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Deployable Dead Time Corrections for Neutron Multiplicity Measurements Accounting for Neutron Correlations and Multiple Detector Chains

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ABSTRACT

Currently, most standard implementations of deadtime corrections for neutron multiplicity counting utilize empirical formulas. The corrections have had success for limited count rates, especially when appropriate deadtime parameters can be determined for a limited range of item properties. However, the corrections often break down outside the intended application range and current dead time corrections are not sufficiently robust to apply to higher order correlations beyond triplets. Sophisticated dead time corrections have been developed by Matthes and Haas (1985) and Hage and Cifarelli (1992) based on the joint probability for detecting correlated neutrons in a paralyzable detector system. Baeton et al. (1997) further modified the joint probability to include the effect of multiple detector chains. Deployable methods are being developed, applying the probability-based approach to standard multiplicity measurement data including multiplicity shift register, time interval analysis and list mode data, including expressions for higher-order correlations. Progress on implementation of the dead time correction in an analysis algorithm and testing of the correction based on simulations and actual data will be presented.

INTRODUCTION

In correlated neutron counting, the measured rates of neutron multiplets are used to determine item properties such as mass and neutron multiplication. The measured multiplet rates is a non-trivial function of the dead time and correlated nature of the neutrons. Most commonly-used dead time corrections include several simplifying assumptions in the dead time model. Many dead time corrections also employ empirically determined factors which can compensate for biases present in the overly simplified dead time model. However, these dead time corrections only hold for limited count rate ranges and are not sufficiently robust to apply to correlations beyond triplets.

Here, we attempt to develop a full and complete dead time model, with the recognition of three facts. First, computing power makes it possible to solve even complex systems of equations in real time. Second, a more accurate dead time model will result in corrections which are applicable over a wider count rate range including high count rates. Finally, there is a deficiency in understanding the true effects of dead time on a neutron pulse train. A comparison of an exact dead time model for correlated neutrons with experimental data and simulated dead time effects will make it possible to rigorously analyze the validity of the assumptions of a paralyzable detector.

The following derivation of an exact dead time correction for correlated neutrons follows the approach of Matthes and Haas [1], Hage and Cifarelli [2] and Baeton et al. [3]. These previous corrections make simplifications early in the derivation. A complete and exact derivation, in terms of infinite series of true multiplicity rates, is provided for singles, doubles, triples and quadruples in an upcoming technical report [4]. However, this paper focuses on the dead time effects on measured singles and doubles rates. The dead time model is then tested against simulated updating dead time applied to a simulated pulse train from a benchmarked Monte Carlo model [5].

JOINT PROBABILITY FUNCTION

In a paralyzable detector with updating dead time δ, the difference between the true pulse train (Π_0) and the measured pulse train (Π_m), is that any neutrons that were detected in Π_0, less than δ after a previous neutron detection is not counted in Π_m. This idea can be formulized through developing the probability (as a function of dead time) of obtaining neutron counts in Π_m. The probability of obtaining a neutron count in Π_m at an arbitrary time t, is equal to the joint probability of having a neutron detection at time t in the true pulse train Π_0 *and* having zero neutron detections in the time interval of δ previous to t. Similarly, the probability of having two neutron counts in Π_m at times t_1 and t_2 is equal to the joint probability of having neutron detections in Π_0 at t_1 and t_2 *and* having zero detections in the dead time intervals prior to t_1 and t_2. Therefore, developing a formula for these joint probabilities in terms of the system dead time and true correlated detection rates provides a relationship between Π_0 and Π_m. The joint probability can become quite complex since each measured multiplicity rate is a non-trivial function of all true multiplicity rates. The relationship between the true pulse train Π_0 and observed pulse train Π_m is conceptually demonstrated in Figure 1.

