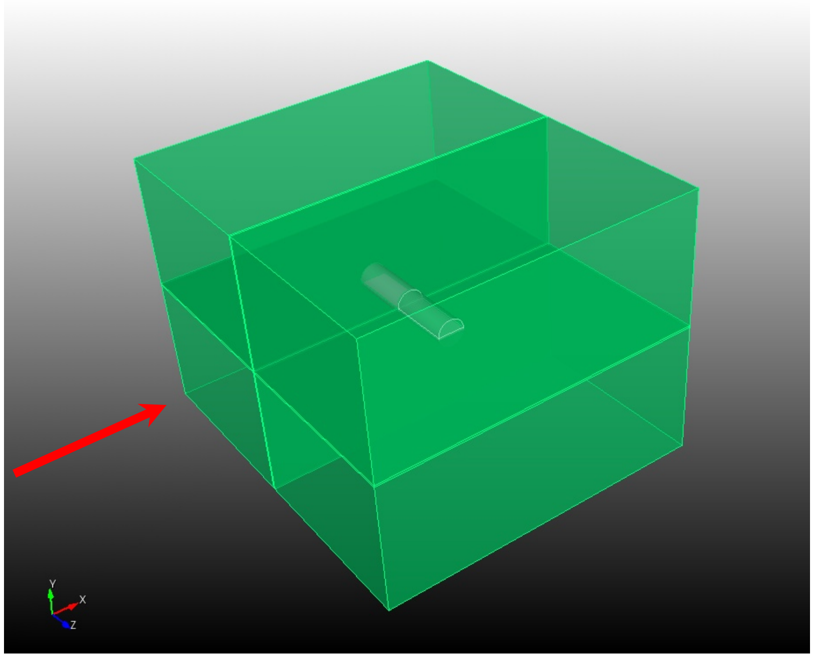
Scattering From Air-Filled Cylinder in Elastic

- **Dimensions:**
  - L_x=51.87 m., L_y=42.32 m., L_z=60 m.
  - Tunnel radius = 2.1373 m.
  - Tunnel length = 20 m.
- **4,882,400 Hexahedral 8-node elements**
  - Elements 0.3 x 0.3 x 0.3 meters



- **Material properties:**
  - Homogenous, isotropic elastic solid
  - Metamorphic rock
    - ρ = 2500 kg/m^{3,} c_p= 4000 m/s, c_s= 2400 m/s
  - Fluid in tunnel:
    - ρ_{air} = 1.2 kg/m^{3,} c_{air}= 343 m/s
- **Resolution:**
  - Solid up to ~ 800 Hz.
  - Fluid up to ~ 114 Hz.

