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Title: An Apropriate Metric to Switch on a Turbulence Model for Rayleigh-Taylor Instability Driven Mixing

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An Apropriate Metric to Switch on a Turbulence Model for Rayleigh-Taylor Instability Driven Mixing

B. Rollin & M. J. Andrews

Abstract Text

Rayleigh-Taylor (RT) instability occurs at a perturbed interface between fluids of different densities, when the lighter fluid is accelerated into the heavier fluid. In time, as the two fluids seek to reduce their combined potential energy, the mixing becomes turbulent. This fundamental instability is observed, and plays a key role, in numerous natural phenomena, e.g. supernovae explosions, and in engineering applications, e.g. Inertial Confinement Fusion (ICF). The importance of initial condition (ICs) effects on the growth and mixing of Rayleigh-Taylor instability open an opportunity for "design" of RT turbulence for engineering, and question our current predictive capability. Indeed, commonly used turbulence models used for engineering applications are tuned for fully developed turbulence, whereas RT instability is a dynamic process that evolves toward turbulence under the influence of ICs. Therefore, our efforts aim at defining a procedure for properly accounting for initial conditions in variable density (RT) turbulence models. Our strategy is to have a model for the "early" evolution of the RT instability that will produce the initial conditions for the turbulence model. We already dispose of a modal model to evolve the RT mixing layer starting from almost any initial conditions. The present work is a first look at determining an appropriate metric for switching from the modal model to a variable density turbulence model.

An Apropriate Metric to Switch on a Turbulence Model for Rayleigh-Taylor Instability Driven Mixing

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LDRD: Turbulence by Design

LA-UR 11-04xxx

ASME-JSME-KSME Joint Fluids Engineering Conference
Hamamatsu – 07/28/11

Importance of Initial Conditions for Turbulence "Design" and Prediction

Work hypothesis:

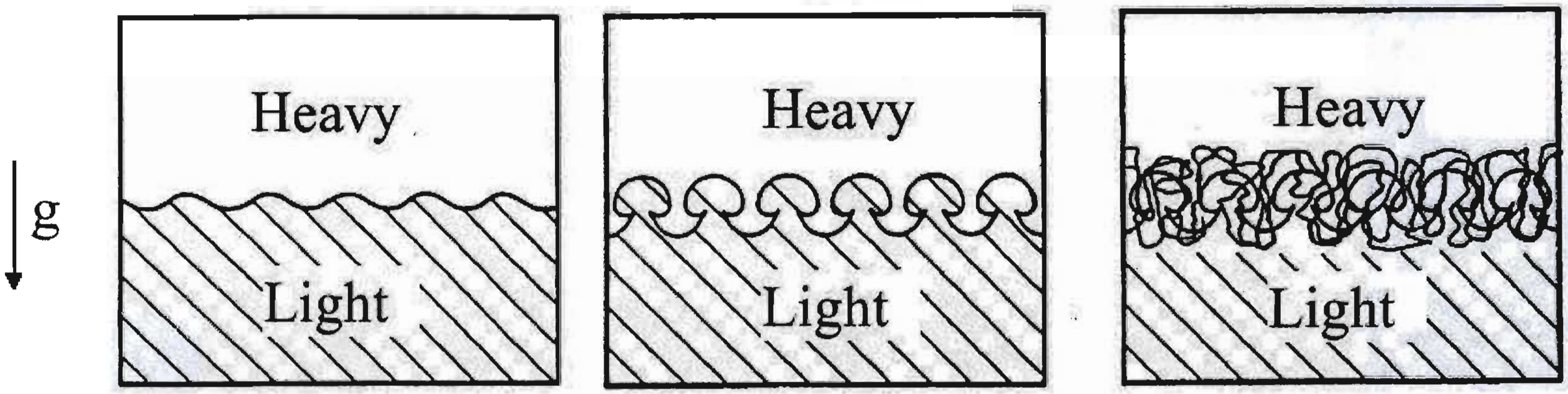
- Initial conditions could affect "late-time" turbulent transport and mixing effectiveness. Hence, a challenge for prediction, but also an opportunity for turbulence "design".

Objective:

- Provide a rational basis for setting up initial conditions in turbulence models.

