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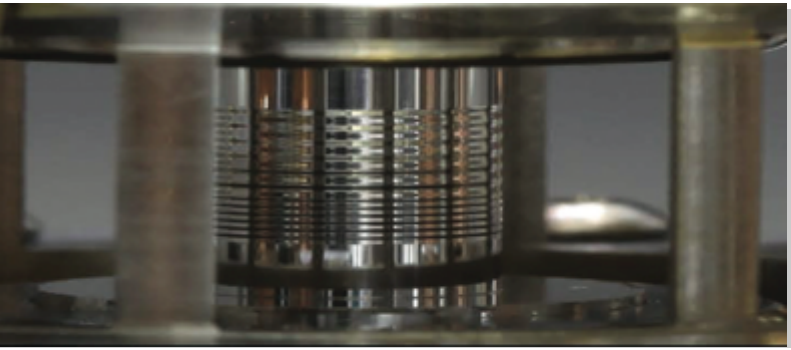

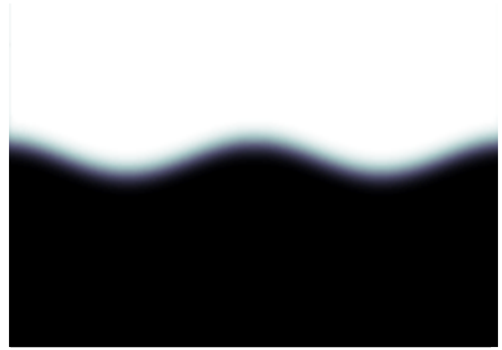

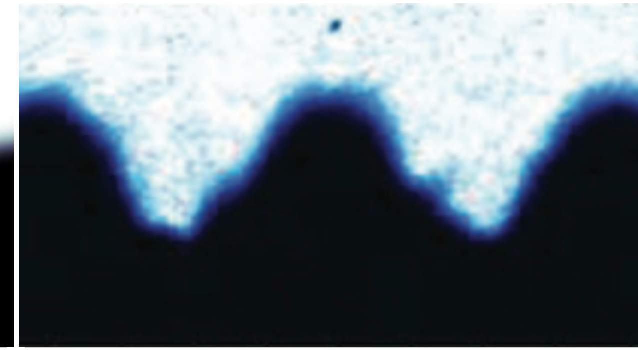

Cylindrical Effects on the Magneto-Rayleigh-Taylor Instability

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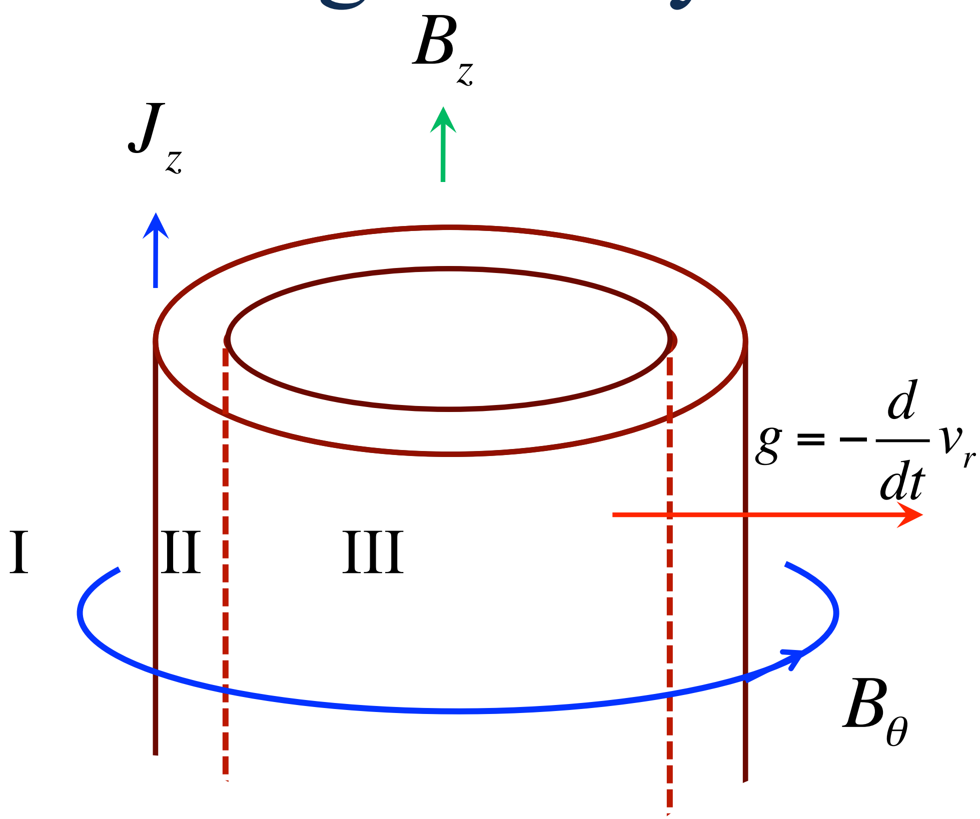
Outline

- Present ideal MHD model for analytic calculation of MRT growth rate and feedthrough in cylindrical coordinates
  - Show results of test problems
- Using 1D/2D Hydra results of seeded aluminum liner implosions to calculate MRT growth rates
  - Calculation from analytic planar/cylindrical model using 1D Hydra data as input
  - Direct calculation from simulated radiographs from 2D Hydra data

Cylindrical geometry instabilities



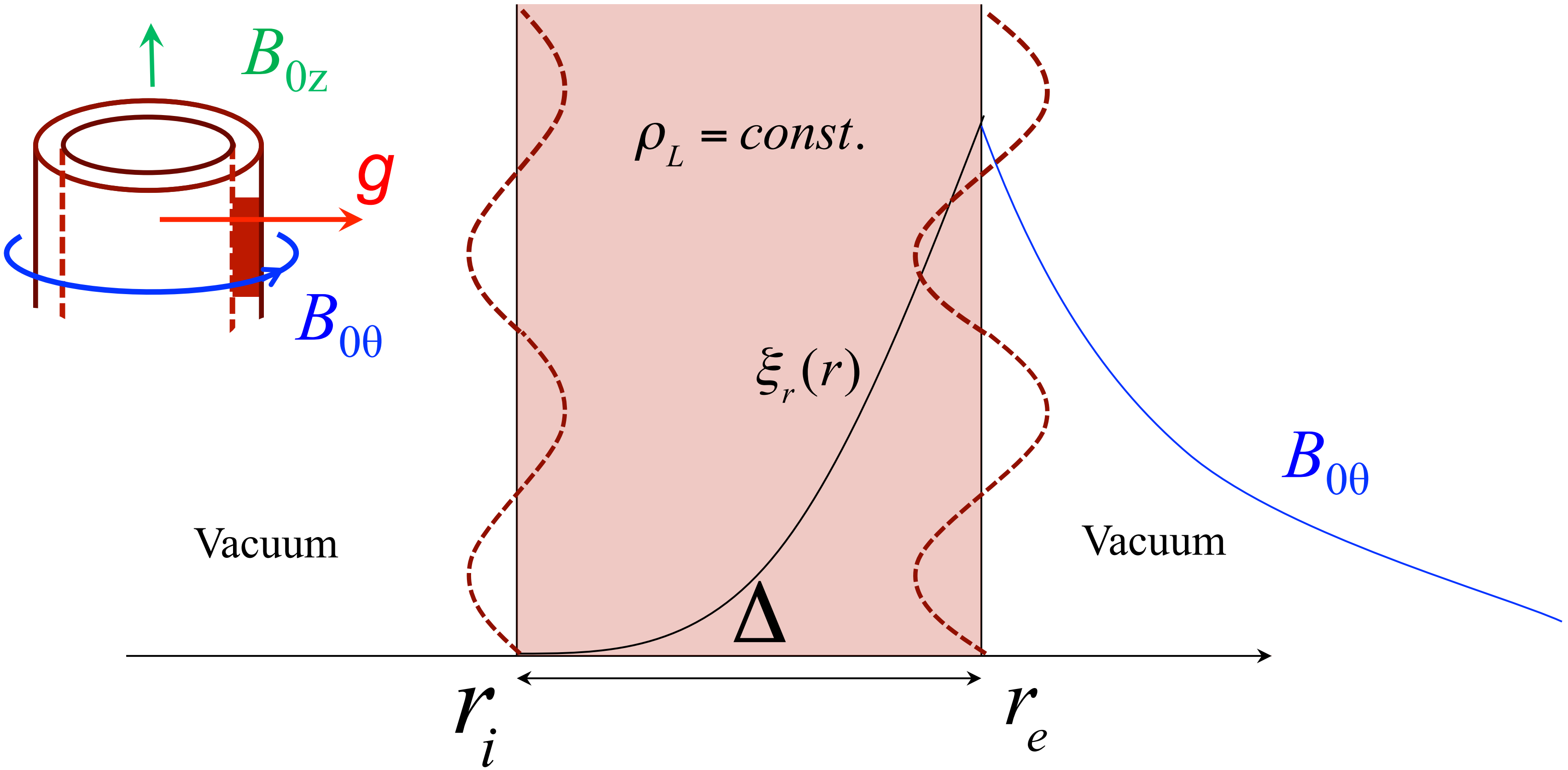
g > 0: Implosion

g < 0: Stagnation

g = effective gravity in rest frame of interface

MRT (acceleration)
Sausage / m=0, Kink / m=1
(present with no acceleration in a cylindrical current carrying plasma)

Sharp boundary model



Aspect ratio:

$$AR = \frac{r_e}{r_e - r_i} = \frac{r_e}{\Delta}$$

The *feedthrough* of instability from the outer to inner surface for a given mode, ω, is defined as:

$$\xi_r(r_i) / \xi_r(r_e) \equiv F(\omega)$$

Model equations

Mass Conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Momentum Conservation:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g}$$

Ampere's Law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Faraday/Ohm Law:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Perturbation of equilibrium

We assume that the time scale for perturbation growth is fast compared to liner dynamics, yielding an approx. instantaneous equilibrium:

$$\frac{dp}{dr}+\frac{1}{\mu_0}\left[B_z\frac{dB_z}{dr}+\frac{B_\theta}{r}\frac{d}{dr}\left(rB_\theta\right)\right]=\rho g$$

We perturb this equilibrium by a small displacement of the form:

$$\vec{\xi}(\vec{r},t)=\left\langle \xi_r(r),\xi_\theta(r),\xi_z(r)\right\rangle e^{i\gamma t+ikz+im\theta}$$

We assume that the perturbed velocity is incompressible:

$$\nabla\cdot\frac{\partial\vec{\xi}}{\partial t}=0$$

The growth rate, ω, is of the form:

Where C includes the effects of azimuthal and current carrying modes

$$\gamma^2\approx kg-\frac{\left(\mathbf{k}\cdot\mathbf{B}\right)^2}{\mu_0\rho}+C(m,r)$$

The dispersion relation is found numerically in cylindrical geometry

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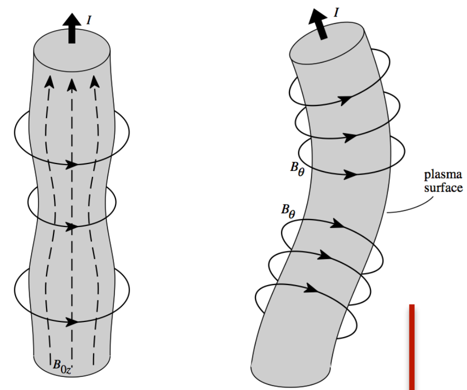
- Continuity of total pressure at each interface gives boundary conditions on the eigenfunction, ξ, and eigenvalue, ω
  - A shooting method is employed to search for the eigenvalues/functions that satisfy the ODE/BCs
  - With no drive field in the liner the solution can be found exactly (m=0)
- Integration is fairly straightforward for constant density and constant Bz in the three regions
  - Challenge is determining what realistic values to use

<u>Normalizations:</u>

$$v_{\theta 0}^2 = \frac{B_{\theta 0}^2}{\mu_0 \rho_0} \qquad \omega^2 = \gamma^2 \left(\frac{r_e}{v_{\theta 0}} \right)^2 \qquad \bar{g} = g \left(\frac{r_e}{v_{\theta 0}^2} \right)$$

