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Title: Adaptive grid Particle-In-Cell methods

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Adaptive grid Particle-In-Cell methods

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Outline

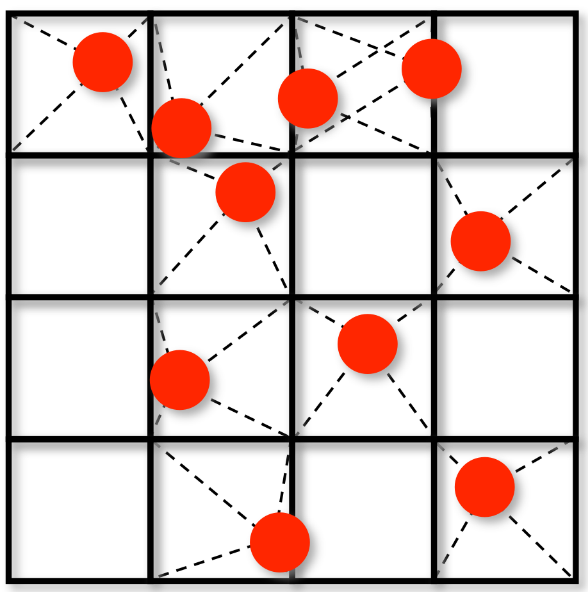
- Motivation: plasma-material interaction
- CPIC:
  - Solver: multigrid
  - Particle mover: hybrid
- Tests
- Conclusions

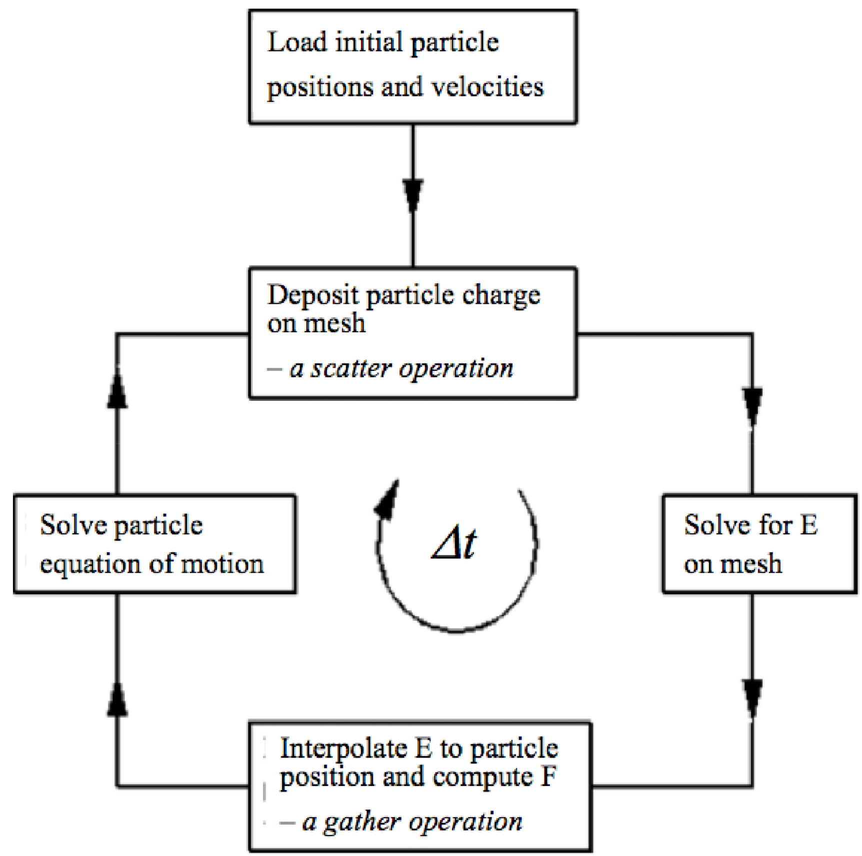
Particle-In-Cell 101

Solves Vlasov's equation with macroparticles:

$$\frac{\partial f_\alpha}{\partial t} + v \cdot \nabla_x f_\alpha + \frac{E + v \times B}{m_\alpha} \cdot \nabla_v f_\alpha = 0$$