Rayleigh-Taylor Instability

Credit: M.J. Andrews



Exponential growth **Nonlinear growth** **Turbulent mixing**

Characteristic non-dimensional number: $A_T = \frac{\rho^h - \rho^l}{\rho^h + \rho^l}$

Interface is unstable if: $\nabla p \cdot \nabla \rho < 0$

Baroclinic generation of vorticity: $\frac{1}{\rho^2} \nabla p \times \nabla \rho$

Inertial Confinement Fusion (ICF)

Some Dramatic Effects of Initial Conditions

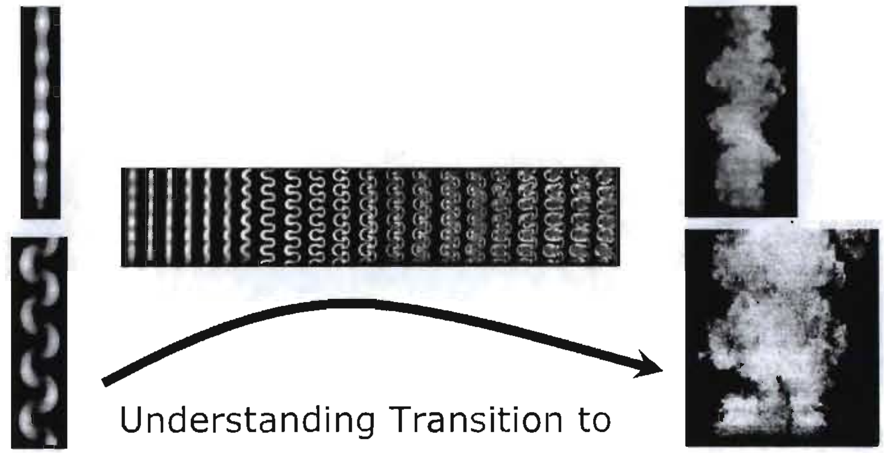
M.J. Andrews, TAMU water channel experiment

Long wavelength initial conditions

Short wavelength initial conditions

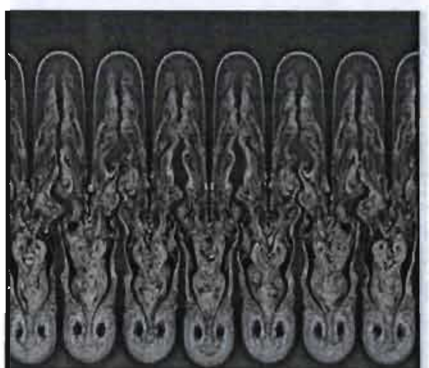
Richtmyer-Meshkov (RM) Transitions From Different Initial Conditions

(from the LANL Gas Shock Tube – K. Prestridge)

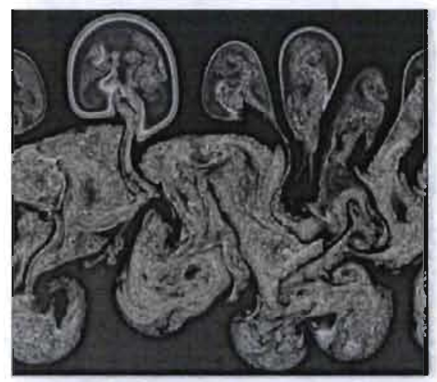


Understanding Transition to Turbulence

Credit: Hjelm & Ristorcelli



No IC noise



With IC noise

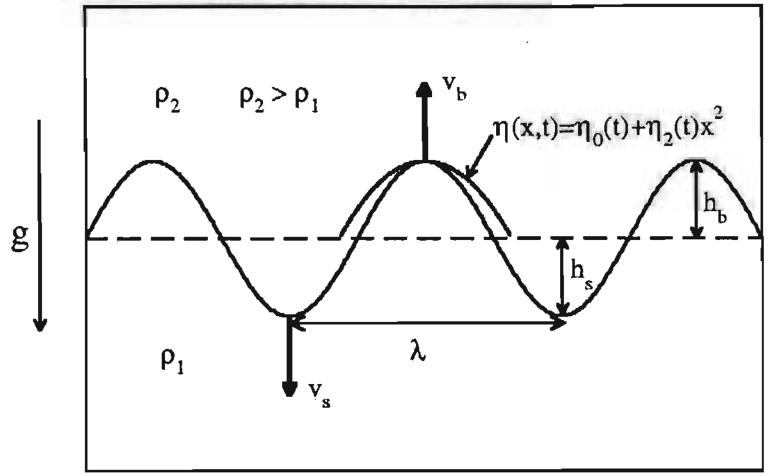
An ODE Model for Multi-mode

Goncharov model:

- **Velocity potentials (3D bubble)**

$$\phi^h = a(t) J_0(kr) e^{-k(z-\eta_0)}$$

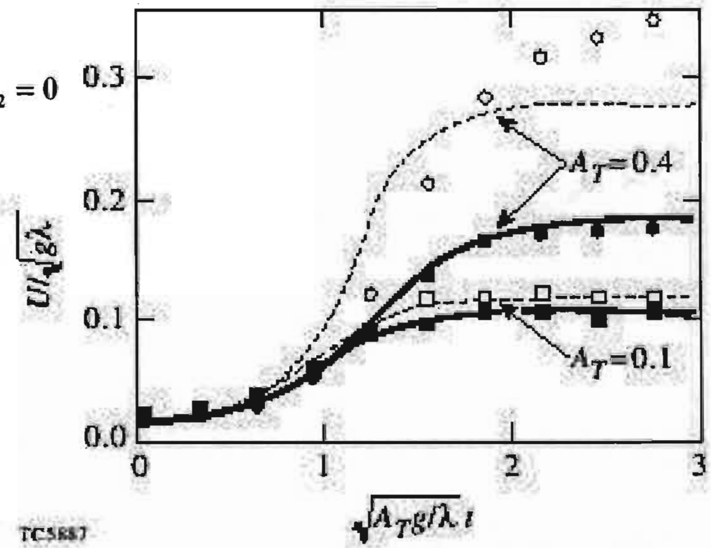
$$\phi^l = b_1(t) J_0(kr) e^{k(z-\eta_0)} + b_2(t) z$$



After substituting the potentials in the boundary conditions:

$$\eta_2 = -\frac{k}{8} + \left(\frac{k}{8} + \eta_2(0)\right) e^{-2k(\eta_0 - \eta_0(0))}$$

$$\ddot{\eta}_0 \frac{k^2 - 4A_T k\eta_2 - 32A_T\eta_2^2}{4(k-8\eta_2)} + \dot{\eta}_0^2 k^2 \frac{(5A_T-4)k^2 + 16(2A_T-3)k\eta_2 + 64A_T\eta_2^2}{8(k-8\eta_2)^2} + A_T g \eta_2 = 0$$

