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Title: Adaptive grid Particle-In-Cell methods

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Adaptive grid Particle-In-Cell methods

Gian Luca Delzanno

Collaborators: J.D. Moulton, E. Camporeale

Outline

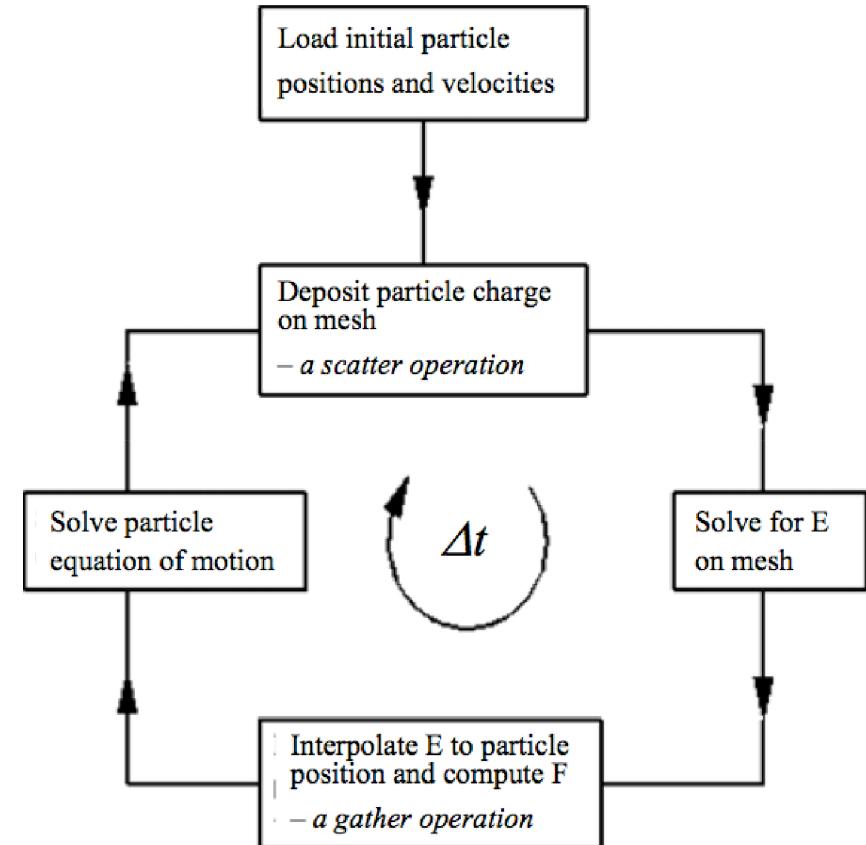
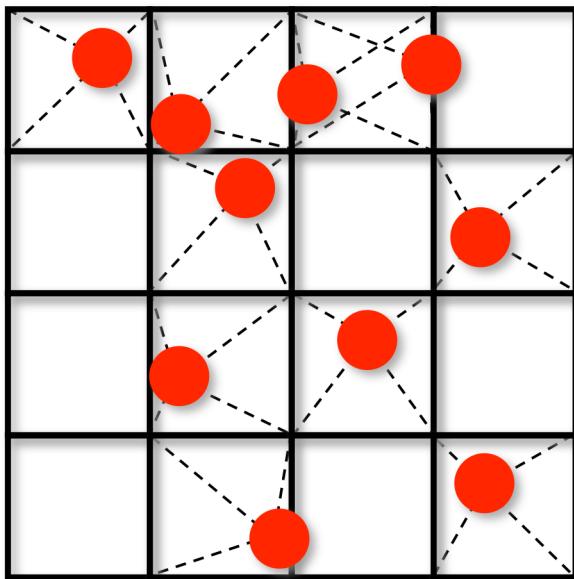
- Motivation: plasma-material interaction
- CPIC:
 - Solver: multigrid
 - Particle mover: hybrid
- Tests
- Conclusions