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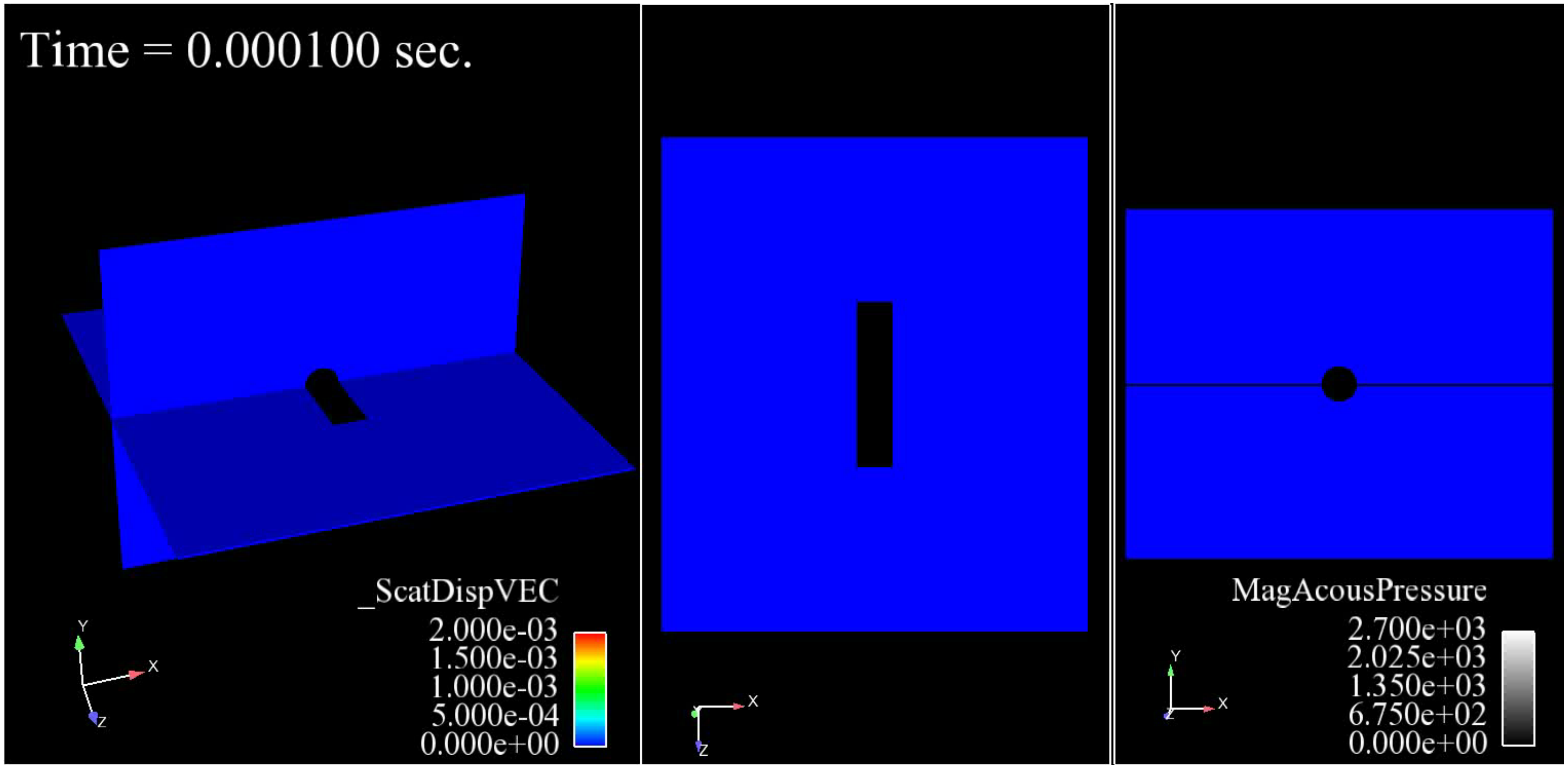
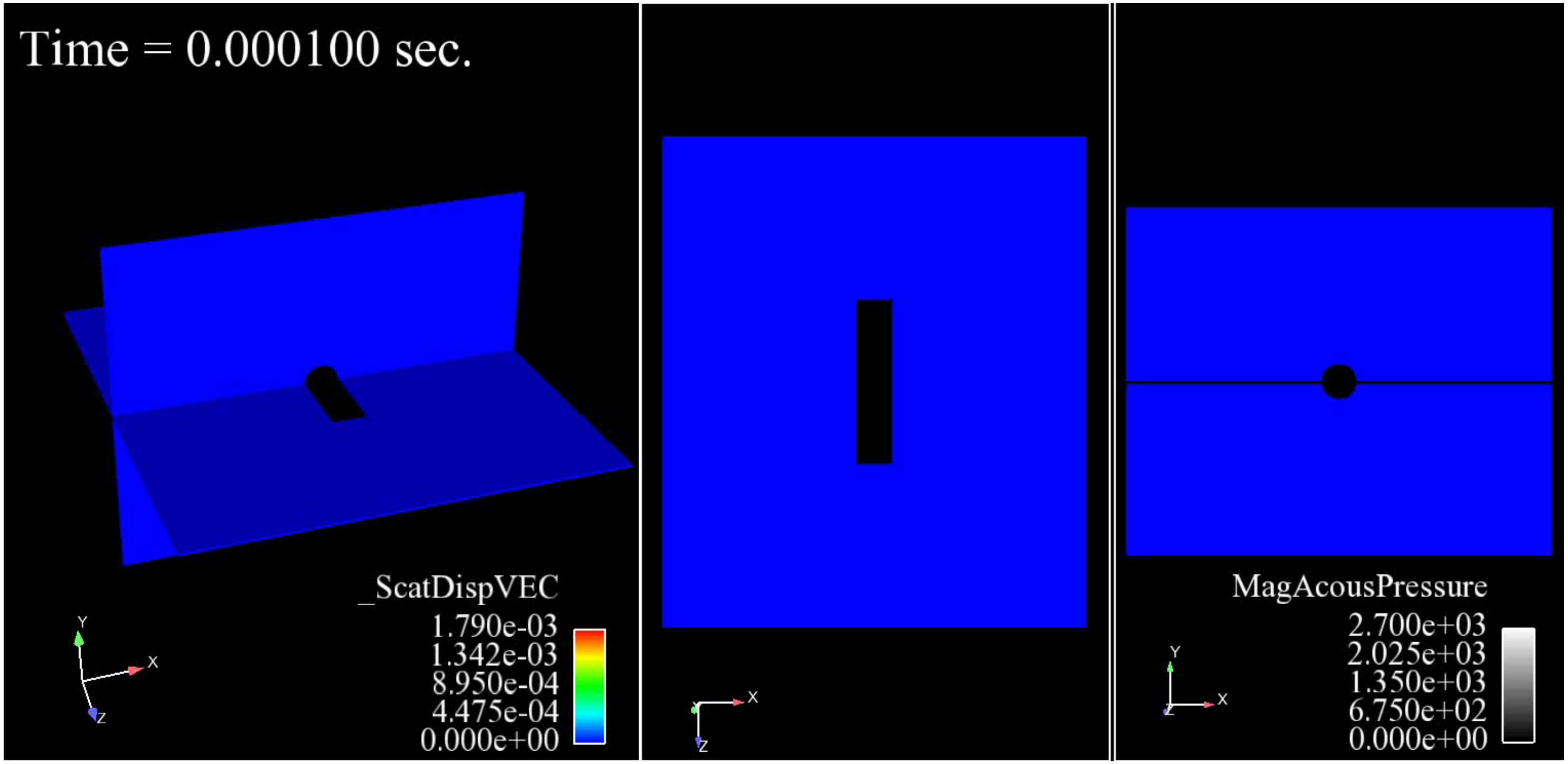
P Wave Excitation

- **Scatter From Air-Filled Tunnel:**



SH Wave Excitation

- **Scatter From Air-Filled Tunnel:**



Quadratic Eigenvalue Problem

- **Eigenanalysis formulation:**

$$\lambda^2 \begin{bmatrix} M_s & 0 \\ 0 & \tilde{M}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} + \lambda \begin{bmatrix} C_s & L \\ L^T & \tilde{C}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & \tilde{K}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

  – Coupling within damping matrix brings about complex eigenvalues for structural acoustics (non-diagonalizable)

- **Solve by converting to state-space form:**

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \{w\} = \begin{bmatrix} 0 & M \\ -M & -C \end{bmatrix} \{\dot{w}\} \text{ where } w = \begin{Bmatrix} \dot{r} \\ r \end{Bmatrix}$$

- **Depending on BC's, must solve both right *and* left eigenvalue problem**

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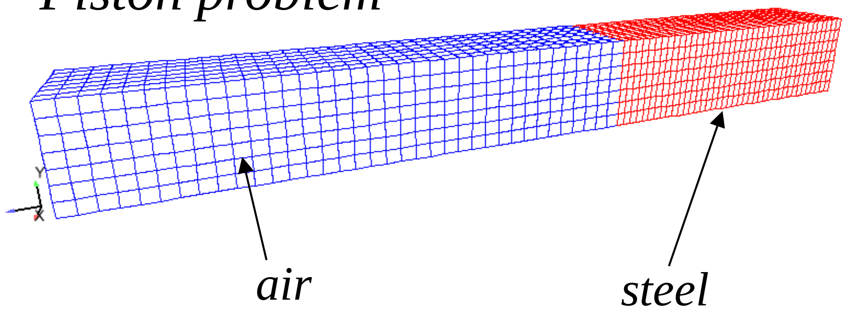
Complex Eigenvalue Modal Analysis