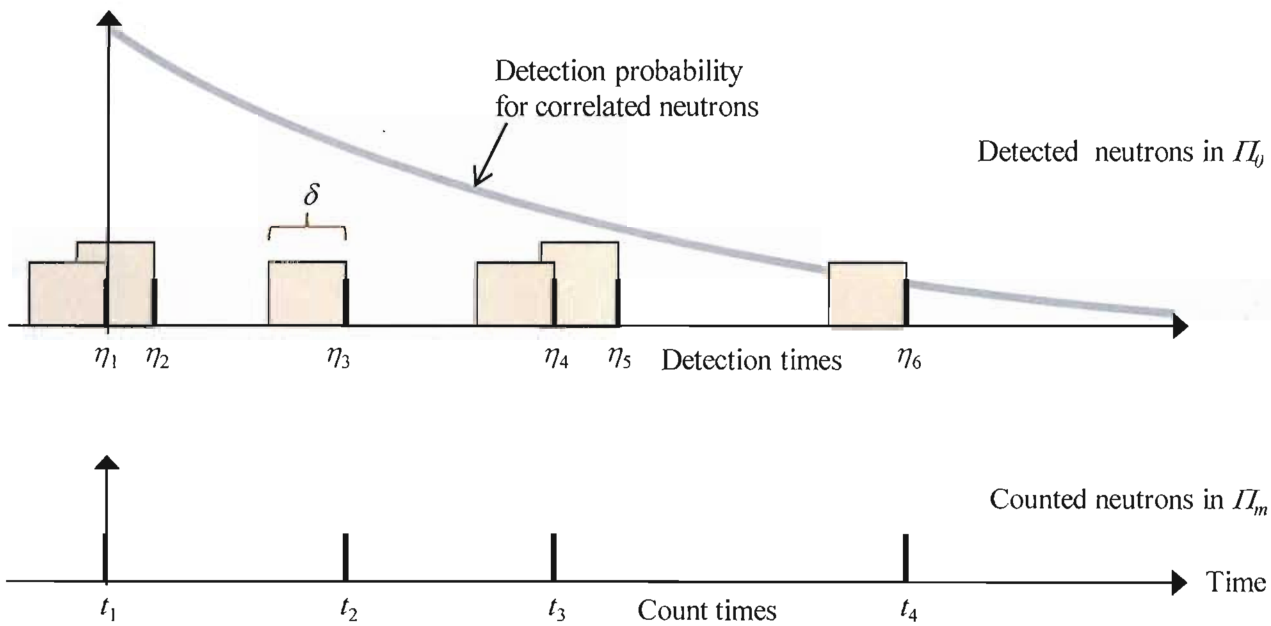

Figure 1. Relationship between the true and observed pulse trains due to dead time effects in a paralyzable detector. Only neutrons that are not detected within a dead time interval d of another neutron are counted in the measured pulse train.

The measured singles rate (d_{1m}) is equal to the probability of observing a count at arbitrary time t in Π_m, which is in turn equal to the joint probability $\mathbb{P}_{01}$ of detecting zero neutrons in the interval $[t-\delta, t]$ and one neutron in the infinitesimal interval $[t, t+dt]$. Therefore, the singles rate is given by

$$d_{1m} = N_0 = \mathbb{P}_{01}(t, \delta, d_1, d_2, d_3, \dots) \qquad \text{Equation 1}$$

Here it is emphasized that the singles joint probability is a function of the dead time as well as all true correlated detection rates including the true singles rate (d_1), doubles rate (d_2), etc. The singles rate is also equal to the zeroth moment (N_0) of the triggered gate histogram (sometimes called the Reals + Accidentals gate).

The probability of observing neutron counts at arbitarty times t_1 and t_2 in Π_m is related to the total number of counts obtained in a series of triggered gates. All derivations in this paper assume that the detector response function has a single exponential (with decay constant λ), and therefore all doubles detection probabilities are dependent only on the time offset between detections $\tau = t_2 - t_1$. In this case, the total number of neutrons detected in a series of triggered measurement gates is the integral of the joint probability over the gate width,

$$N_1 = T_M \int_{PD}^{PD+G} \mathbb{P}_{0101}(\tau, \delta, d_1, d_2, d_3)\, dt_2 \qquad \text{Equation 2}$$

where T_M is the total measurement time, PD is the gate predelay and G is the gate width. The total number of neutrons in a series of triggered gates is also equal to the first moment of the triggered gate histogram N_1. The first moment can be written in terms of the first two true correlated detection rates d_1 and d_2 [6], as

$$N_1 = \left(d_{1m}{}^2 G + d_{2m} f_2\right) T_M \qquad \text{Equation 3}$$

where f_2 is the gate utilization factor for doubles. Rearranging, the measured doubles rate can be written as a function of the true multiplicity rates as

$$d_{2m} = \frac{1}{f_2}\left(\frac{N_1}{T_M} - d_{1m}{}^2 G\right) \qquad \text{Equation 4}$$

Equations 1 and 2 can be inverted numerically (including numerical integration of Equation 2) up to arbitrary desired accuracy to determine the true multiplicity rates in terms of measured rates when two true multiplicities (d_1 and d_2) are included in the model. Extension to the measured triples rate (discussed in the technical report [4]) is required for a closed inversion solution with three multiplicities in the model. The increased accuracy by including three multiplicities is considered in this paper. Full implementation will be discussed in future publications.

