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Title: A Model for Mixing Layer Growth of the Rayleigh-Taylor Instability

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A Model for Mixing Layer Growth of the Rayleigh-Taylor Instability

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It is now well accepted in the "Rayleigh-Taylor research community" that relatively late time RT turbulent mixing can still be influenced by the initial conditions that set off the instability. This property surely opens an opportunity for design of the initial perturbation in order to fit a given application's need (e.g. improve efficiency of ICF by minimizing RT mixing). However, it also implies that one has little chance of giving a meaningful prediction for an application where RT plays an important role, unless one can somehow reasonably predict the effects of initial conditions on the RT instability development. We are trying to capture the RT mixing layer evolution and important initial conditions effects from the initial moment of the RT instability until late time, to be able to provide profiles of turbulence model variables (turbulence model initial conditions) at any time a turbulence model user would want to start his model. I will present a modal model predicting the growth of the RT instability mixing layer. Based on the model predictions and observed characteristics of the RT mixing layer, I will show how initial profiles for turbulence model variables could be extracted.

A Model for Mixing Layer Growth of the Rayleigh-Taylor Instability

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LDRD: Turbulence by Design

LA-UR 11-xxxxx

CoMuEx – Los Alamos – 07/?/11

Outline

- Motivation
- Modal Model
- Initial profile for turbulence model
- Summary

Importance of Initial Conditions for Turbulence "Design" and Prediction

Premise:

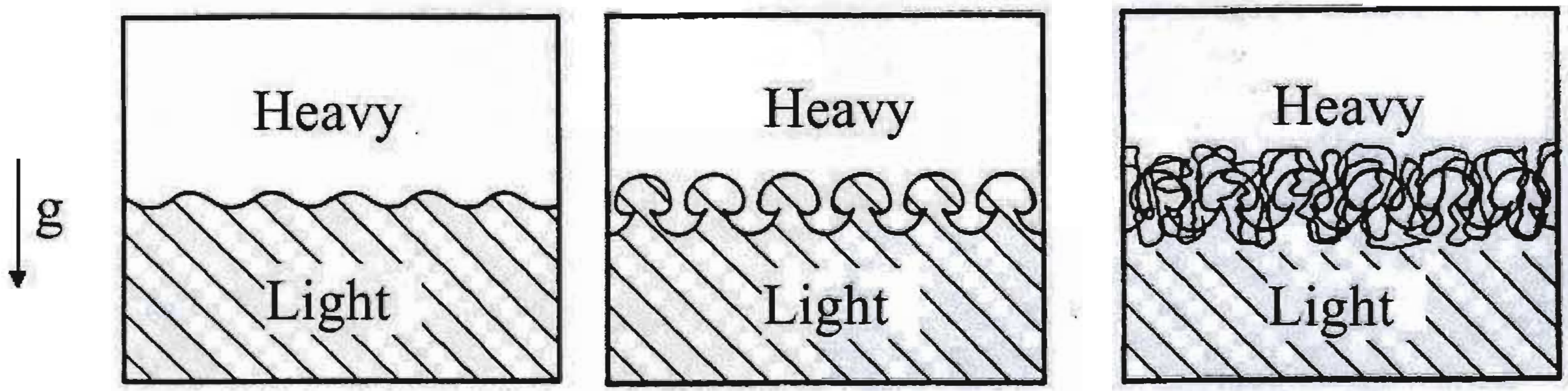
- Initial conditions could affect "late-time" turbulent transport and mixing effectiveness. Hence, a challenge for prediction, but also an opportunity for turbulence "design".

Objective:

- Provide a rational basis for setting up initial conditions in turbulence models.

Rayleigh-Taylor Instability

Credit: M.J. Andrews

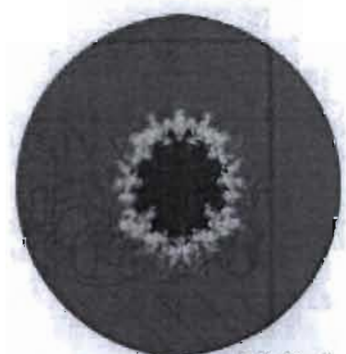


Exponential growth **Nonlinear growth** **Turbulent mixing**

Characteristic non-dimensional number: $A_T = \frac{\rho^h - \rho^l}{\rho^h + \rho^l}$

Interface is unstable if: $\nabla p \cdot \nabla \rho < 0$

Baroclinic generation of vorticity: $\frac{1}{\rho^2} \nabla p \times \nabla \rho$



Inertial Confinement Fusion (ICF)

Some Dramatic Effects of Initial Conditions

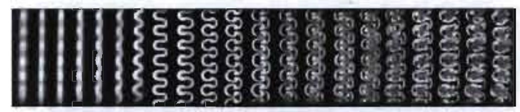
M.J. Andrews, TAMU water channel experiment

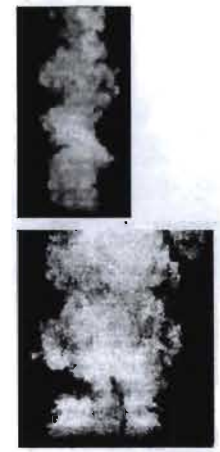
Long wavelength initial conditions

Short wavelength initial conditions

Richtmyer-Meshkov (RM) Transitions From Different Initial Conditions

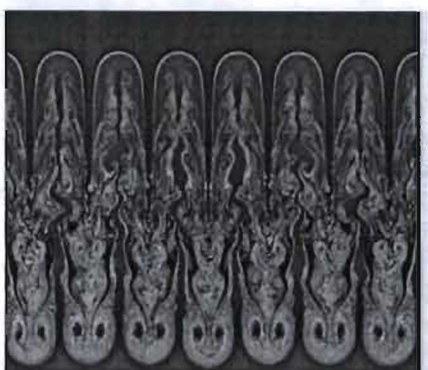
(from the LANL Gas Shock Tube – K. Prestridge)



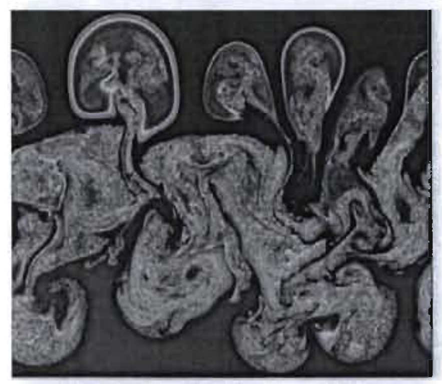


Understanding Transition to Turbulence

Credit: Hjelm & Ristorcelli



No IC noise

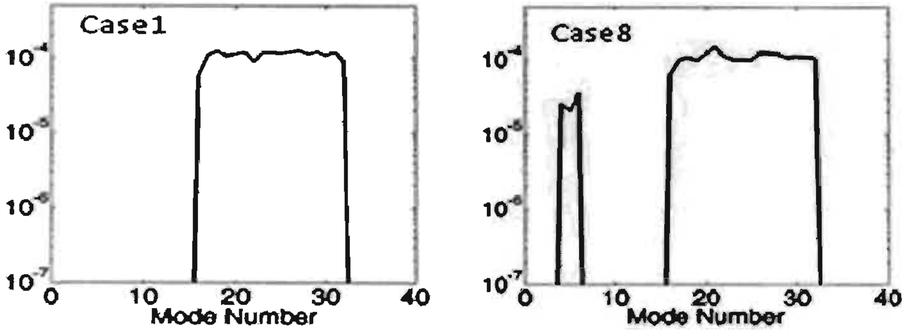


With IC noise

Why a Modal Model?

