

Tools for Evaluating Risk of Terrorist Acts Using Fuzzy Sets and Belief/Plausibility

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Abstract— Terrorist acts are intentional and therefore differ significantly from “dumb” random acts that are the subject of most risk analyses. There is significant epistemic (state of knowledge) uncertainty associated with such intentional acts, especially for the likelihood of specific attack scenarios. Also, many of the variables of concern are not numeric and should be treated as purely linguistic (words).

Epistemic uncertainty can be addressed using the belief/plausibility measure of uncertainty, an extension of the traditional probability measure of uncertainty. Fuzzy sets can be used to segregate a variable into purely linguistic values. Linguistic variables can be combined using an approximate reasoning rule base to map combinations of fuzzy sets of the constituent variables to fuzzy sets of the resultant variable.

We have implemented the mathematics of fuzzy sets, approximate reasoning, and belief/plausibility into Java software tools. The PoolEvidence© software tool combines evidence (pools) from different experts. The LinguisticBelief© software tool evaluates the risk associated with scenarios of concern using the pooled evidence as input.

The tools are not limited to the evaluation of terrorist risk; they are useful for evaluating any decision involving significant epistemic uncertainty and linguistic variables. Sandia National Laboratories’ analysts have applied the tools to: risk of terrorist acts, security of nuclear materials, cyber security, prediction of movements of plumes of hazardous materials, and issues with nuclear weapons. This paper focuses on evaluating the risk of acts of terrorism.

Keywords—risk of terrorist acts, fuzzy sets, epistemic uncertainty, approximate reasoning, belief and plausibility, LinguisticBelief software

I. INTRODUCTION

A. Summary of Approach

The belief/plausibility measure of uncertainty from the Dempster/Shافر Theory of Evidence is an extension of the probability measure of uncertainty that can better capture epistemic uncertainty. Belief/plausibility is a superset of probability and, under certain conditions, belief and plausibility both become probability. Under other conditions belief/

plausibility become necessity/possibility, respectively.¹ Belief/plausibility addresses a type of uncertainty called ambiguity. The uncertainty associated with predicting an event in the future is ambiguity.

A simple example illustrates the difference between aleatory (random or stochastic) and epistemic uncertainty, and the use of a belief/plausibility measure. Consider a fair coin, heads on one side, tails on the other, with each side equally likely. The uncertainty as to the outcome of a toss—heads or tails—is aleatory. The probability of heads is one half and the probability of tails is one half. The uncertainty is due to the randomness of the toss. Suppose, however, that we do not know the coin is fair; the coin could be biased to come up heads, or the coin could even be two-tailed. Now we have epistemic uncertainty; our state of knowledge is insufficient to assign a probability to heads or tails: all we can say is the likelihood of heads (or tails) is somewhere between 0 and 1. To consider epistemic uncertainty as well as aleatory uncertainty, belief/plausibility can be used as the measure of uncertainty. With total ignorance about the coin, the belief that the toss will be heads is 0 and the plausibility that the toss will be heads is 1; similarly, the belief that the toss will be tails is 0 and the plausibility that the toss will be tails is 1. Belief/plausibility form an interval that can be interpreted as giving the lower and upper bound of probability. If we have enough information, both belief and plausibility reduce to a single value, probability. Epistemic uncertainty can be reduced with more information. If we toss the coin a few times and a heads and a tails occur, we know the coin is two-sided; with more tosses we can evaluate the fairness of the coin. Aleatory uncertainty cannot be reduced with more information.

B. Using Fuzzy Sets

In addition to ambiguity, we have another type of uncertainty called vagueness. We have vagueness when we use linguistics (words) to classify events; for example, yesterday was “sunny”, public confidence in the stock market is “low”, etc. Vagueness is uncertainty as to how to classify a *known* event. For example, assume we know how tall John is, but instead of saying John is 6 feet 2 inches tall, we categorize John as “tall” without a precise definition of “tall”. The

¹ To be precise, if the focal elements are singletons, belief/plausibility both become probability. If the focal elements are nested, belief/plausibility become necessity/possibility, respectively.

linguistic (word) “tall” is vague. Vagueness can be addressed using the mathematics of fuzzy sets.