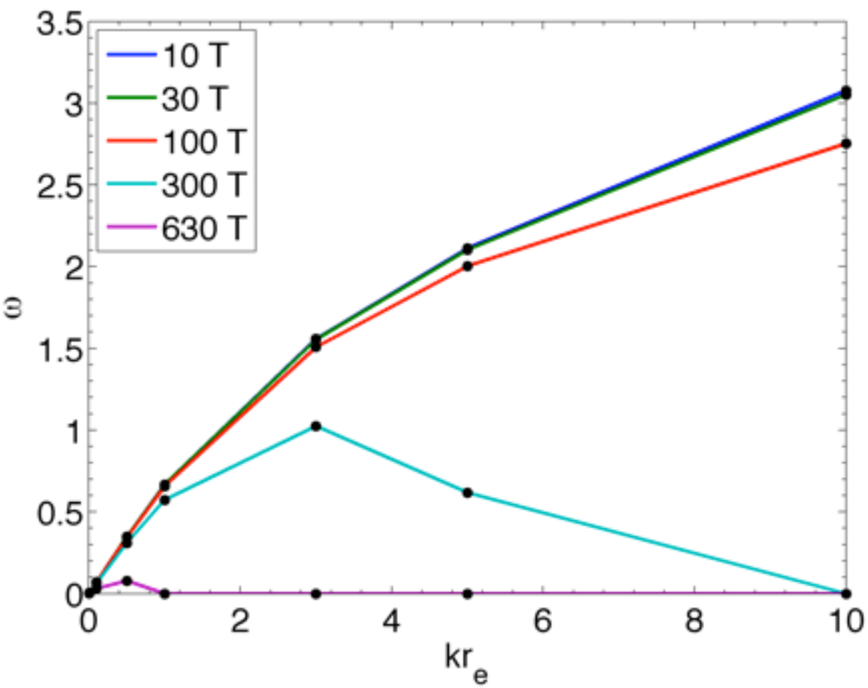
Sausage and kink modes are successfully recovered

- For **g = 0** and AR = 1 (solid plasma column undergoing no acceleration) give well known test problem

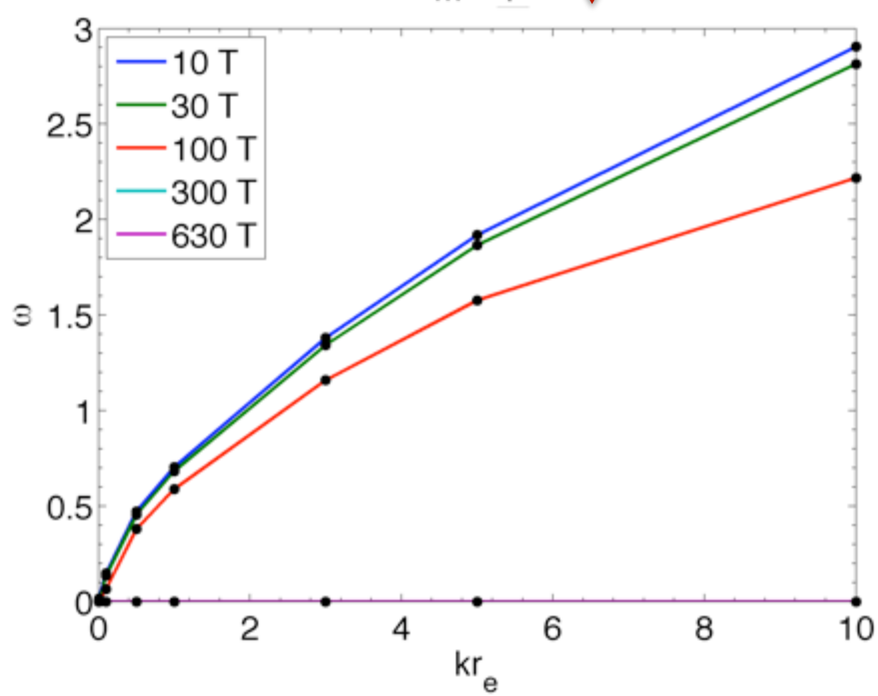


*

m = 0

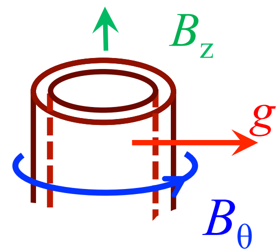


m = 1

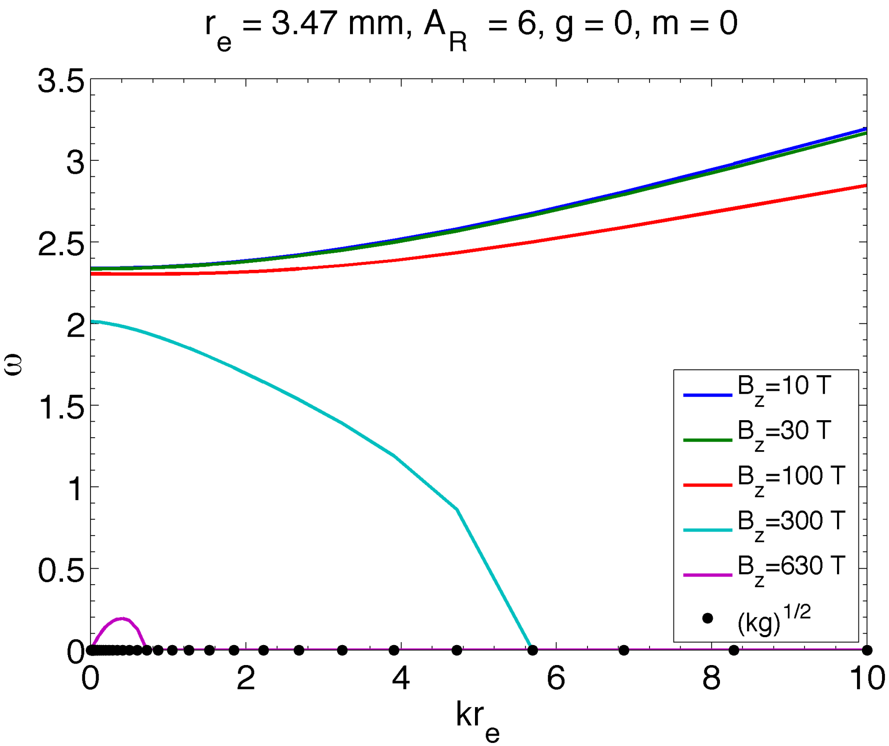


* Boyd and Sanderson. *The Physics of Plasmas*, Cambridge Press, 2003

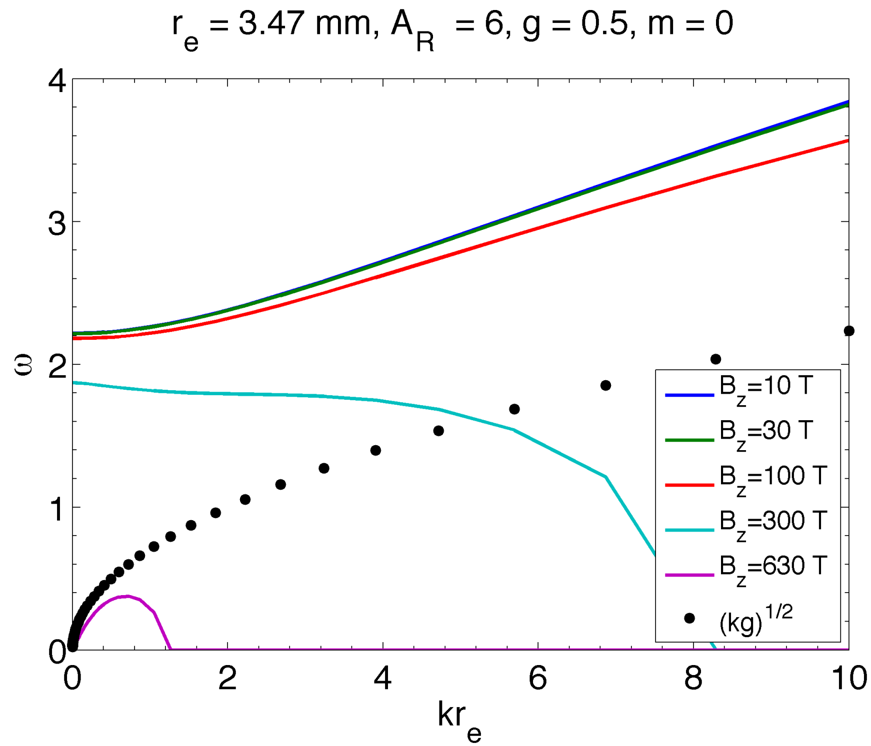
AR = 6 liners show stabilization with Bz as well



No acceleration

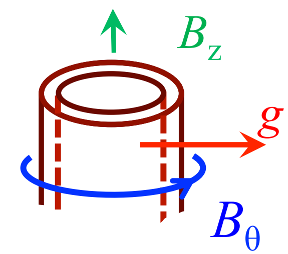


Implosion acceleration



Results look similar to current carrying modes, but longer wavelengths are not cut off (kr <<1)

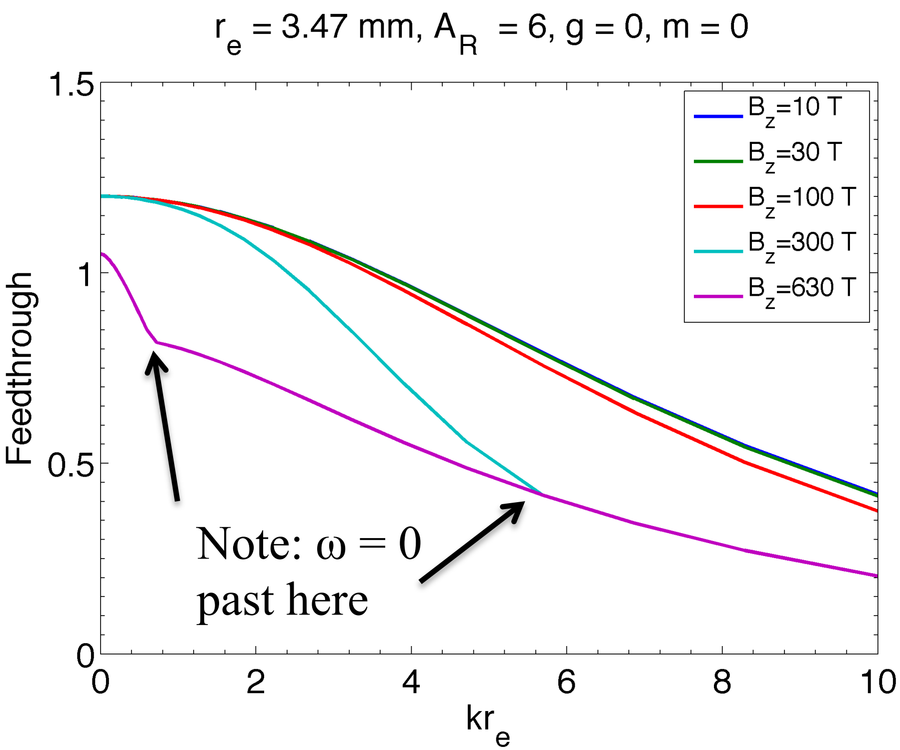
AR=6 liners show feedthrough reduction with Bz as expected