A Rayleigh-Taylor Multi-Mode Study with Banded Spectra and Their Effect on Late-Time Mix Growth (Banerjee & Andrews, 2009)

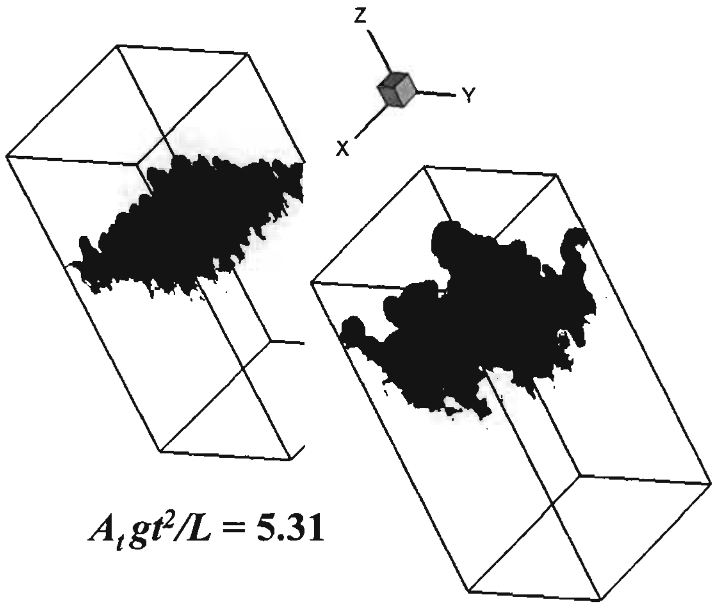
Initial Spectrum



$$\frac{\overline{h_0'^2}}{2} = \int_{k_{\min}}^{k_{\max}} E_{h0}(k)dk = \int_{k_{\min}}^{k_1} E_{h1}(k)dk + \int_{k_2}^{k_{\max}} E_{h2}(k)dk = \frac{\overline{h_1'^2}}{2} + \frac{\overline{h_2'^2}}{2}$$

$$\Lambda = \overline{h_2'^2} / \overline{h_1'^2}$$

3-D ILES Simulations

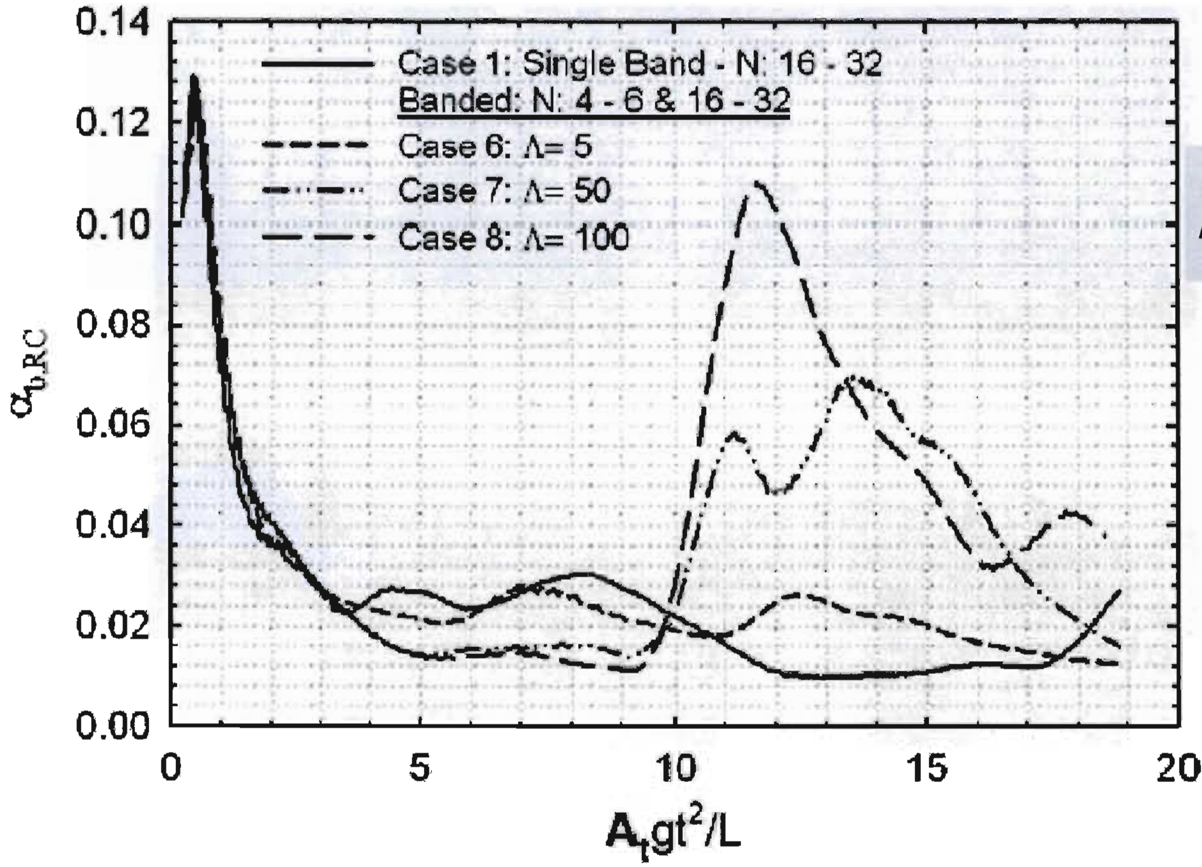


$A_t g t^2/L = 5.31$

$A_t g t^2/L =$ 19.62

Why a Modal Model?

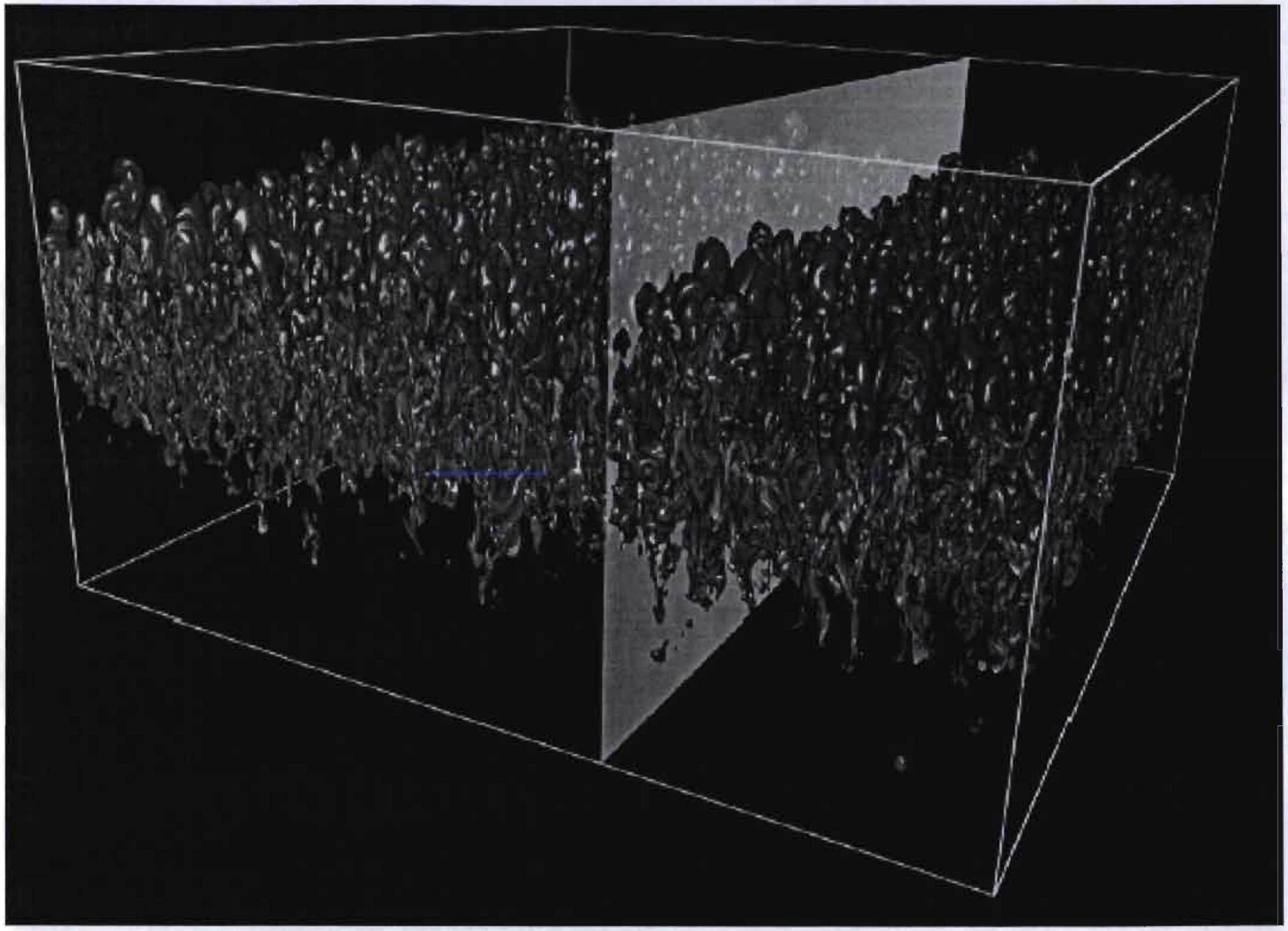
3-D ILES Simulations of Banded Spectra and Late-Time Appearance of Long Wavelengths (Banerjee & Andrews 2009)



$$h_b = \alpha A_t g t^2$$

$$\alpha_b = \frac{\dot{h}_b^{\,2}}{4 A_t g h_b}$$

Why a modal model ?