JOINT PROBABILITY FUNCTIONS FOR A SINGLE DETECTOR CHAIN

Each of the previous developments [1-3] of dead time corrections based on the joint probability function make simplifications to the equations early in the derivation and the complete results are never given. The complete joint probability functions for singles, doubles, triples and quadruples have been derived in terms of infinite series of the true multiplicity rates $d_1, d_2, d_3, \dots$ in a technical report [4]. In the interest of space, the results for observed singles and doubles rates are reported here.

Singles Rate

The measured singles rate is given by

$$d_{1m} = \mathbb{P}_{01} = A_1 e^{B_1} \qquad \text{Equation 5}$$

where the coefficients are given by

$$\begin{aligned} B_1 &= \sum_{n_1=1}^{\infty} (-1)^{n_1} d_n \delta \left(1 + \sum_{k=1}^{n-1} \binom{n-1}{k} \frac{(-1)^k}{\lambda k \delta} \left(1 - e^{-\lambda k \delta}\right) \right) \\ &= -d_1\delta + d_2\delta\left[1 - \tfrac{1}{\lambda\delta}\left(1 - e^{-\lambda\delta}\right)\right] - d_3\delta\left[1 - \tfrac{2}{\lambda\delta}\left(1 - e^{-\lambda\delta}\right) + \tfrac{1}{2\lambda\delta}\left(1 - e^{-2\lambda\delta}\right)\right] \end{aligned} \qquad \text{Equation 6}$$

$$\begin{aligned} A_1 &= dt_1 \sum_{N=1}^{\infty} (-1)^{n_1} d_N \left(1 - e^{-\lambda\delta}\right)^{n1} \\ &= dt_1\left[d_1 - d_2\left(1 - e^{-\lambda\delta}\right) + d_3\left(1 - e^{-\lambda\delta}\right)^2 - \cdots\right] \end{aligned} \qquad \text{Equation 7}$$

Note that with one multiplicity accounted for, the measured singles rate becomes

$$d_{1m} = d_1 e^{-d_1\delta} dt_1 \qquad \text{Equation 8}$$

which is identical to the dead time correction obtained from the Poisson distribution assumption. A plot of the measured singles rate as a function of true singles rate is shown in Figure 2 using up to one, two and three multiplicities. The functional relationship was calculated using detector parameters from a Monte Carlo model of Pu metal in a neutron counter with a 1000 cps emission rate. The simulated dead time effect on the measured singles rate is also plotted in Figure 2.

Doubles Rate

The joint probability for observing two neutrons in the measured pulse train is given by

$$\mathbb{P}_{0101} = (A_{21} + A_{22}A_{23})e^{B_2} \qquad \text{Equation 9}$$

where

$$A_{21} = \sum_{N=2}^{\infty} \sum_{n_1=0}^{N-2} (-1)^N G_{(n_1,1,n_3,1)} = G_{(0,1,0,1)} - \left(G_{(1,1,0,1)} + G_{(0,1,1,1)}\right) + \cdots \qquad \text{Equation 10}$$

Each coefficient $G_{(n_1,1,n_3,1)}$ corresponds to the contribution from n1 neutron detections in the first interval and n3 detections in the third interval, for a total of N = n1+n3+2 detections. The coefficients are derived in the technical report [4] and reported here

$$G_{(0,1,0,1)} = d_2 \lambda e^{-\lambda\tau}$$
$$G_{(1,1,0,1)} = d_3 \lambda e^{-\lambda\tau}\left(1 - e^{-2\lambda\delta}\right)$$
$$G_{(0,1,1,1)} = 2 d_3 \lambda e^{-2\lambda\tau}\left(e^{\lambda\delta} - 1\right)$$
$$G_{(n_1,1,n_3,1)} = \frac{(N-1)!}{(n_1-1)!\,n_3!} d_N \lambda e^{-(n_3+1)\lambda\tau}\left(e^{\lambda\delta} - 1\right)^{n3} \sum_{k=N-n1}^{N-1} \binom{n_1 - 1}{k - (N - n_1)} \frac{1}{k}\left(1 - e^{-k\lambda\delta}\right)$$

