58918-00-105

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Proceedings of the 3rd International Symposium on Metallic Multilayers, June 14-19, 1998, Vancouver, BC, Canada

Work supported by the Ministry of Science and Technology of Korea, the Korea Research

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Abstract

Expressions are presented for various magneto-optical Kerr effects in the ultrathin film limit with arbitrary magnetization direction by considering the multiple reflections within an optically thin film. The Kerr effect of p- and s-polarization consists of products of two factors: the prefactor, dependent only on the optical parameters of the system, and the main factor of the polar Kerr effect for normal incidence in the ultrathin limit.

PACS : 78.20.L, 33.55.-b, 78.66.-w, 75.70.-i

Keywords : Magneto-optical Kerr effect, ultrathin magnetic film, multilayers, analytic expressions, magneto-optical Fresnel coefficients

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Ultrathin magnetic films have attracted much attention due to a wealth of interesting physical properties [1–4]. To understand their magnetic properties, the magneto-optical Kerr effect (MOKE) has been widely used because it is sensitive to the spin-polarized electronic band structure which causes magnetism. MOKE manifests itself as a change of polarization and/or intensity of incident linearly polarized light when it is reflected from the surface of a magnetized medium.

Ultrathin magnetic films exhibit only weak signals due to the small number of magnetic atoms and the possible effects of surface oxidation. To overcome these difficulties, one should perform measurements on a sample in ultrahigh vacuum with a sensitive tool, such as MOKE, which can be employed satisfactorily even in the monolayer regime, with relatively simple *in situ* instrumentation [5]. Based on the fact that MOKE is sensitive to the surface magnetization and proportional to the projection of the magnetization in the incident beam direction, Haijar *et al.* [6] and Purcell *et al.* [7] have discussed MOKE as a method to study the magnetic anisotropy energy of a sample having perpendicular magnetic anisotropy. Weller *et al.* [8] applied this method to obtain the magnetic anisotropy energy of Co/X multilayers (X= Pd, Pt and Ni). These analyses were essentially qualitative in nature in employing the simple proportionality of the magnetization and MOKE signal. The quantitative analysis of the MOKE signal for the general case, where the direction of the magnetization is arbitrary and the direction of the incident beam is not normal, is not simple because of the complicated optical relations [9–11]. Recently, we have reported [12], the derivation of simplified analytic formulae for the MOKE of an optically **thick** magnetic film. In this paper, we present simplified analytic expressions for various MOKE's of an optically **thin** magnetic film.

As depicted in Fig.1, when a beam of light is passed from a non-magnetic medium 0 to another non-magnetic medium 2 through a magnetic medium 1 having an arbitrary magnetization direction and thickness d_1, the dielectric tensor ϵ of the medium 1 can be expressed by [9,11]

$$\epsilon = \epsilon_{xx} \begin{pmatrix} 1 & -iQm_z & iQm_y \\ iQm_z & 1 & -iQm_x \\ -iQm_y & iQm_x & 1 \end{pmatrix}, \tag{1}$$

using Euler's angle. For generality, we treat all physical quantities as complex numbers. For simplicity we assume $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz}$. In Eq.(1), m_x, m_y and m_z are the direction cosines of the magnetization vector $\mathbf{M}_s$. The magneto-optical constant Q in Eq.(1) is defined by

$$Q = i\frac{\epsilon_{xy}}{\epsilon_{xx}}\,. \tag{2}$$

Here, we take the same sign convention for Q as proposed by Atkinson [13]. Solving Maxwell's equations for the above dielectric tensor with the optical system as shown in Fig. 1, the magneto-optical Fresnel reflection matrix $\hat{\Re}$ is

$$\hat{\Re} = \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix}, \tag{3}$$

where r_{ij} is the ratio of the incident j polarized electric field and reflected i polarized electric field.