Livescu, ADTSC Science Highlights 2011

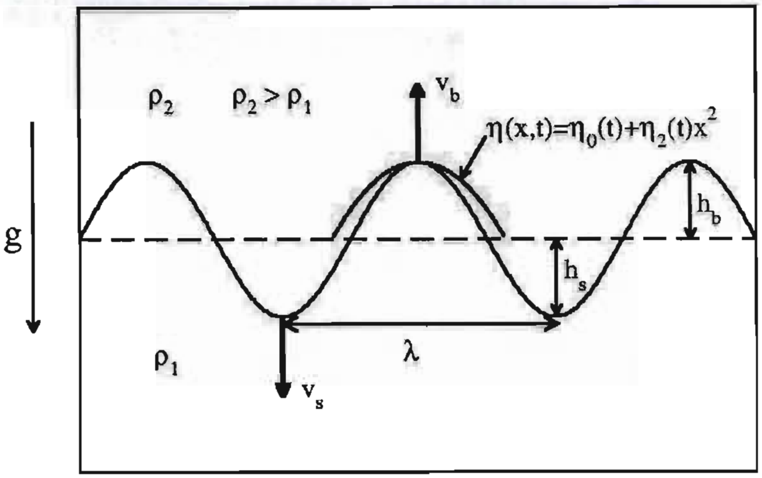
A Potential Flow Model for Single Mode Perturbation

Goncharov model:

$$\Delta\phi^{h/l} = 0$$

$$\phi^h = a(t)J_0(kr)e^{-k(z-\eta_0)}$$

$$\phi^l = b_1(t)J_0(kr)e^{k(z-\eta_0)} + b_2(t)z$$



$$\partial_t\eta + v_r^{h/l}\partial_r\eta = v_z^{h/l} \qquad \left[v_z - v_r\partial_r\eta\right] = 0 \qquad [Q] = Q^h - Q^l$$

$$\left[\rho\left(\partial_t\phi + \frac{1}{2}v^2 + g\eta\right)\right] = P$$

- **The velocity potential are expended to 2nd order and plugged in the interfacial conditions**

Single Mode Model Results

$$\eta_2 = -\frac{k}{8} + \left(\frac{k}{8} + \eta_2(0)\right) e^{-2k(\eta_0 - \eta_0(0))}$$

$$\ddot{\eta}_0 \frac{k^2 - 4A_T k\eta_2 - 32A_T\eta_2^2}{4(k - 8\eta_2)} + \dot{\eta}_0^2 k^2 \frac{(5A_T - 4)k^2 + 16(2A_T - 3)k\eta_2 + 64A_T\eta_2^2}{8(k - 8\eta_2)^2} + A_T g\eta_2 = 0$$

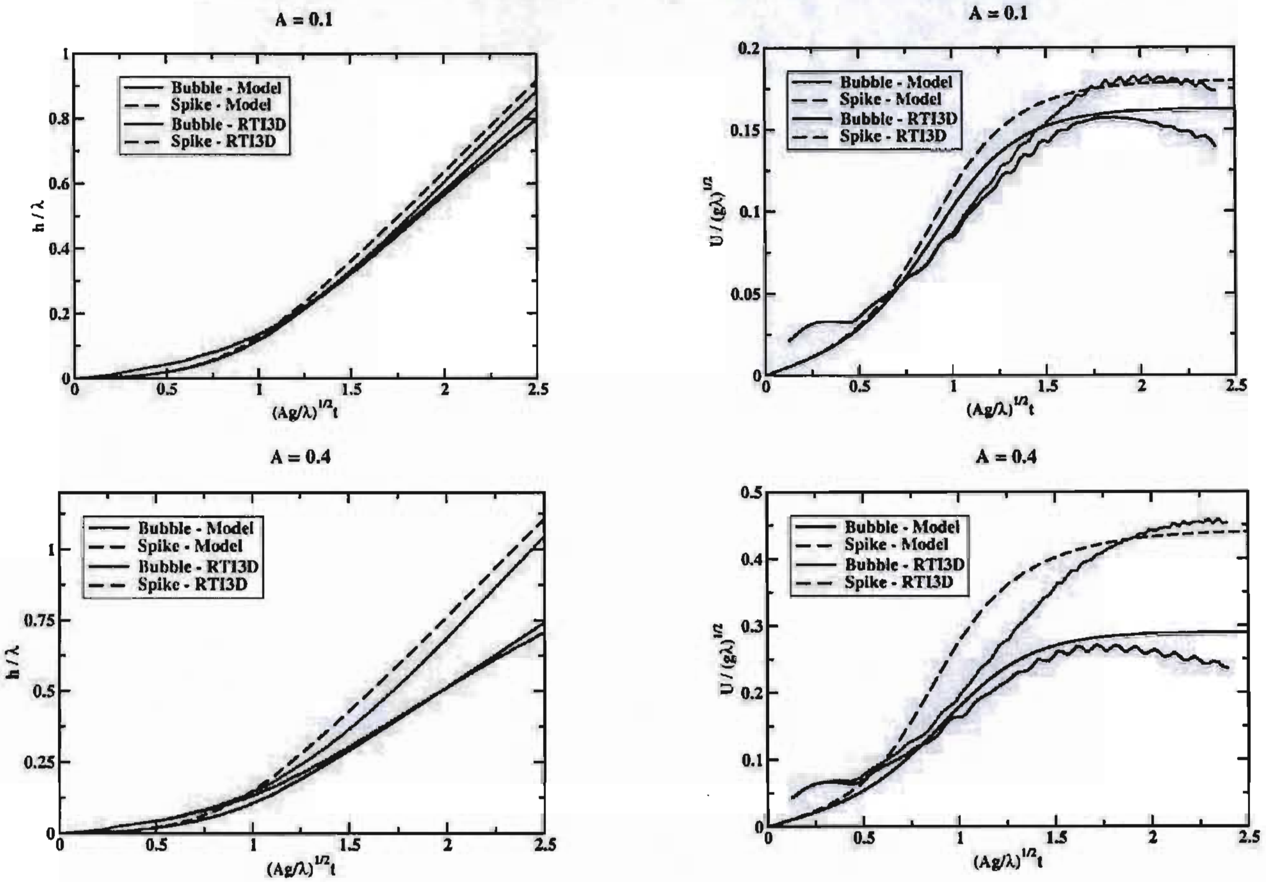
$$\eta_2(t \to \infty) = -\frac{k}{8} \qquad U = \sqrt{\frac{2A_T}{1 + A_T}\frac{g}{k}}$$