+ Maxwell's equations



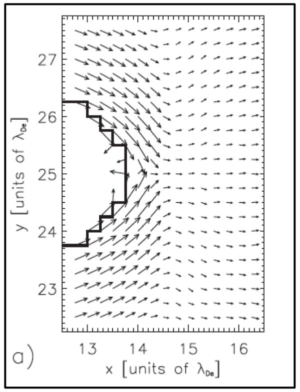


General motivation: plasma-material interaction

- Traditional PIC: uniform Cartesian grid, explicit time stepping
- Explicit time stepping (leap-frog)
  - Must resolve shortest length scale and fastest frequency
  - Plasmas can be multiscale $\lambda_D, \ \rho_e, \ \rho_i$
- Plasma-material interaction:
  - Additional scales due to the object: 'more' multiscale!

Example: geosynchronous satellite
- Spacecraft size ~1-10 m
- Debye length ~300 m
- Electron gyroradius ~ 800 m
- Ion gyroradius ~ 30 km

A uniform grid would require at least 10^5 cells in each spatial direction UNFEASIBLE!

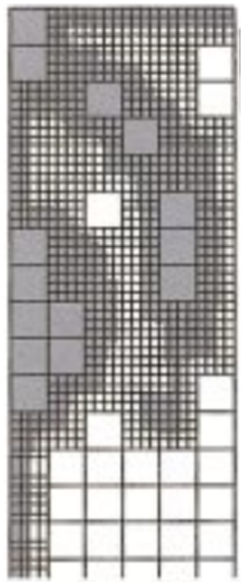


Miloch et al, PRE 08

General motivation: plasma-material interaction (2)

- Needs some kind of adaptivity

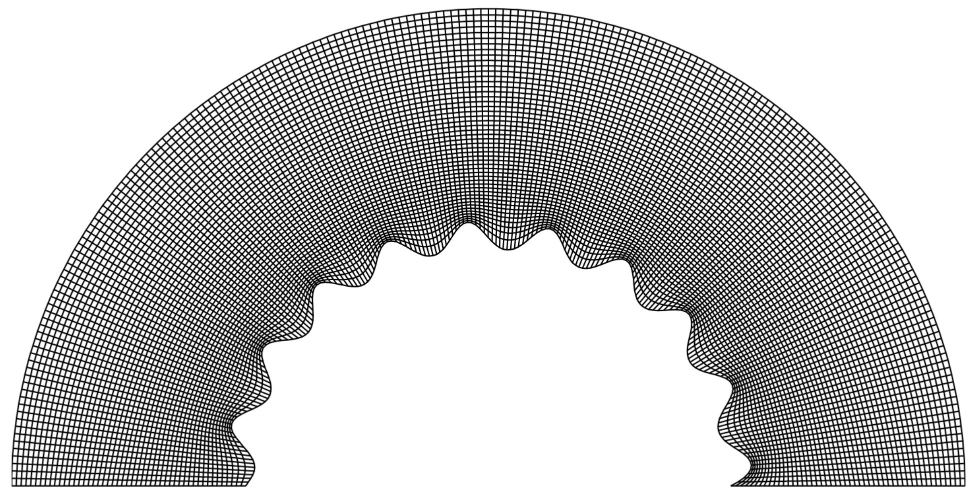
1) AMR:



Vay et al, POP 2004, Fujimoto&Sydora, 2008; Usui et al, 2010; …

2) Body-fitted: our choice

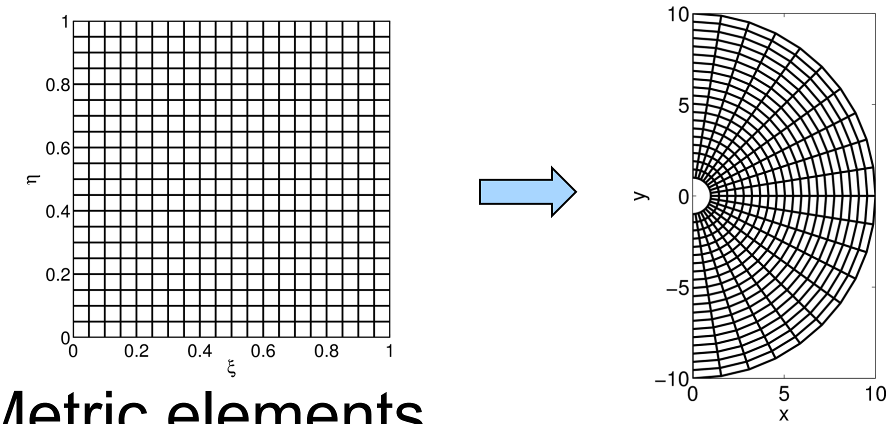
Westerman, 1994; Eastwood et al.1995, Wang et al. 1999
Spacecraft charging community



- Other design constraints:
  - Flexible, robust
  - Geometry independent, i.e. curvilinear
  - Performance

Body-fitted logical to physical space mapping

- Coordinate transformation: $\mathbf{x} = \psi(\boldsymbol{\xi})$
  - Physical space variables $\mathbf{x} = (x, y, z)$
  - Logical space variables $\boldsymbol{\xi} = (\xi, \eta, \zeta)$



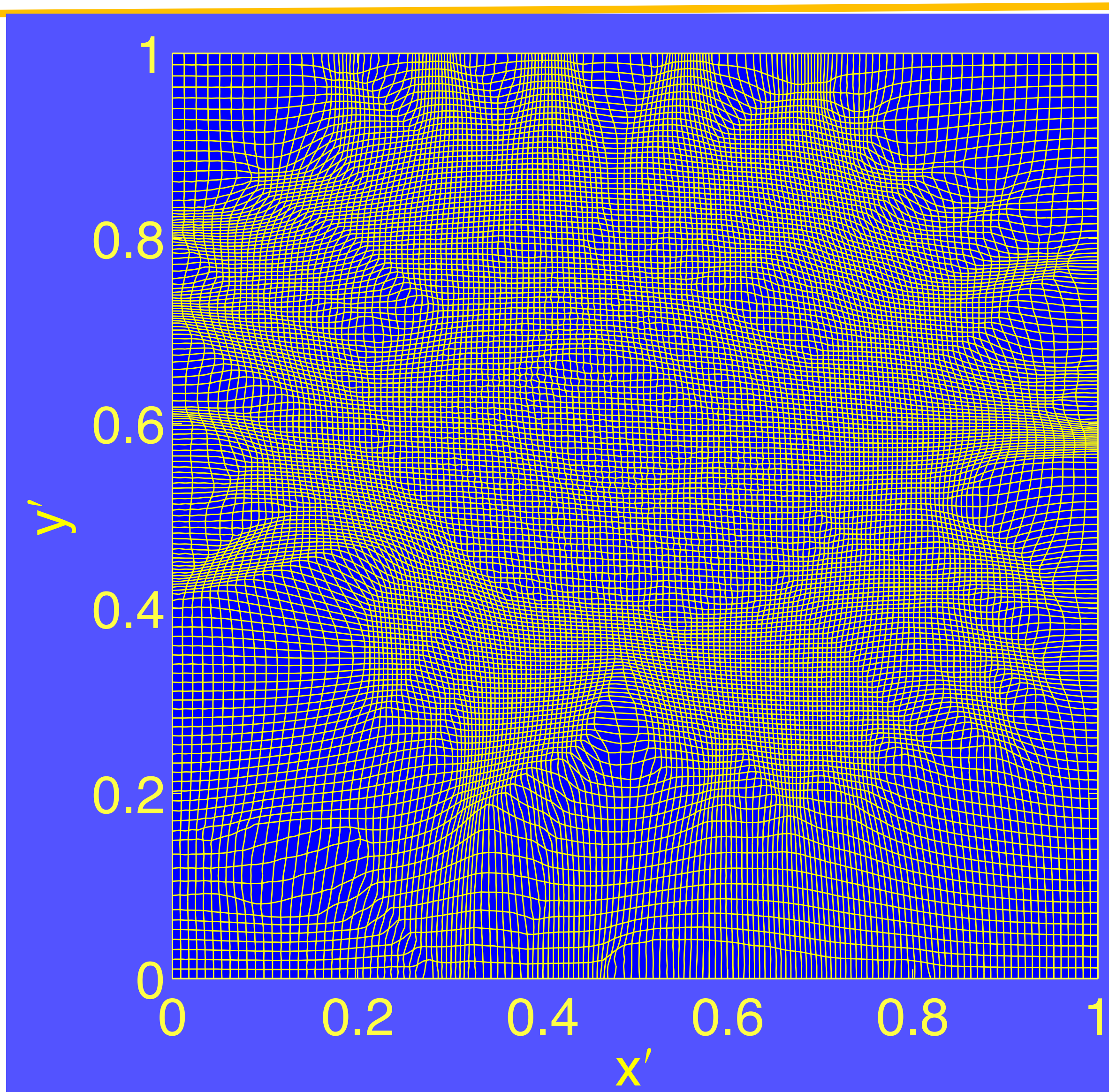
$$x = [r_1 + (r_2 - r_1)\,\xi]\sin(\pi\eta)$$
$$y = [r_1 + (r_2 - r_1)\,\xi]\cos(\pi\eta)$$
$$z = \zeta$$