Particle-In-Cell 101

Solves Vlasov's equation with **macroparticles**:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla_x f_\alpha + \frac{\mathbf{E} + \mathbf{v} \times \mathbf{B}}{m_\alpha} \cdot \nabla_v f_\alpha = 0$$

+ Maxwell's equations



General motivation: plasma-material interaction

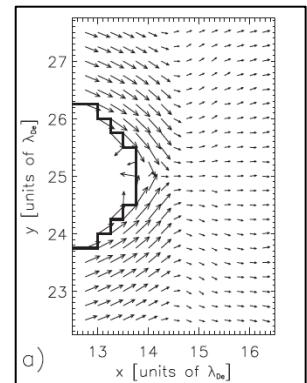
- Traditional PIC: uniform Cartesian grid, explicit time stepping
- **Explicit time stepping (leap-frog)**
 - Must resolve shortest length scale and fastest frequency
 - Plasmas can be **multiscale** $\lambda_D, \rho_e, \rho_i$
- Plasma-material interaction:
 - Additional scales due to the object: **'more' multiscale!**

Example: **geosynchronous satellite**

- Spacecraft size $\sim 1\text{-}10$ m
- Debye length ~ 300 m
- Electron gyroradius ~ 800 m
- Ion gyroradius ~ 30 km



A uniform grid would require at least 10^5 cells in each spatial direction
UNFEASIBLE!

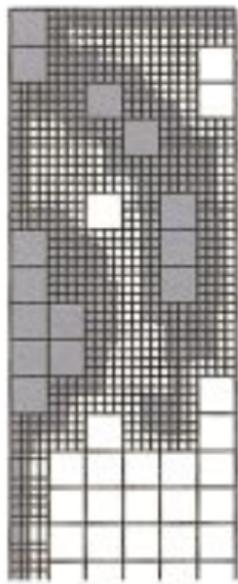


Miloch et al, PRE 08

General motivation: plasma-material interaction (2)

- Needs some kind of **adaptivity**

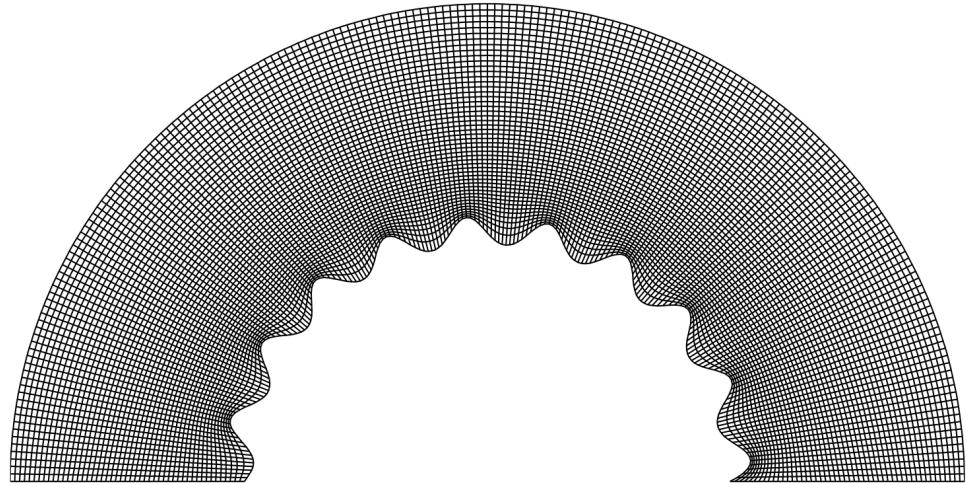
1) AMR:



Vay et al, POP 2004,
Fujimoto&Sydora, 2008;
Usui et al, 2010; ...