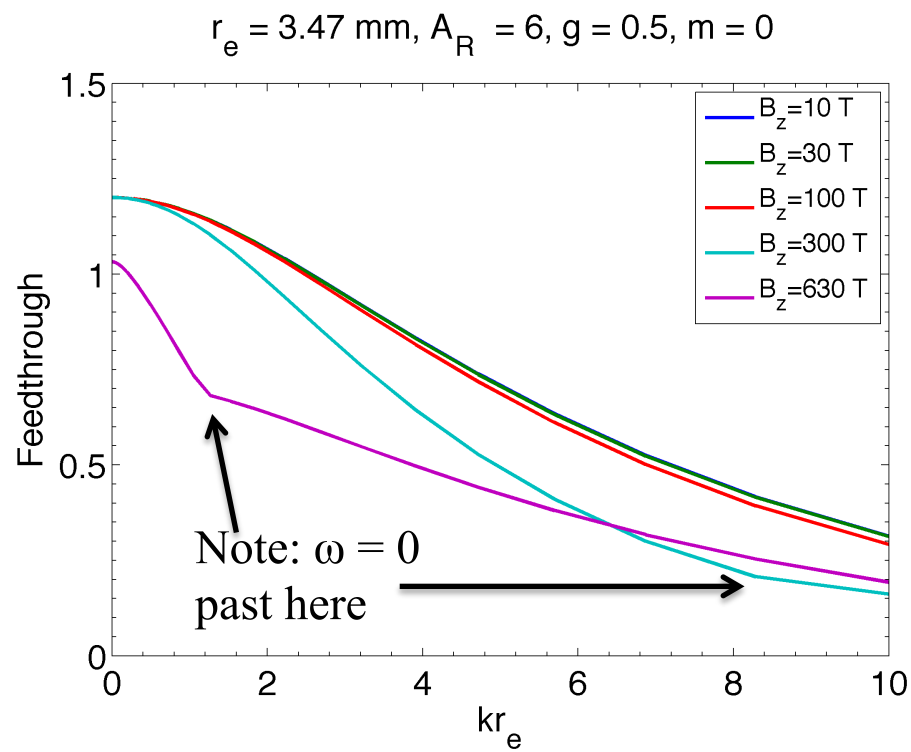
Reverse feedthrough also exists for small kr

- **This is not present in planar results!**
- **Increasing g reduces this effect**

No acceleration

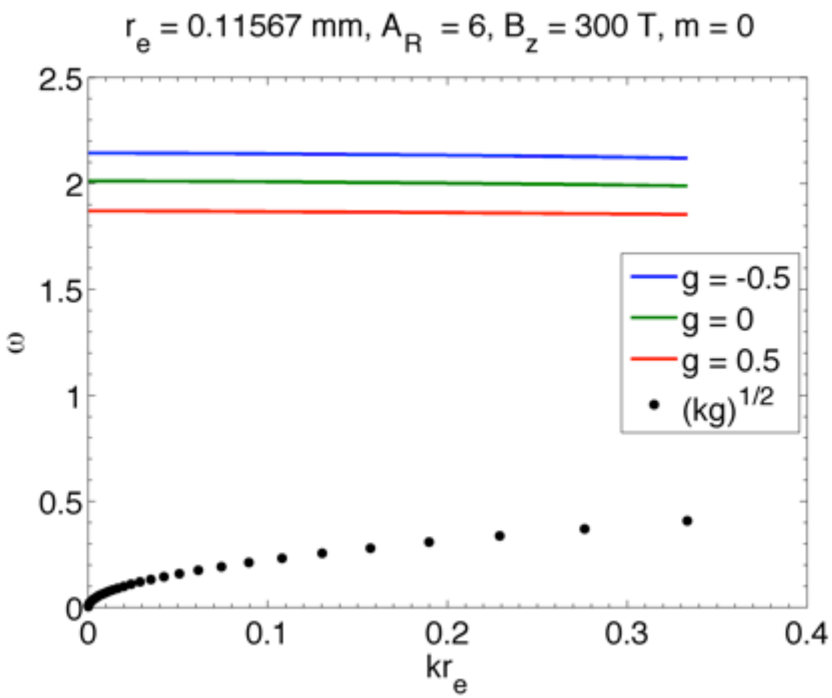


Implosion acceleration

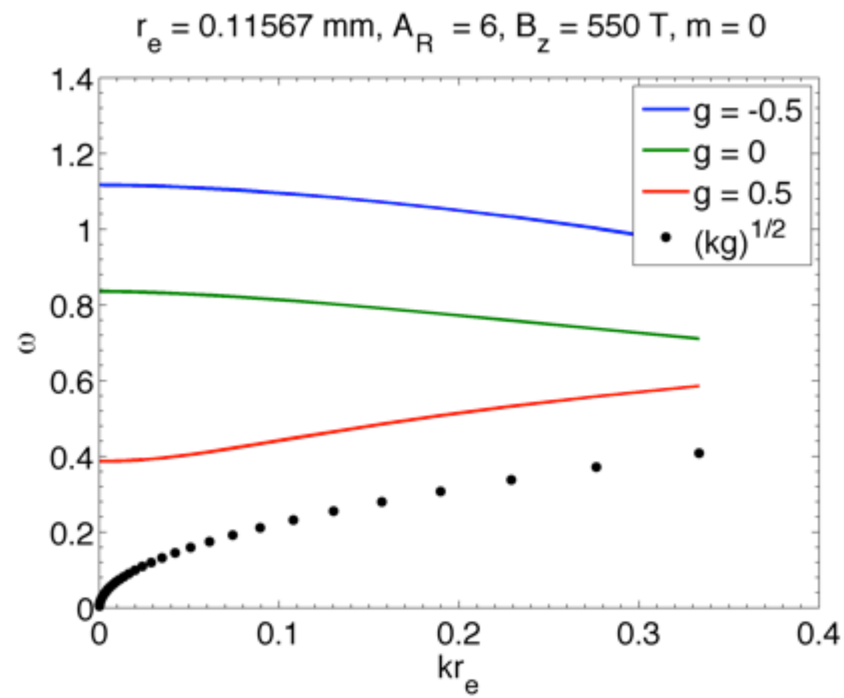


For smaller radii, significant Bz is required for stabilization

We have kept kr_e = const. but reduced the radius by a factor of 30 (high convergence)



Bz = 30% of drive field



Bz = 55% of drive field

For smaller radii, stagnation type acceleration is also more dangerous

Using realistic data as input into linearized model

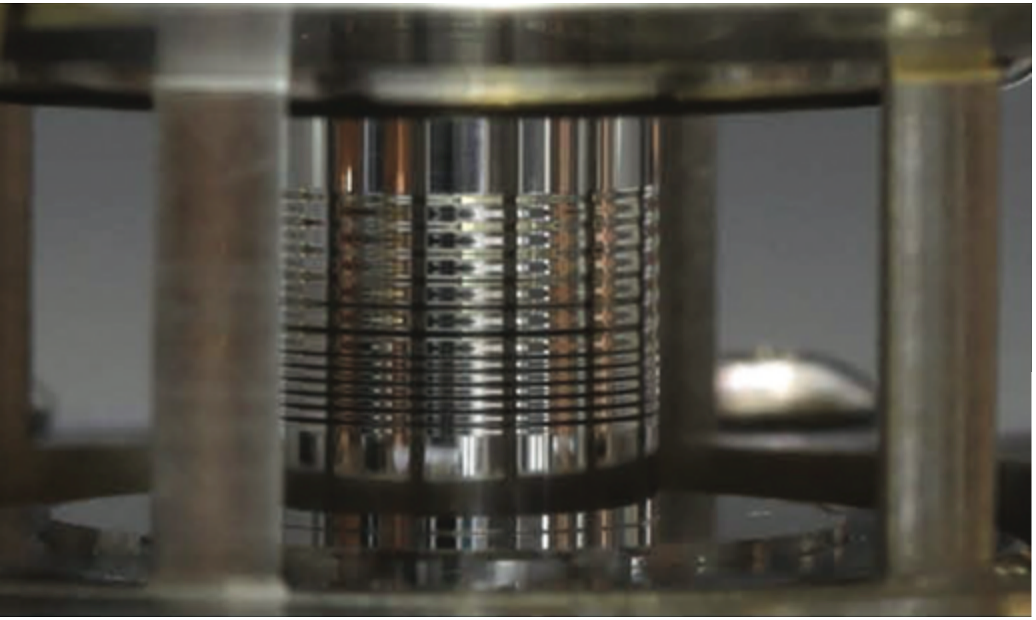
- Average physical quantities from 1D Hydra data in each 'region'
  - Running Lagrangian zones can be used to find liner/vacuum interfaces and, hence, the boundaries for averaging
- For a given wavelength we can calculate the instantaneous growth rate, $\omega(t)$ for each time step
  - The amplitude, η, of the instability is then determined by

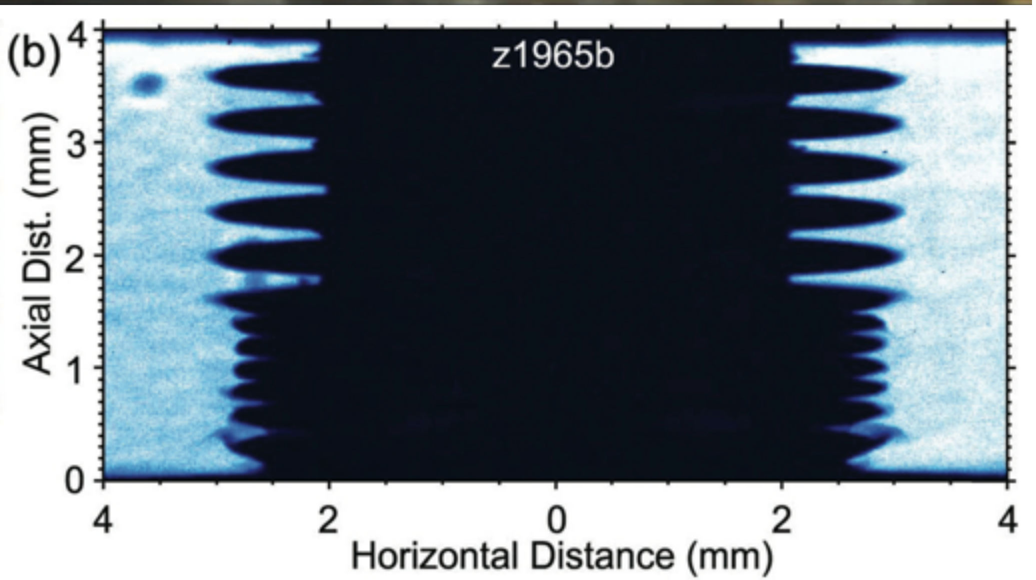
$$\frac{d^2}{dt^2}\eta(t) = \omega(t)^2 \eta(t)$$

  - The feedthrough between interfaces is just the ratio of the eigenfunction at the inner and outer interface