- Metric elements
  - Jacobi matrix and its inverse: $j_{\alpha\beta}(\boldsymbol{\xi}) = \dfrac{\partial x^\alpha}{\partial \xi^\beta}, \quad k^{\alpha\beta}(\mathbf{x}) = \dfrac{\partial \xi^\alpha}{\partial x^\beta}$
  - Metric tensors: $g_{\alpha\beta}(\boldsymbol{\xi}) = \dfrac{\partial x^\gamma}{\partial \xi^\alpha}\dfrac{\partial x^\gamma}{\partial \xi^\beta}, \quad g^{\alpha\beta}(\mathbf{x}) = \dfrac{\partial \xi^\alpha}{\partial x^\gamma}\dfrac{\partial \xi^\beta}{\partial x^\gamma}$
  - Operators in logical space:

$$\nabla^2_{\mathbf{x}}\Phi(\mathbf{x}) = \frac{1}{J}\frac{\partial}{\partial \xi^\alpha}\left(J g^{\alpha\beta}\frac{\partial \Phi(\boldsymbol{\xi})}{\partial \xi^\beta}\right)$$

Grid adaptation: PDE based

Delzanno et al, JCP08

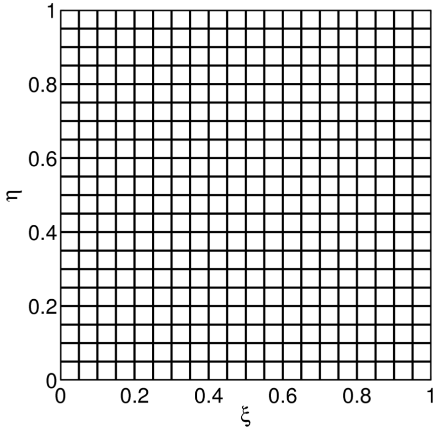


Curvilinear PIC (CPIC) @ a glance

- Solves collisionless Vlasov-Poisson equations for a plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}} f_\alpha + \frac{q_\alpha\left(-\nabla\phi + \mathbf{v}\times\mathbf{B_0}\right)}{m_\alpha}\cdot\nabla_{\mathbf{v}} f_\alpha = 0 \qquad \nabla^2\phi = \int (f_e - f_i)\, d\mathbf{v}$$

