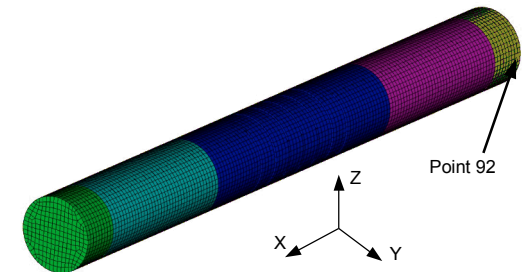
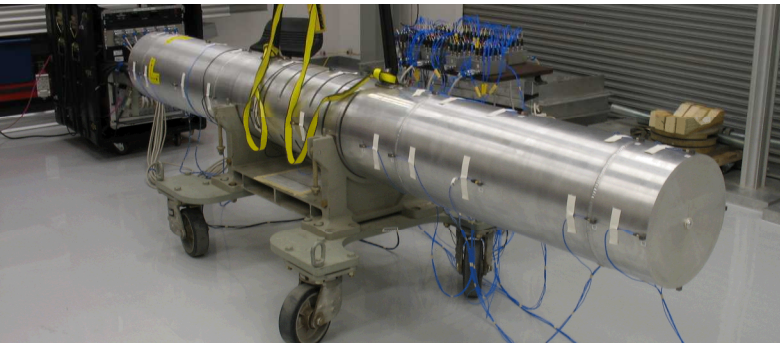


*Exceptional service in the national interest*



# Force Reconstruction Using SWAT (Sum Weighted Accelerations Technique)

Tyler Schoenherr  
HOCWOG31 – 5/20/2013

# Acknowledgements

“If I have seen a little further, it is by standing on the shoulders of giants.” –Sir Isaac Newton



Tom Carne



Randy Mayes

- Gregory, Priddy, Smallwood, 1986. SAE #861791

# Outline

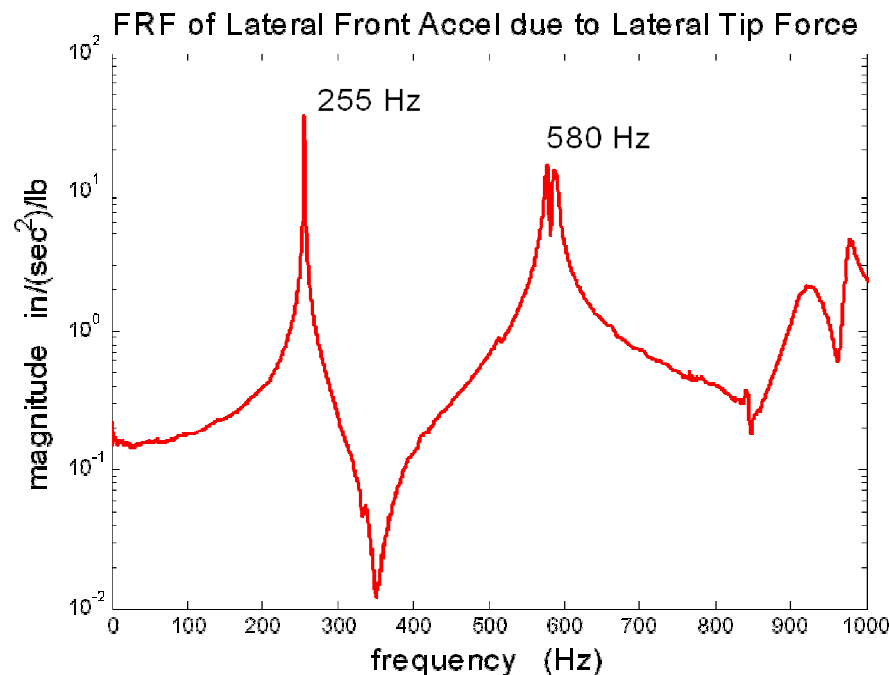
- Background on Force Reconstruction
- Theory of SWAT
- Test Object/Scenario
- Lab Test
- Field test
- How do we know if it worked and how well?
- Sum it all up

# Background on Force Reconstruction

Force Reconstruction is an inverse problem.

$$\vec{F}(\omega) = \mathbf{H} \cdot \vec{X}(\omega)$$

$$\vec{X}(\omega) = \mathbf{H}^{-1} \cdot \vec{F}(\omega)$$



- Modal substitution of the Equation of Motion

$$\mathbf{M}\ddot{\bar{x}} + \mathbf{C}\dot{\bar{x}} + \mathbf{K}\bar{x} = \bar{F}$$

$$\bar{x} = \sum_{i=1}^n \bar{\phi}_i q_i = \Phi \bar{q}$$

$$\Phi^T \mathbf{M} \Phi \ddot{\bar{q}} + \Phi^T \mathbf{C} \Phi \dot{\bar{q}} + \Phi^T \mathbf{K} \Phi \bar{q} = \Phi^T \bar{F}$$