Piston problem

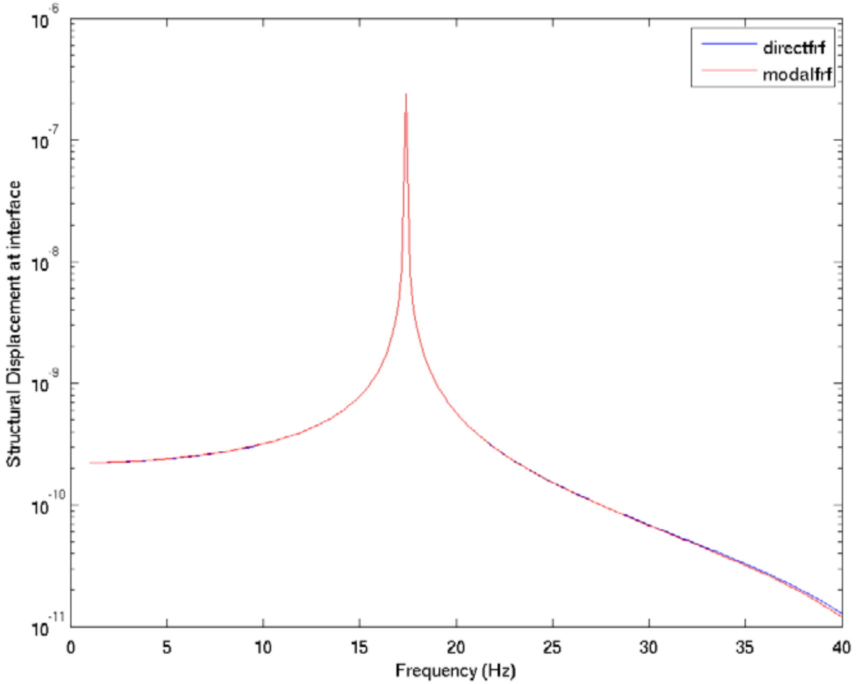


- **DirectFRF:**

$$u(\omega) = \frac{F(\omega)}{-\omega^2[M] + i\omega[C] + [K]}$$

- **ComplexModalFRF:**
  - Use complex modes from quadratic eigenvalue solution

A comparison of structural displacement from directFRF vs CmodalFRF



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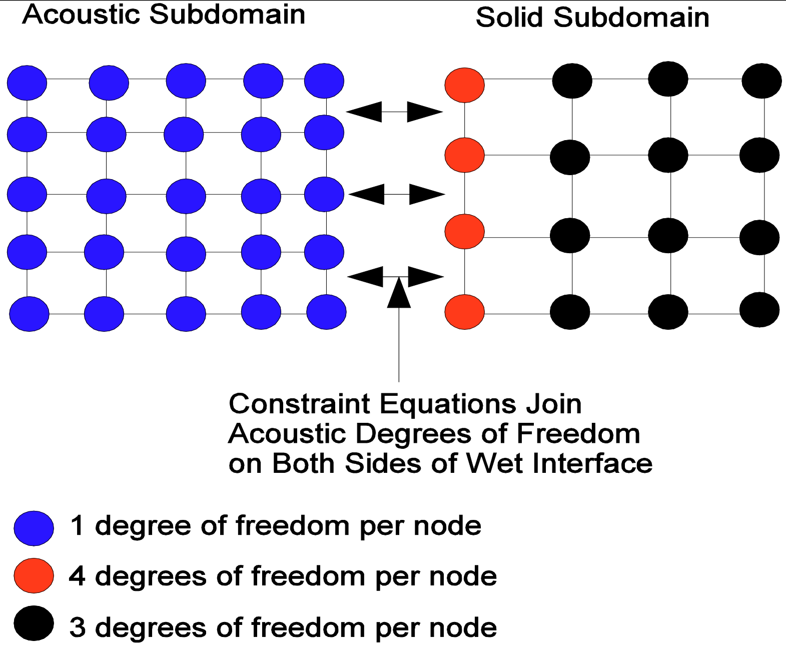
Mismatched Acoustic/Solid Meshes



- **Mesh density requirement inconsistency**
  - Acoustic phase speed < structural (typically)
- **Solution: tying/mortars**
  - Use ghost acoustic d.o.f. on solid nodes at interface, conforming coupling to solid
  - Couple the acoustic d.o.f. now on both sides of wet interface using constraint equations

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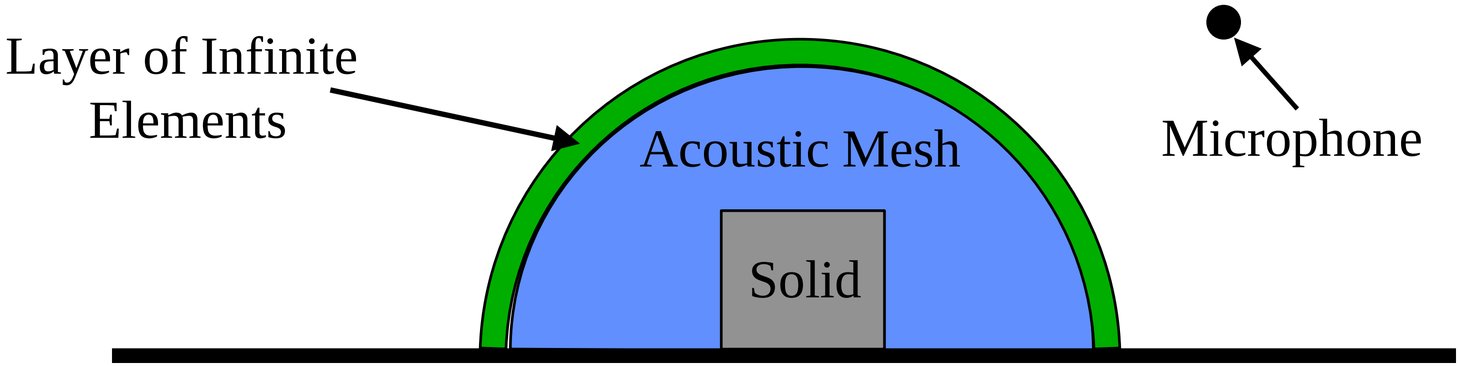
Infinite Elements Capability