- 3D, parallel
- Most operations are performed in logical space
  - P to G, G to P, field solver
- Optimal, scalable Poisson solver: black box multigrid
- Hybrid particle mover: 'leap-frog like'
  - Particle position in logical space
  - Particle velocity in physical space



$$\frac{d\xi_p}{dt} = \hat{K}\cdot v_p$$

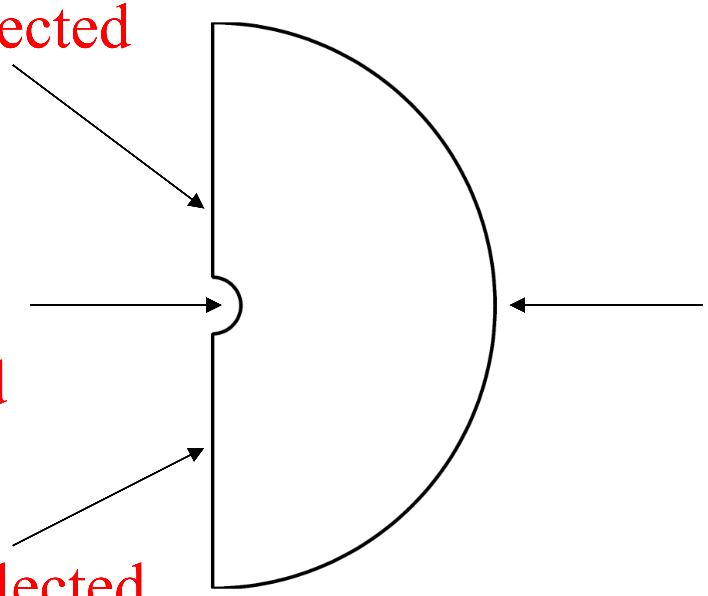
$$m_p \frac{dv_p}{dt} = q_p\left[E + v_p \times B_0\right]$$

Delzanno et al, under review

TEST: charging of a sphere, unmagnetized plasma

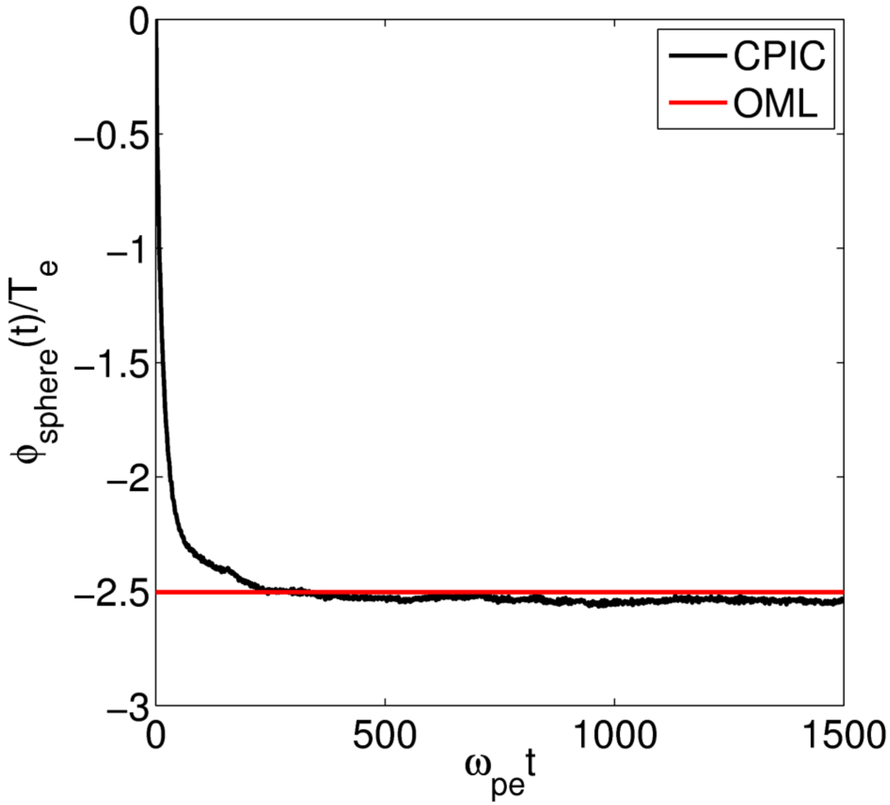
- Cylindrical geometry + azimuthal symmetry
- Parameters: $r_1 = 1,\ r_2 = 10,\ 128 \times 128\,\text{grid},\ \omega_{pe}\Delta t = 0.1,\ \mathbf{B} = 0$