- For the rigid body mode shapes, there is no internal deflections or damping forces

$$\mathbf{C}\bar{\phi}_r = \mathbf{K}\bar{\phi}_r = 0$$

$$\mathbf{M}_r \ddot{\bar{q}}_r = \bar{\phi}_r^T \bar{F}$$

NEWTON'S LAW FOR FLEXIBLE BODIES

$$\boxed{\mathbf{M}_r \ddot{q}_r = \bar{\phi}_r^T \bar{F}}$$

- Define the SWAT dof (s) as a weighted sum of the measured accelerations and perform modal substitution

$$s = \sum_{k=1}^n w_k \ddot{x}_k = \bar{w}^T \ddot{\bar{x}}$$

$$s = \bar{w}^T \sum_{i=1}^n \bar{\phi}_i \ddot{q}_i = \sum_{i=1}^n (\bar{w}^T \bar{\phi}_i) \ddot{q}_i$$

- Finally define w for the  $i^{\text{th}}$  elastic mode shape and the  $r^{\text{th}}$  rigid body mode shape  $\bar{w}^T \bar{\phi}_i = 0 \ \& \ \bar{w}^T \bar{\phi}_r = 1$
- Because of this definition  $s = \ddot{q}_r$  and s can be substituted back into Newton's Law to get

$$\mathbf{M}_r s = \bar{\phi}_r^T \bar{F}$$

# But, Where is the Inverse?

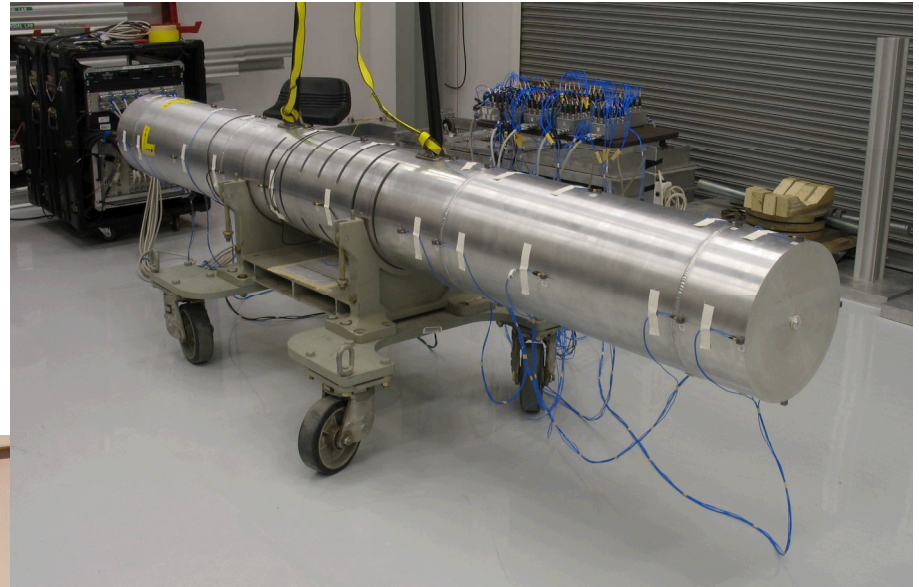
- The solution for the weight vectors contains the inverse when we defined  $\bar{\mathbf{w}}$  to solve  $\bar{\mathbf{w}}^T \bar{\phi}_i = 0$  &  $\bar{\mathbf{w}}^T \bar{\phi}_r = 1$

$$\mathbf{w}^T \Phi = \bar{\delta}$$

$$\mathbf{w}^T = \bar{\delta} \Phi^{-1}$$

- The Delta Function is actually a modal filter. In this case we are filtering out the elastic motion and keeping the rigid body motion
- Solution gives the sum of the forces at the center of gravity and no spatial distribution. This is acceptable if we know or can assume the locations of the external forces on the system