Equation 11-14

To summarize, A21 is given by

$$A_{21} = d_2 \lambda e^{-\lambda\tau} - d_3\left[\lambda e^{-\lambda\tau}\left(1 - e^{-2\lambda\delta}\right) + 2\lambda e^{-2\lambda\tau}\left(e^{\lambda\delta} - 1\right)\right] + \cdots$$

Equation 15

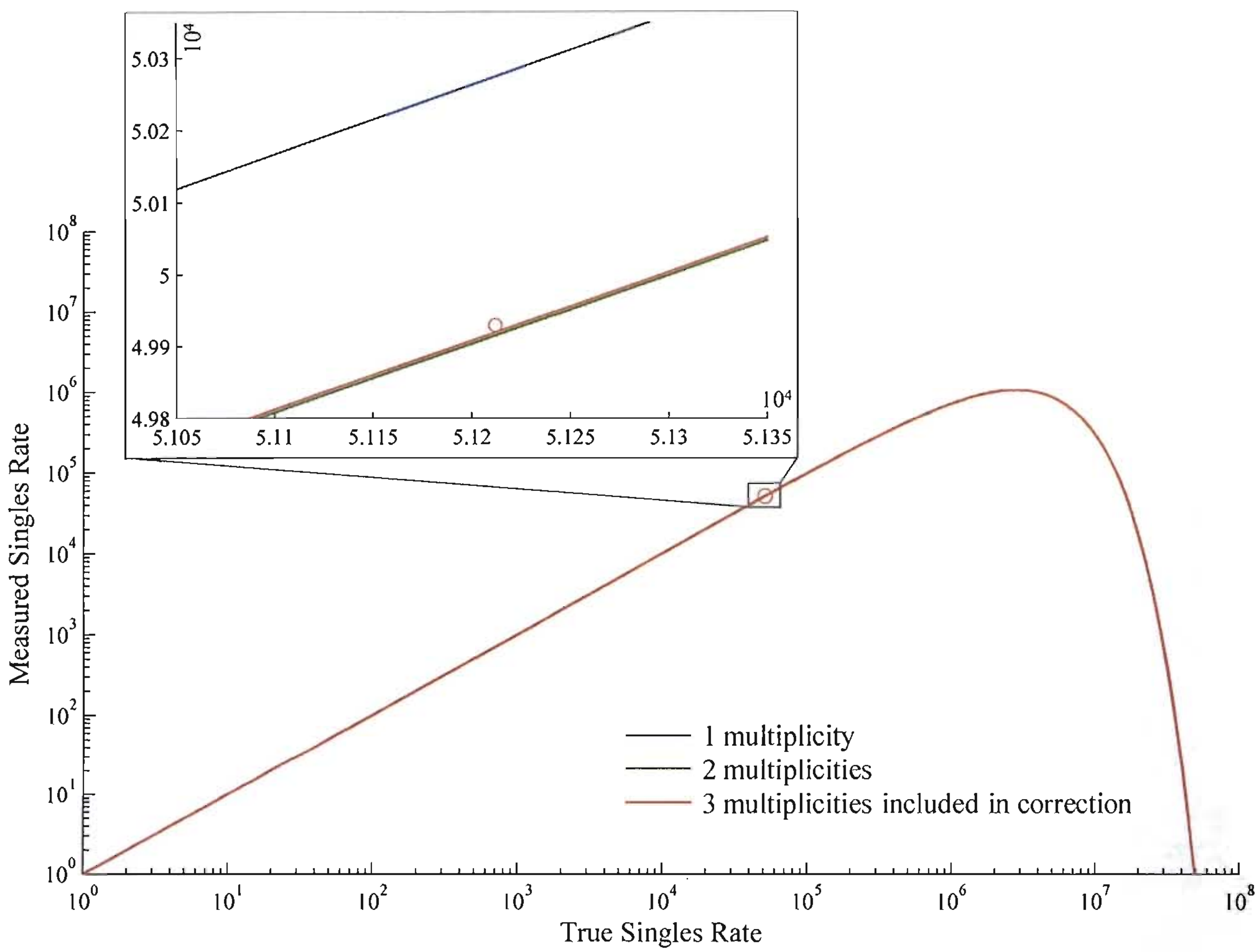

Figure 2. Effects of dead time on measured singles rate. The comparison data point (red circle) represents simulated dead time on a neutron pulse stream of Pu metal from a Monte Carlo model [6]. Greater accuracy results from the inclusion of a greater number of true multiplicity rates.