For spikes

$$\eta \to -\eta \qquad A_T \to -A_T \qquad g \to -g$$

- **The Goncharov model predicts the bubble growth up to first saturation**

Single Mode Model Results

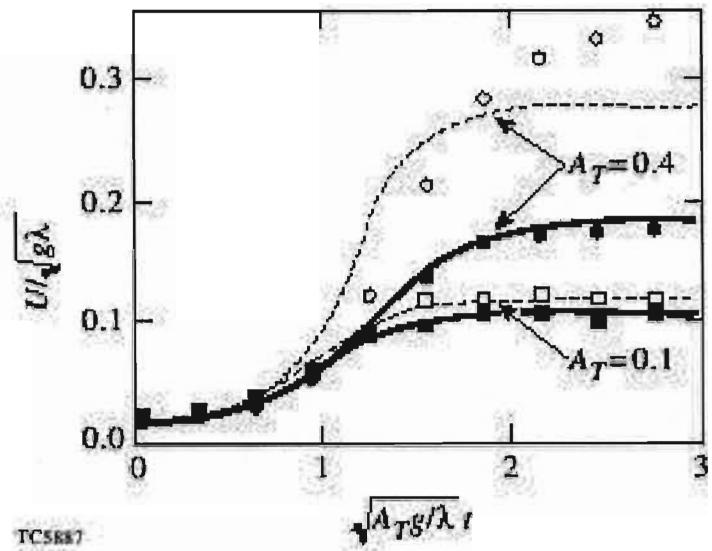


- The Goncharov model performs well for low Atwood numbers

Single Mode Model Summary

- 👍 Nonlinear model
- 👍 Valid on a large range of A_T ($0 \leq A_T \leq 0.4$)
- 👍 Good prediction for bubble

- 👎 Spike inaccurate for high A_T



Goncharov, PRL, **88**, 2002

A Weakly Nonlinear Model for Multimode Perturbation

Haan's model:

$$\Delta\phi^{h/l} = 0$$

$$\partial_t Z + \partial_x Z \cdot \partial_x \phi\big|_Z + \partial_y Z \cdot \partial_y \phi\big|_Z = \partial_z \phi\big|_Z$$

$$\left[\rho\left(\partial_t \phi + \frac{1}{2}v^2 + gZ\right)\right] = P$$

$$Z(\vec{x},t) = \sum_{\vec{k}} Z_k(t) e^{i\vec{k}\cdot\vec{x}}$$

$$\phi^h(\vec{x},z,t) = \sum_{\vec{k}} \phi_k^h(t) e^{-kz} e^{i\vec{k}\cdot\vec{x}}$$

$$\phi^l(\vec{x},z,t) = \sum_{\vec{k}} \phi_k^l(t) e^{kz} e^{i\vec{k}\cdot\vec{x}}$$

$$\ddot{Z}_k = \gamma(k)^2 Z_k + \underbrace{A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right) \right\}}_{\text{Mode coupling term}}$$

Mode coupling term

$$\vec{n} = \vec{k} - \vec{m} \qquad \gamma(k) = \sqrt{A_T g k}$$

- **Haan's model allow mode generation**

Weakly Nonlinear Model Summary

- 👍 Nonlinear model
- 👍 Valid for all Atwood number
- 👍 Multimode model, i.e., handle mode coupling

- 👎 Valid until early transition to nonlinear behavior

A Modal Model for Multimode RT: Linear regime

An modal model for multimode RT built from the "fusion" between a potential flow model for single mode and a weakly nonlinear model:

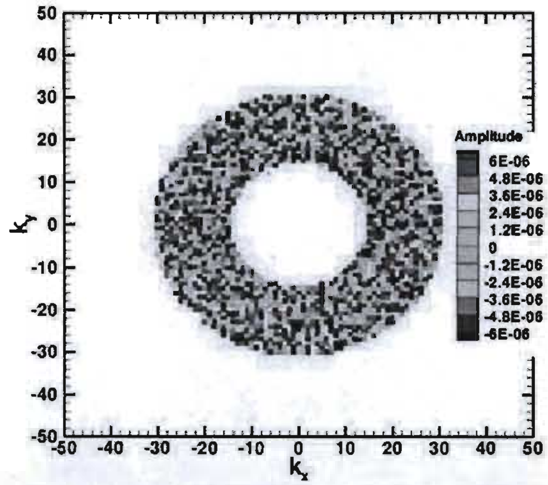
For all k,

$$\ddot{Z}_k = \frac{4(k-8\eta_2)}{k^2 - 4A_T k\eta_2 - 32A_T\eta_2^2}\left(-\dot{Z}_k^2 k^2 \frac{(5A_T-4)k^2 + 16(2A_T-3)k\eta_2 + 64A_T\eta_2^2}{8(k-8\eta_2)^2} - A_T g\eta_2\right.$$

$$\left. + A_T k\sum_{\vec{m}}\left\{\ddot{Z}_m Z_n\left(1-\hat{m}\cdot\hat{k}\right) + \dot{Z}_m\dot{Z}_n\left(\frac{1}{2}-\hat{m}\cdot\hat{k}-\frac{1}{2}\hat{m}\cdot\hat{n}\right)\right\}\right)$$

$$k = \sqrt{k_x^2 + k_y^2}$$

Initial perturbation in wave space

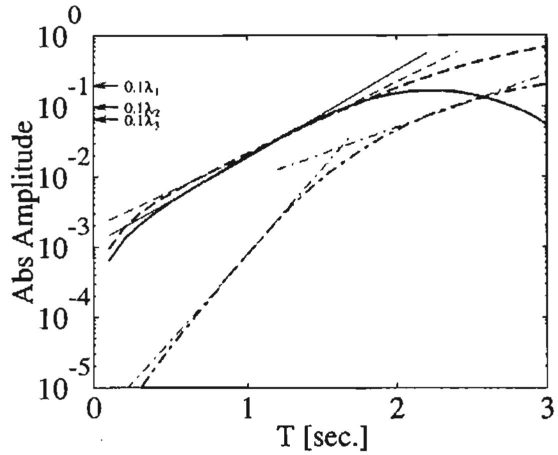


Using a two dimensional initial perturbation spectrum for the model allow a one-to-one match with ICs for 3D simulations

Post-saturation treatment

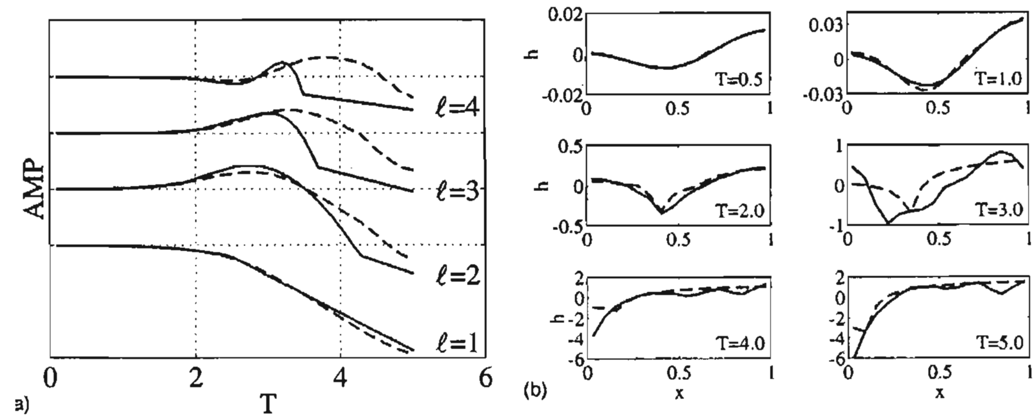
Ofer *et al.*, Phys. Plasmas, **3** (1996)

Evolution of a two mode initial perturbation, modes 2 & 3



A saturated mode cease to contribute to mode coupling

Evolution of a two mode initial perturbation, modes 1 & 2



A saturated mode k can only be affected by two lower-k modes. Its velocity can never exceed its saturation velocity.