# Application: Test Article



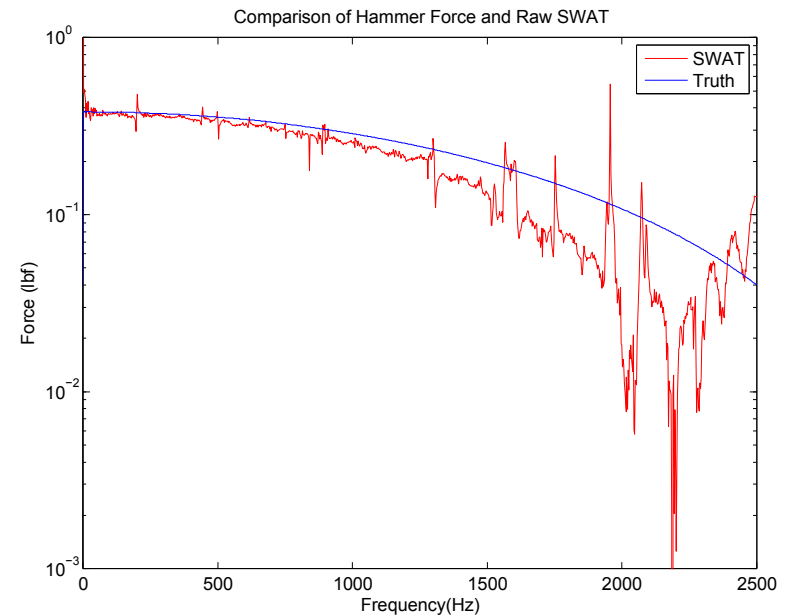
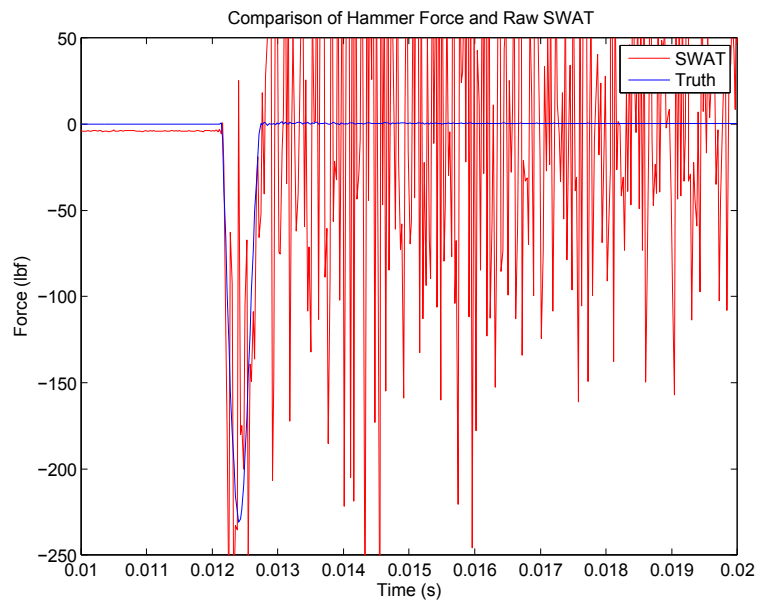


# Laboratory Test

- Function #1: Measure and calculate mode shapes
  - Not all mode shapes are of equal importance and a subset can be used at the analyst's discretion
  - If the force is primarily in the axis of the cylinder, then only the axial modes are necessary under most circumstances
- Function #2: Test your mode shapes and ability of SWAT
  - Because performing a modal test requires a measured force and measured responses, the measured responses can be used to recalculate the measured force using SWAT and a check on its performance

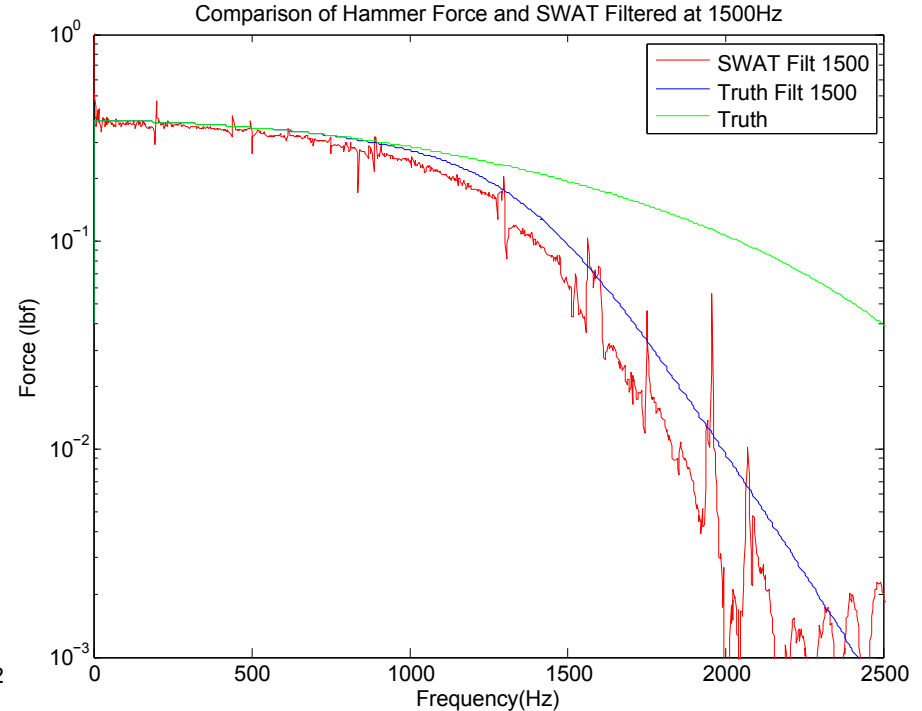
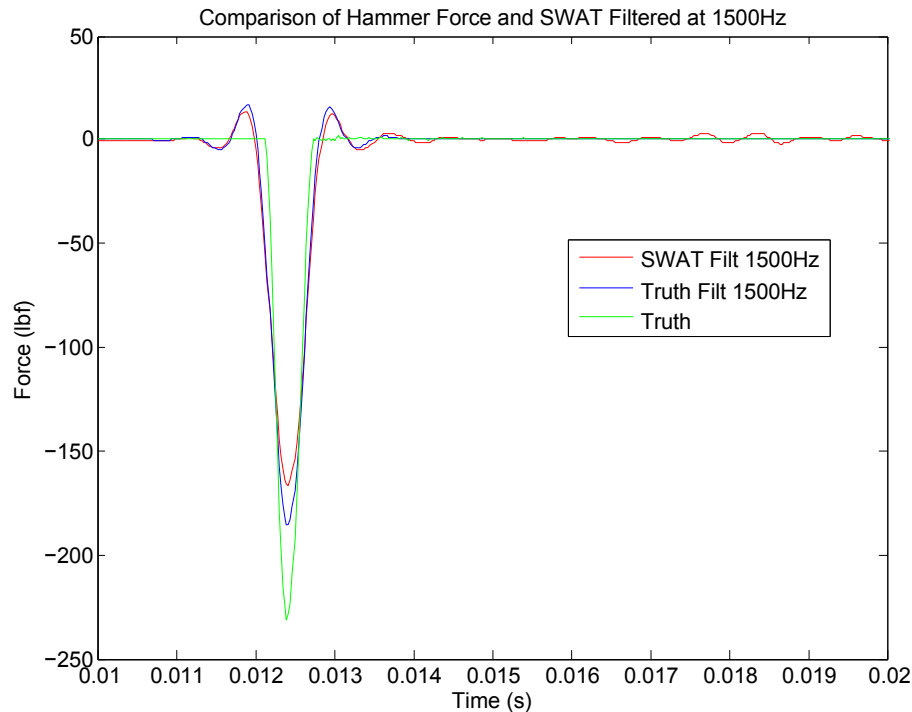
# Laboratory Test