Many applications use fuzzy sets for a numeric variable, specifically, fuzzy numbers. Some variables cannot be adequately described numerically. For example, consider the variable “Quality of Life”. We do not know the appropriate numeric scale for “Quality of Life”; does it range from 0 to 1, 1 to 100, -10 to 10, ...? The problem of arbitrary scale is exacerbated when variables are combined. Suppose we combine “Quality of Life” and “Outlook on Life” into “Happiness”. If we use arbitrary numeric scales for “Quality of Life” and “Outlook on Life” we do not know what the resultant numeric answer for “Happiness” means.

For such situations, it is better to reason on purely linguistic fuzzy sets for variables—since the linguistic themselves convey more information than any arbitrary number—and combine the variables using approximate reasoning. Here, approximate reasoning is a rule base for combining purely linguistic fuzzy sets.

NOTE: The references provide details on the mathematics of belief/plausibility, fuzzy sets, and approximate reasoning. [1] through [4] Also, the references discuss our implementation of these techniques. [5] through [10]

II. APPLICATION

A. Defining Risk and Threat

We use expert judgment to create the risk model, specify approximate reasoning rules, and assign evidence to variables for specific scenarios.

We define the risk of a terrorist scenario as:

$$Risk = Threat \times Vulnerability \times Consequence \quad (1)$$

where “ \times ” denotes convolution per an approximate reasoning rule base, not algebraic multiplication.

A physical security scenario includes adversary resources, the attack plan, and the target. Threat is the likelihood of the scenario. Vulnerability is the likelihood that the Threat is successful in causing Consequence. Consequence is the result of a successful scenario.

We evaluate Threat from the perspective of the adversary (the terrorists) and Vulnerability and Consequence from the perspective of the defender (us). The adversary and defender each have different uncertainty. For example, the adversary has more uncertainty than the defender for Vulnerability, since

the adversary has less knowledge of the possible security measures in place. The defender has significant epistemic uncertainty for the Threat, but the adversary has no uncertainty for Threat as the adversary *is* the Threat.

Since the adversaries have a choice of scenarios, they select a scenario based on their perception of the combination of Vulnerability and Consequence.

$$Threat = \frac{Adversary\ Perception\ of\ Vulnerability \times Adversary\ Perception\ of\ Consequence}{Adversary\ Perception\ of\ Consequence} \quad (2)$$

B. Variables Used in LinguisticBelief

For ease of illustration, we will limit our example to the variables in equations 1 and 2. In practice, the variables of concern are broken down into numerous constituent variables. For example, Consequence can be further developed as a combination of: Fatalities, Injuries, Economic Loss, Damage to National Morale, Fear in the Populace, etc.

Note that Fatalities, Injuries, and Economic Loss are “hard” consequences, meaning they can be defined numerically. However, Damage to National Morale and Fear in the Populace are “soft” consequences in that we do not know the appropriate numeric scale to use; for these variables a purely linguistic description is better than the forced use of an arbitrary numeric scale. Since we will be combining variables, many of which cannot be appropriately described numerically, we will treat all variables linguistically.

Each variable is either a basic or a rule-based variable. Basic variables are not developed further, and rule-based variables are formed by combinations of other variables, either rule-based or basic. For our simple example the basic variables are: Vulnerability, Consequence, Adversary Perception of Vulnerability, and Adversary Perception of Consequence. The rule-based variables are: Threat and Risk.

For each variable, we define linguistic fuzzy sets. For example, for Threat we define the fuzzy sets as {Unlikely, Credible, and Likely}. For Vulnerability we define the fuzzy sets {Low, Marginal, and High} and for Consequence we define the fuzzy sets {Low, Moderate, Major, and Catastrophic}. The rest of the variables are similarly described with fuzzy sets.

C. Creating a LinguisticBelief Model

To create the model for Risk, the variables and their fuzzy sets are entered into LinguisticBelief with the result indicated in Figure 1.