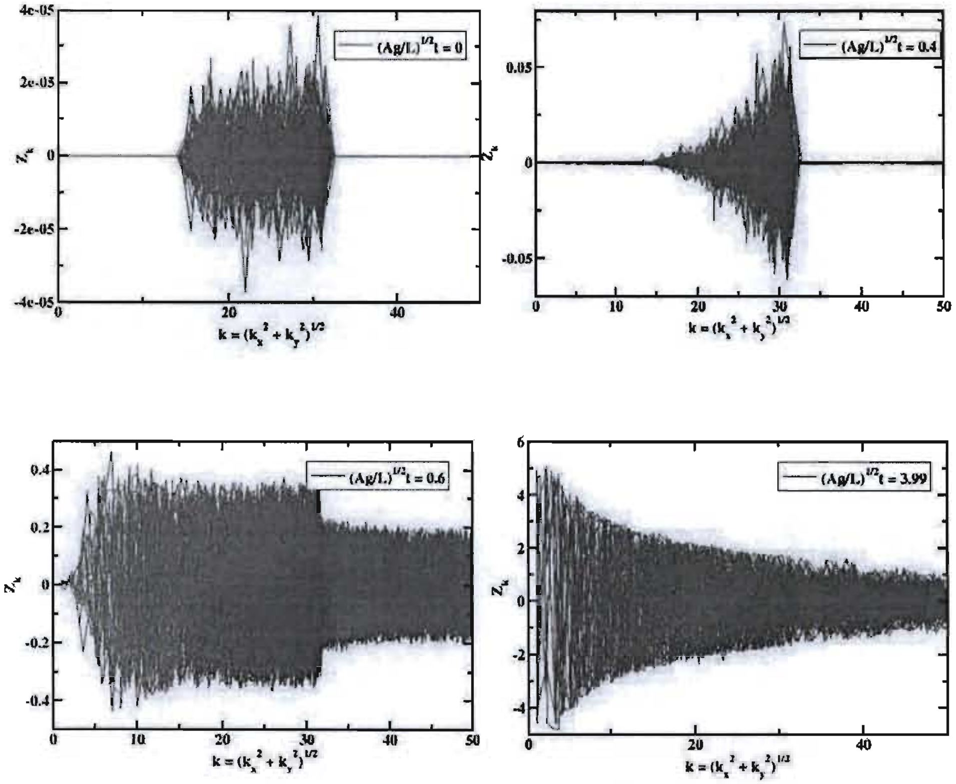
A Modal Model for Multimode RT mixing layer Growth: summary

For all k,

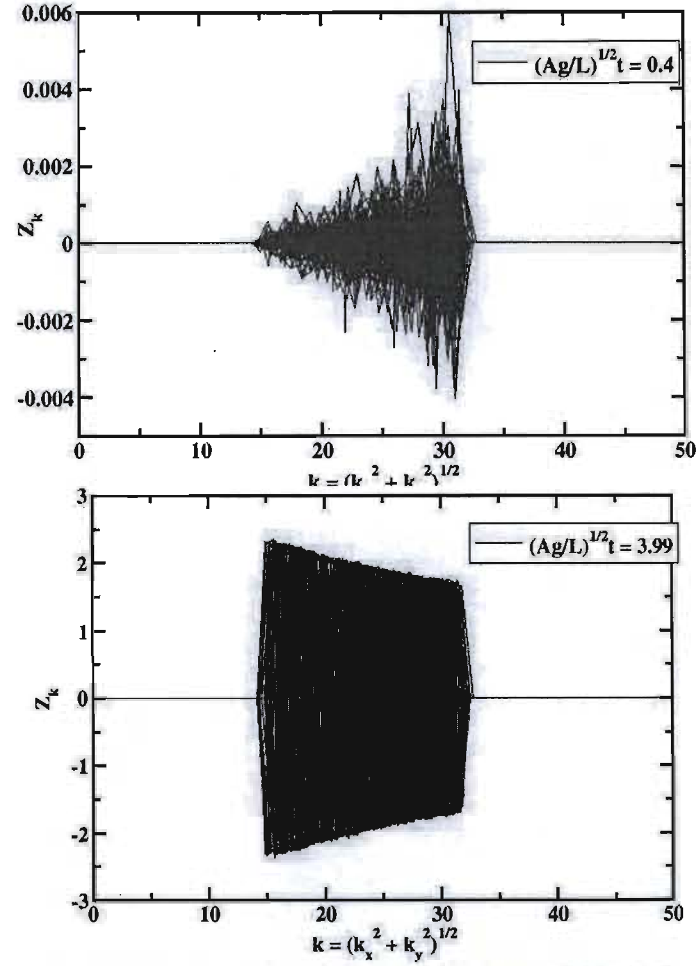
$$\begin{cases} \ddot{Z}_k = G(k) + A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right) \right\} & \text{Before k saturates} \\ \ddot{Z}_k = A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right) \right\} & \text{After k has saturated } |m|,|n| < |k| \end{cases}$$