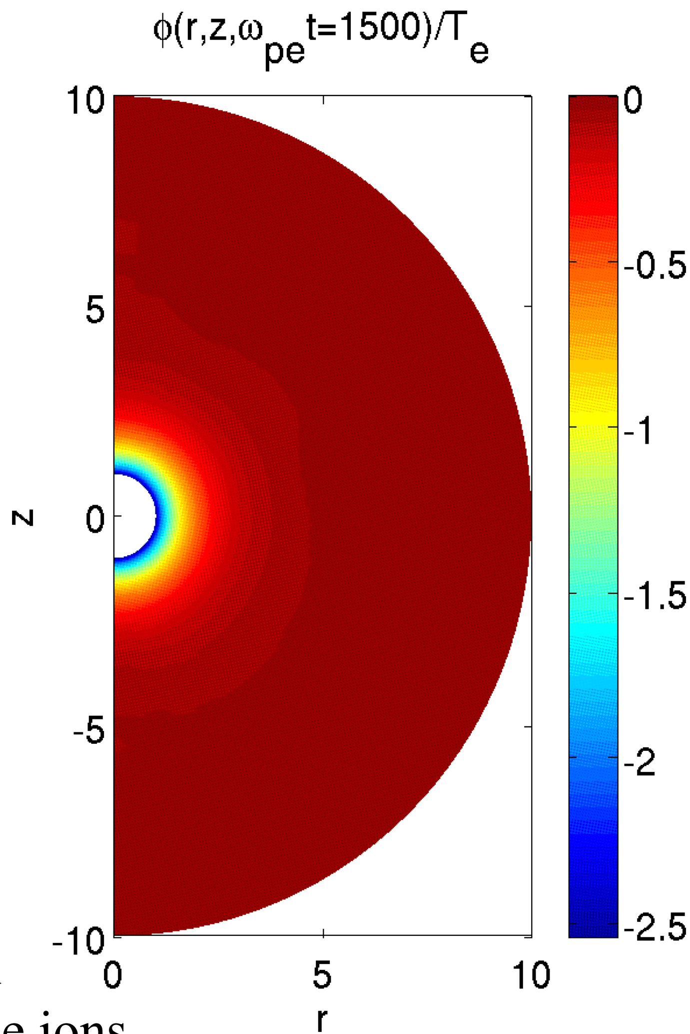
$$\frac{T_e}{T_i} = 1,\ \frac{m_i}{m_e} = 1836$$



- Benchmark: OML theory + 1D PIC [Delzanno PRL04, POP05]