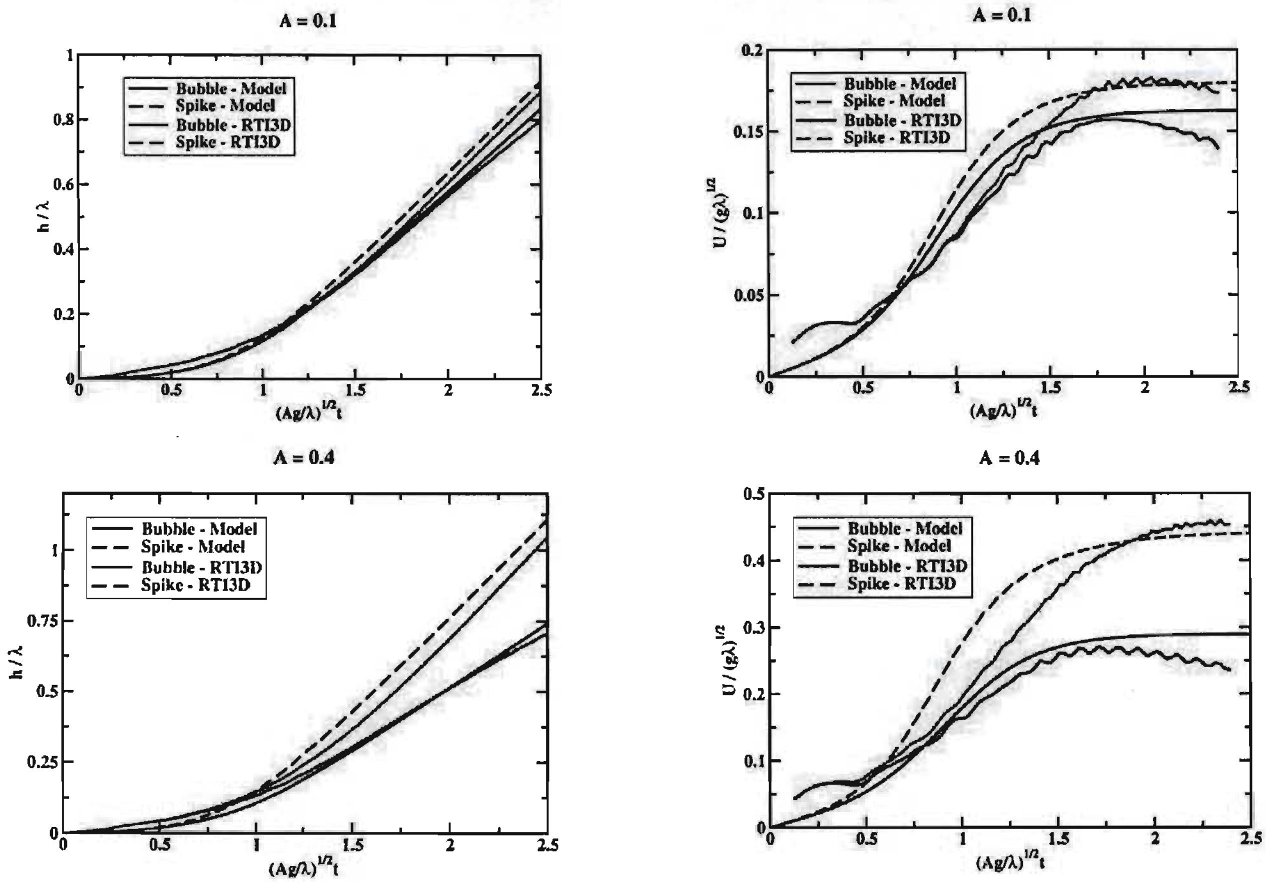


- **Non-linear model**
- **Valid for moderate A_T**
- **Good prediction for bubble**

- **Single mode model**
- **Spike inaccurate for high A_T**

Goncharov, PRL, **88**, 2002

Single Mode Model Results



- **The Goncharov model performs well for low to moderate Atwood numbers**

A Weakly Nonlinear Model for Multimode Perturbation

Haan's model:

$$\Delta\phi^{h/l} = 0$$

$$\partial_t Z + \partial_x Z \cdot \partial_x \phi\big|_Z + \partial_y Z \cdot \partial_y \phi\big|_Z = \partial_z \phi\big|_Z$$

$$\left[\rho\left(\partial_t \phi + \frac{1}{2}v^2 + gZ\right)\right] = P$$

$$Z(\vec{x},t) = \sum_{\vec{k}} Z_k(t) e^{i\vec{k}\cdot\vec{x}}$$

$$\phi^h(\vec{x},z,t) = \sum_{\vec{k}} \phi_k^h(t) e^{-kz} e^{i\vec{k}\cdot\vec{x}}$$

$$\phi^l(\vec{x},z,t) = \sum_{\vec{k}} \phi_k^l(t) e^{kz} e^{i\vec{k}\cdot\vec{x}}$$

$$\ddot{Z}_k = \gamma(k)^2 Z_k + \underline{A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right)\right\}}$$

Mode coupling term

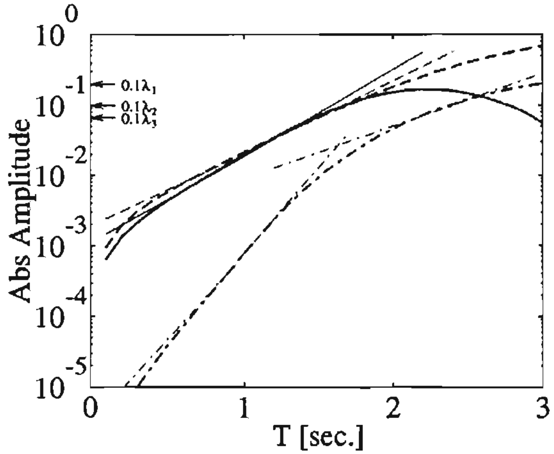
$$\vec{n} = \vec{k} - \vec{m} \qquad \gamma(k) = \sqrt{A_T g k}$$

- **Haan's model allow mode generation, but is valid only until ealy transition to nonlinear behavior**

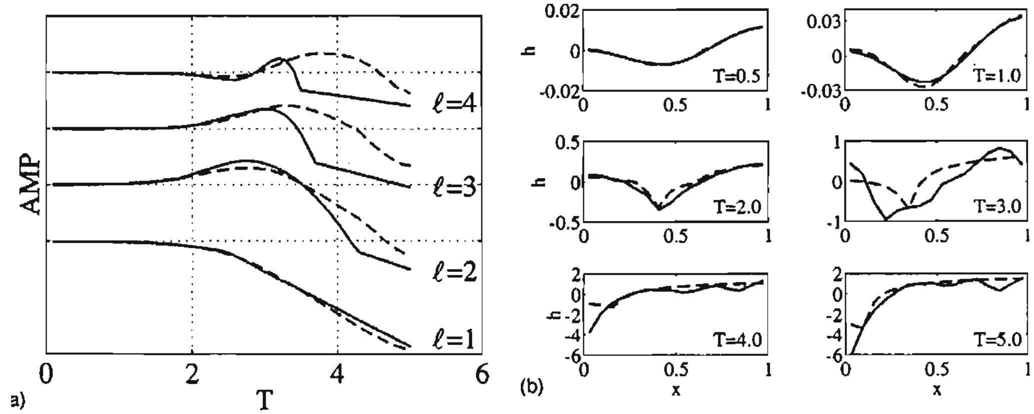
Modes Post-saturation Behavior

Ofer *et al.*, Phys. Plasmas, **3** (1996)

Evolution of a two mode initial perturbation, modes 2 & 3



Evolution of a two mode initial perturbation, modes 1 & 2



A saturated mode cease to contribute to mode coupling

A saturated mode k can only be affected by two lower-k modes. Its velocity can never exceed its saturation velocity.