2) **Body-fitted**: our choice

Westerman, 1994; Eastwood et al. 1995, Wang et al. 1999
Spacecraft charging community

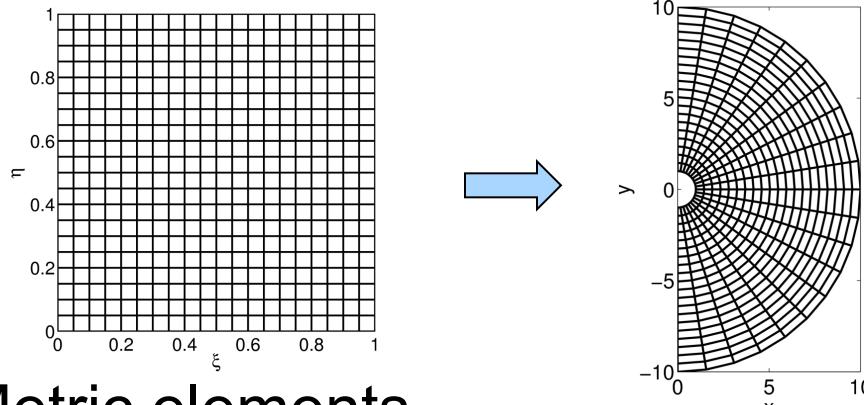


- Other design constraints:
 - **Flexible, robust**
 - **Geometry independent, i.e. curvilinear**
 - **Performance**

Body-fitted logical to physical space mapping

- Coordinate transformation: $\mathbf{x} = \psi(\xi)$

- Physical space variables $\mathbf{x} = (x, y, z)$
- Logical space variables $\xi = (\xi, \eta, \zeta)$



$$\begin{aligned}x &= [r_1 + (r_2 - r_1) \xi] \sin(\pi \eta) \\y &= [r_1 + (r_2 - r_1) \xi] \cos(\pi \eta) \\z &= \zeta\end{aligned}$$

- Metric elements

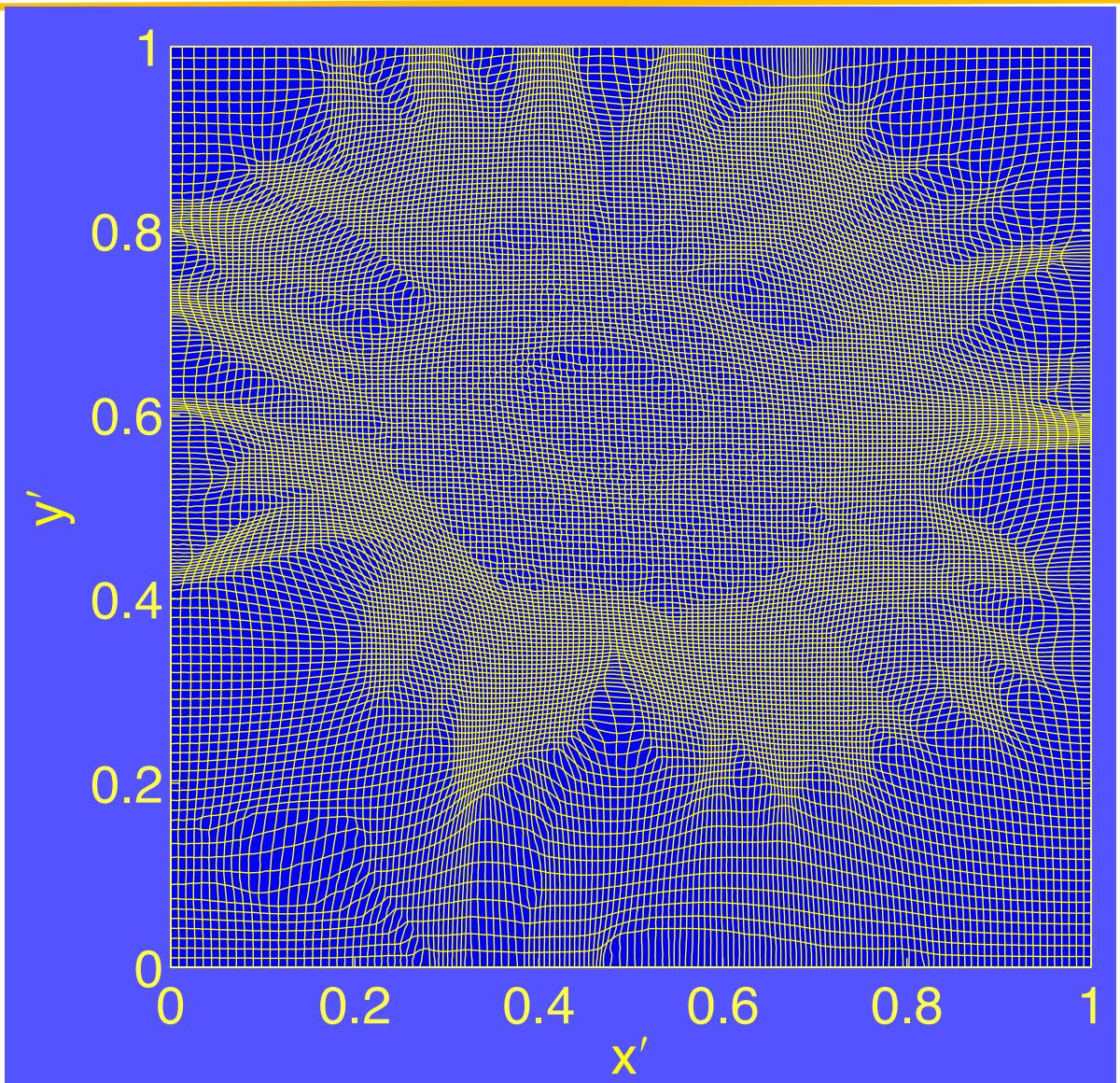
- Jacobi matrix and its inverse: $j_{\alpha\beta}(\xi) = \frac{\partial x^\alpha}{\partial \xi^\beta}$, $k^{\alpha\beta}(\mathbf{x}) = \frac{\partial \xi^\alpha}{\partial x^\beta}$
- Metric tensors: $g_{\alpha\beta}(\xi) = \frac{\partial x^\gamma}{\partial \xi^\alpha} \frac{\partial x^\gamma}{\partial \xi^\beta}$, $g^{\alpha\beta}(\mathbf{x}) = \frac{\partial \xi^\alpha}{\partial x^\gamma} \frac{\partial \xi^\beta}{\partial x^\gamma}$
- Operators in logical space:

$$\nabla_{\mathbf{x}}^2 \Phi(\mathbf{x}) = \frac{1}{J} \frac{\partial}{\partial \xi^\alpha} \left(J g^{\alpha\beta} \frac{\partial \Phi(\xi)}{\partial \xi^\beta} \right)$$

Grid adaptation: PDE based



Delzanno et al, JCP08

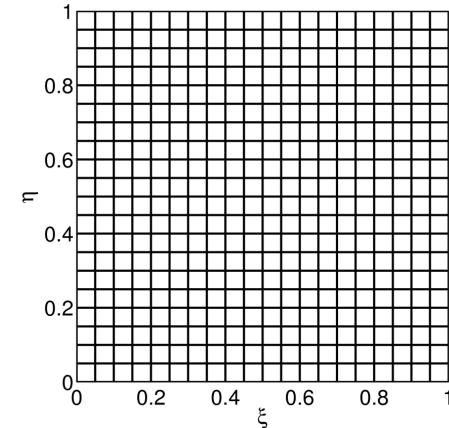


Curvilinear PIC (CPIC) @ a glance

- Solves collisionless Vlasov-Poisson equations for a plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_\alpha + \frac{q_\alpha (-\nabla \phi + \mathbf{v} \times \mathbf{B}_0)}{m_\alpha} \cdot \nabla_{\mathbf{v}} f_\alpha = 0 \quad \nabla^2 \phi = \int (f_e - f_i) d\mathbf{v}$$

- 3D, parallel
- Most operations are performed in logical space
 - P to G, G to P, field solver
- Optimal, scalable Poisson solver: black box multigrid
- Hybrid particle mover: 'leap-frog like'
 - Particle position in logical space
 - Particle velocity in physical space