- **Provides an asymptotically exact boundary condition for exterior problems**
- **Allows for computing response at far-field points outside of acoustic mesh**
- **Currently implementing time-domain, conjugated version of "mapped wave envelope" elements of Astley et al.**

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Nonlinear Acoustics

- **Linear (first-order) acoustic wave equation:**

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = 0$$

- **Nonlinear (second-order) wave equation[2]:**

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = \frac{1}{c^2}\frac{\partial}{\partial t}\left[(\nabla\varphi)^2 + \frac{B/A}{2c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2 + \frac{b}{\rho_o}\nabla^2\varphi\right]$$

where $b = \kappa\left(\frac{1}{c_v} - \frac{1}{c_p}\right) + \frac{4}{3}\eta + \zeta$

- **Not yet implemented for structural acoustics**

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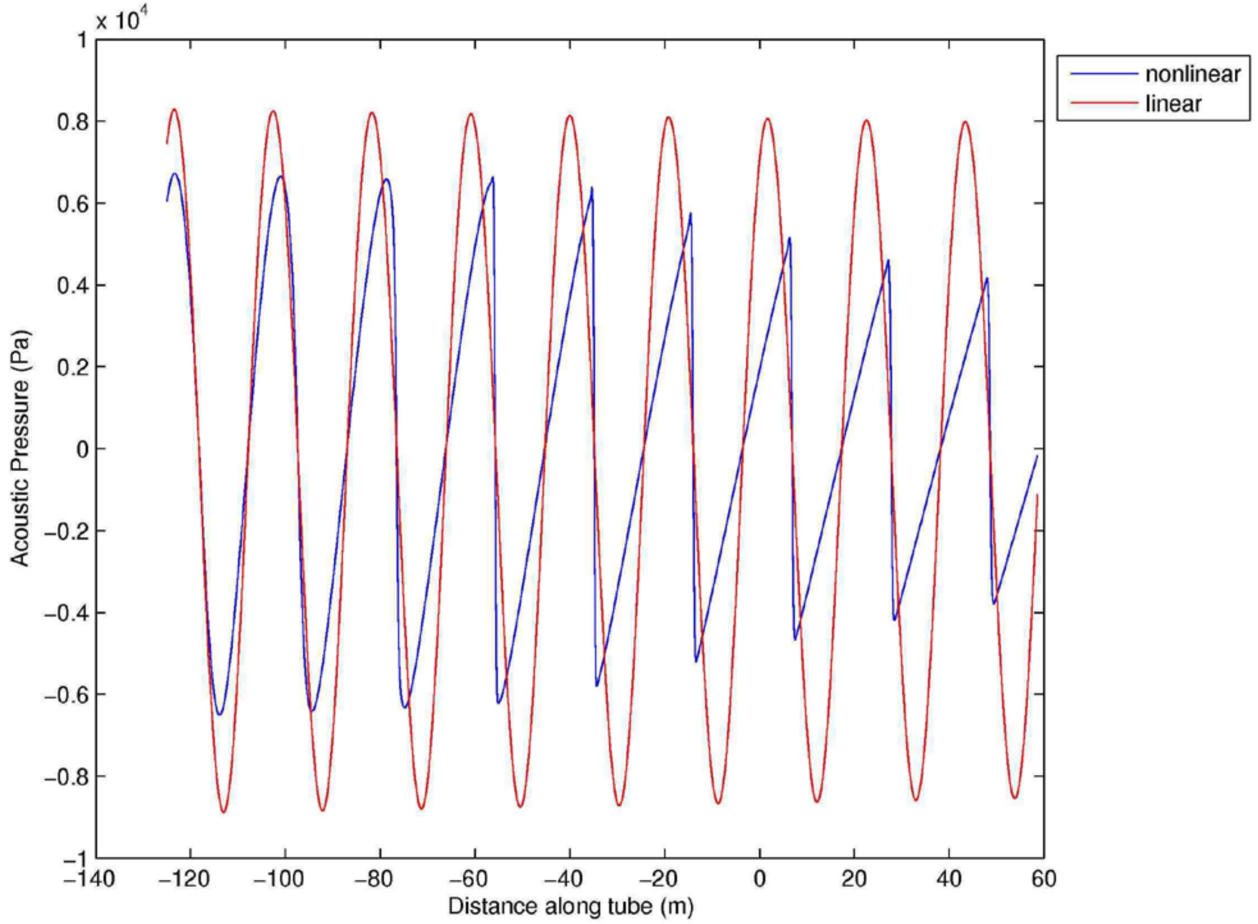
[2] V. P. Kuznetsov, "Equations of Nonlinear Acoustics," *Sov. Phys. Acoust.* **16**: 467-470, (1971).