One has to consider the multiple reflections to treat optically thin magnetic films. We have used the medium boundary matrices and medium propagation matrices [10,14] to take this into account. The medium boundary matrix A_j for a jth layer having arbitrary magnetization direction is

$$A_j = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{iQ\alpha_{yj}^2}{2}\left(m_y\frac{1+\alpha_{zj}^2}{\alpha_{yj}\alpha_{zj}} - m_z\right) & \alpha_{zj} & \frac{-iQ\alpha_{yj}^2}{2}\left(m_y\frac{1+\alpha_{zj}^2}{\alpha_{yj}\alpha_{zj}} + m_z\right) & -\alpha_{zj} \\ \frac{-in_jQ}{2}(m_y\alpha_{yj} + m_z\alpha_{zj}) & -n_j & \frac{-in_jQ}{2}(m_y\alpha_{yj} - m_z\alpha_{zj}) & -n_j \\ n_j\alpha_{zj} & \frac{-in_jQ}{2}\left(m_y\frac{\alpha_{yj}}{\alpha_{zj}} - m_z\right) & -n_j\alpha_{zj} & \frac{in_jQ}{2}\left(m_y\frac{\alpha_{yj}}{\alpha_{zj}} + m_z\right) \end{pmatrix}, \tag{4}$$

where $\alpha_{yj} = \sin\theta_j$ and $\alpha_{zj} = \cos\theta_j$. Here the complex refractive angle θ_j is determined by Snell's law. The medium propagation matrix D_j for a jth layer is

$$D_j = \begin{pmatrix} U\cos\delta^i & U\sin\delta^i & 0 & 0 \\ -U\sin\delta^i & U\cos\delta^i & 0 & 0 \\ 0 & 0 & U^{-1}\cos\delta^r & U^{-1}\sin\delta^r \\ 0 & 0 & -U^{-1}\sin\delta^r & U^{-1}\cos\delta^r \end{pmatrix}, \quad (5)$$

where U, δ^i, δ^r, g^u and g^r are defined by

$$U = \exp(-i\frac{2\pi}{\lambda}n_j\alpha_{zj}d_j)\ , \quad (6)$$

$$\delta^i = -\frac{\pi n_j Q d_j g^i}{\lambda\alpha_{zj}}, \quad (7)$$

$$\delta^r = -\frac{\pi n_j Q d_j g^r}{\lambda\alpha_{zj}}, \quad (8)$$

$$g^i = m_z\alpha_{zj} + my\alpha_{yj}, and \quad (9)$$

$$g^r = m_z\alpha_{zj} - my\alpha_{yj}. \quad (10)$$

Here, d_j denotes the thickness of the jth layer. To obtain the magneto-optical Fresnel reflection matrix, one computes the matrix M defined by

$$M = A_0^{-1}A_1D_1A_1^{-1}A_2\ . \quad (11)$$

The 4×4 matrix M can be expressed in the form of 2×2 block matrices as follows;

$$M = \begin{pmatrix} G & H \\ I & J \end{pmatrix}\ . \quad (12)$$

Then, the block matrices G, H, I and J are related to the magneto-optical Fresnel reflection coefficients as follows [10,14]:

$$\begin{pmatrix} r_{ss} & r_{sp} \\ r_{ps} & r_{pp} \end{pmatrix} = IG^{-1}\ . \quad (13)$$

In principle, one can obtain the analytic expressions for the magneto-optical Kerr effects from Eq.(13), even though these expressions are too complicated to provide any physical insight.

Since MOKE is utilized to probe magnetism in ultrathin magnetic films, we now consider the limiting case where the thickness of magnetic medium 1 satisfies the condition, $\frac{2\pi|n_1|d_1}{\lambda} \ll 1$. In this limit one can expand the block matrices G and I to within the first order of $\frac{2\pi|n_1|d_1}{\lambda}$ as follows:

$$G = \begin{pmatrix} \frac{1}{2} + \frac{n_s \cos\theta_2}{2n_0 \cos\theta_0} - \frac{id\pi(n_0 n_s \cos\theta_0 \cos\theta_2 + n_1^2 \cos^2\theta_1)}{\lambda n_0 \cos\theta_0} & -\frac{n_1 d_1 \pi Q(m_z n_1 \cos\theta_2 + m_y n_s \sin\theta_1)}{\lambda n_0 \cos\theta_0} \\ \frac{n_1 d_1 \pi Q(m_z n_1 \cos\theta_0 + m_y n_0 \sin\theta_1)}{\lambda n_0 \cos\theta_0} & \frac{n_s}{2n_0} + \frac{\cos\theta_2}{2\cos\theta_0} - \frac{id\pi(n_0 n_s \cos^2\theta_1 + n_1^2 \cos\theta_0 \cos\theta_2)}{\lambda n_0 \cos\theta_0} \end{pmatrix}, \tag{14}$$

$$I = \begin{pmatrix} \frac{1}{2} - \frac{n_s \cos\theta_2}{2n_0 \cos\theta_0} - \frac{id\pi(n_0 n_s \cos\theta_0 \cos\theta_2 - n_1^2 \cos^2\theta_1)}{\lambda n_0 \cos\theta_0} & \frac{n_1 d_1 \pi Q(m_z n_1 \cos\theta_2 + m_y n_s \sin\theta_1)}{\lambda n_0 \cos\theta_0}, \\ \frac{n_1 d_1 \pi Q(m_z n_1 \cos\theta_0 - m_y n_0 \sin\theta_1)}{\lambda n_0 \cos\theta_0} & \frac{n_s}{2n_0} - \frac{\cos\theta_2}{2\cos\theta_0} + \frac{id\pi(n_0 n_s \cos^2\theta_1 - n_1^2 \cos\theta_0 \cos\theta_2)}{\lambda n_0 \cos\theta_0} \end{pmatrix}. \tag{15}$$

Here, n_0, n_1, n_s, θ_0, θ_1 and θ_2 are the refractive indices and complex refractive angles of non-magnetic media 0 and 2, and magnetic medium 1. We assume the layer 2 is substrate. Using Eqs.(13) $\sim$ (15), one can obtain the expressions for r_{ij} as follows;

$$r_{pp} = \frac{n_s \cos\theta_0 - n_0 \cos\theta_2}{n_s \cos\theta_0 + n_0 \cos\theta_2} + \frac{4\pi i n_0 d_1 \cos\theta_0 (n_s^2 \cos^2\theta_1 - n_1^2 \cos^2\theta_2)}{\lambda(n_0 \cos\theta_2 + n_s \cos\theta_0)^2} \tag{16}$$

$$r_{sp} = \frac{4\pi n_0 n_1 Q d_1 \cos\theta_0 (m_z n_1 \cos\theta_2 + m_y n_s \sin\theta_1)}{\lambda(n_0 \cos\theta_0 + n_s \cos\theta_2)(n_0 \cos\theta_2 + n_s \cos\theta_0)}, \tag{17}$$

$$r_{ss} = \frac{n_0 \cos\theta_0 - n_s \cos\theta_2}{n_0 \cos\theta_0 + n_s \cos\theta_2} + \frac{4\pi i n_0 d_1 \cos\theta_0 (n_1^2 \cos^2\theta_1 - n_s^2 \cos^2\theta_2)}{\lambda(n_0 \cos\theta_0 + n_s \cos\theta_2)^2}, \tag{18}$$

$$r_{ps} = \frac{4\pi n_0 n_1 Q d_1 \cos\theta_0 (m_z n_1 \cos\theta_2 - m_y n_s \sin\theta_1)}{\lambda(n_0 \cos\theta_0 + n_s \cos\theta_2)(n_0 \cos\theta_2 + n_s \cos\theta_0)}. \tag{19}$$