A Modal Model for Multimode RT mixing layer Growth

A modal model for multimode RT built from the "fusion" between a potential flow model for single mode and a weakly nonlinear model:

For all k,

$$\begin{cases} \ddot{Z}_k = G(k) + A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n} \right) \right\} & \text{Before k saturates} \\ \ddot{Z}_k = A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n} \right) \right\} & \text{After k has saturated} \end{cases}$$

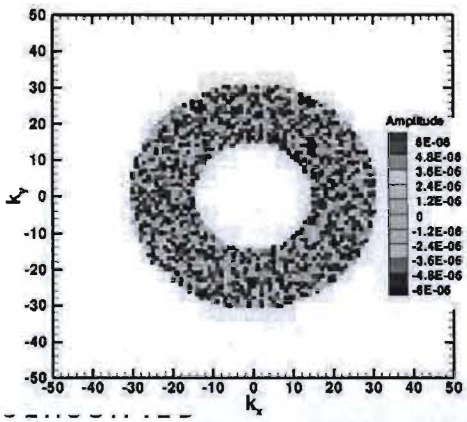
$|m|, |n| < |k|$

$$G(k) = \frac{4(k - 8\eta_2)}{k^2 - 4A_T k \eta_2 - 32 A_T \eta_2^2} \left(-\dot{Z}_k^2 k^2 \frac{(5A_T - 4)k^2 + 16(2A_T - 3)k\eta_2 + 64 A_T \eta_2^2}{8(k - 8\eta_2)^2} - A_T g \eta_2 \right)$$

$$k = \sqrt{k_x^2 + k_y^2}$$

$$\eta_2 = -\frac{k}{8} + \left(\frac{k}{8} + \eta_2(0) \right) e^{-2k(\eta_0 - \eta_0(0))}$$