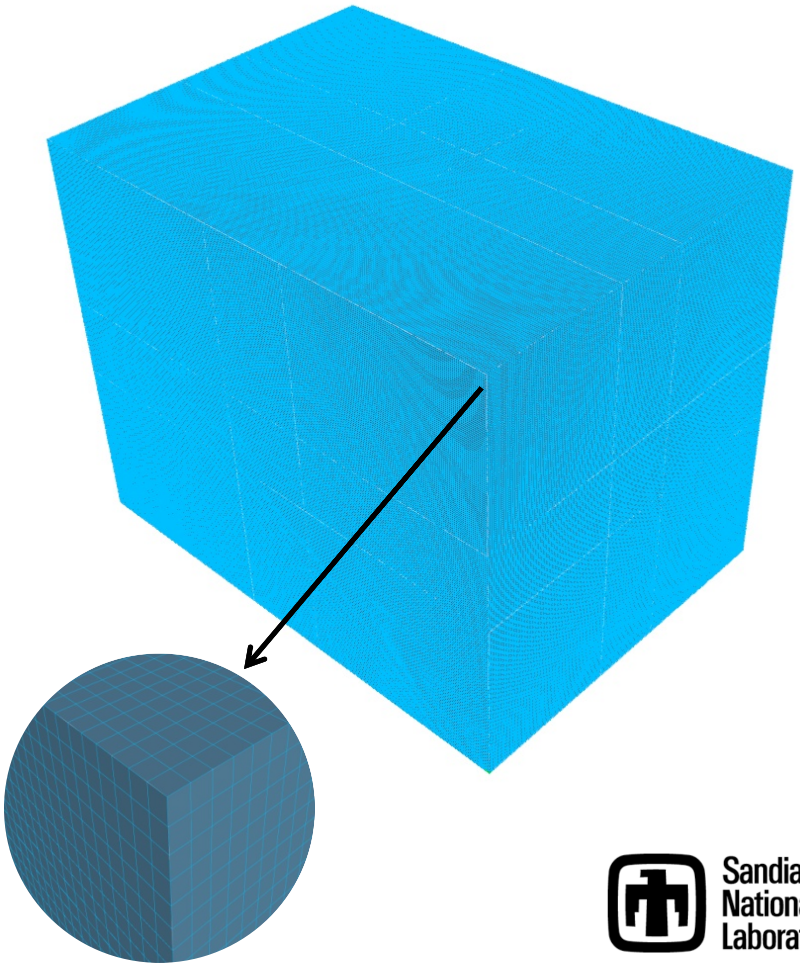
Linear vs Nonlinear Acoustics

Shock-Tube Simulation



Transient Excitation of Reverb Chamber

- **16,000 ft^3 reverb chamber**
  - Wall BCs consistent with real chamber
- **Meshed 10 ele / λ at 1 kHz**
  - ~ 11.33 million nodes
- **Excited with 1 kHz sine**
  - 1000 time steps at dt = 0.0001 s
- **Used 800 processors**
  - Took 15 minutes to complete