- Force was reconstructed using mode shapes up to 1900 Hz
- Some high frequency modes were not filtered well



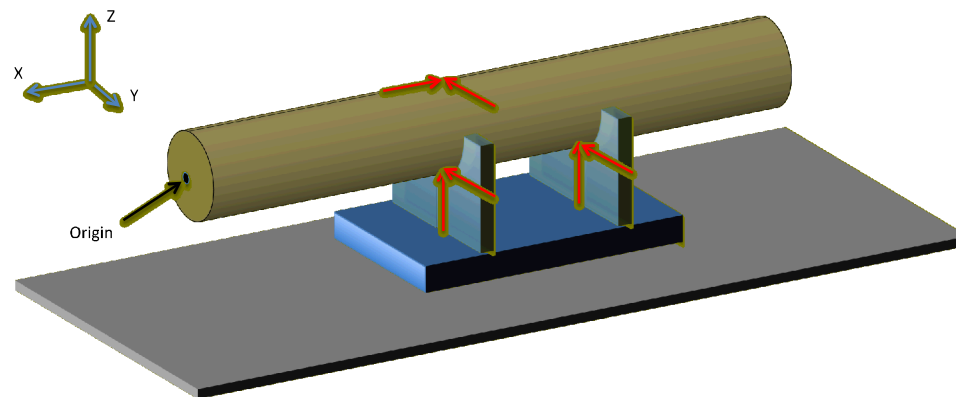
# Laboratory Test

- Used 1500 Hz filter to eliminate high frequency response
- Necessary to eliminate unfiltered mode shapes. Frequency limit was identified for the road test



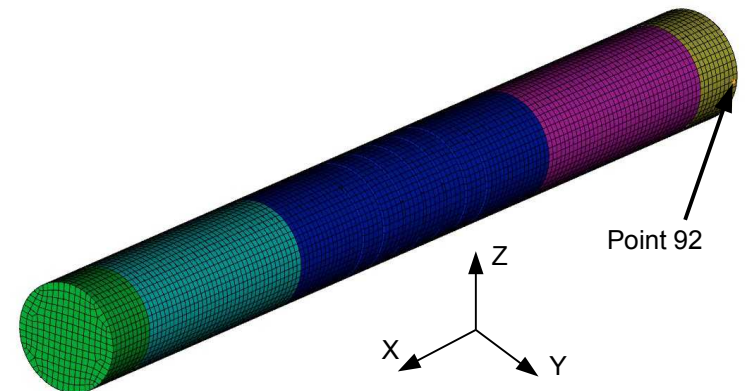
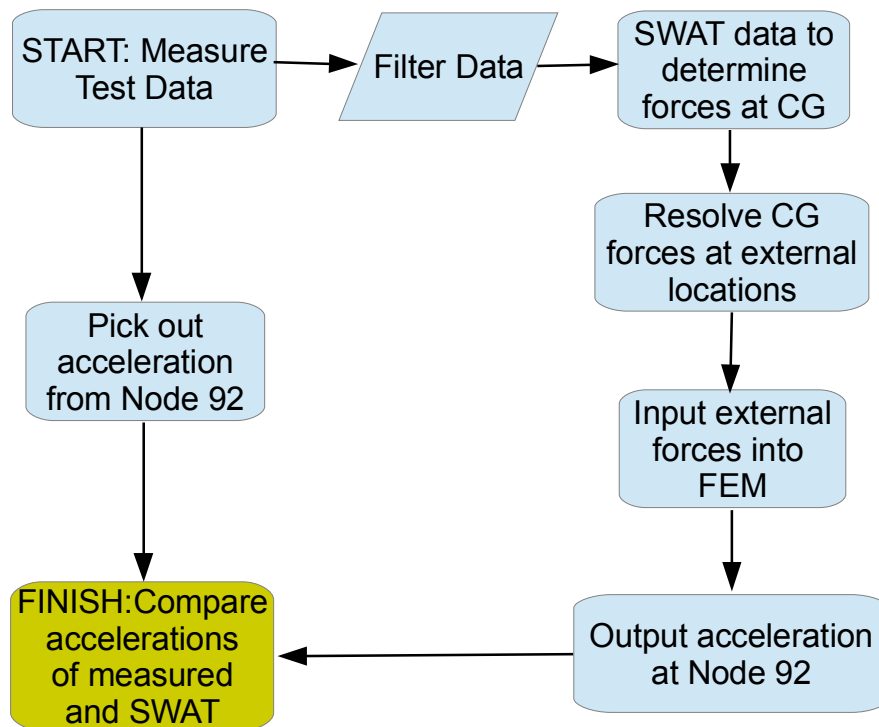
# Field Test