Similarly expressions exist for the remaining terms in $\mathbb{P}_{0101}$, including A22;

$$A_{22} = \sum_{N=1}^{\infty}\sum_{n_1=0}^{N-1}(-1)^{N-1}\,G_{(n_1,1,n_3,0)}$$

$$= G_{(0,1,0,0)} - \left(G_{(1,1,0,0)} + G_{(0,1,1,0)}\right) + \left(G_{(2,1,0,0)} + G_{(1,1,1,0)} + G_{(0,1,2,0)}\right) - \cdots$$

$$= dt_1\left\{d_1 - d_2\left[\left(1-e^{-\lambda\delta}\right) + e^{-\lambda\tau}\left(e^{\lambda\delta}-1\right)\right] + d_3\left[\left(1-e^{-\lambda\delta}\right)^2 + e^{-\lambda\tau}\left(1-e^{-2\lambda\delta}\right)\left(e^{\lambda\delta}-1\right) + e^{-2\lambda\tau}\left(e^{\lambda\delta}-1\right)^2\right] + \cdots\right\}$$

Equation 16

$$G_{(0,1,0,0)} = d_1 dt_1$$
$$G_{(n1,1,0,0)} = d_N dt_1\left(1-e^{-\lambda\delta}\right)^{n_1}$$
$$G_{(0,1,n3,0)} = d_3 dt_1 e^{-n_3\lambda\tau}\left(e^{\lambda\delta}-1\right)^{n_3}$$
$$G_{(n_1,1,n_3,0)} = \frac{(N-1)!}{(n_1-1)!\,n_3!} d_N dt_1 e^{-n_3\lambda\tau}\left(e^{\lambda\delta}-1\right)^{n_3}\sum_{k=N-n_1}^{N-1}\binom{n_1-1}{k-(N-n_1)}\frac{(-1)^k}{k}\left(1-e^{-k\lambda\delta}\right)$$

Equation 17-20

the term A23;

$$A_{23} = \sum_{N=1}^{\infty}\sum_{n_1=0}^{N}(-1)^{N-1}\,G_{(n_1,0,n_3,1)}$$

$$= G_{(0,0,0,1)} - \left(G_{(1,0,0,1)} + G_{(0,0,1,1)}\right) + \left(G_{(2,0,0,1)} + G_{(1,0,1,1)} + G_{(0,0,2,1)}\right) - \cdots$$

$$= dt_2\left\{d_1 - d_2\left[e^{-\lambda\tau}\left(1-e^{-\lambda\delta}\right) + \left(1-e^{-\lambda\delta}\right)\right] + d_3\left[e^{-\lambda\tau}\left(1-e^{-\lambda\delta}\right)^2 + e^{-2\lambda\tau}\left(e^{\lambda\delta}-1\right)\left(1-e^{-2\lambda\delta}\right) + \left(1-e^{-\lambda\delta}\right)^2\right] + \cdots\right\}$$

Equation 21

$$G_{(0,0,0,1)} = d_1 dt_2$$
$$G_{(0,0,n3,1)} = d_N\left(1-e^{-\lambda\delta}\right)^{n1} dt_2$$
$$G_{(n1,0,0,1)} = d_N e^{-\lambda\tau}\left(1-e^{-\lambda\delta}\right)^{n_1} dt_2$$
$$G_{(n_1,0,n_3,1)} = \frac{(N-1)!}{(n_1-1)!\,n_3!} d_N e^{-(n_3+1)\lambda\tau}\left(e^{\lambda\delta}-1\right)^{n3} dt_2\sum_{k=1}^{n1}\binom{n_1-1}{k-1}\frac{(-1)^k}{N-k}\left(1-e^{-(N-k)\lambda\delta}\right)$$