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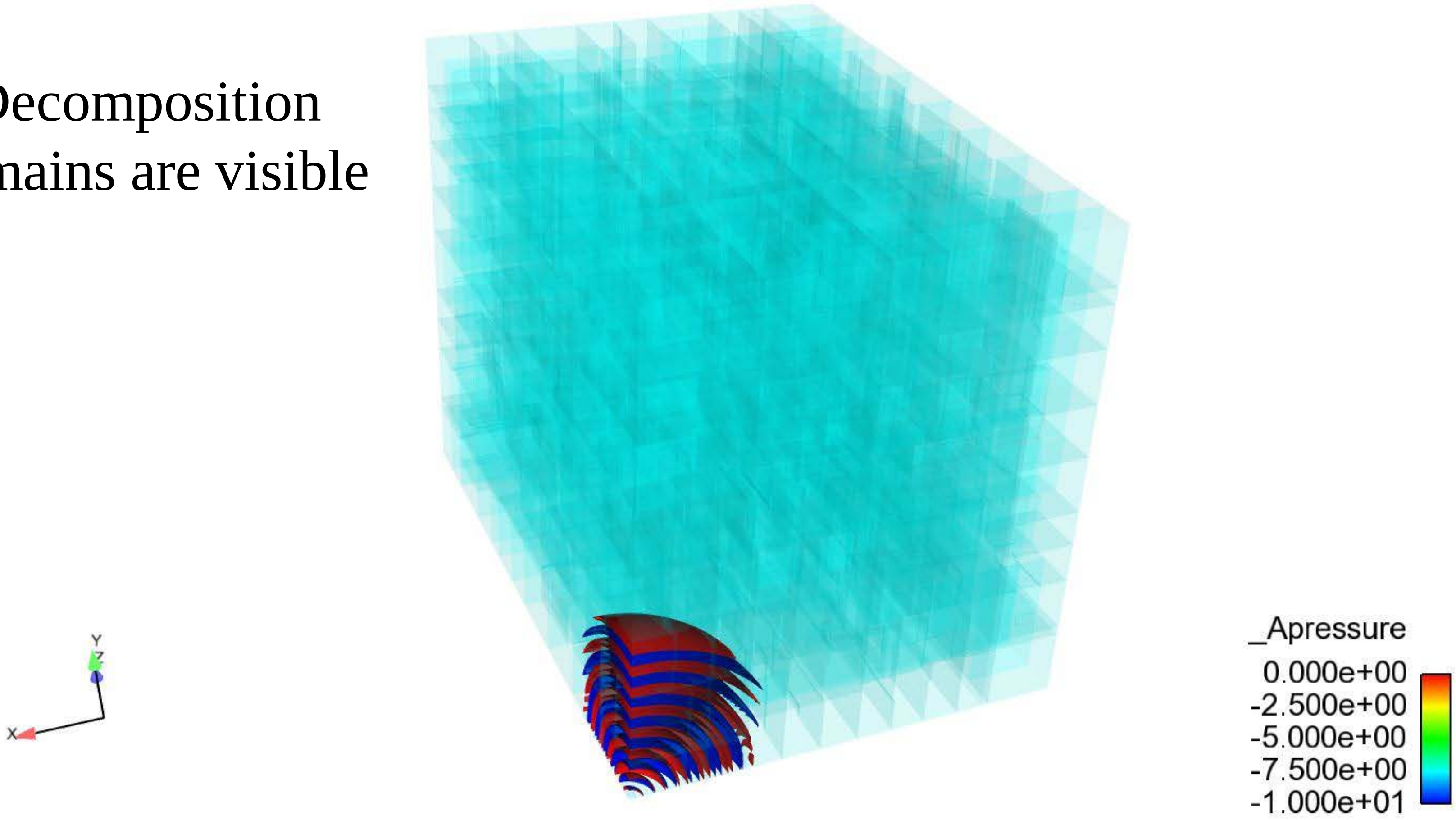
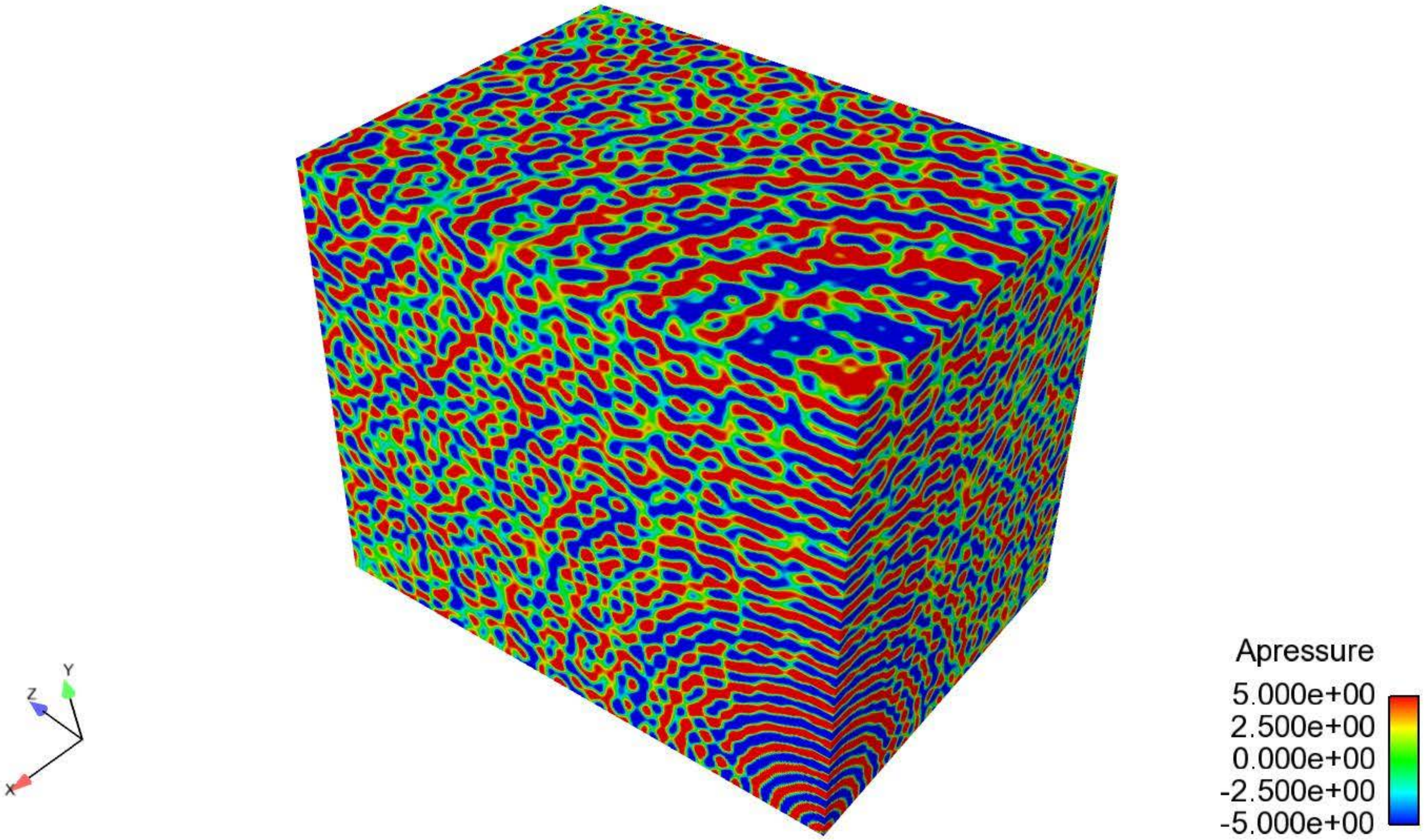
Transient Excitation of Reverb Chamber