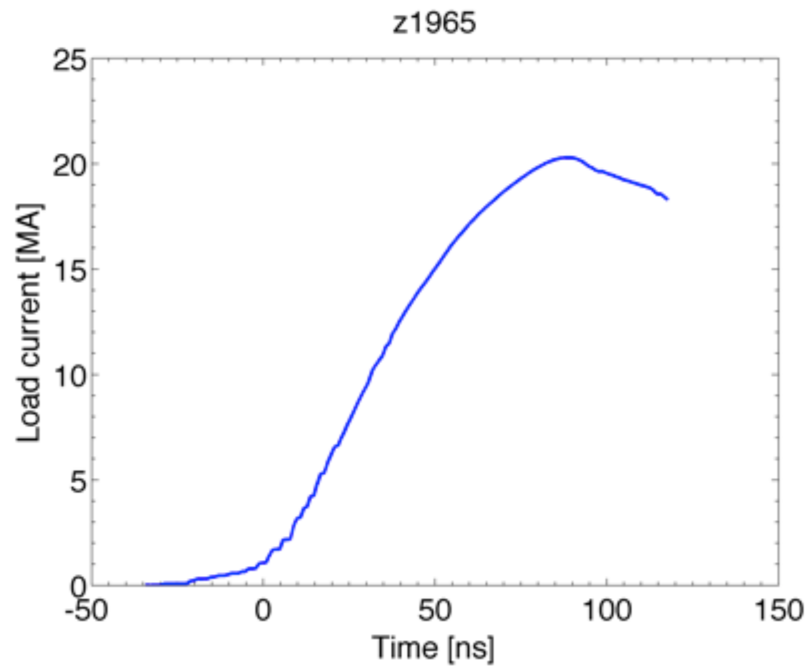
$$F(\omega) = \xi(r_i) / \xi(r_e)$$

Aluminum liner experiments on Z with seeded MRT *





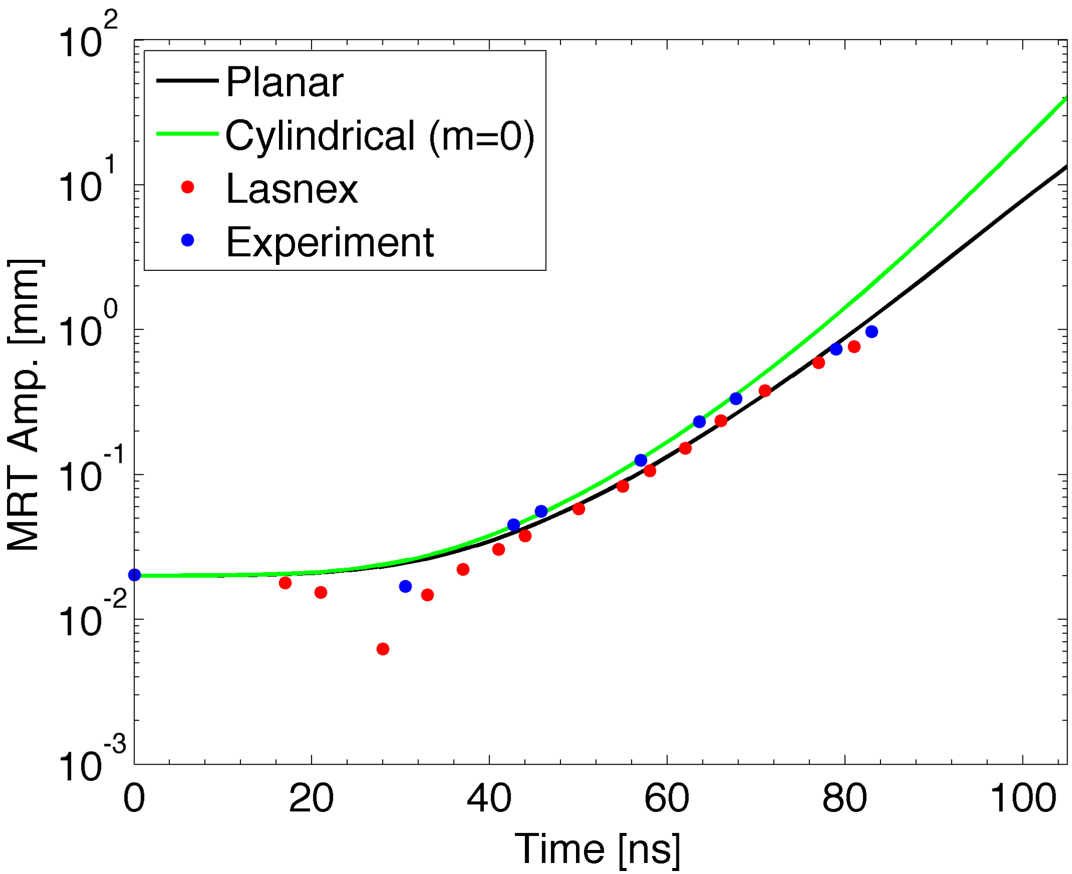
A 1D simulation with Hydra can be driven with the measured load current from which we can extract our averaged physical quantities

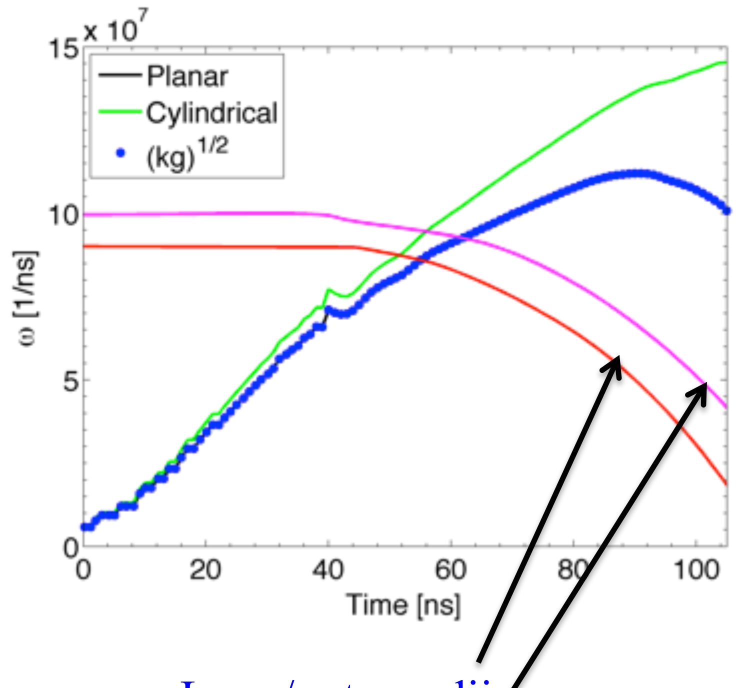


* Sinars et. al. Phys. Plasmas **18**, 056301 (2011)

Applying linearized model to Sinars et. al. * experiments shows good agreement while convergence is low

- Aluminum liner seeded with 400 um surface perturbation



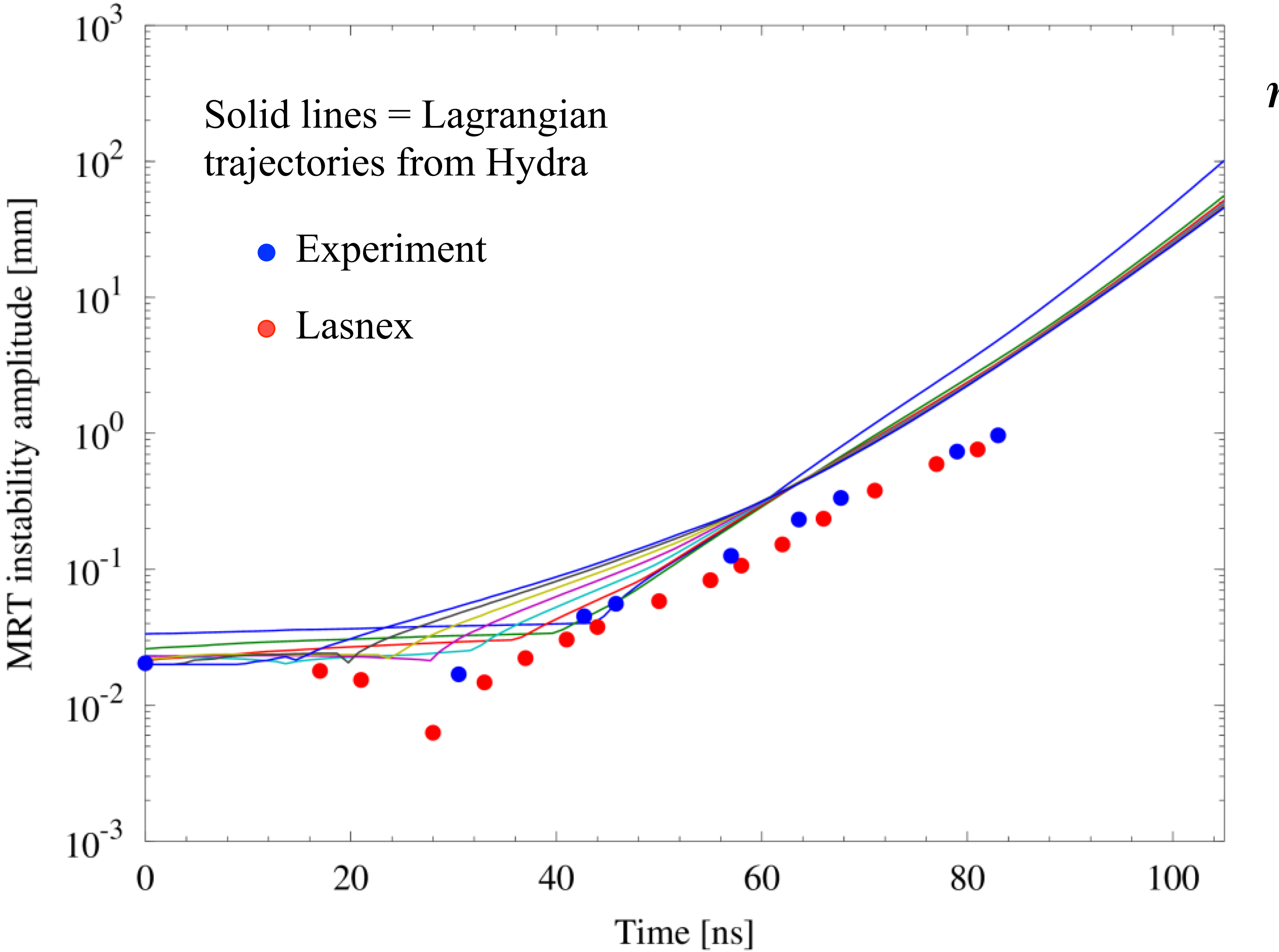


Inner/outer radii

As convergence increases, growth rate becomes more complicated

* Sinars et. al. Phys. Plasmas **18**, 056301 (2011)

Another simple MRT scaling model also represents the data well

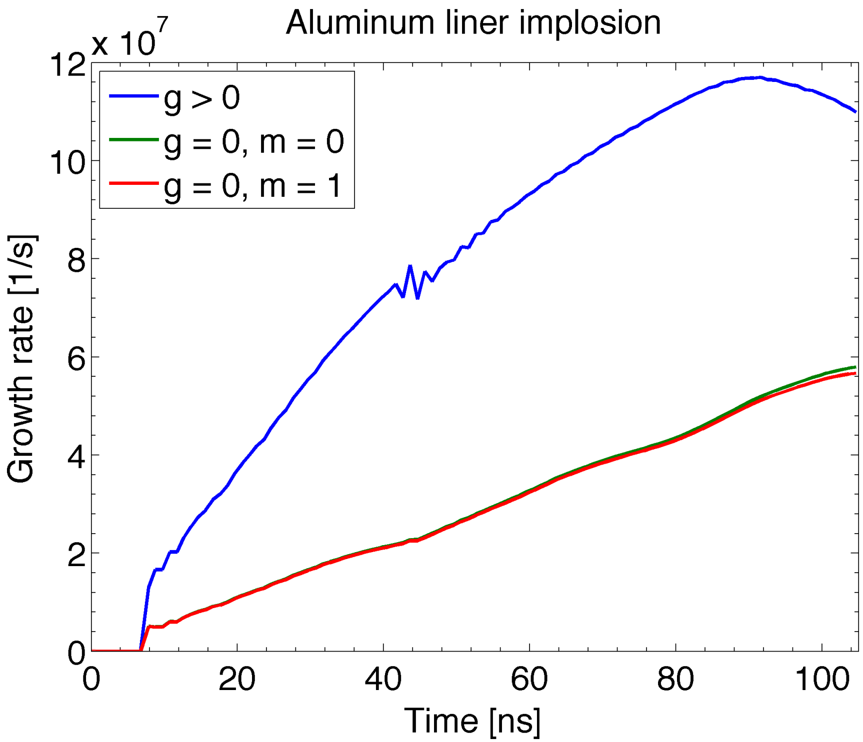


$$\eta(t) = \eta_0 e^{\sqrt{2k(r-r_0)}}$$

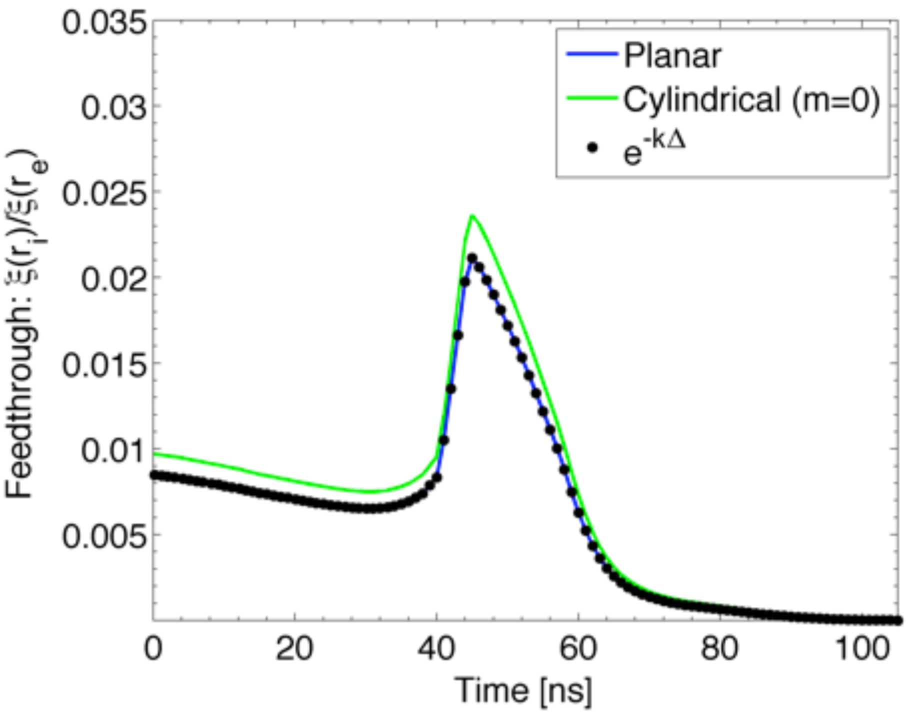
Where $r - r_0$ is the distance the liner has moved