Then, to the first-order approximation in the quantity $\frac{2\pi|n_1|d_1}{\lambda}$ the complex Kerr rotation angles can be expressed by

$$\Theta_K^p \equiv \frac{r_{sp}}{r_{pp}} = \frac{4\pi n_0 n_1 Q d_1 \cos\theta_0 (m_z n_1 \cos\theta_2 + m_y n_s \sin\theta_1)}{\lambda(n_s \cos\theta_0 - n_0 \cos\theta_2)(n_0 \cos\theta_0 + n_s \cos\theta_2)}, \tag{20}$$

$$\Theta_K^s \equiv \frac{r_{ps}}{r_{ss}} = \frac{4\pi n_0 n_1 Q d_1 \cos\theta_0 (m_z n_1 \cos\theta_2 - m_y n_s \sin\theta_1)}{\lambda(n_0 \cos\theta_0 - n_s \cos\theta_2)(n_0 \cos\theta_2 + n_s \cos\theta_0)}. \tag{21}$$

The denominator of Eq. (20) can be expanded as follows:

$$\lambda\left((n_s^2 - n_0^2)\cos\theta_0\cos\theta_2 + n_0 n_s(\cos^2\theta_0 - \cos^2\theta_2)\right). \tag{22}$$

Using the relation $\cos^2\theta_0 - \cos^2\theta_2 = \sin^2\theta_2 - \sin^2\theta_0$ and Snell's law, $n_0\sin\theta_0 = n_s\sin\theta_2$, the second term of the denominator can be simplified as follows:

$$\begin{aligned} n_0 n_s(\cos^2\theta_0 - \cos^2\theta_2) &= n_0 n_s(\sin^2\theta_2 - \sin^2\theta_0) \\ &= n_0 n_s \sin^2\theta_2(1 - \frac{n_s^2}{n_0^2}) \\ &= \sin\theta_0\sin\theta_2(n_0^2 - n_s^2)\ . \end{aligned} \tag{23}$$

By substituting this result in Eq.(22), the denominator can be simplified as follows:

$$\lambda(n_s^2 - n_0^2)(\cos\theta_0\cos\theta_2 - \sin\theta_0\sin\theta_2) = \lambda(n_s^2 - n_0^2)\cos(\theta_0 + \theta_2)\ . \tag{24}$$

Substituting Eq.(24) in Eq. (20), and one can simplify Eqs.(20) and (21) as follows:

$$\Theta_K^p = \frac{\cos\theta_0}{\cos(\theta_0 + \theta_2)}\left(m_y\frac{\sin^2\theta_1}{\sin\theta_2} + m_z\cos\theta_2\right)\Theta_n, \tag{25}$$

$$\Theta_K^s = \frac{\cos\theta_0}{\cos(\theta_0 - \theta_2)}\left(m_y\frac{\sin^2\theta_1}{\sin\theta_2} - m_z\cos\theta_2\right)\Theta_n. \tag{26}$$

where Θ_n is the complex polar Kerr effect for normal incidence in the ultrathin film limit given by [14–16]

$$\Theta_n \equiv \frac{4\pi n_0 d n_1^2 Q}{\lambda(n_s^2 - n_0^2)}. \tag{27}$$

It is interesting to note that the expressions for the magneto-optical Kerr effect in an ultrathin magnetic film can also be expressed as a product of the same two factors as an optically thick film [12]. The prefactor is a simple function of the optical parameters and the direction of magnetization. The main factor, Θ_n, is the complex polar Kerr effect for normal incidence in the ultrathin film limit. Also the first part, $\frac{\cos\theta_0}{\cos(\theta_0\pm\theta_2)}$, is similar to the first part of the simplified expressions for the optically thick film, except for the interchange of θ_2 and θ_1, since the optical properties of the system is governed by those of the substrate for and ultrathin magnetic medium.

In conclusion, we have derived simplified analytic formulae for various MOKE's in a ultrathin magnetic film. Considering the effect of the multiple reflections in the ultrathin magnetic film, it was found that they could be described as a product of two factors, similar to those of the optically thick magnetic film case. The prefactor is a function of the optical parameters of the system and the main factor is the well-known complex polar Kerr effect for normal incidence in a ultrathin magnetic film. These simplified formulae should be useful in using MOKE to study the magnetic properties of ultrathin magnetic films having arbitrary magnetization direction.