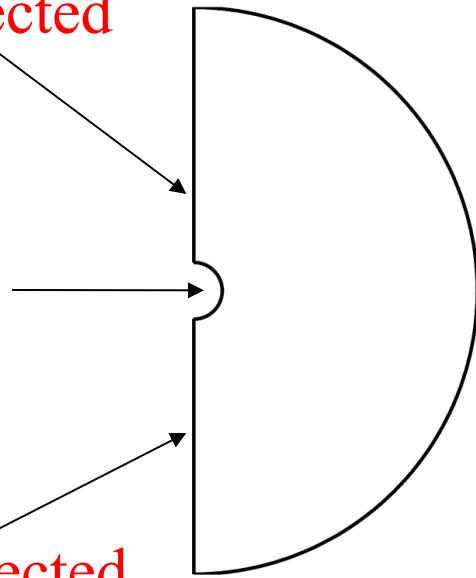
$$\frac{d\xi_p}{dt} = \hat{\mathbf{K}} \cdot \mathbf{v}_p$$

$$m_p \frac{d\mathbf{v}_p}{dt} = q_p [\mathbf{E} + \mathbf{v}_p \times \mathbf{B}_0]$$

TEST: charging of a sphere, unmagnetized plasma

- Cylindrical geometry + azimuthal symmetry
- Parameters: $r_1 = 1, r_2 = 10, 128 \times 128$ grid, $\omega_{pe}\Delta t = 0.1, \mathbf{B} = 0$

Neumann, reflected



$$\frac{T_e}{T_i} = 1, \frac{m_i}{m_e} = 1836$$

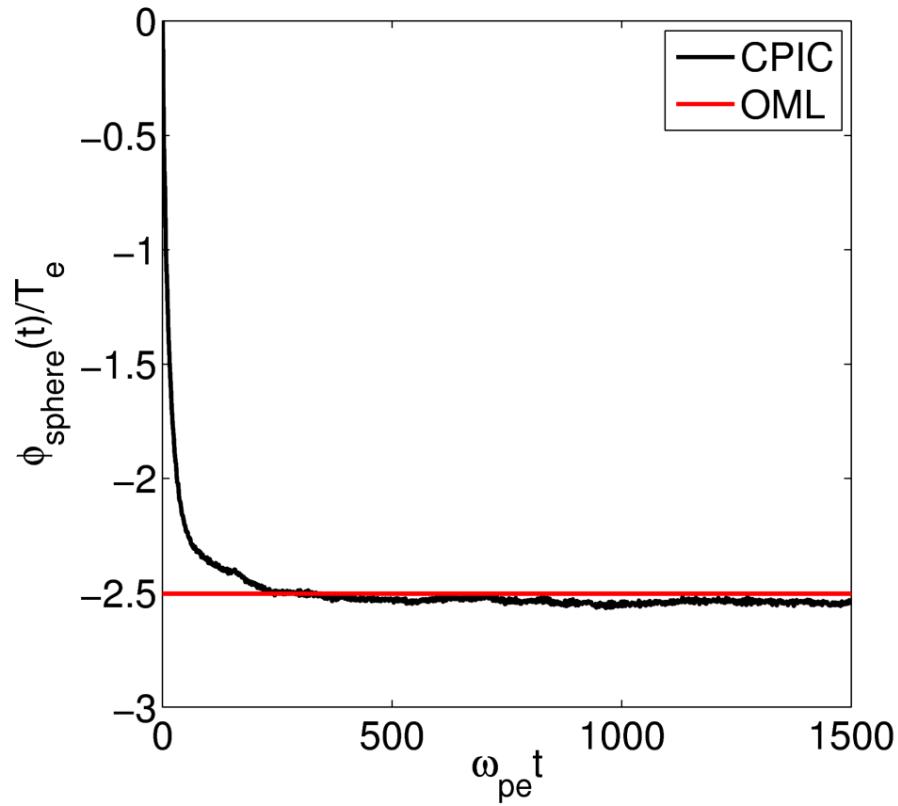
SPHERE:
Gauss' law,
particles absorbed

Dirichlet,
particles exit + injection
of Maxwellian fluxes

Neumann, reflected

- Benchmark: OML theory + 1D PIC [Delzanno PRL04, POP05]