While g is large and convergence is small, growth is dominated by classical Rayleigh-Taylor growth rate:

$$\omega^2 \approx kg >> -\frac{(\mathbf{k}\cdot\mathbf{B})^2}{\mu_0 \rho} + C(m,r)$$



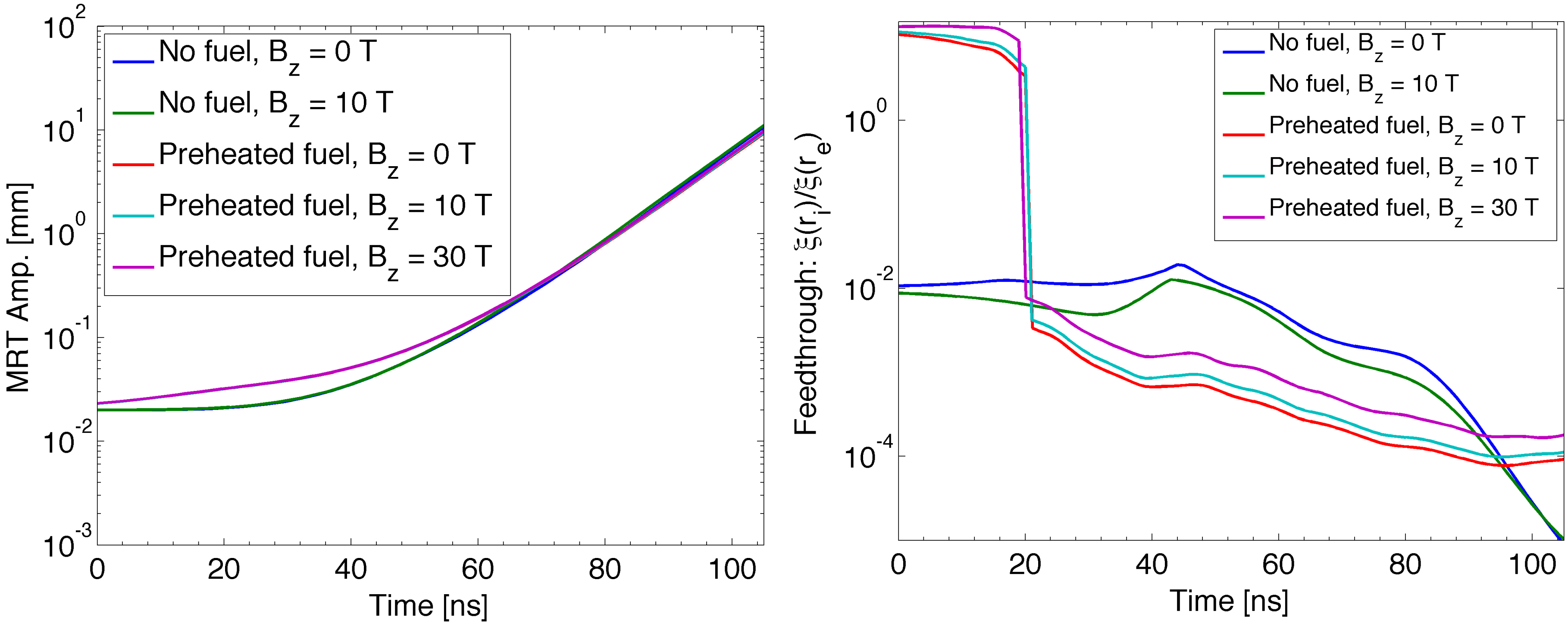
If we remove g for the same problem, we see the remaining physics gives much lower growth



Feedthrough is similarly dominated by the classical expression

Our planar model can include a fuel *

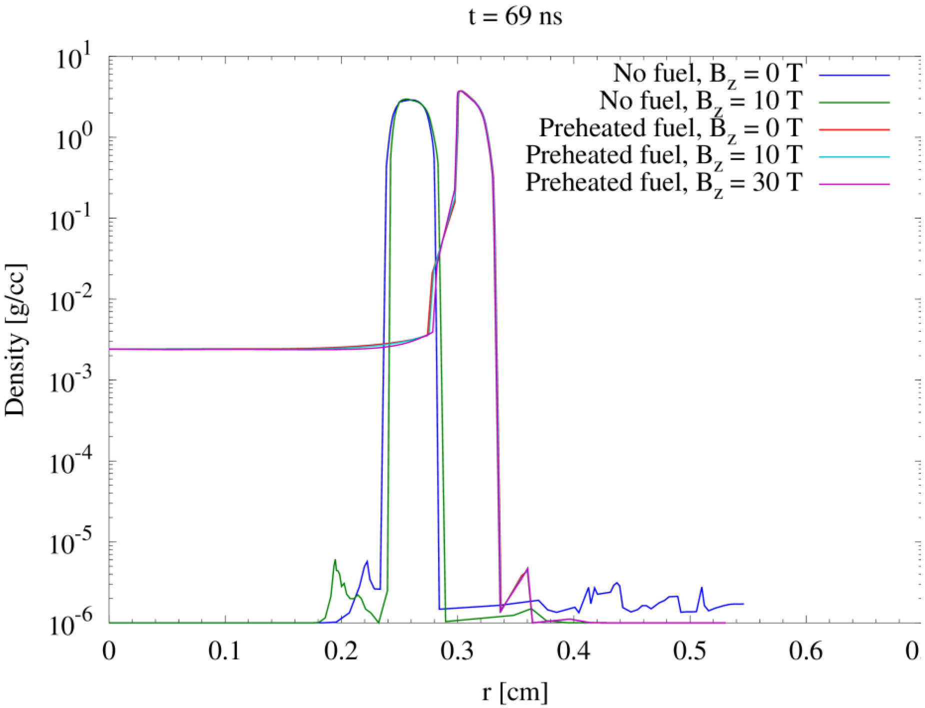
Re-ran same problem in Hydra but with various combinations of Bz and hydrogen fuel and input results into planar model

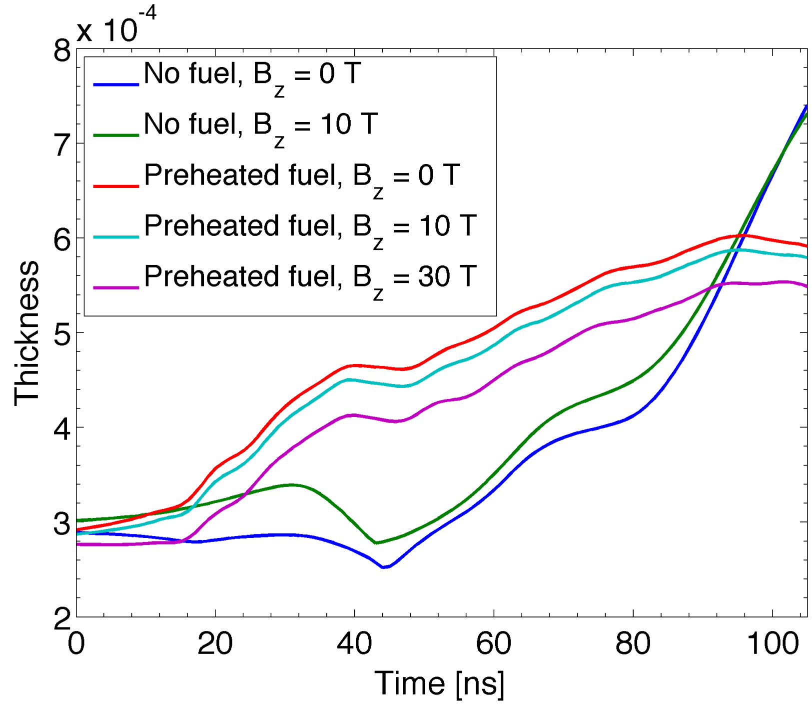


- **Feedthrough is more significantly modified by adding a fuel and Bz (to a smaller extent)**
  - **This is due to the strong relationship between feedthrough and liner thickness**

* Y. Y. Lau, et al., **Phys. Rev. E** 83, 066405 (2011)

Differences in feedthrough come from liner thickness





- **We know Bz can reduce feedthrough and stabilize MRT, but Bz must be in the ballpark of the drive field which requires high convergence**
  - **This corresponds to a thinner liner which will have worse feedthrough**

Future work with the linearized model

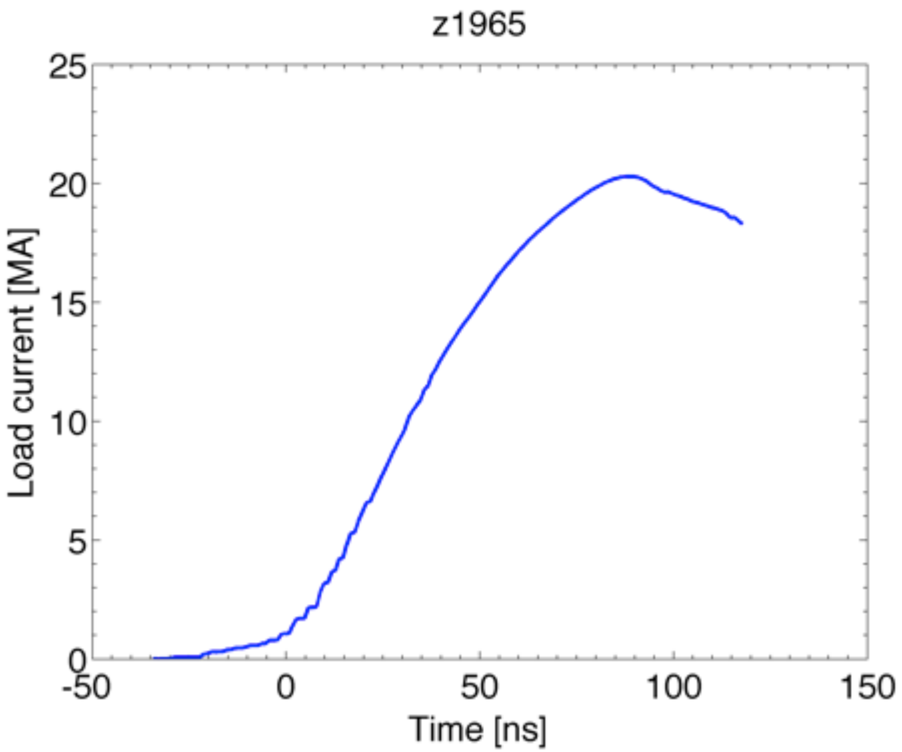
- Current work involves:
  - Determining the best way to use 1D Hydra data in our model
  - Adding a fuel in cylindrical coordinates
  - Adding a model of the diffusion of the drive field into the liner which can be de-stabilizing.
  - Add adiabatic compressibility (important for fuel) which is stabilizing

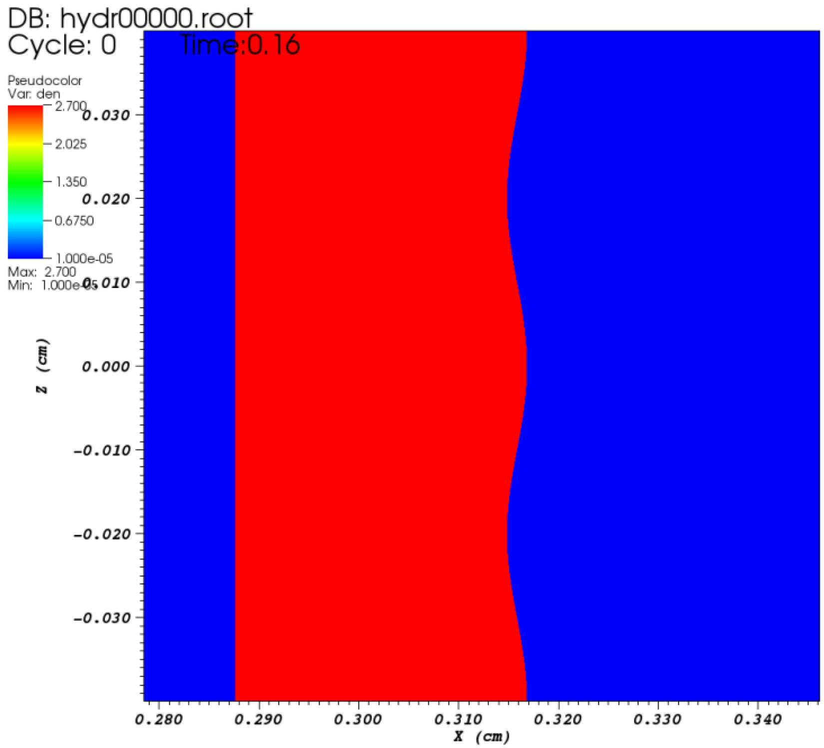
Summarizing:

- Large R and g seem to make $\sqrt{kg}$ a good approximation
- Feedthrough is mostly dependent upon liner thickness
  - Can be reduced by strong Bz
- Bz is relatively unimportant for stabilization until high convergence

Hydra has been used to model Al liner implosions with seeded MRT

- A sinusoidal perturbation of λ=400 um was applied to the surface of an Al liner and an implosion was driven using the load current on shot z1965 in attempt to replicate the MRT growth rates shown earlier *



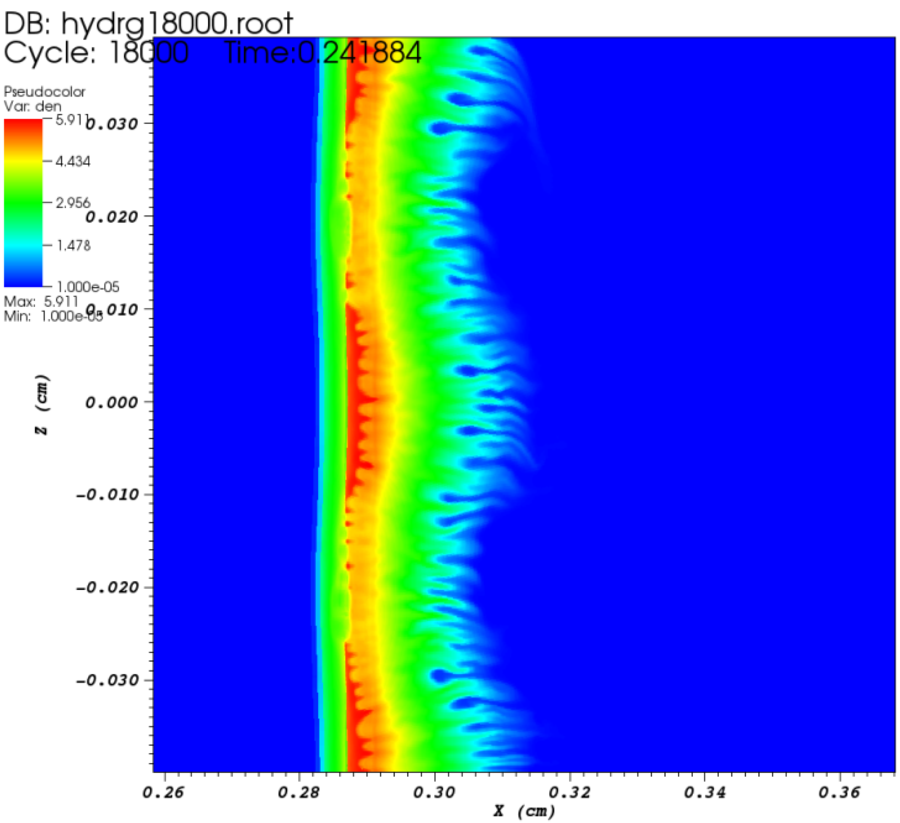


* Sinars et. al. Phys. Plasmas **18**, 056301 (2011)

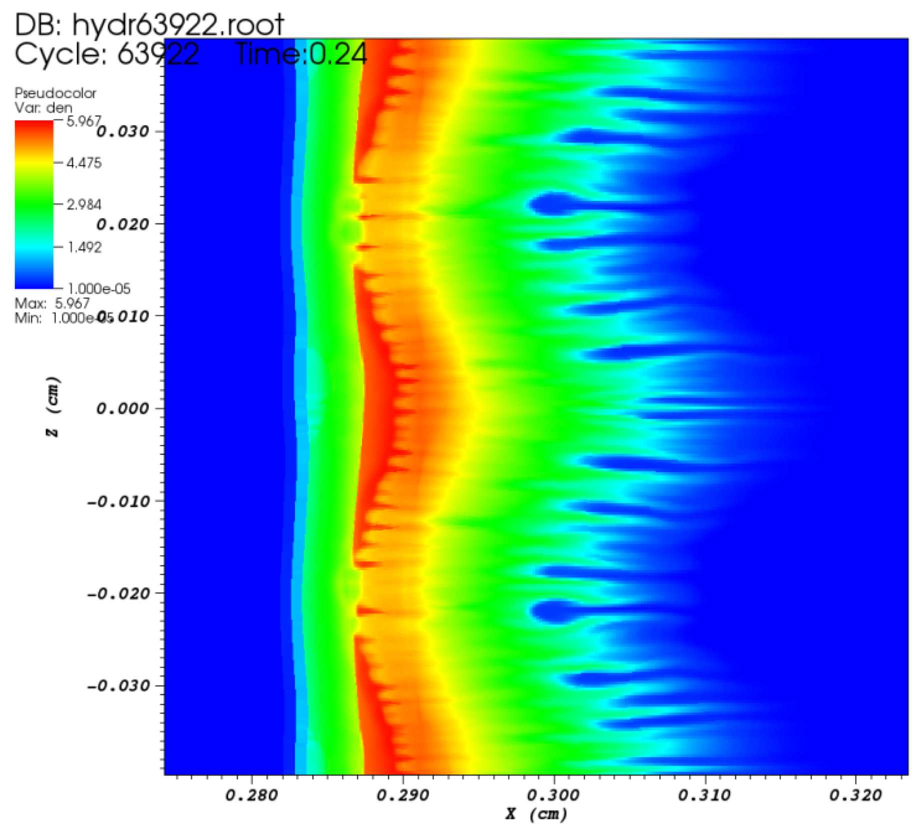
Two MHD force calculation methods were run

- Calculation of J x B at nodes (Kyle Peterson has had best growth rate results with this)
  - No hydro sub-cycling allowed (problems run slowly)
- Alternate method that allows hydro sub-cycling
  - Problems run much faster

Sub-cycling



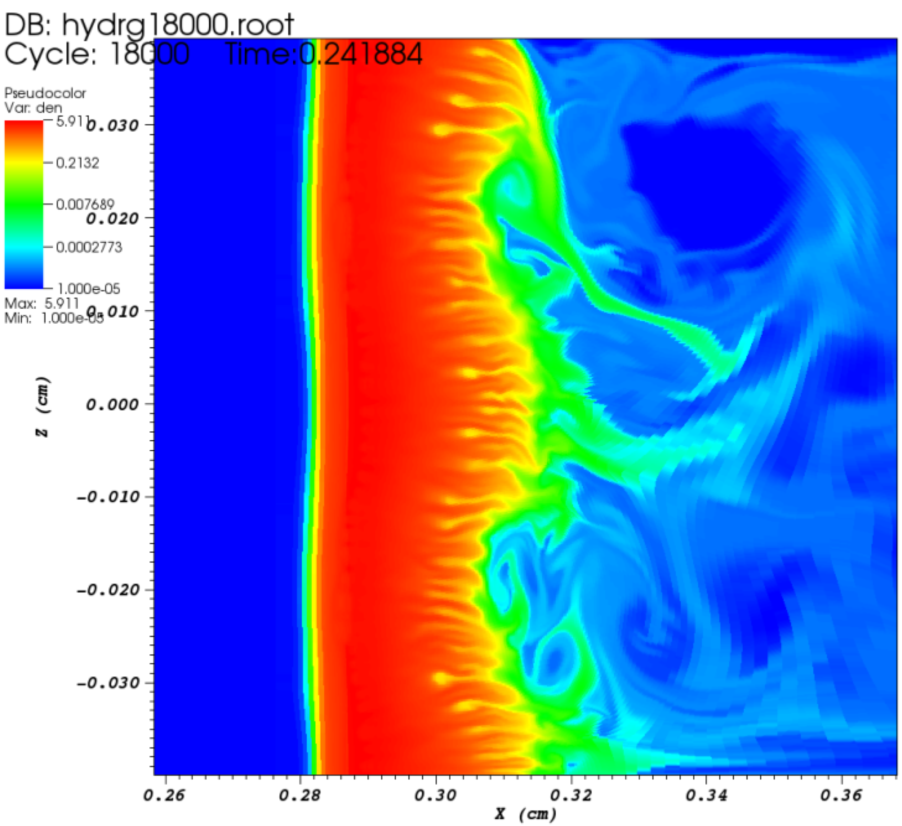
No sub-cycling



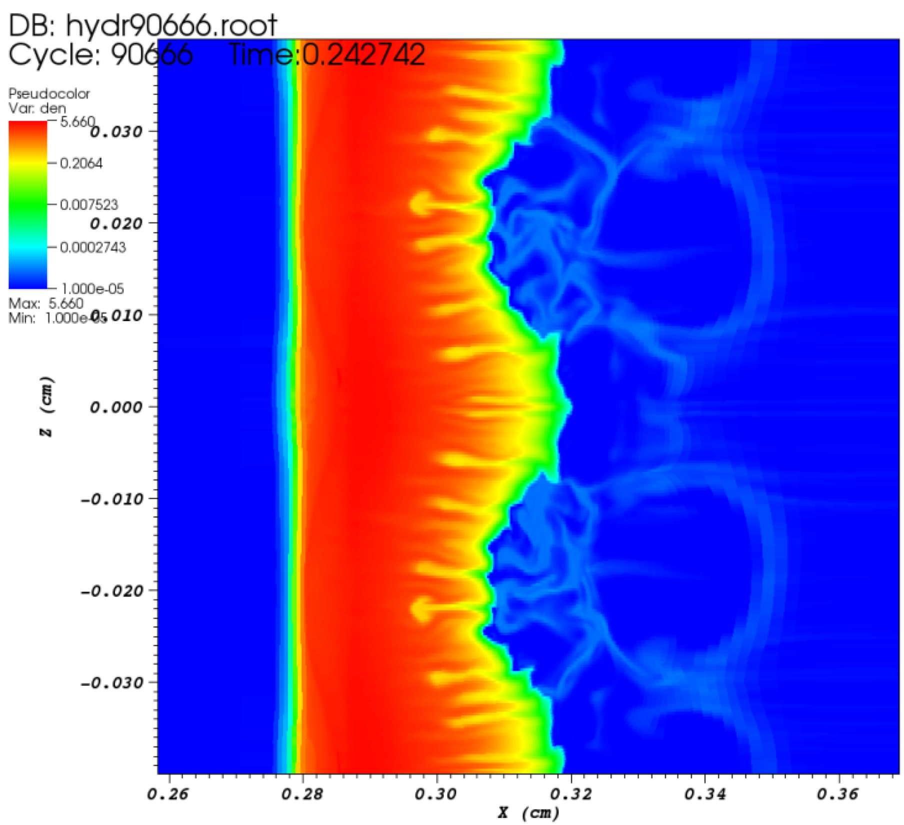
Behavior becomes much different near lower densities

Much more structure is present in the sub-cycling case at low density ($\rho < 0.7$ g/cc)

Sub-cycling (log plot)