Modal Model Behavior

Mode Coupling

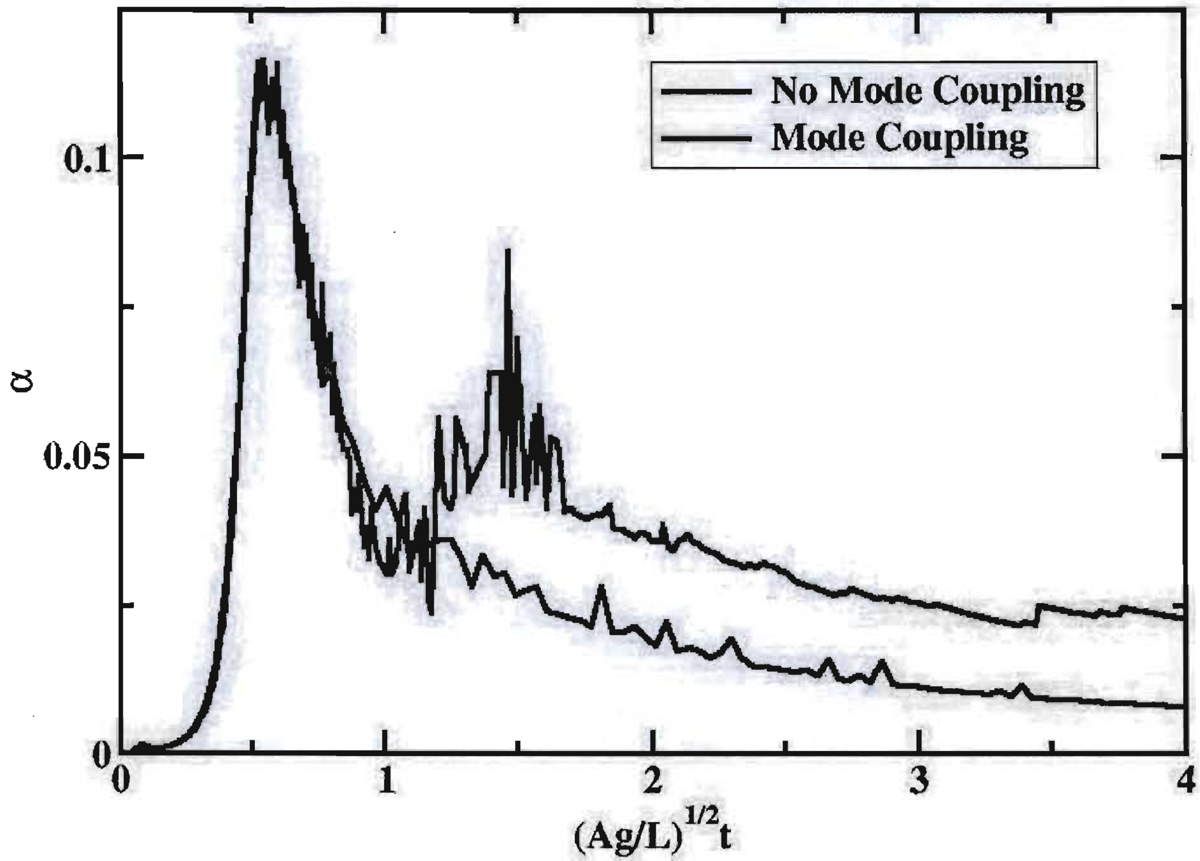


No Mode Coupling



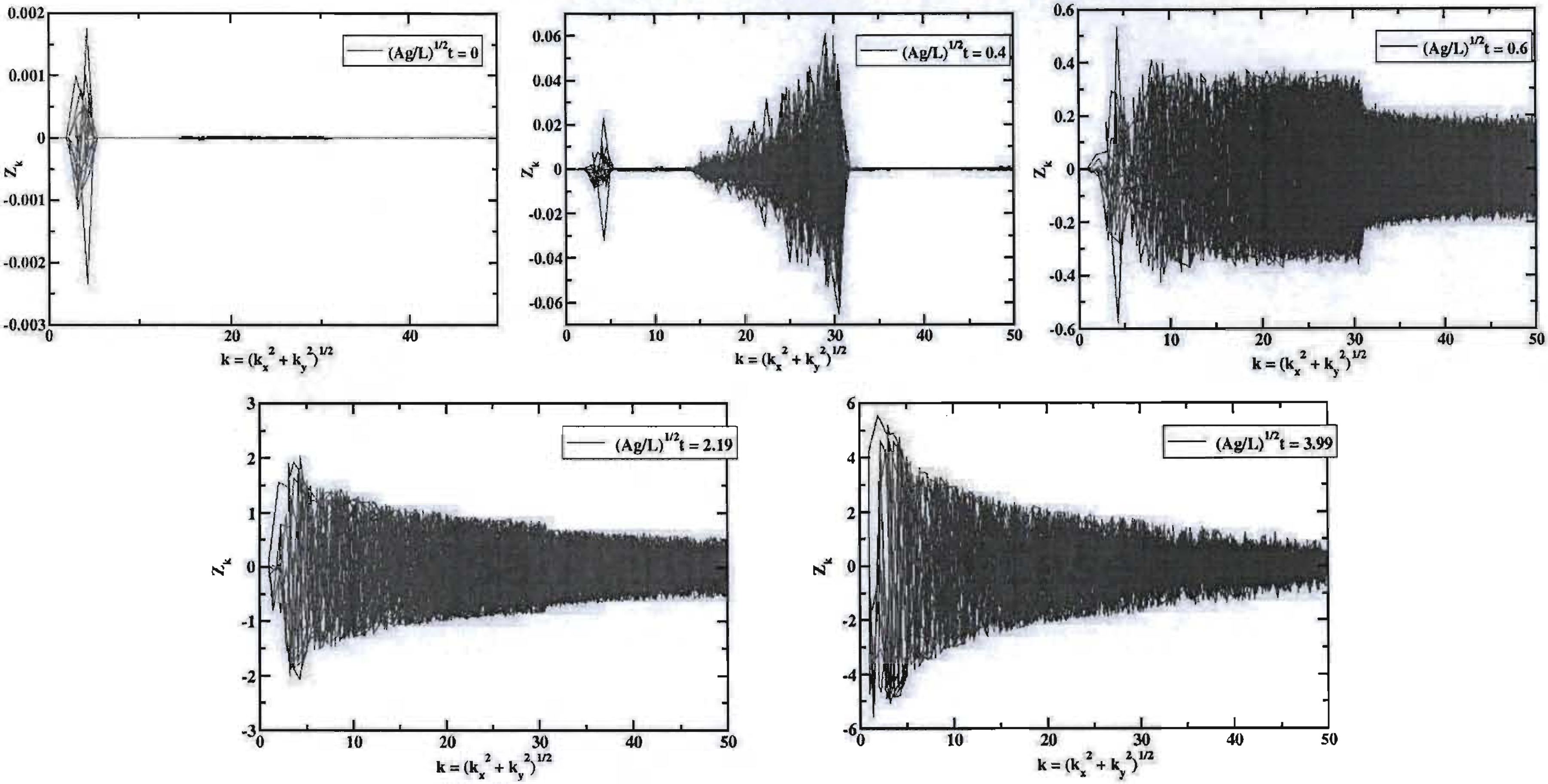
The mode coupling function will "populate" the entire spectrum

Modal Model Behavior



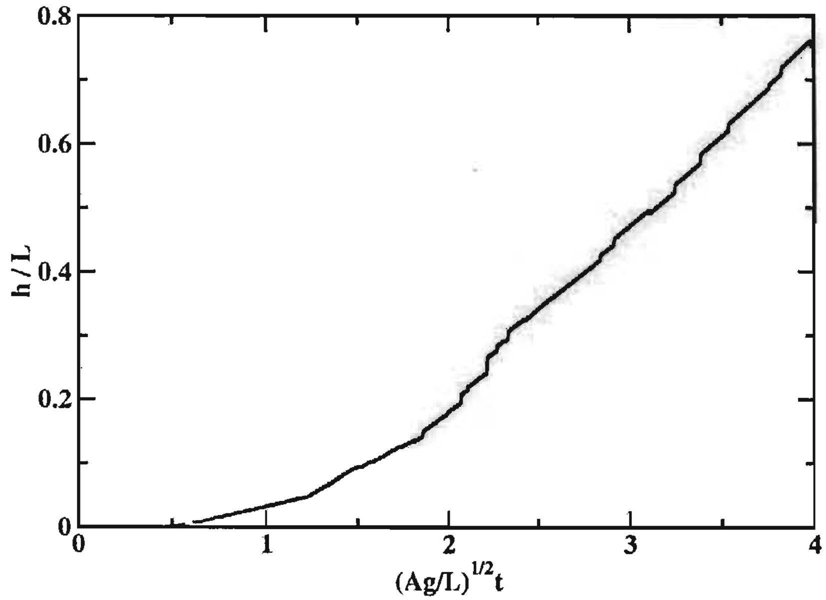
Mode coupling is at the origin of self similarity

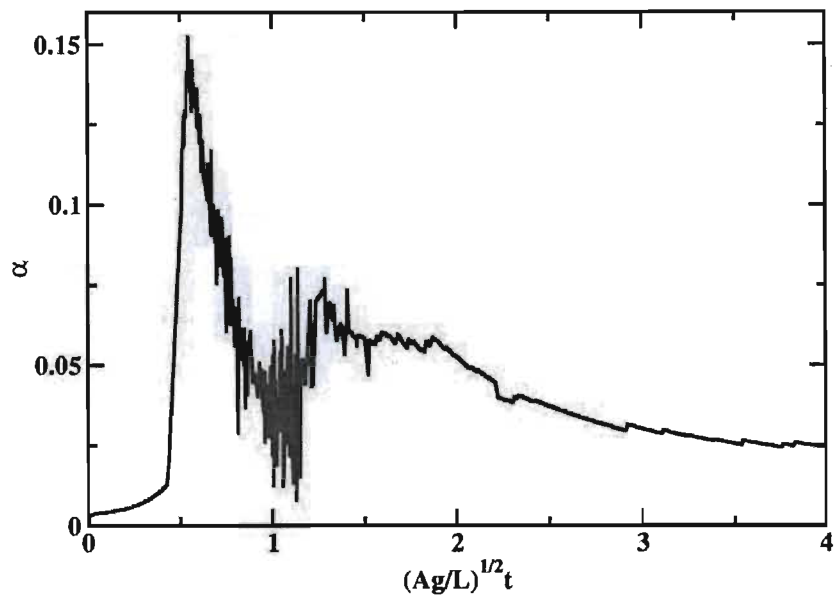
Banded Spectrum Case



Existing long wavelength in the initial spectrum is not washed out by mode coupling

Banded Spectrum Case





The mixing layer expansion experience an "extra kick" when the long wavelengths of the initial pertubation become the dominant modes

BHR Turbulence Model for RT Instability

Besnard-Harlow-Rauenzhan (BHR) turbulence model:

- **Single-point turbulent transport model**
- **Designed for variable density turbulence**

D. Besnard, F. H. Harlow, R. Rauenzhan, LA-10911-MS (1987)

Model Variables:

$$k = \frac{1}{2}\overline{u_i'u_i'} \qquad a_i = \frac{\overline{\rho' u_i'}}{\overline{\rho}} \qquad b = -\overline{\rho' v'} \qquad S = \frac{k^{3/2}}{\varepsilon} \qquad \nu_t = C_\mu k^{1/2} S$$

Governing equation for the variable S:

$$\partial_t S = \left(\frac{3}{2} - C_4\right) a_z g \frac{S}{k} + \frac{1}{\rho}\partial_z\left(\rho \frac{\nu_t}{\sigma_S}\partial_z S\right) - \left(\frac{3}{2} - C_2\right) k^{1/2}$$