Decomposition domains are visible



Transient Excitation of Reverb Chamber



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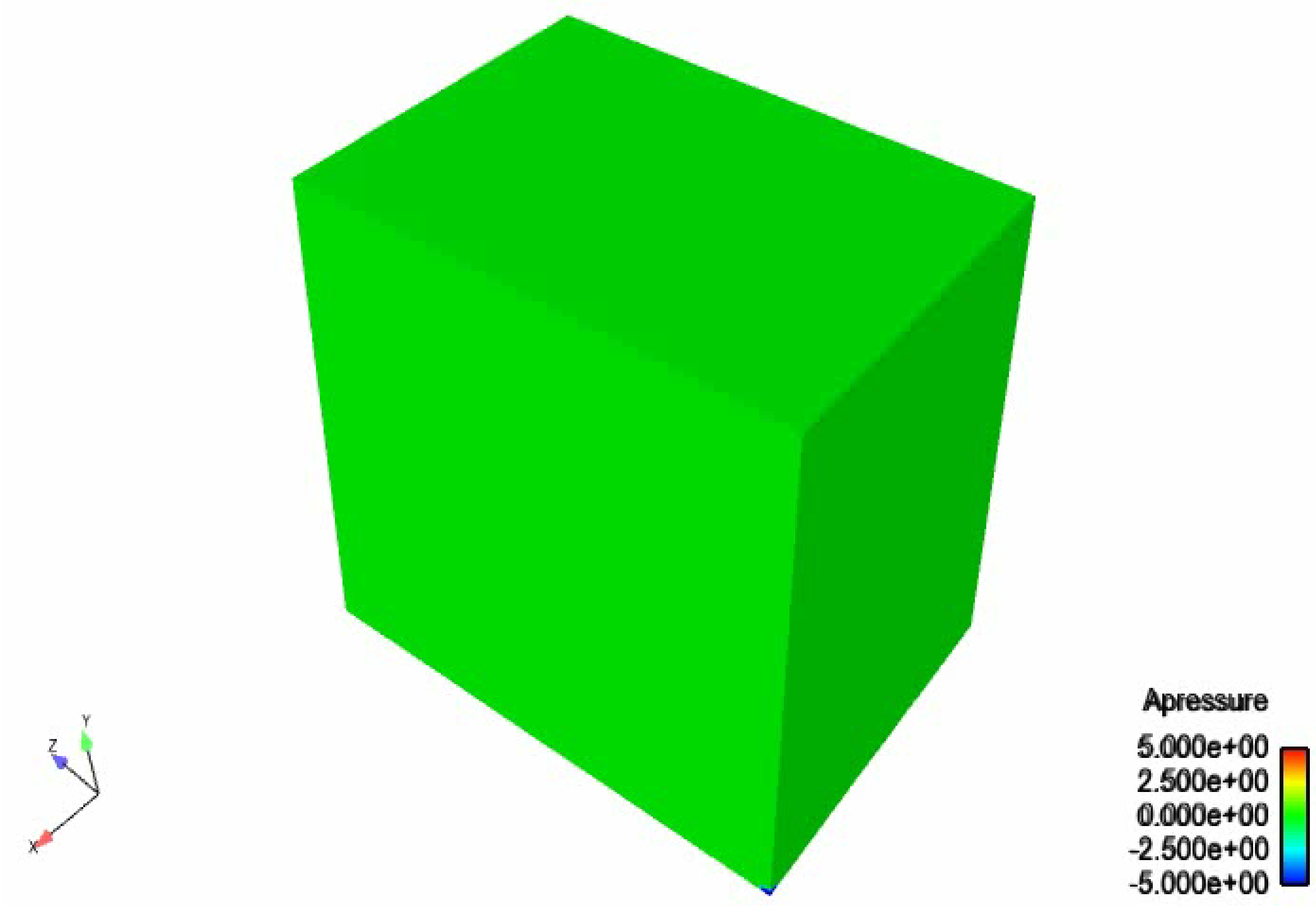
Transient Excitation of Reverb Chamber



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Inverse Capabilities

- **Joint work with Wilkins Aquino, Duke University**
- **Emerging capabilities aimed at providing force, shape and material inversion capabilities**
  - All capabilities are parallelized
- **Current capabilities:**
  - Shape inversion using topological derivatives
  - Material inversion for elastics in frequency domain
  - Fource/source inversion for acoustics

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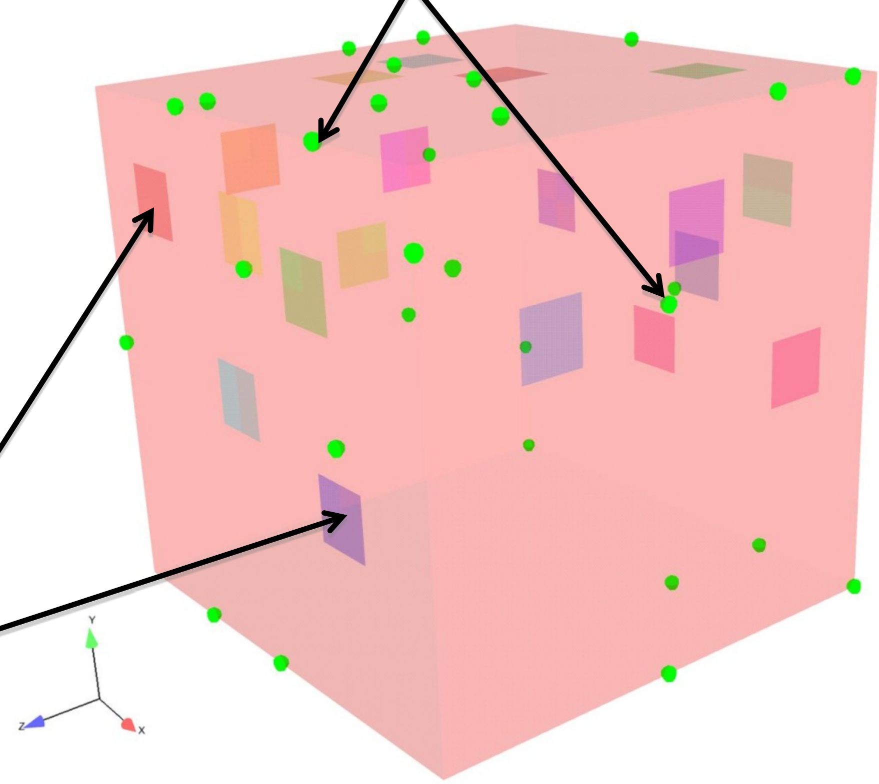
Acoustic Source Inversion Test

- **Model of acoustic reverb chamber**
  - All boundaries rigid
- **18 unknown speaker inputs**
- **29 internal microphones**
- **Microphone data generated by running forward problem**
  - Randomly chosen amplitudes
- **Forward simulation: FRF at 4 Hz**

Microphone locations (green)