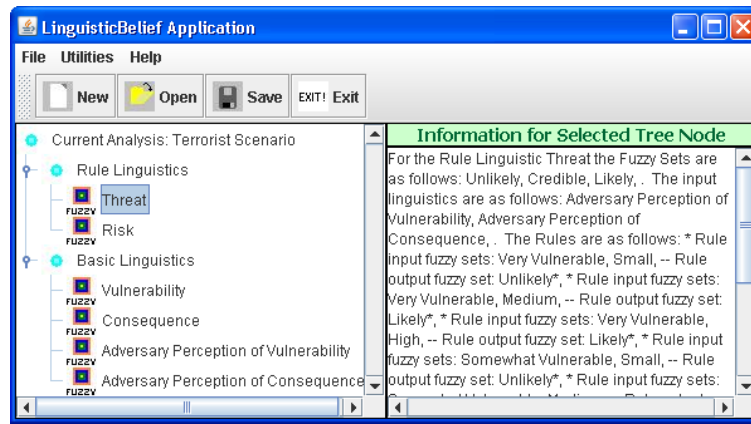


Figure 1. Simple Model in LinguisticBelief

The left panel above shows the variables in a tree structure. The right panel displays the current state of a selected variable, a node in the tree in the left panel. In Figure 1, the current state of Threat is displayed.

For each rule-based variable, the variable is defined in terms of its constituent variables and the approximate reasoning rule base is defined. Figure 2 shows the approximate reasoning rule base for Threat partially completed. The rules are completed using expert judgment.

Rules for RuleLinguistic: Threat		
Fuzzy Set for Input Linguistic: Adversary Perception of Vulnerability	Fuzzy Set for Input Linguistic: Adversary Perception of Consequence	Output Fuzzy Set
Very Vulnerable	Small	
Very Vulnerable	Medium	
Very Vulnerable	High	Likely
Somewhat Vulnerable	Small	Unlikely
Somewhat Vulnerable	Medium	Credible
Somewhat Vulnerable	High	
Not Vulnerable	Small	Unlikely
Not Vulnerable	Medium	
Not Vulnerable	High	

Specify Output Fuzzy Set for Selected Rule

Choices Are: Unlikely

Accept Rules as Shown Cancel

Figure 2. Approximate Reasoning Rule Base for Threat Partially Completed

Once all the rules have been created, the model is complete. A specific scenario is evaluated by assigning evidence (focal elements) to each basic variable. The evidence is assigned

using expert judgment. Figure 3 is an example of evidence assigned to Consequence by one expert.

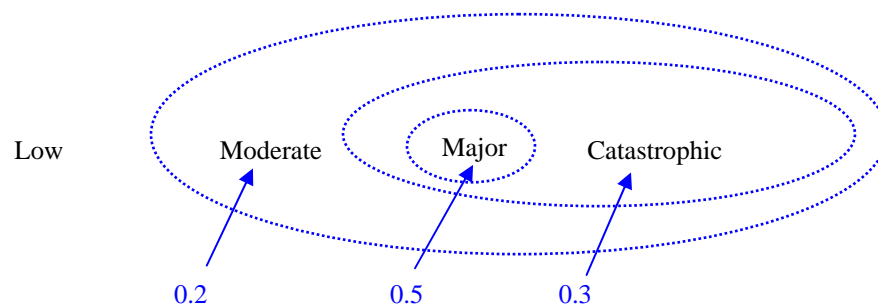


Figure 3. Example of Evidence Assigned to Consequence

The experts may not assign the same evidence, and the PoolEvidence code is used to pool the evidence into one set of evidence. The pooling weights each expert equally. Figure 4

is an example of pooled evidence from two experts for Consequence.

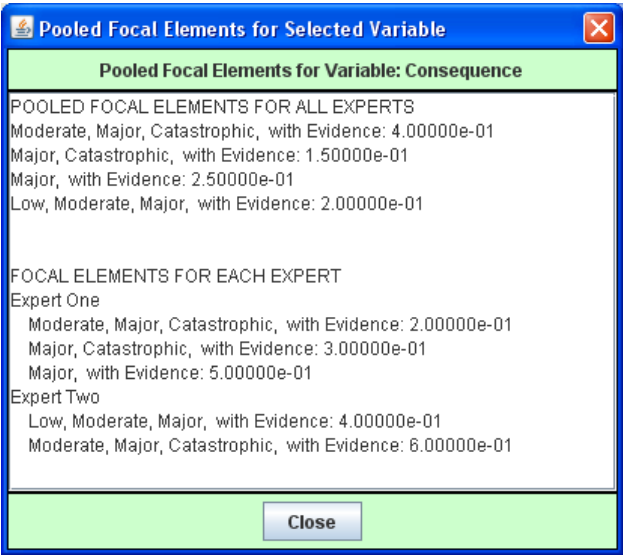


Figure 4. Pooled Evidence for Consequence

After all the pooled evidence is entered into LinguisticBelief, the scenario may be evaluated. Using the mathematics of belief/plausibility, LinguisticBelief convolutes the evidence for the basic variables to produce evidence for each rule-based variable. The variables are assumed to be non-interacting (independent). The belief and plausibility of any variable (basic or rule-based) can then be evaluated. Figure 5