Equation 22-25

and finally the term B2;

$$B_2 = \sum_{N=1}^{\infty}\sum_{n_1=0}^{N}(-1)^N G_{(n_1,0,n_3,0)}$$

$$= -\left(G_{(0,0,1,0)} + G_{(1,0,0,0)}\right) + \left(G_{(0,0,2,0)} + G_{(1,0,1,0)} + G_{(2,0,0,0)}\right) - \left(G_{(0,0,3,0)} + G_{(1,0,2,0)} + G_{(2,0,1,0)} + G_{(3,0,0,0)}\right) + \cdots$$

$$
\begin{aligned}
&= -2d_1\delta + d_2\left[2\delta\left(1 - \tfrac{1}{\lambda\delta}(1 - e^{-\lambda\delta})\right) + \frac{1}{\lambda}e^{\lambda\delta}(1 - e^{-\lambda\delta})^2 e^{-\lambda\tau}\right] \\
&\quad -d_3\left[2\delta\left(1 - \tfrac{2}{\lambda\delta}(1 - e^{-\lambda\delta}) + \tfrac{1}{2\lambda\delta}(1 - e^{-2\lambda\delta})\right) + \frac{1}{\lambda}e^{-2\lambda\tau}(e^{\lambda\delta} - 1)^2(1 - e^{-2\lambda\delta}) \right. \\
&\qquad \left. + 2\frac{1}{\lambda}e^{-\lambda\tau}(e^{\lambda\delta} - 1)\left((1 - e^{-\lambda\delta}) - \tfrac{1}{2}(1 - e^{-2\lambda\delta})\right)\right] + \cdots
\end{aligned}
$$

Equation 26

$$
\begin{aligned}
G_{(0,0,1,0)} \text{ or } G_{(1,0,0,0)} &= -d_1\delta \\
G_{(0,0,n,0)} \text{ or } G_{(n,0,0,0)} &= d_n\delta\left(1 + \sum_{k=1}^{n-1}\binom{n-1}{k}\frac{(-1)^k}{k\lambda\delta}(1 - e^{-k\lambda\delta})\right) \\
G_{(n_1,0,n_3,0)} &= \frac{(N-1)!}{(n_1-1)!\,n_3!}\frac{d_N}{\lambda}e^{-n_3\lambda\tau}(e^{\lambda\delta} - 1)^{n3}\sum_{k=1}^{n_1}\binom{n_1-1}{k-1}\frac{(-1)^k}{N-k}(1 - e^{-(N-k)\lambda\delta})
\end{aligned}
$$

Equation 27-29

Figure 3 shows the effect of dead time on the measured doubles rate, taking into account up to two and three true multiplicity rates. The simulated dead time effects are more accurately modeled by including higher order multiplicity rates. The doubles rate curve is compared to the simulated dead time effect on the measured doubles rate from a Monte Carlo model of a Pu metal item.