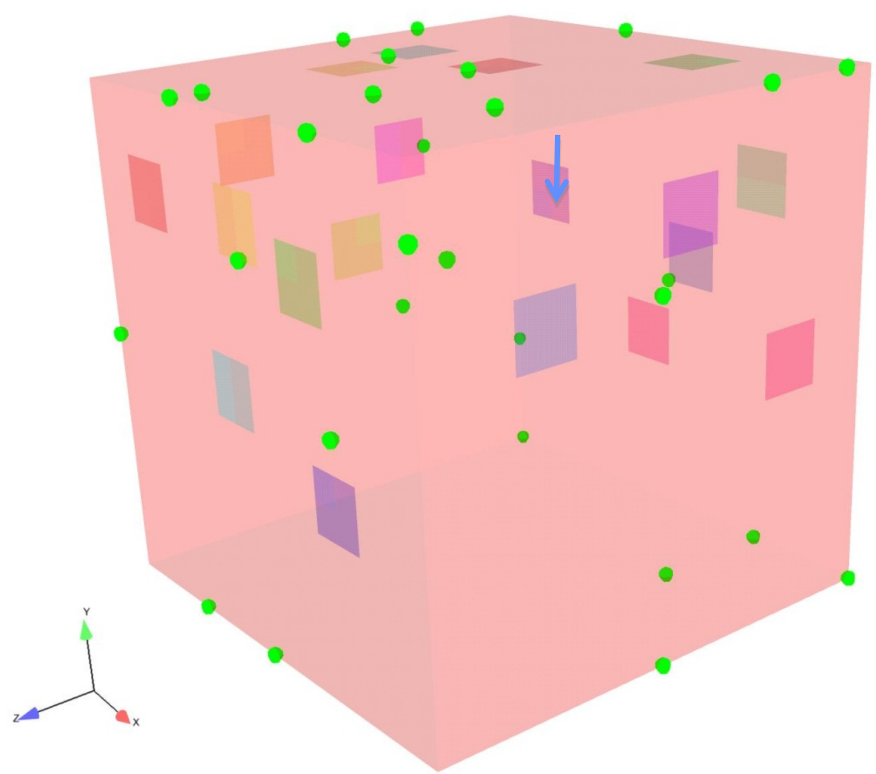
Unknown speaker inputs
(square patches on boundary)

Acoustic Source Inversion Test

Inverse solution results:

	Exact Solution from Synthetic Data	Predicted (from inverse solve)	Initial Guess
Speaker 1	10	10.002	0
Speaker 2	10	10.001	0
Speaker 3	10	10.01	0
Speaker 4	10	9.998	0
Speaker 5	10	10.01	0
Speaker 6	10	10.00	0
Speaker 7	10	10.02	0
Speaker 8	20	19.99	0
Speaker 9	1	1.002	0
Speaker 10	1	1.03	0
Speaker 11	1	1.00	0
Speaker 12	1	0.836	0
Speaker 13	1	1.585	0
Speaker 14	1	1.269	0
Speaker 15	1	0.942	0
Speaker 16	1	1.484	0
Speaker 17	1	0.966	0
Speaker 18	1	0.890	0

29 Measured microphone data (green spheres)
18 Unknown speaker inputs (square patches)

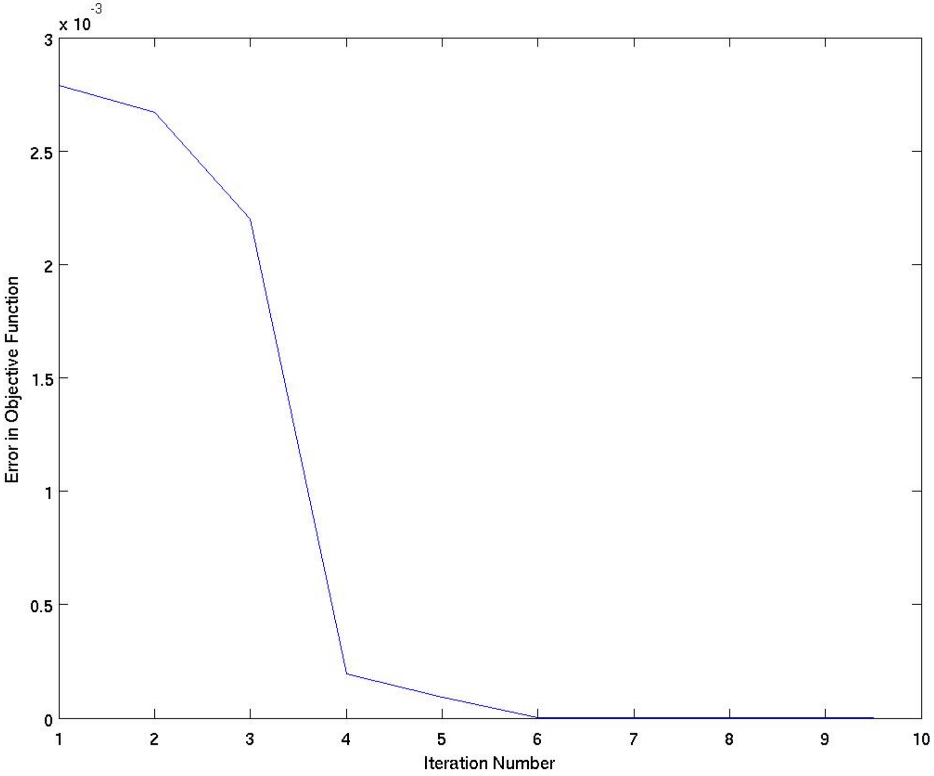


Acoustic Source Inversion Test

Observations:

- **Convergence not sensitive to initial guess, as expected. The inverse problem is quadratic.**
- **Lower amplitude inputs converge more slowly (lower sensitivity), as expected.**

Objective Function



Future Capabilities

- Develop parallel solver for structural acoustic Helmholtz equation
- Extend inverse methods to structural acoustics for both time and frequency domain
- Explore special elements for high frequency acoustics
- GDSW three-level parallel solver for problems requiring over 100,000 processors (available now)

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Conclusions

- **Massively Parallel FEM**
- **Fully Coupled Structural Acoustics**
- **Quadratic Eigenvalue Solver**
- **Structural Acoustic Tying/Mortars**
- **Infinite Elements**
- **Inverse Methods**

- **Salinas is an export controlled code. Shared with other US Government Labs for use.**
- **For Inquiries:**

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