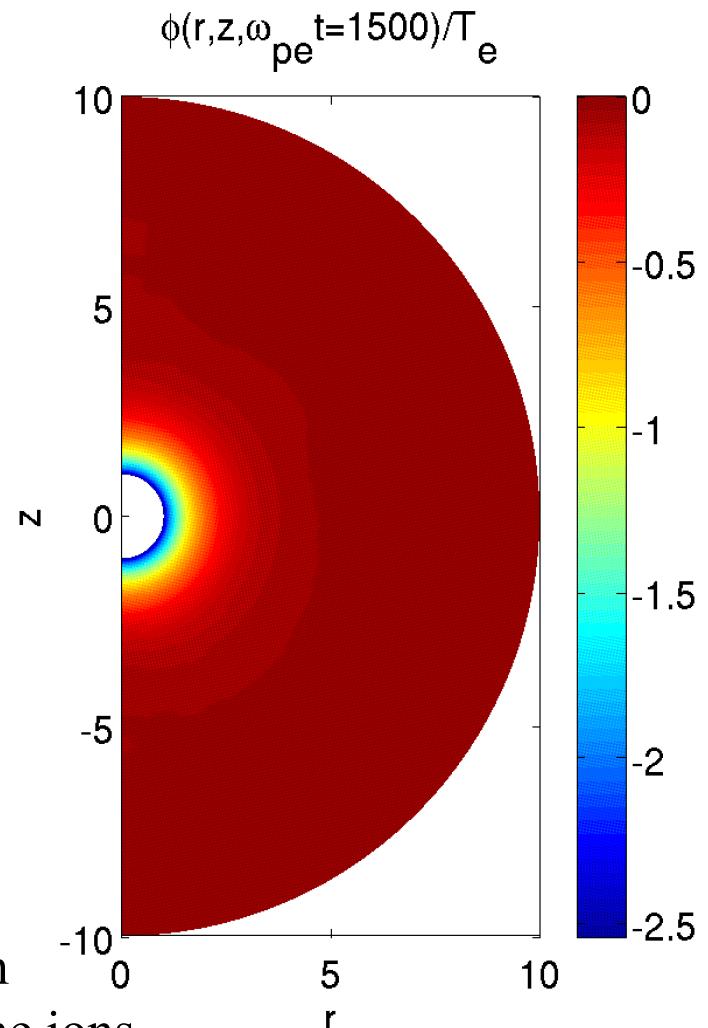
TEST: charging of a sphere, unmagnetized plasma