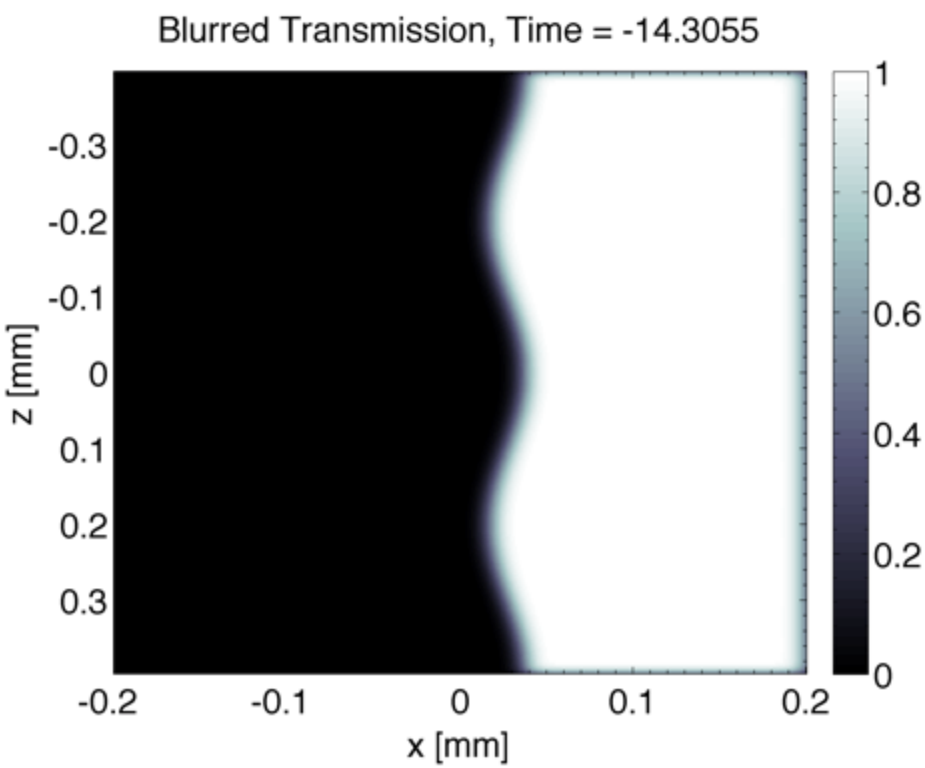
No sub-cycling (log plot)

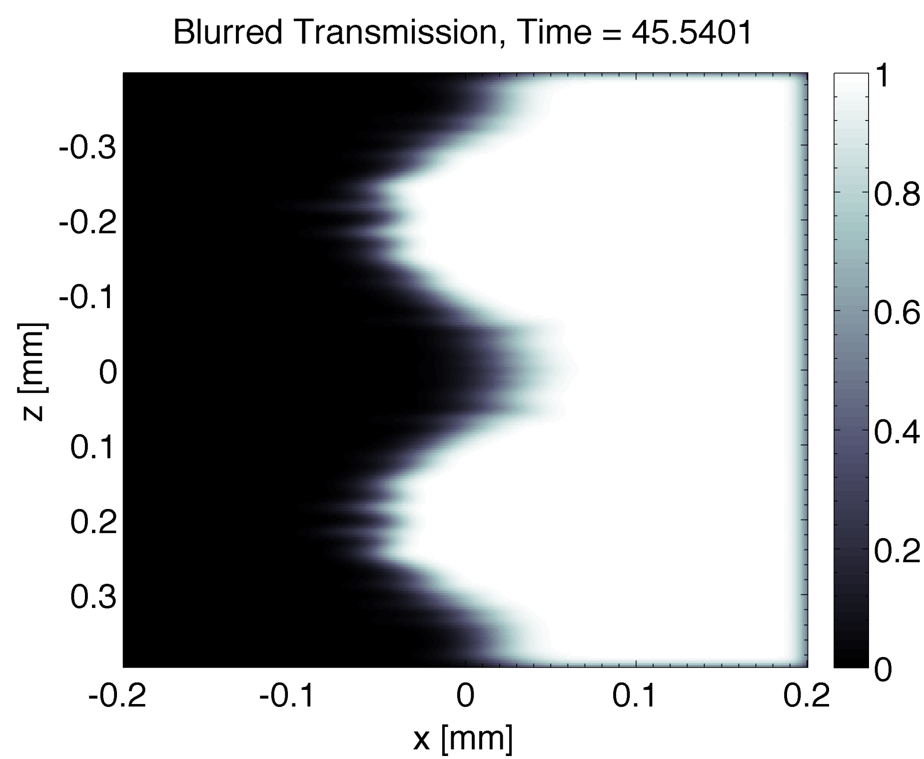


Determining MRT growth rates can be done many ways using 2D data

- An FFT can be applied to the mass per unit length and the amplitude at 400 um can be tracked as a function of time
  - The more wavelengths included in the simulation the better, but this may not be computationally feasible
- The bubble and spike radii can also be tracked
  - The diffuse nature of the ‘interface’ makes this prone to error depending on the amount of ablation
  - Bounds on the radii can be determined by tracking density contours around high gradients
- Simulated radiographs can be computed using Spect3D and compared visually to the experimental data, as well as be analyzed similarly to the above

Simulated radiographs are generated from X-ray transmission through plasma onto a submicron resolution detector

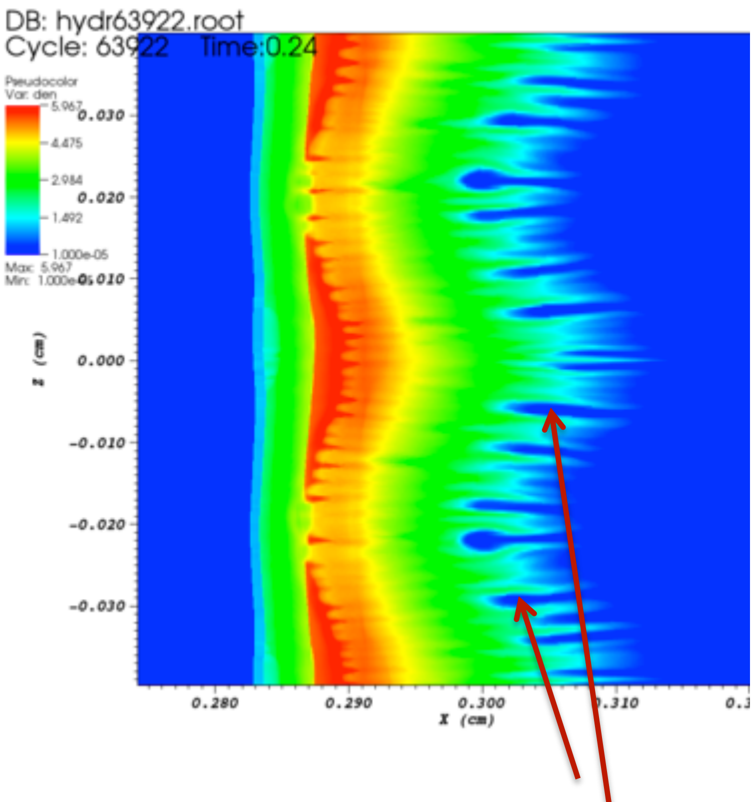




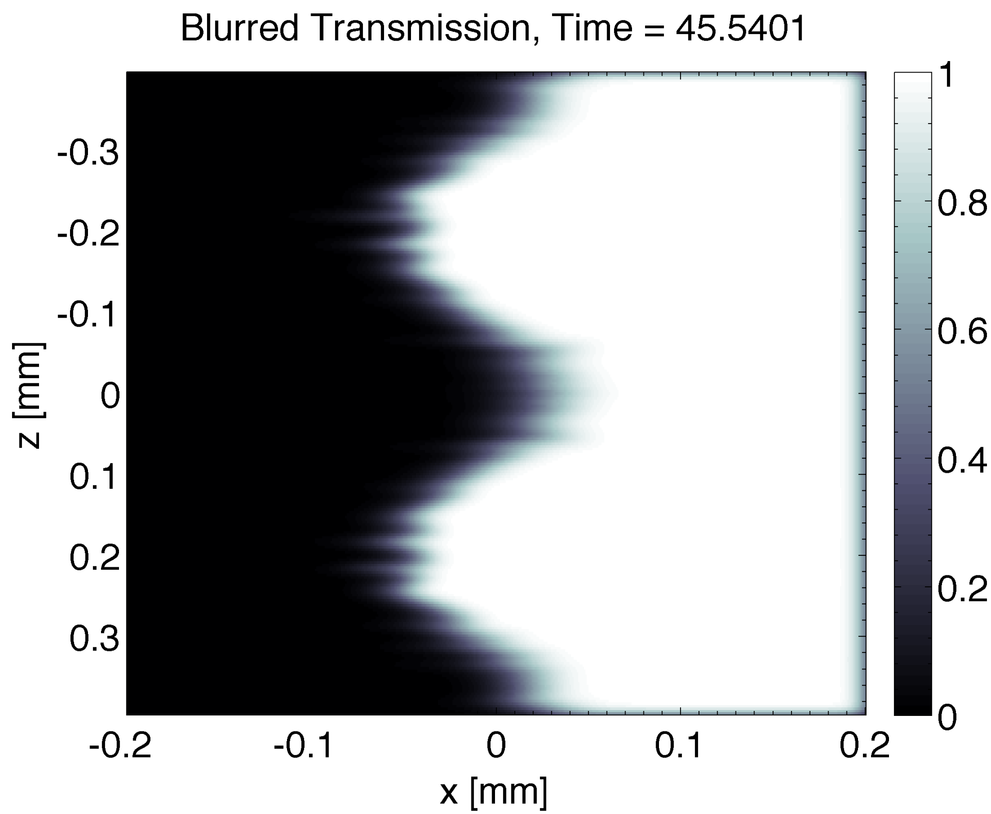
15 micron Gaussian blur is added to model ZBL resolution

Some features are lost in the conversion to radiographs

Hydra density (no sub-cyc.)