- Test article was driven around in/on several vehicles and secured by a bolster and the location of the forces were assumed



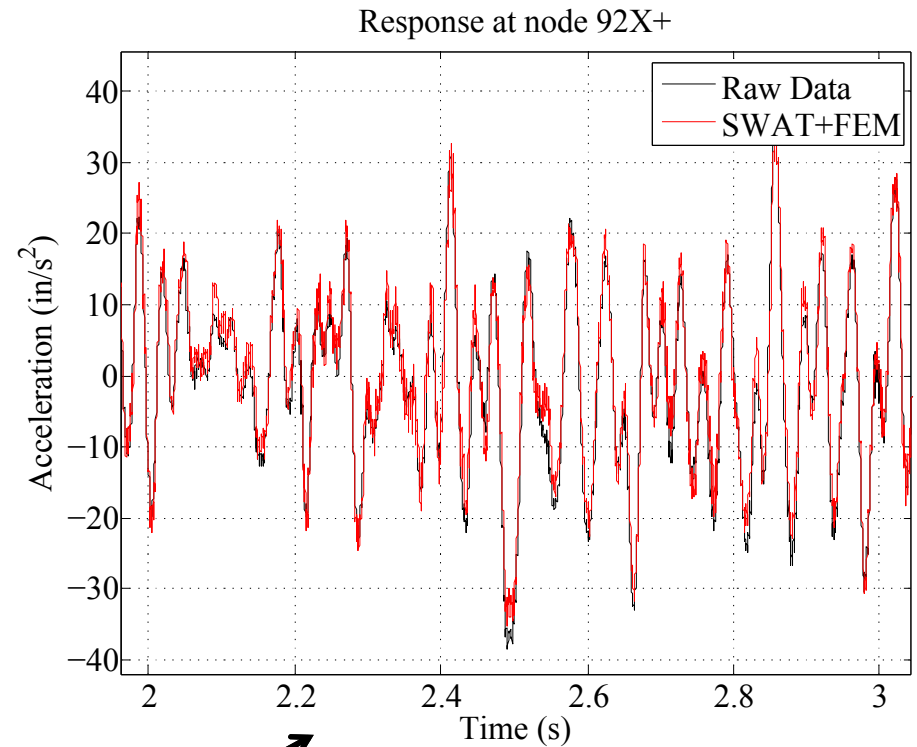
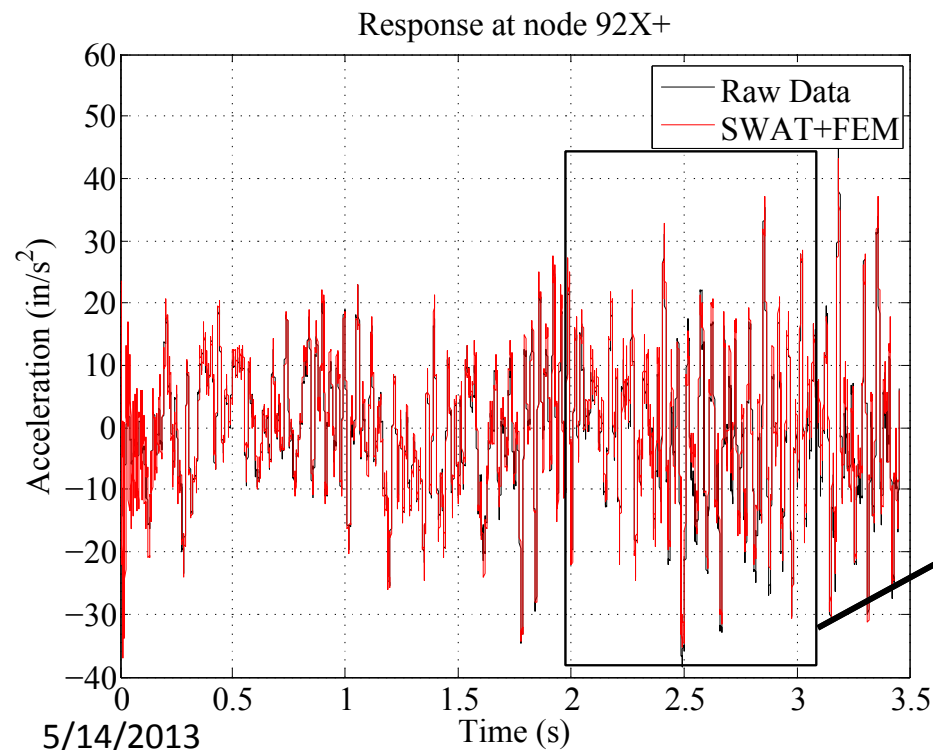
# Did it Work?