TEST: charging of a sphere, unmagnetized plasma



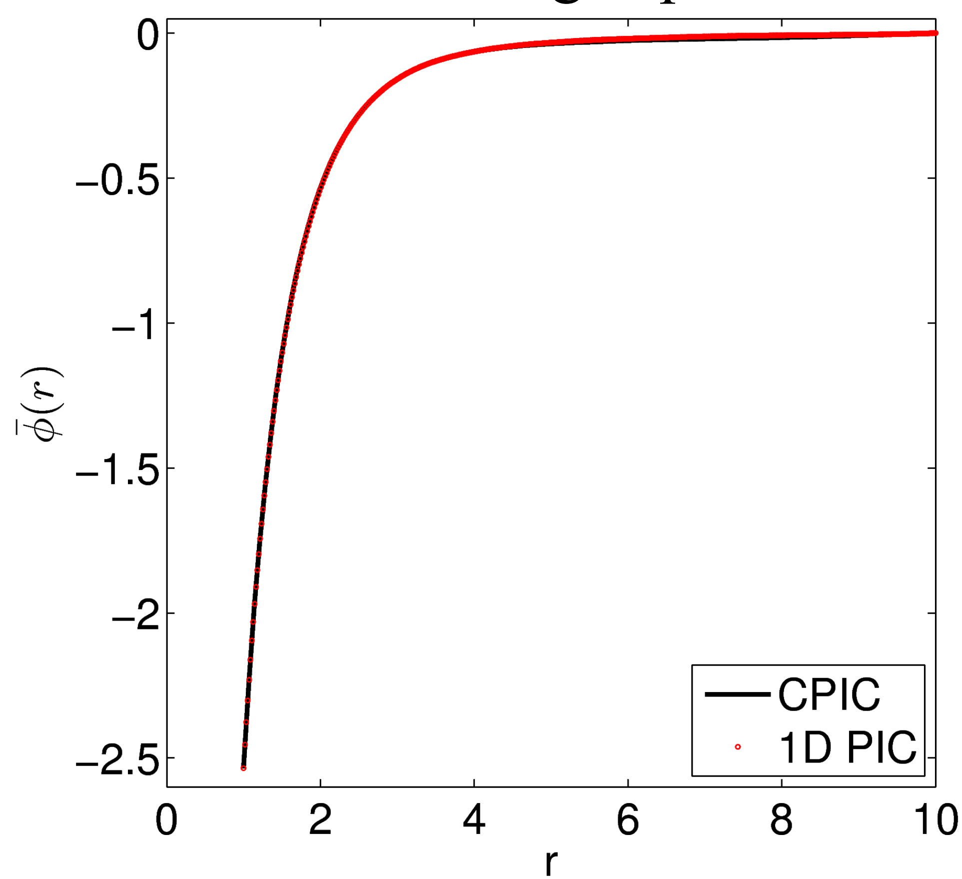


$$\phi_{\rm sphere}^{\rm CPIC} \simeq -2.54$$
$$\phi_{\rm sphere}^{\rm OML} \simeq -2.50$$

Consistent with formation of absorption radius for the ions [Daugherty et al., JAP 1992]

TEST: charging of a sphere, unmagnetized plasma

CPIC: averaged potential



Screening captured correctly!