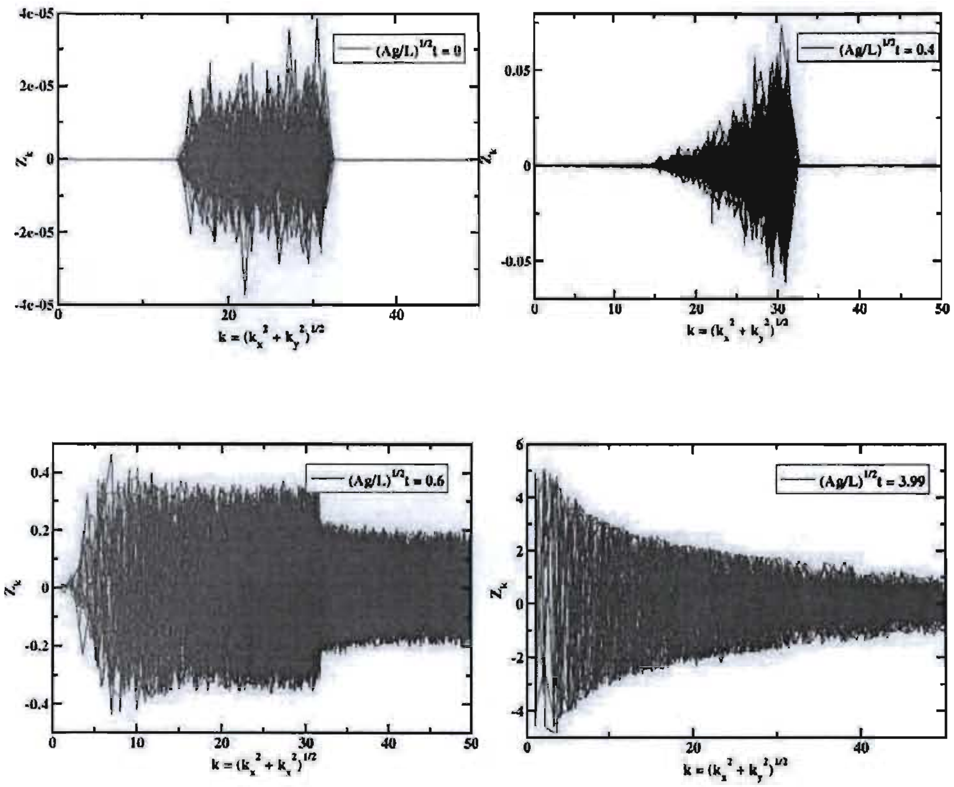
Initial perturbation in wave space



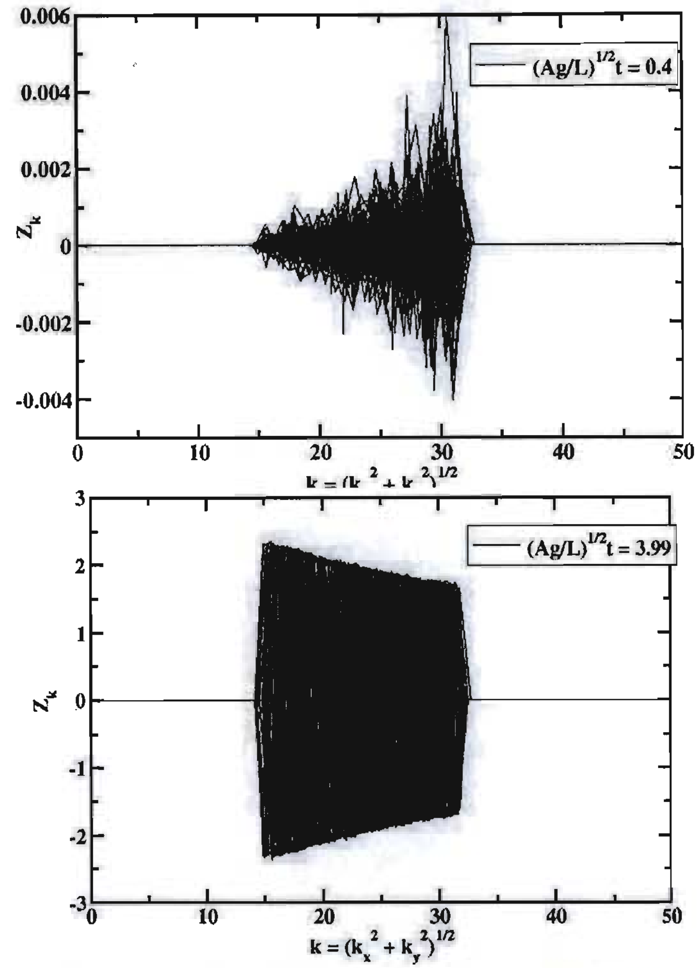
Using a two dimensional initial perturbation spectrum for the model allow a one-to-one match with ICs for 3D simulations