- A method of determining the effectiveness of SWAT filtered at 1500 Hz compares calculated acceleration and measured acceleration



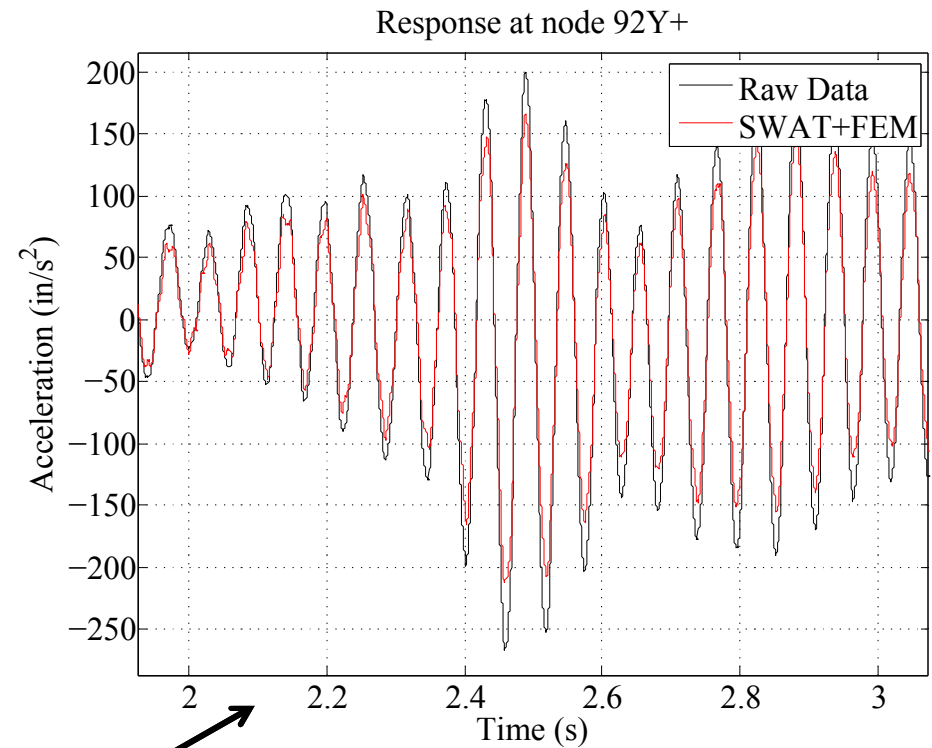
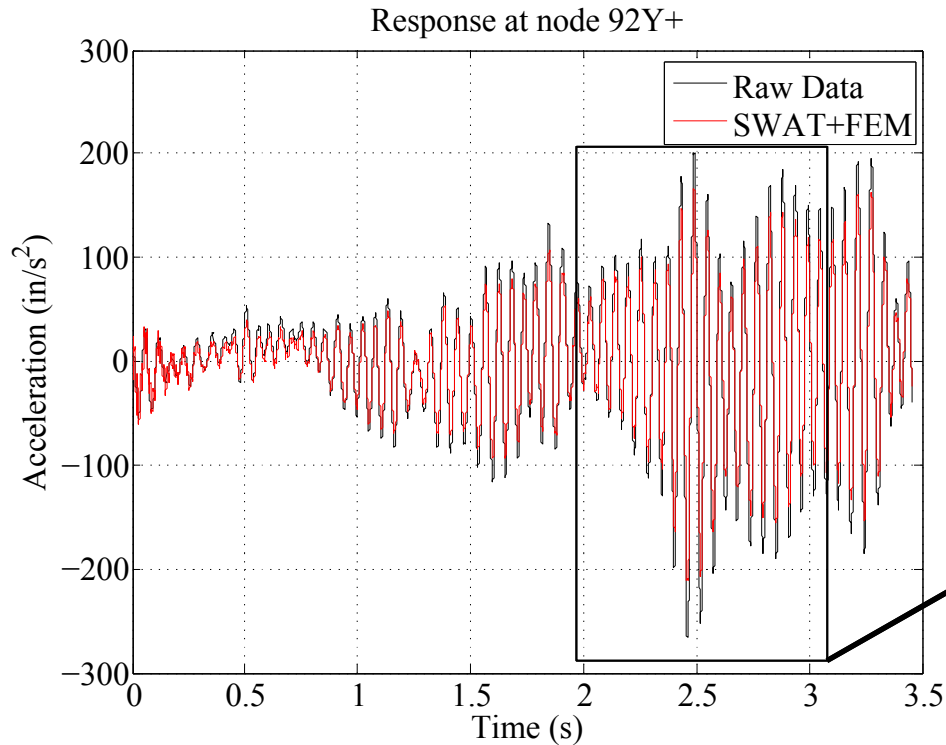
# Results