ACKNOWLEDGMENTS

This work was supported by the Ministry of Science and Technology of Korea, and one author (CYY) wishes to acknowledge the financial support of the Korea Research Foundation made in program Year 1997, and the hospitality of Argonne National Laboratory. Argonne was supported the U.S. Department of Energy, BES-Material Science, under contract No. W-31-109-ENG-38.

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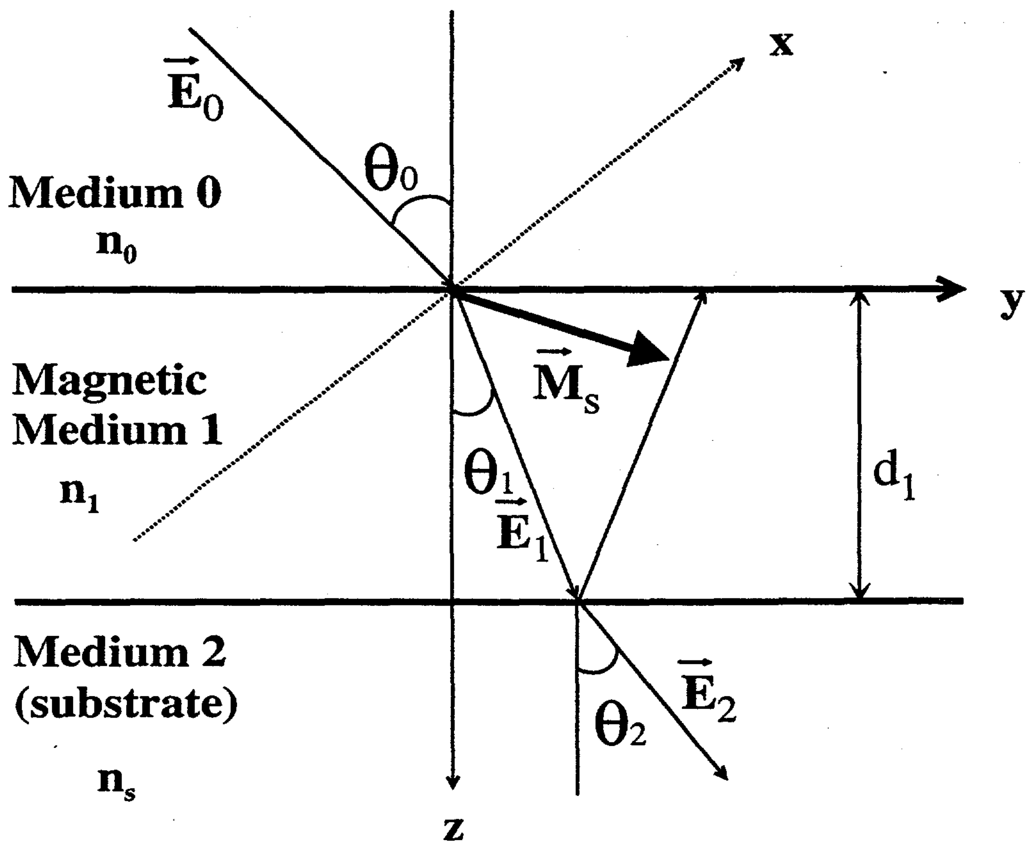
FIGURES

FIG. 1. The coordinate system of a non-magnetic medium 0, a magnetic medium 1, and a non-magnetic medium 2. The thickness of the medium 1 is d_1. The magnetization direction of medium 1 is arbitrary.



$\vec{E}_0$
x
θ_0
Medium 0
n_0
y
Magnetic
Medium 1
$\vec{M}_s$
θ_1
$\vec{E}_1$
d_1
n_1
Medium 2
(substrate)
$\vec{E}_2$
θ_2
n_s
z

Fig. 1