Modal Model Behavior

Mode Coupling

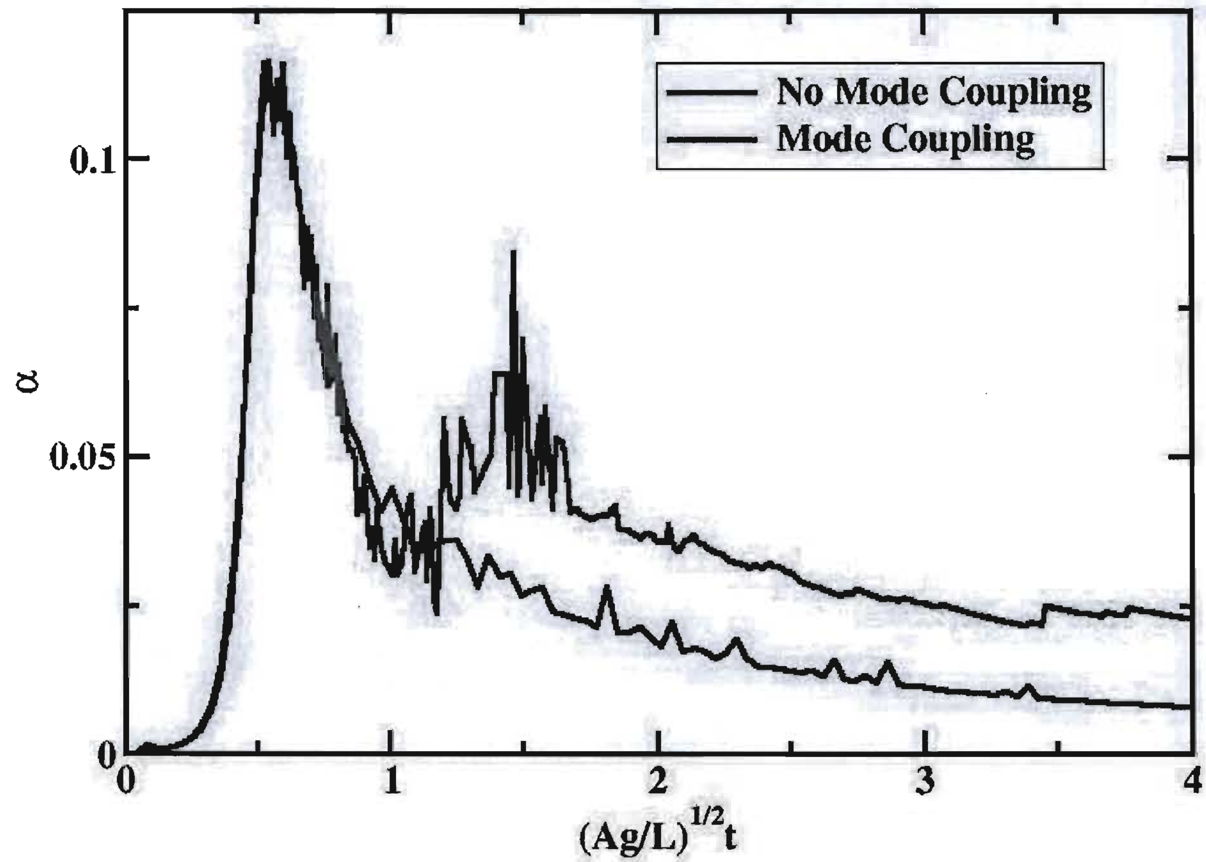


No Mode Coupling



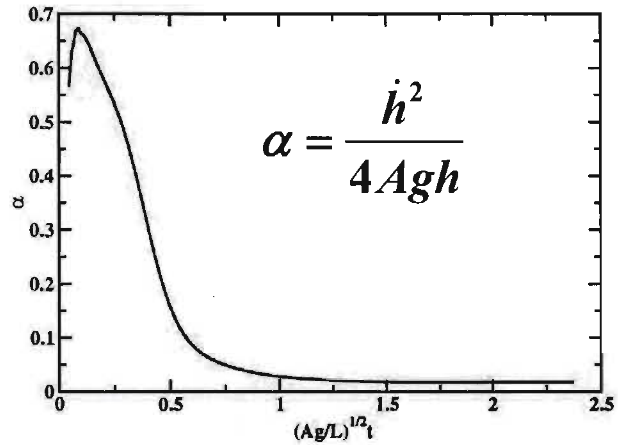
The mode coupling function will "populate" the entire spectrum