BHR initiated with:

- **Profiles for:** $k \quad a_i \quad b \quad S$
- **Values for:** $C_4 \quad C_2 \quad C_\mu \quad \sigma_S \quad \ldots$

Two-Fluid Formulation for BHR Variables

$$\overline{\rho} = f_l \rho_l + f_h \rho_h \qquad \overline{\mathbf{u}} = f_l \mathbf{u}_l + f_h \mathbf{u}_h$$

$$k = C_k \frac{3}{2}\left(\overrightarrow{v_b} - \overrightarrow{v_s}\right)^2 \frac{f_h f_l \rho_h \rho_l}{\left(f_h \rho_h + f_l \rho_l\right)^2}$$

Isotropy hypothesis

$$a_z = C_{a_z} \frac{f_h f_l}{f_h \rho_h + f_l \rho_l}\left(\rho_h - \rho_l\right)\left(\overrightarrow{v_s} - \overrightarrow{v_b}\right)$$

$$b = C_b \frac{f_h f_l \left(\rho_h - \rho_l\right)^2}{\rho_h \rho_l}$$

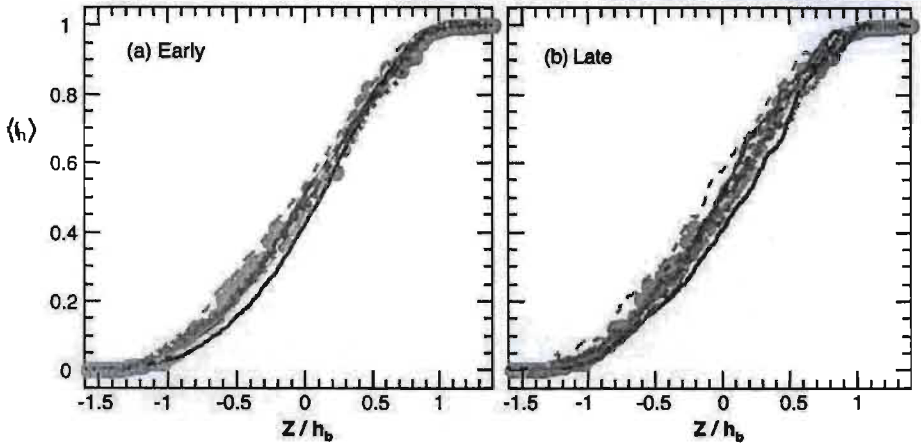
$$S = C_S \left(h_b + h_s\right)\left(4 f_h f_l\right)^{1/2}$$

- **Self-similarity hypothesis**
- **Derived for low Atwood number**

$$C_k = C_S = C_b = C_{a_z} = 1$$

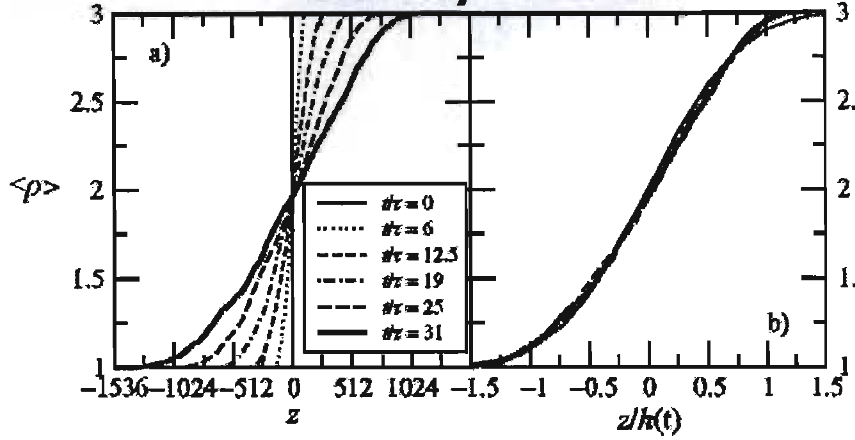
Approximation for Density Profile

Heavy Fluid Volume Fraction Profile



Dimonte *et al.*, Phys. of Fluids, **16** (2004)

Density Profile



Livescu *et al.*, J. Turbulence, **10** (2009)

$$\rho = f_l \rho_l + f_h \rho_h$$

$$\begin{cases} f_l = \dfrac{\rho - \rho_h}{\rho_l - \rho_h} \\ f_h = 1 - f_l \end{cases}$$

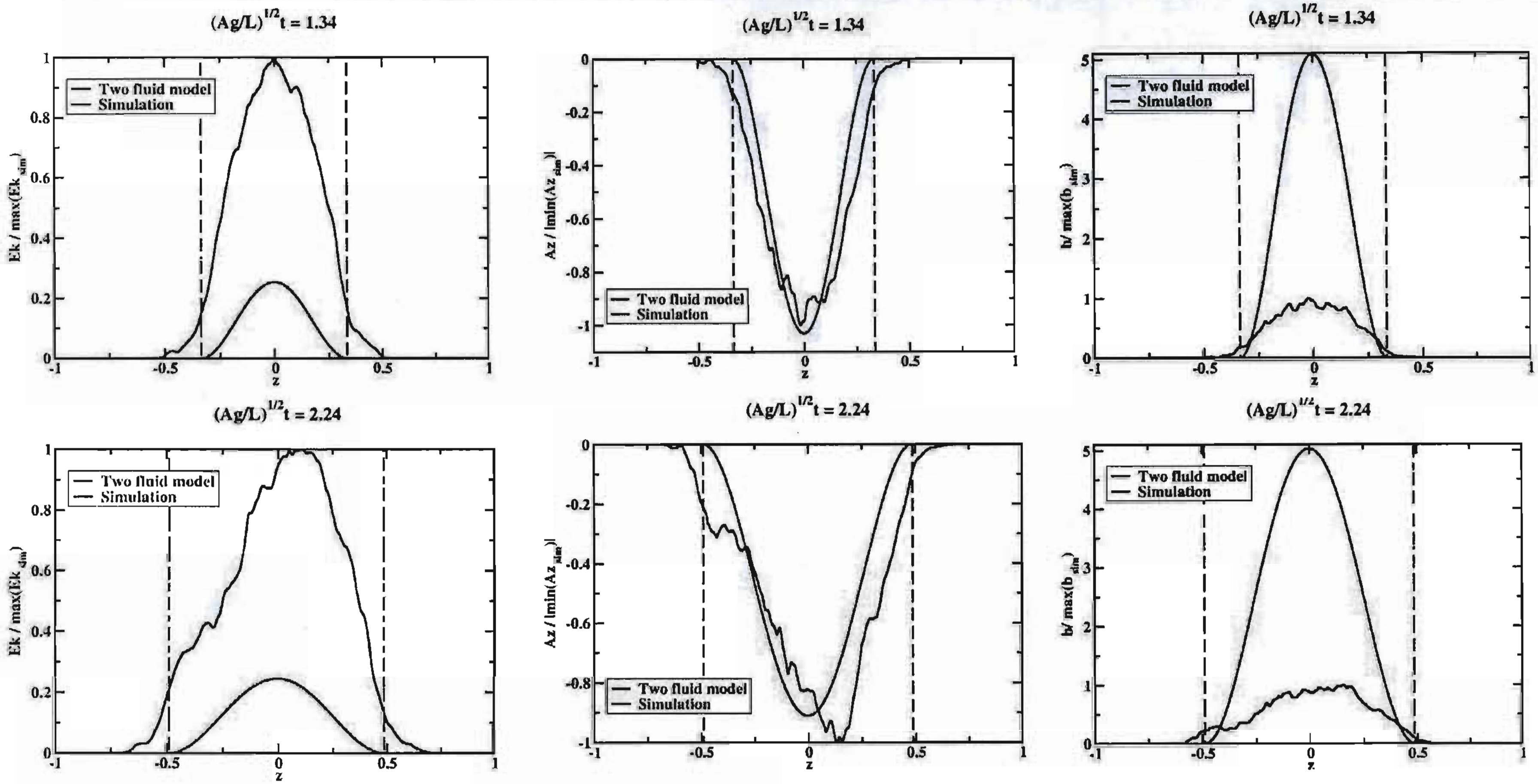
$$\begin{cases} f_h = 0 & if \quad z < -h_s \\ f_h = 0.5\dfrac{z + h_s}{h_s} & if \quad -h_s \le z < 0 \\ f_h = 0.5\dfrac{z}{h_b} + 0.5 & if \quad 0 \le z \le h_b \\ f_h = 1 & if \quad z > h_b \end{cases}$$