provides example results for Risk for a scenario using dummy data. Two graphs are provided in Figure 5. The top is the likelihood of a fuzzy set provided as a belief to plausibility interval. The bottom graph is the likelihood of exceedance of a fuzzy set, with the fuzzy sets ordered from “best” to “worst” in the view of the Defender.

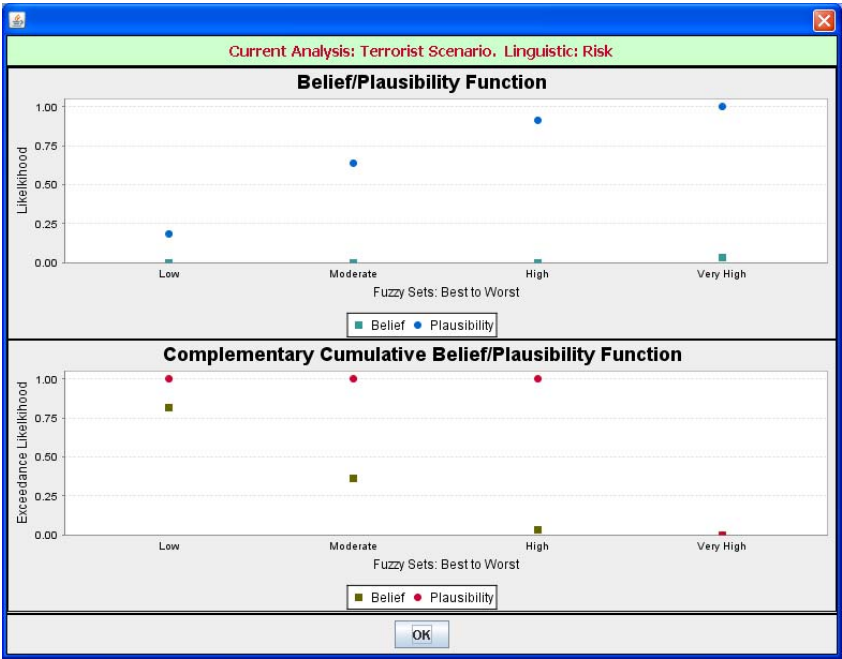


Figure 5. Risk for a Scenario

Scenarios are ranked from highest to lowest concern from the view of the defender as follows. Scenarios are ranked by non-zero plausibility of exceeding the “worst” fuzzy set (decreasing). For scenarios with equal ranking by plausibility, these scenarios are sub-ranked by belief of exceeding the fuzzy set (decreasing). The scenario in Figure 5 has a ranking “Exceeds High with Plausibility 1.0 and Belief 0.03”. If another scenario had plausibility/belief of exceeding High of 1.0/0.4 it would be ranked higher. If another scenario had zero plausibility of exceeding High, but plausibility/belief of exceeding Moderate of 1.0/0.9 it would be ranked lower.

Dominant contributors to Risk for a scenario can be identified by examining the belief/plausibility of lower level variables; for example, Threat, and its constituent variables. We plan to add importance and sensitivity measures into LinguisticBelief to automate the evaluation of dominant contributors.

III. CONCLUSIONS

Evaluation of the risk of intentional terrorist acts requires new techniques not available in traditional probabilistic-based risk assessment approaches. Both adversary and defender must be considered. The significant epistemic uncertainty—especially for the defender related to threat—should be addressed. It is necessary to evaluate and combine purely linguistic variables that have unknown numeric scales.

To address these needs for evaluating the risk of terrorist acts, we have implemented the mathematics of fuzzy sets, approximate reasoning, and the belief/plausibility measure of uncertainty into software tools: LinguisticBelief and PoolEvidence.

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