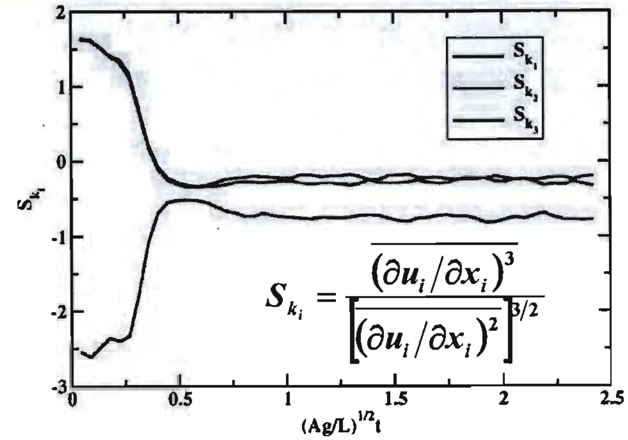
Modal Model Behavior

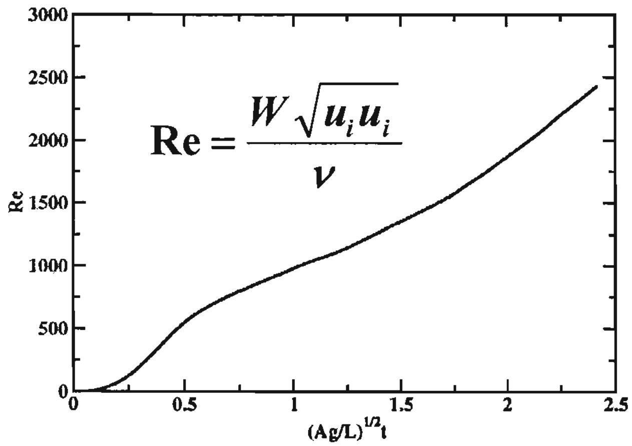


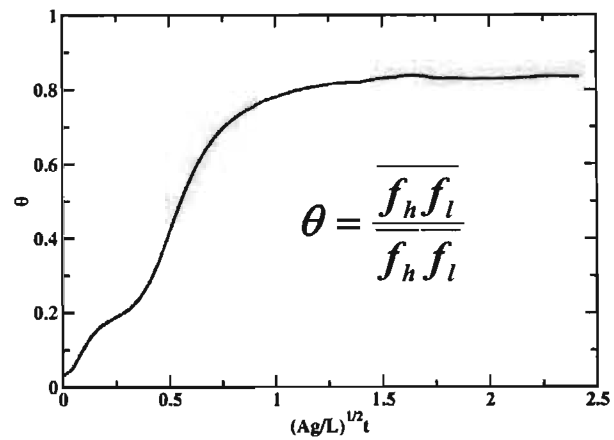
Mode coupling is at the origin of self similarity

Establishment of a nonlinear cascade process and mixing







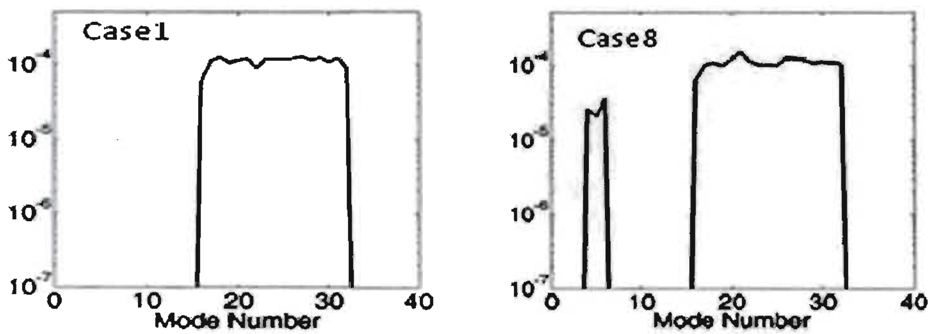


It appears that the establishment of a nonlinear cascade process occurs at about the same time as the mixing layer growth becomes self-similar

Complex initial spectrum

A Rayleigh-Taylor Multi-Mode Study with Banded Spectra and Their Effect on Late-Time Mix Growth (Banerjee & Andrews, 2009)

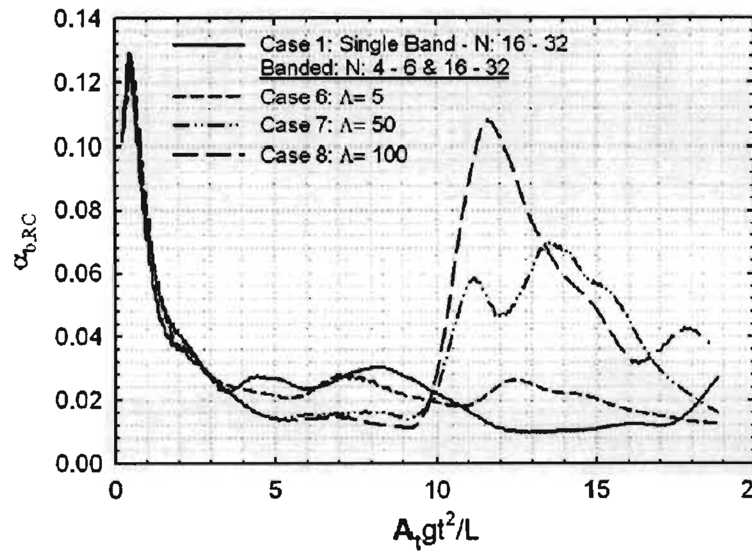
Initial Spectrum



$$\frac{\overline{h_0'^2}}{2} = \int_{k_{min}}^{k_{max}} E_{h0}(k)dk = \int_{k_{min}}^{k_1} E_{h1}(k)dk + \int_{k_2}^{k_{max}} E_{h2}(k)dk = \frac{\overline{h_1'^2}}{2} + \frac{\overline{h_2'^2}}{2}$$

$$\Lambda = \overline{h_2'^2} \Big/ \overline{h_1'^2}$$

Bubble front growth rate



What metric can we use to define a reasonable starting point for a turbulence model based on the initial perturbation spectrum?

Summary

- We have a model for multimode RT
- Model track development of perturbation spectrum
- The center of the mixing layer appears sufficiently turbulent as the mixing layer growth turns self-similar

- **Next steps:**
  - Fine tuning our model
  - Improve the model to a larger range of Atwood number
  - Define a metric indicating that using a turbulence model is appropriate

- **Acknowledgements:**

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