- Calculated 3.5 seconds of force data
- X-dir Performed the best in lab test



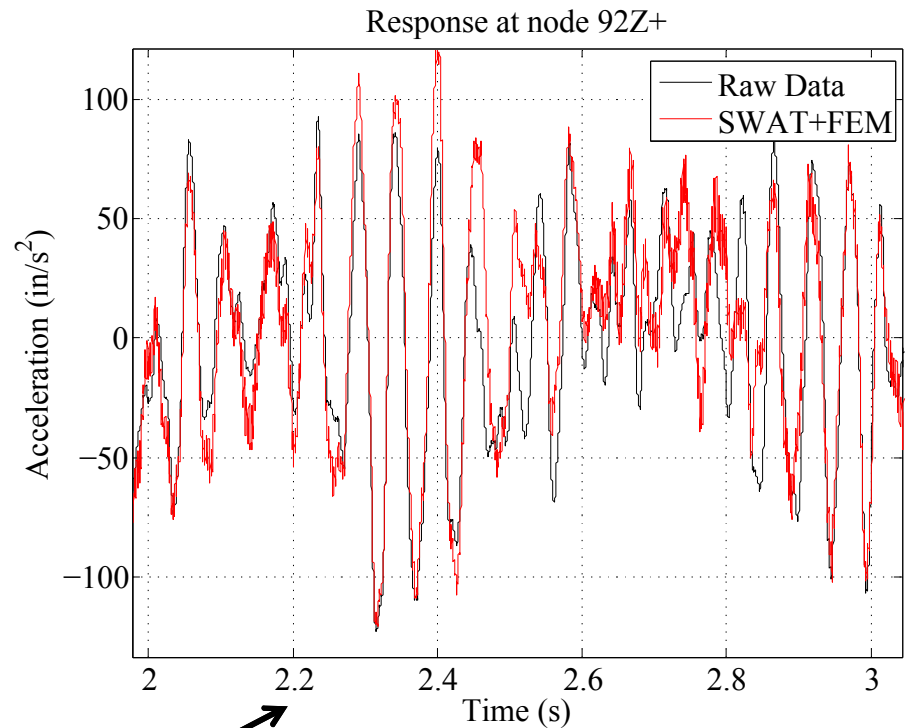
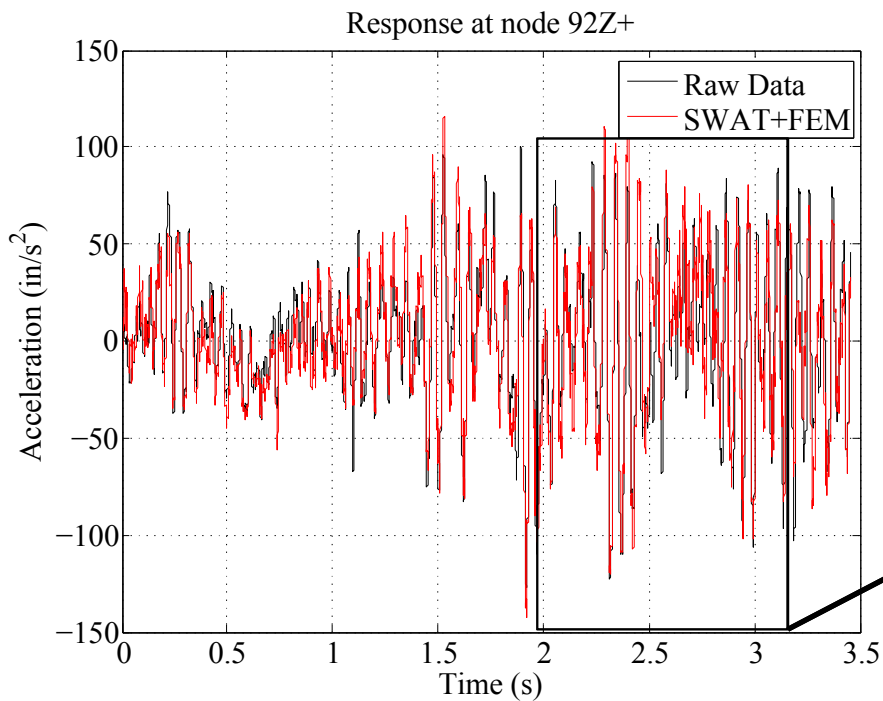
# Results