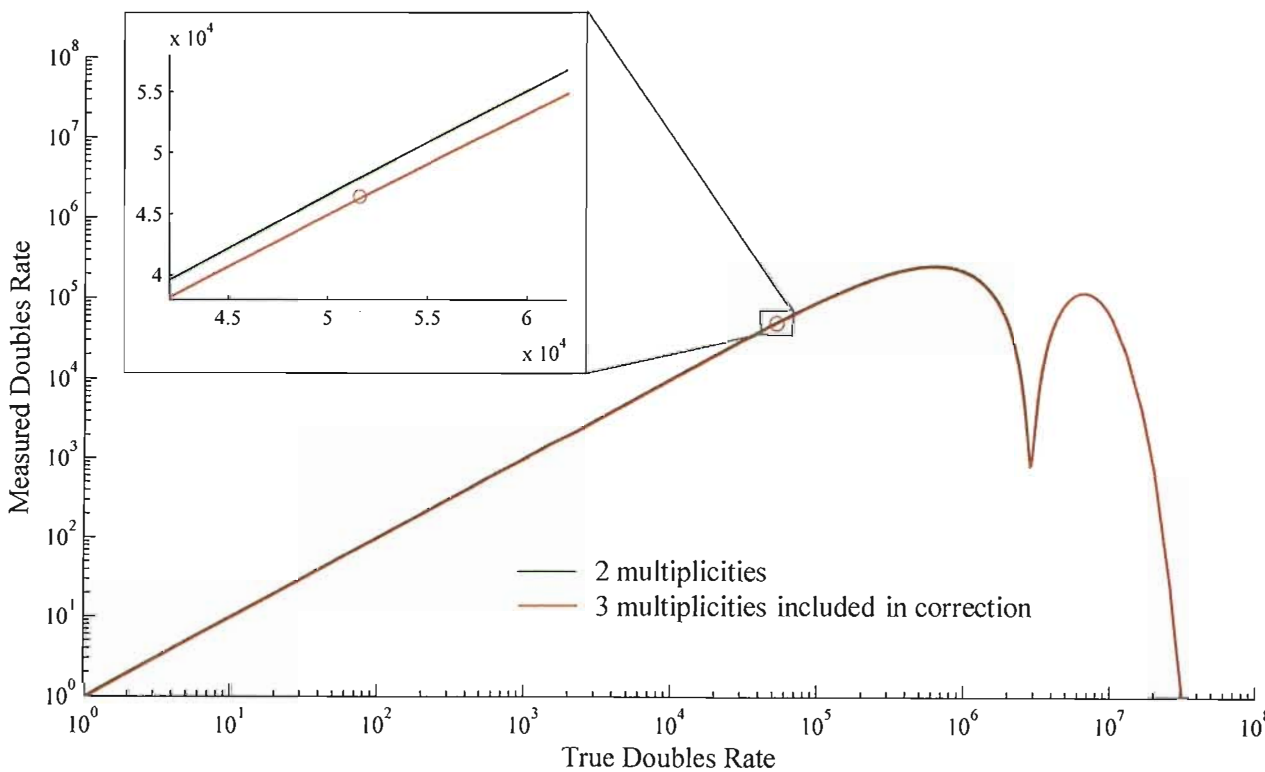

Figure 3. Dead time effects on the measured doubles rate. The comparison data point (red circle) represents simulated dead time on a neutron pulse stream of Pu metal from a Monte Carlo model [6]. The physical portion of the curve ends at the singularity.

TESTING

The dead time model was tested against a simulated updating dead time applied to a pulse train from a Monte Carlo model of various items in a neutron multiplicity counter [6]. Emission count rate and dead time were varied to supply testing capability across a range of measurement scenarios. The ability to accurately characterize the singles and doubles correction factors using up to one, two and three multiplicities was tested. The dead time model was compared to two other dead time correction methods; the empirical method often used conventionally [7] and a recently developed dead time correction factor which is a variation of Dytlewski [5]. The results from selected test cases are given in Table 1.

Table 1. Relative accuracy of the dead time correction. Accuracy expressed as percent recovery, defined as $Recovery = 1 - Corrected\ Rate/Measured\ Rate$. The first column lists the simulated dead time for each case and the second column gives the simulated measured rates after dead time effects.

δ		Measured Rates	Empirical Correction	Croft-Dytlewski	Based on Joint Probability Function: 1 Component	2 Components	3 Components
Cf-252		Rate = 1000 cps	Singles = 996833	Doubles = 491144			
90	S	910527	-0.190	-0.038	0.081	-0.001	-0.001
	D	338672	-1.469	-9.106		0.476	0.221
180	S	831679	-0.898	-0.109	0.173	0.007	0.007
	D	229190	-6.870	-18.365		0.945	0.408
360	S	694253	-4.815	-0.326	0.288	-0.005	-0.005
	D	99270	-29.365	-35.763		0.001	-1.256
Pu Metal		Rate = 1000 cps	Singles = 51211	Doubles = 51615			
90	S	50886	-0.178	-0.063			0.000
	D	50184	-0.875	-0.941			0.107
180	S	50563	-0.355	-0.113			-0.008
	D	48868	-1.750	-1.869			0.049
360	S	49929	-0.704	-0.223	0.691	-0.020	-0.020
	D	46332	-3.419	-3.660		3.396	-0.078
Pu Oxide		Rate = 1000 cps	Singles = 1029765	Doubles = 164889			
90	S	938394	-0.133	-0.018	0.025	-0.003	-0.003
	D	110279	-3.555	-11.314		2.240	2.173
180	S	855208	-0.807	-0.043	0.042	-0.016	-0.016
	D	75742	-7.859	-19.738		0.212	-0.314
360	S	710201	-4.924	-0.225	0.080	-0.016	-0.016
	D	30450	-35.101	-41.554		2.392	2.106

The dead time correction developed here out performs the two comparison techniques in its ability to match the simulated dead time effects, even for scenarios with relatively high neutron multiplication. In the infinite limit of the inclusion of all higher multiplicity rates, the dead time correction developed here is exact for correlated neutrons for an updating dead time, and in this sense should well represent the simulated dead time. However, the dead time correction assumes that the detector response has a single exponential, which is not necessarily true in the modeled neutron detector system. This may explain the small differences between the simulated measured rates and corrected rates. A rigorous investigation of the effect of multiple dieaway times in the detector response on dead time corrections is underway and will be presented in future publications.