$$\phi_{\text{sphere}}^{\text{CPIC}} \simeq -2.54$$

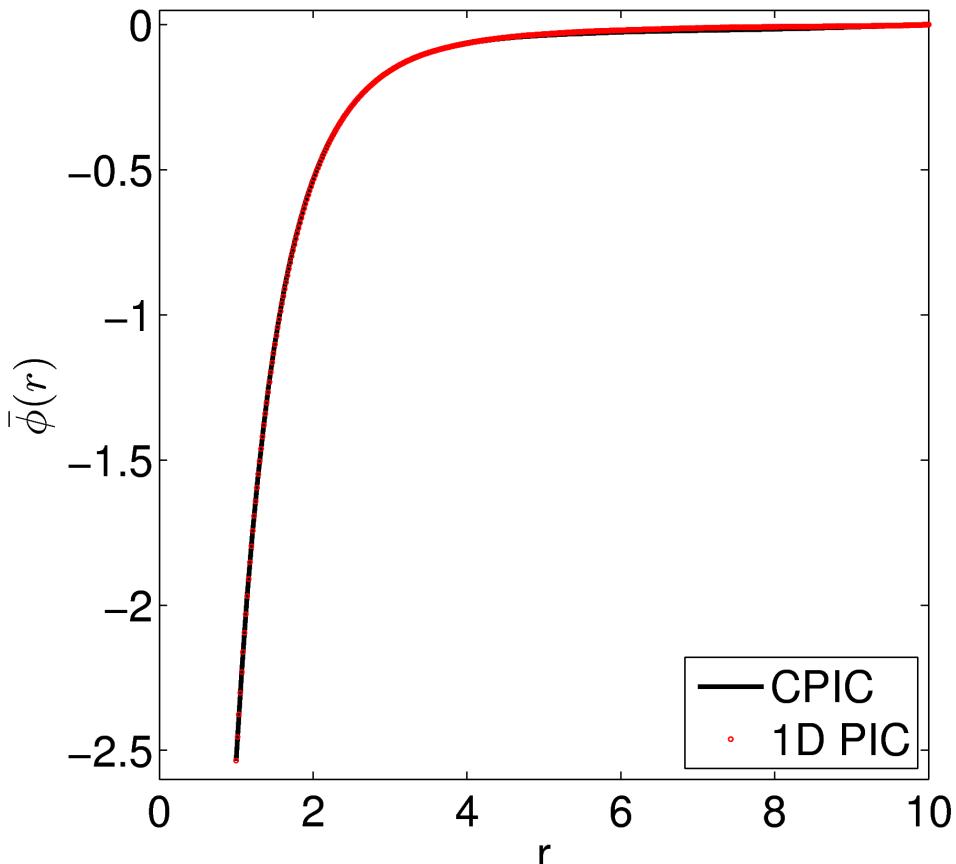
$$\phi_{\text{sphere}}^{\text{OML}} \simeq -2.50$$

Consistent with formation
of absorption radius for the ions
[Daugherty et al., JAP 1992]



TEST: charging of a sphere, unmagnetized plasma

CPIC: averaged potential



Screening captured correctly!