For a smooth mixture fraction description

$$\tilde{f}_h(z) = \int_{h_s}^{h_b} (z - h_s)^{a-1} (h_b - z)^{b-1} dz$$

$$f_h(z) = \frac{\tilde{f}_h(z)}{\tilde{f}_h(h_b)}$$

Two fluid model predictions



Two-fluid formulation produces reasonable profiles that need to be adjusted correction coefficients

Summary

- We have a model for multimode RT
- Model track development of perturbation spectrum
- Two-fluid formulation for BHR variables profiles

- **Next steps:**
  - Fine tuning our model
  - Determine coefficients for two fluid formulation
  - Introduce 3rd order terms in model ?
  - Improve the model to a larger range of Atwood number

- **Acknowledgements:**

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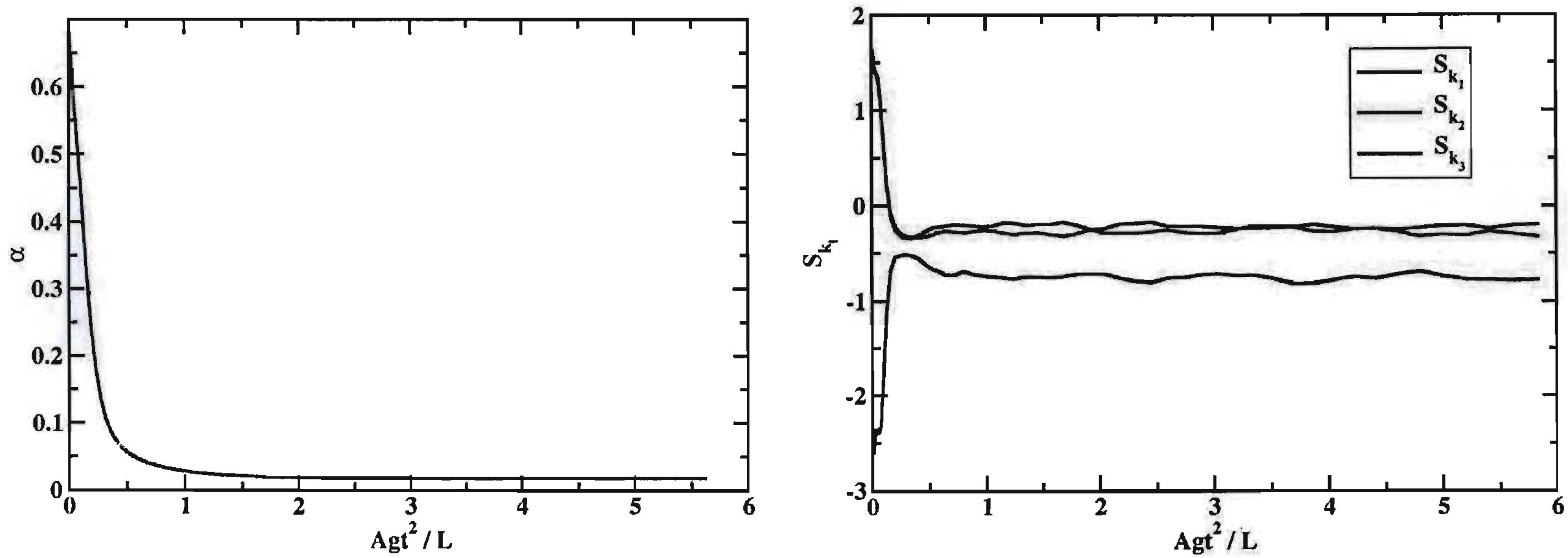
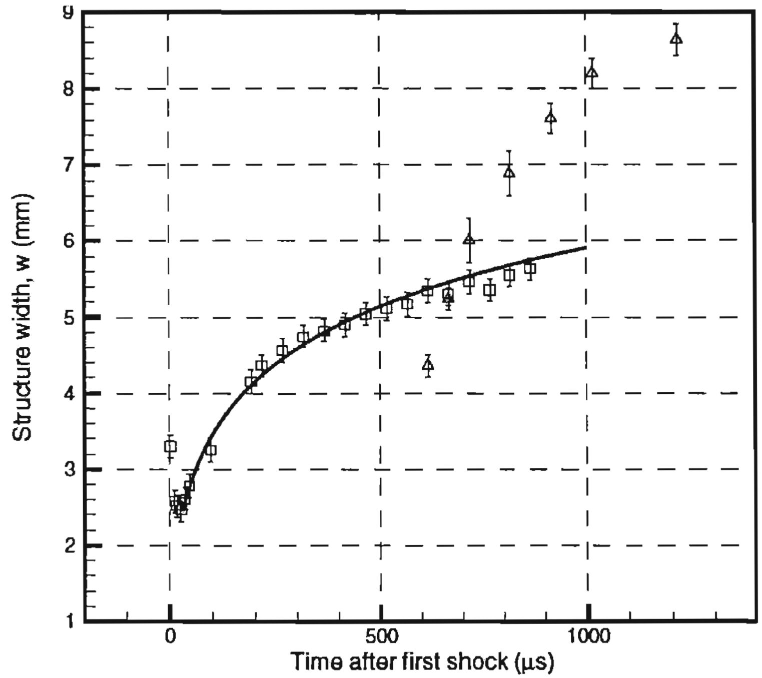
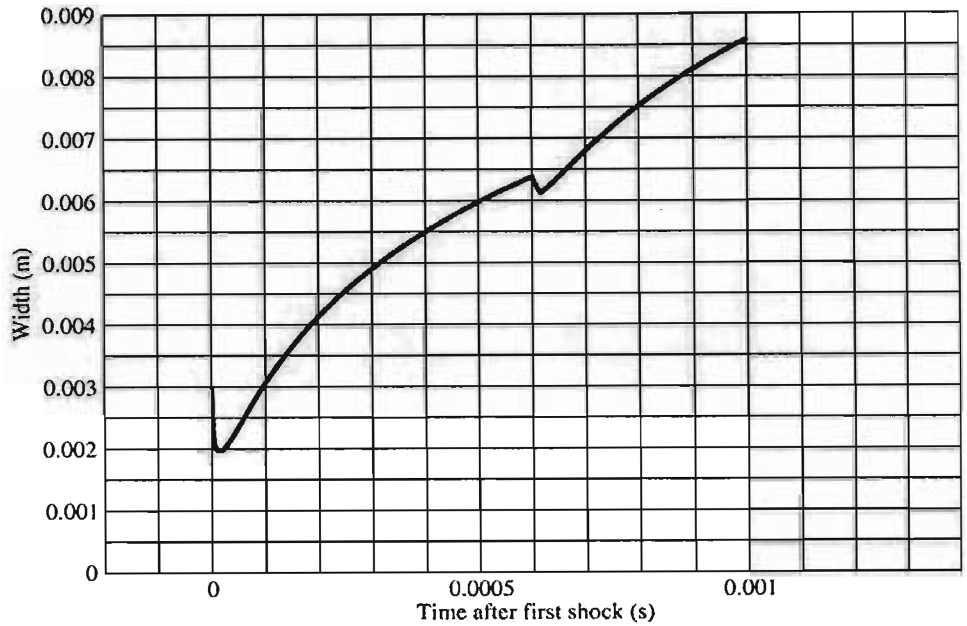
Establishement of a nonlinear cascade process



It appears that the establishment of a nonlinear cascade process occurs at about the same time as the mixing layer growth becomes self-similar

Application to RM instability





The Goncharov model applied to the gas curtain experiment produces a very close result