- Calculated 3.5 seconds of force data



# Results

- Calculated 3.5 seconds of force data



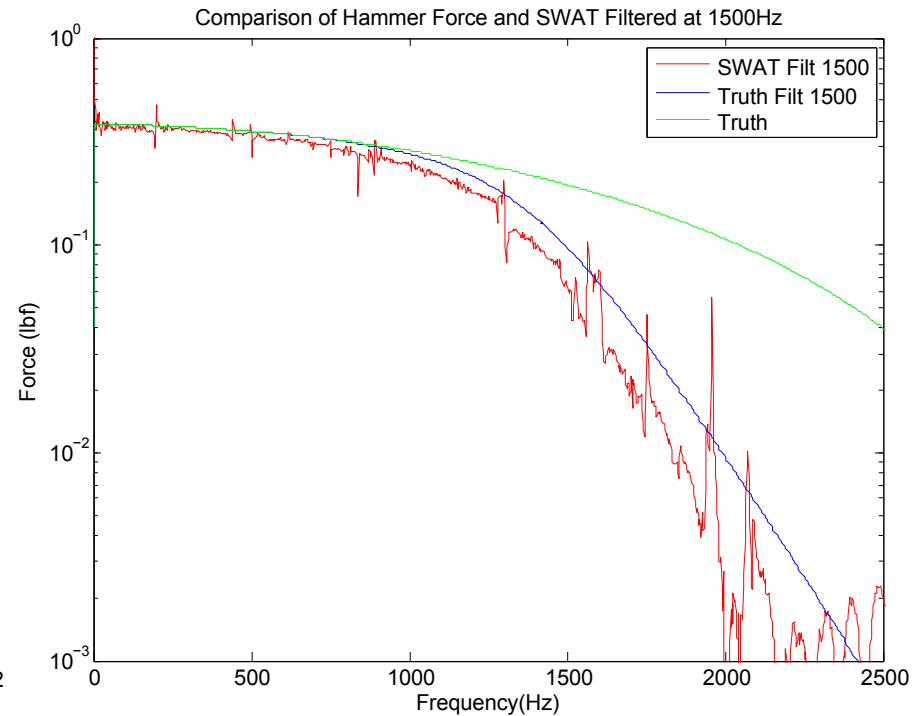
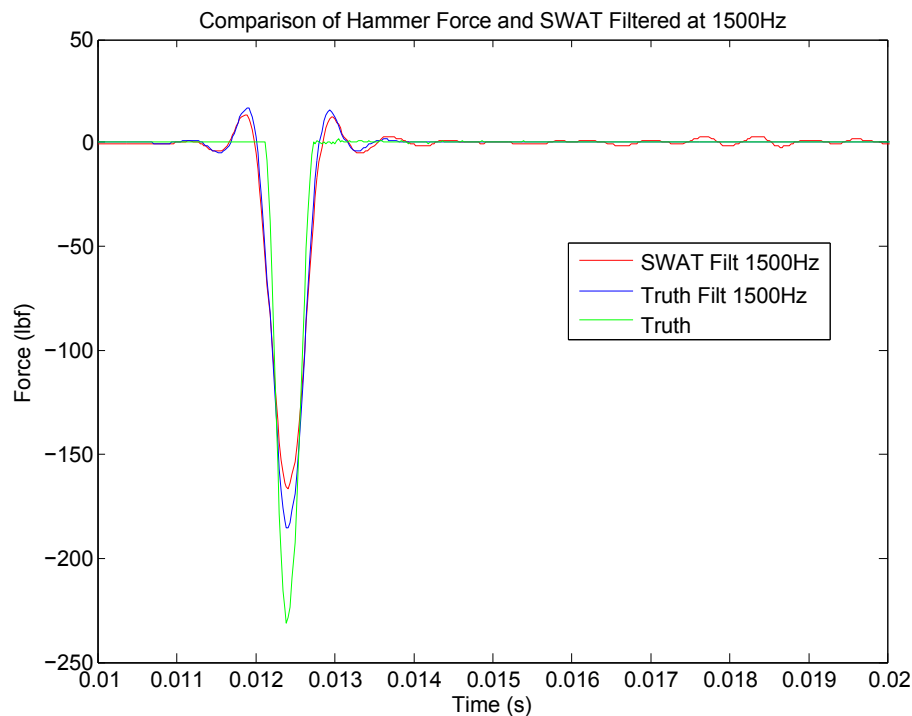


# Conclusions

- The inverse problem is controlled by using a well conditioned, robust matrix such as mode shapes or time domain responses
- No spatial representation is given. The location of the applied forces must be known or assumed.
- SWAT works on a 6 dof problem when the location and profile of the incoming forces are not obvious.
- There is room for improvement and expanding applications.

# SWAT-TEEM Laboratory Test

- TEEM – Time Eliminated Elastic Motion
- Takes the decayed response (sum of the mode shapes) from a free free unit and filters the motion to get the rigid body forces.



# SWAT-TEEM Laboratory Test

- Low level accelerometer noise an issue at higher frequencies.