The dead time correction presented here can be numerically inverted with the inclusion of two detection multiplicities to obtain a relationship between true multiplicity detection rates in terms of measured rates. Hence, the accuracies presented in Table 1 for 2 multiplicities are legitimately obtainable using only the measured singles and doubles rates for neutron counter systems that closely approximate a paralyzable system. The dead time correction with three multiplicities (which is a more accurate model) requires knowledge of the measured triples rate to be inverted. In this case, a numerical inversion can be performed, although this will require multiple numerical double integrations. Preliminary testing shows that this can be done in a reasonable amount of time with current standard computational power.

JOINT PROBABILITY FUNCTIONS FOR MULTIPLE DETECTOR CHAINS

Most detector systems utilize multiple detector chains, in which only a few detector tubes are connected to each preamplifier and discriminator. The dead time effects result in statistical modification to the pulse streams coming from each individual detector chain. The individual pulse trains are then combined through a derandomizer or OR gate into a single pulse train. The total dead time effects are minimized by having multiple detector chains. However, the statistical effects of dead time are altered. Baeton et al [3] derived a formulism for calculating the joint probability function for a multi-chain system, based on the relative singles rates in each chain and the single-chain joint probability function. The multi-chain joint probabilities can then be used as discussed, in place of the single chain functions derived here. Planned work includes extension of the exact dead time model developed here to a multi-chain system following Baeton's approach.

CONCLUSIONS

The dead time correction developed here closely followed the approach of Matthes and Haas [1], Hage and Cifarelli [2], and Baeton et al. [3]. The differences in the dead time corrections lie in the number of multiplicities included and the type of simplifications that are made to the equations. Here, we recognize that current computing power makes it possible to leave equations in their exact form. Additionally, further improvement to dead time corrections will require rigorous understanding of dead time effects and the relationship between reality and any physical model that is used. For this purpose it is necessary to have an exact description of the paralyzable dead time model.

The dead time model based on the joint probability function, although published previously, has not been widely adopted, and its importance has not been fully acknowledged. This is due partially to the availability of implemented dead time corrections in neutron counting software. One major contribution of the neutron counting project which supported this work will be to implement selected dead time algorithms in accessible analysis software.

FUTURE WORK

Future work will focus on evaluating the legitimacy of the assumptions included in the developed dead time correction, and where needed, extension to more realistic physical models. Most dead time models (including the one presented here) assume a single time constant in the detector response function. Deviations from this assumption and the effects on the dead time correction accuracy are currently being investigated. The dead time model presented here is exact for correlated neutrons within the paralyzable assumption. The next planned step is to experimentally test how well reality conforms to the paralyzable model with a large neutron counter with a single detector chain. The counter system will be modified to include multiple detector chains to test a multi-chain extension based on Baeton's approach.

ACKNOWLEDGMENTS

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REFERENCES

[1] Matthes, W. and R. Haas (1985) Deadtime Correction for 'Updating Deadtime' Counters. *Annals of Nuclear Energy* (12) 693-698.

[2] Hage, W. and D. M. Cifarelli (1992) Correlation analysis with neutron count distribution for a paralyzing dead-time counter for the assay of spontaneous fissioning material. *Nuclear Science and Engineering* (112) 136–158.

[3] Baeten, P., M. Bruggeman and R. Carchon (1997) Single- and multi-deadtime parameter corrections of one- and two-dimensional Rossi-alpha distributions for time interval analysis in neutron coincidence counting. *Nuclear Instruments and Methods in Physics Research A* (390) 345-358.

[4] Hauck, D. K. (2011) An Exact Dead Time Model for Correlated Neutron Counting based on the Joint Probability Function. *LANL Technical Report*, LA-UR pending.

[5] Evans, L. G, M. T. Swinhoe, S. Croft, D. K. Hauck, and P. A. Santi (2011) Extension of ESARDA NDA Multiplicity Benchmark Simulations to Validate Dead Time Correction Algorithms, *33rd ESARDA Annual Meeting, Budapest, Hungary.*

[6] Hage, W. and D. M. Cifarelli (1985) Correlation analysis with neutron count distributions in randomly or signal triggered time intervals for assay of special fissile materials. *Nuclear Science and Engineering* (89) 159-176.

[7] Ensslin, N., W. C. Harker, M. S. Krick, D. G. Langner, M. M. Pickrell and J. E. Steward (1998) Application Guide to Neutron Multiplicity Counting. *LANL Technical Report*, LA-13422-M.