CPIC: solver

Grid	Iterations
32^2	8.0
64^2	8.0
128^2	8.0
256^2	9.0
512^2	9.0

Solver performance remains fairly flat

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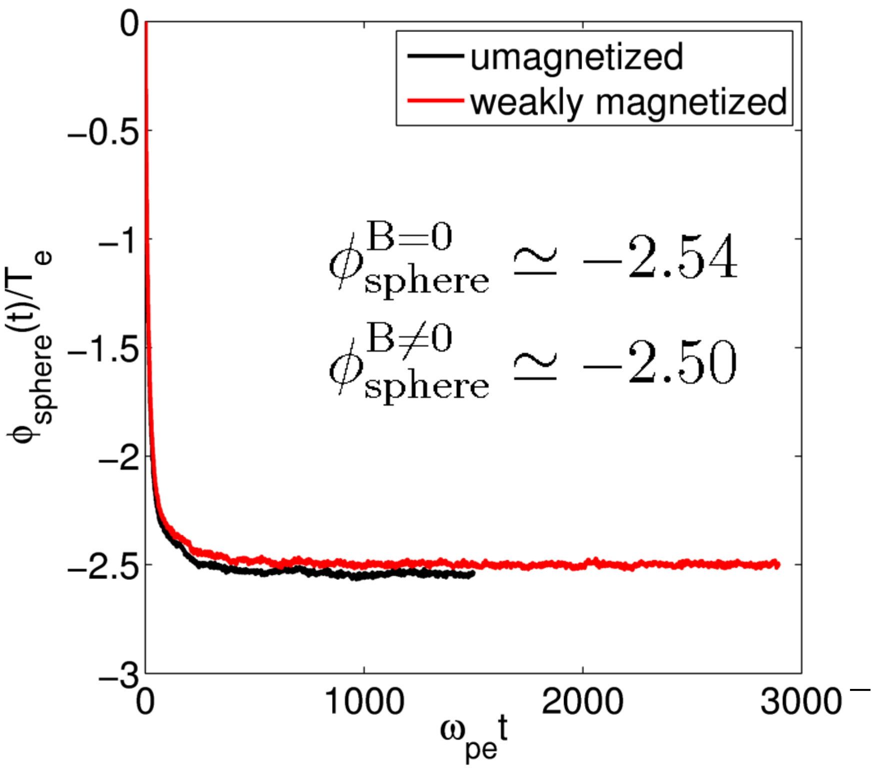
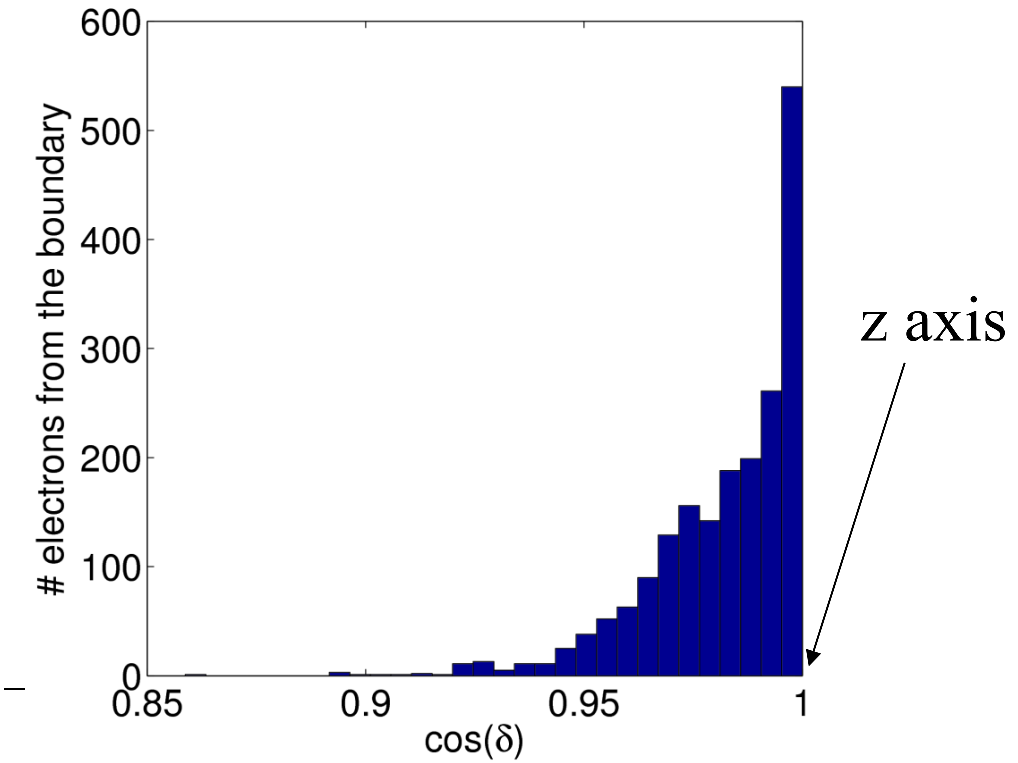
TEST: charging of a sphere, magnetized plasma

- Same setting as before, $\omega_{ce}\Delta t = 0.1$ B along z axis
- Parameters: $\rho_e = 0.5,\ \lambda_{De} = 1,\ \rho_i = 21,\ \omega_{ce} = 2,\ \omega_{pe} = 1,\ \omega_{ci} = 0.001$
- Benchmark: PTetra [Marchand, IEEE 2012].



Electron collection restricted by B



Conclusions:

- CPIC: fully kinetic, 3D electrostatic PIC code in general curvilinear geometry
- Features:
  - ✓ Curvilinear: can handle any geometry by defining the metric transformation from logical to physical space
  - ✓ Body-fitted mapping: objects treated accurately with no stair-stepping
  - ✓ P-G, G-P, field solver in logical space
  - ✓ Optimal, scalable solver with black box multigrid
  - ✓ Hybrid mover: second order accurate and efficient
  - ✓ Parallelized