Simulated radiograph

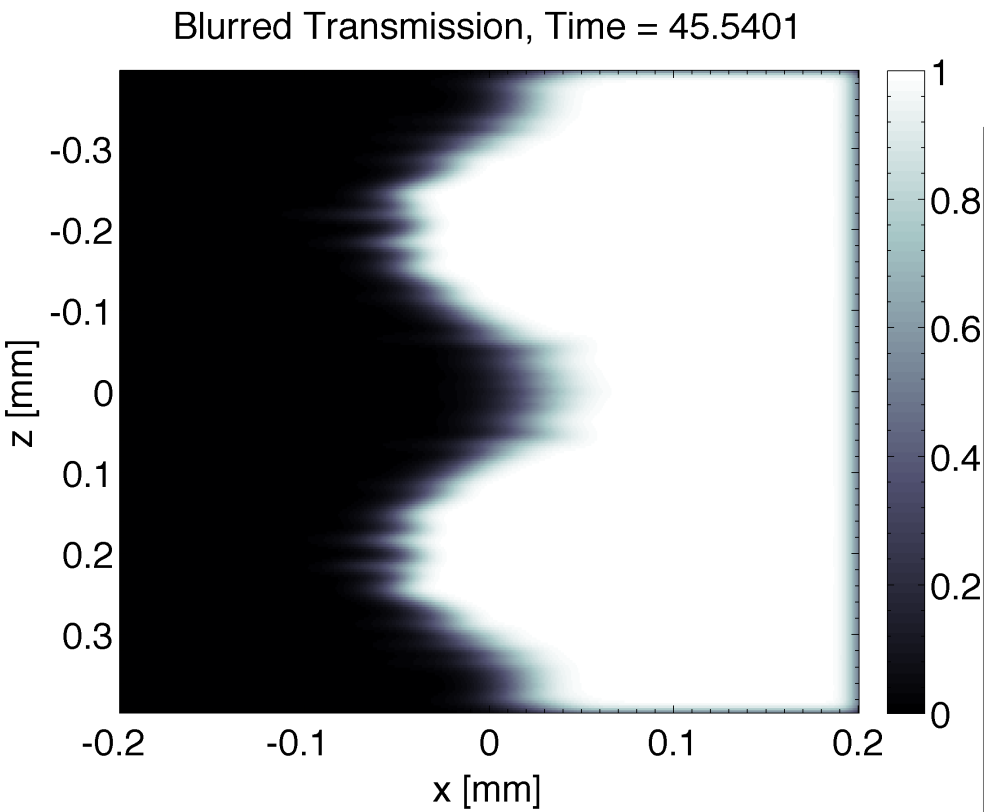


Density in between small fingers ($\lambda \sim 20$ um) is ~ 0.7 g/cc

Fingers are just barely visible due to resolution

Example of FFT calculation from radiograph

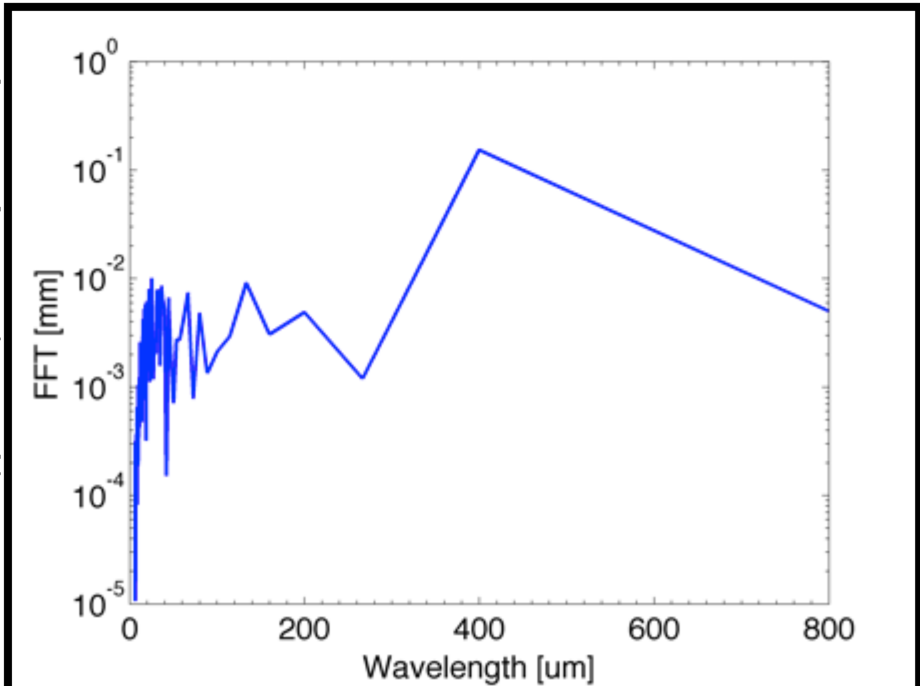
Hydra simulated radiograph (no sub-cyc.)



Axial FFT of result

$$\int \rho(r,z) r\, dr = m_L(z)$$

$$m_L(k) \approx \int m_L(z) e^{-ikz}\, dz$$



Hydra clearly shows similar MRT growth but interpretation has a significant impact on results

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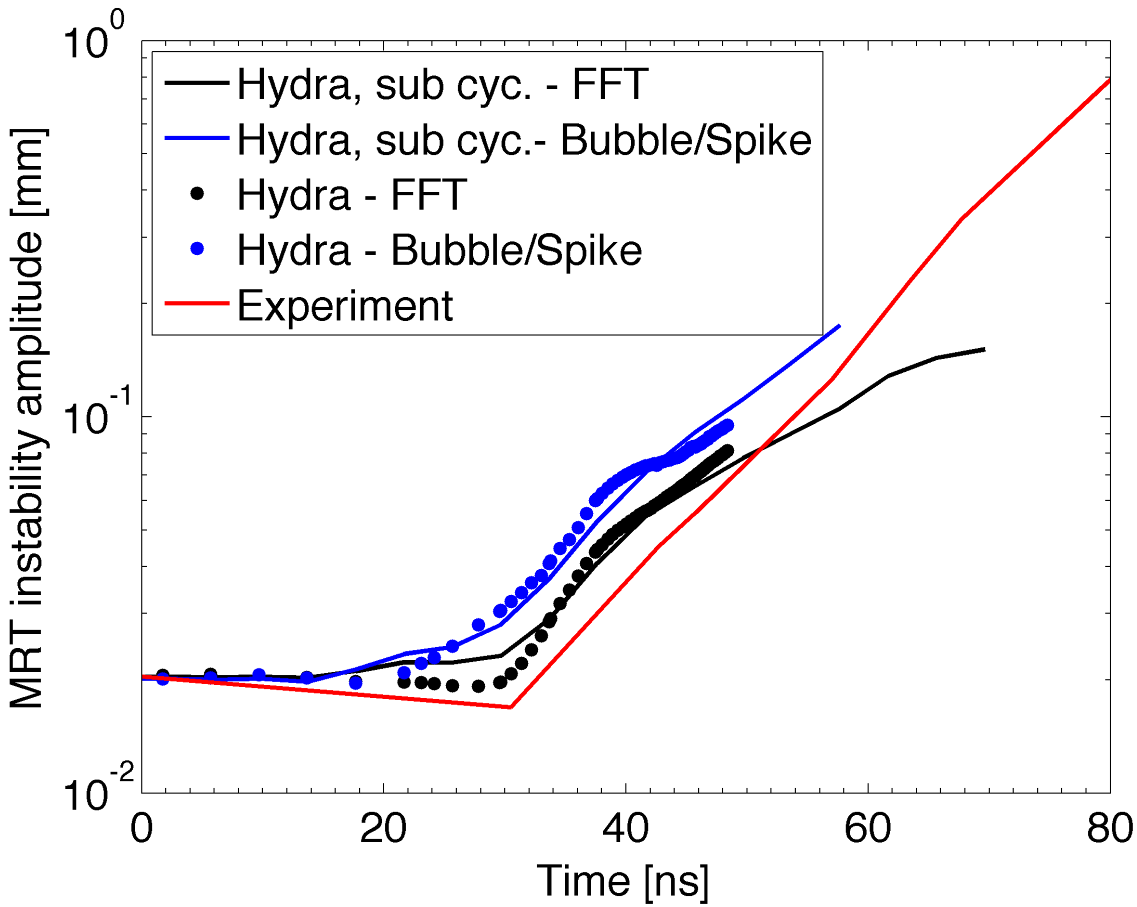
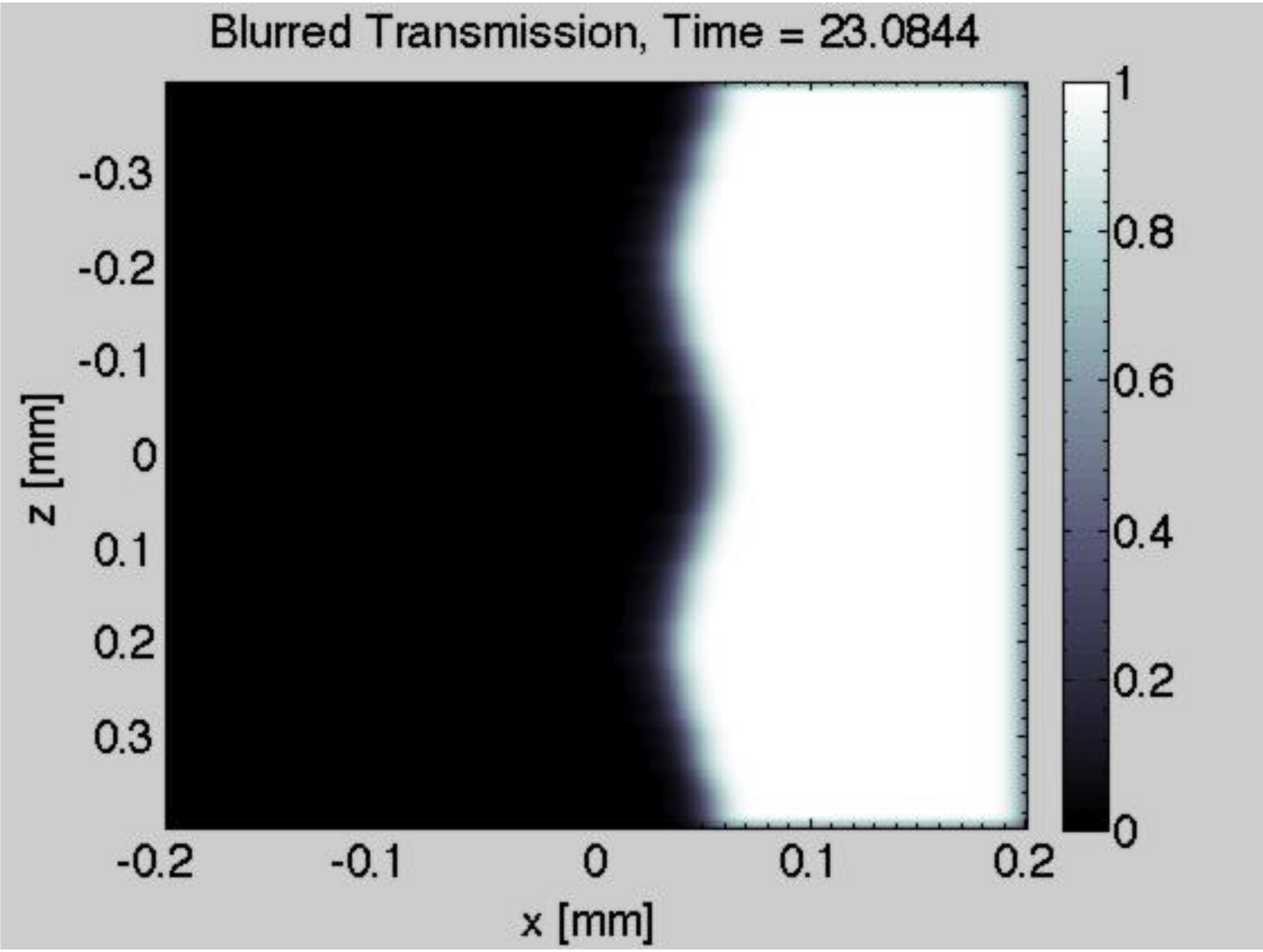


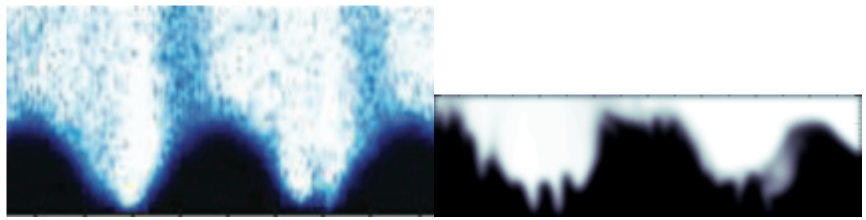
- MRT amp. dip at ~ 27 ns is more noticeable without sub-cycling
- Bubble/Spike gives larger growth
- FFT calculation becomes worse as the 400 um becomes less isolated in λ space

Hydra at least seems to qualitatively capture dip in MRT amplitude from ablation



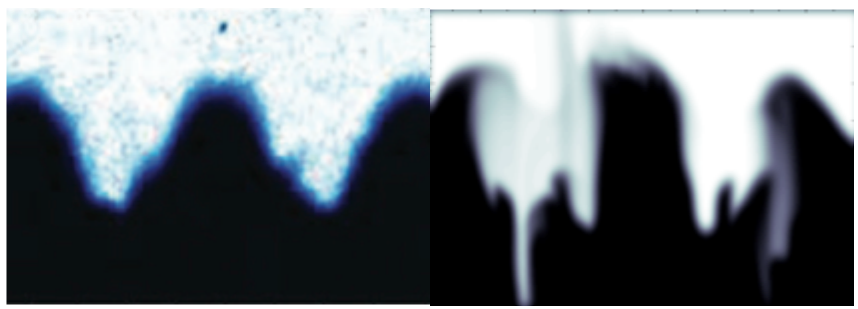
Comparing to the radiographs from sub-cycling calculation shows poor agreement

57 ns



Only sub-cycle runs made it this far in sufficient real time (1.5 weeks)

63ns



- Excess structure may be because of force calculation method, or too much sub-cycling
  - Direct comparison of force calculation methods without sub-cycling will be next

Summary of Hydra results

- Hydra seems to get the gross MRT features correct but the details get fuzzy depending calculation methods
  - FFT requires higher number of wavelengths for accuracy, as MRT grows, seeded wavelength becomes less isolated
  - Bubble/Spike method should be fairly robust but perhaps overestimates growth
- More runs will have to be made, altering the run parameters…

Future work (near term forecast)

- Upgrade analytic results to MagLIF problem
  - Add fuel, magnetic diffusion (model)
  - Perhaps relax some assumptions on physical quantities (i.e. density not constant)
- Get better agreement between Sinars et. al. data and Hydra
  - Start by tinkering with force calculation method and sub-cycling
- Assuming the above gets sorted out, we can next use Hydra output to characterize feedthrough and compare to theory
  - An interesting note: it takes roughly 40 ns for the shock to reach the inner liner surface, before this feedthrough should not occur
  - Using beryllium we can also compare to experiment

Thank you!

Questions?