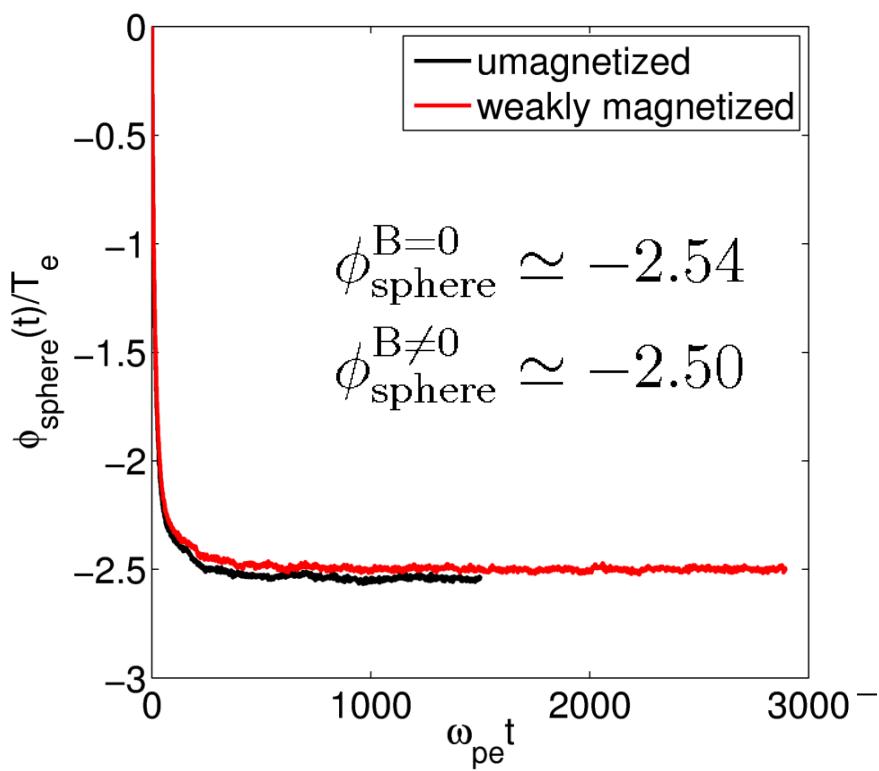
CPIC: solver

Grid	Iterations
32^2	8.0
64^2	8.0
128^2	8.0
256^2	9.0
512^2	9.0

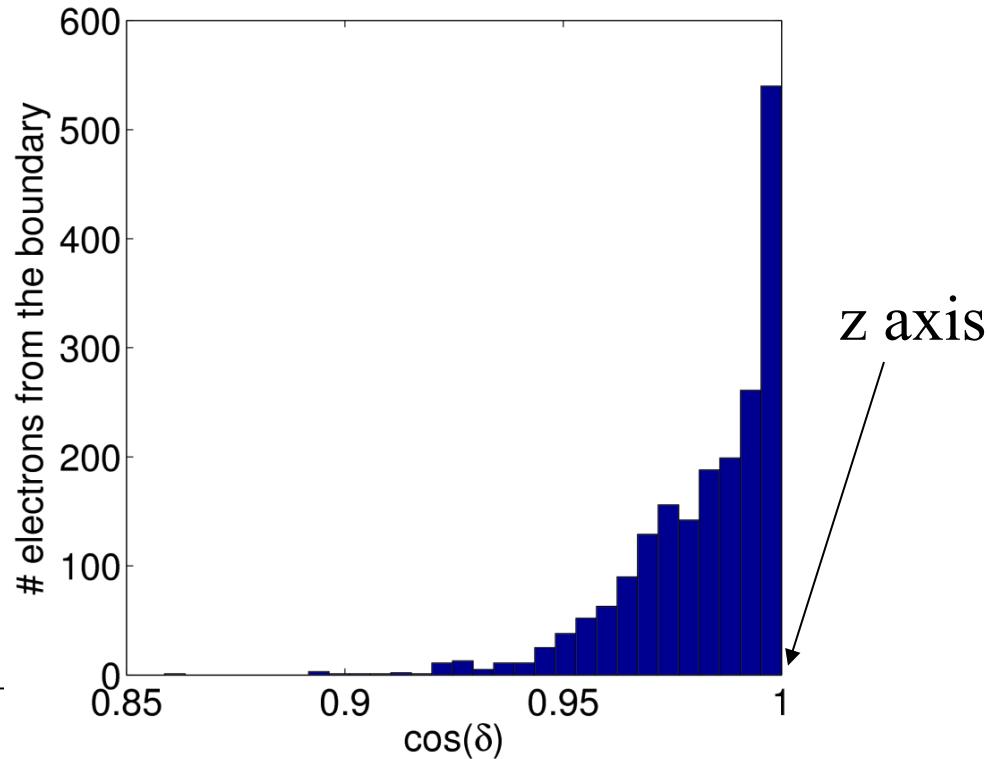
Solver performance remains fairly flat

TEST: charging of a sphere, magnetized plasma

- Same setting as before, $\omega_{ce}\Delta t = 0.1$ B along z axis
- Parameters: $\rho_e = 0.5, \lambda_{De} = 1, \rho_i = 21, \omega_{ce} = 2, \omega_{pe} = 1, \omega_{ci} = 0.001$
- Benchmark: PTetra [Marchand, IEEE 2012].



Electron collection restricted by B



Conclusions:

- **CPIC**: fully kinetic, 3D electrostatic PIC code in general curvilinear geometry
- Features:
 - ✓ **Curvilinear**: can handle any geometry by defining the metric transformation from logical to physical space
 - ✓ **Body-fitted** mapping: objects treated accurately with no stair-stepping
 - ✓ P-G, G-P, field solver in logical space
 - ✓ **Optimal, scalable solver** with black box multigrid
 - ✓ **Hybrid mover**: second order accurate and efficient
 - ✓ **Parallelized**