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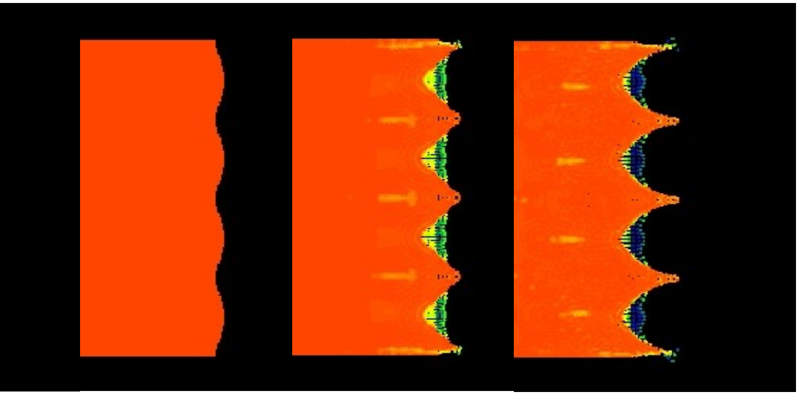

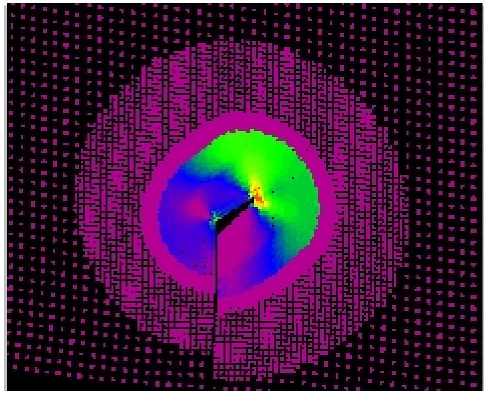

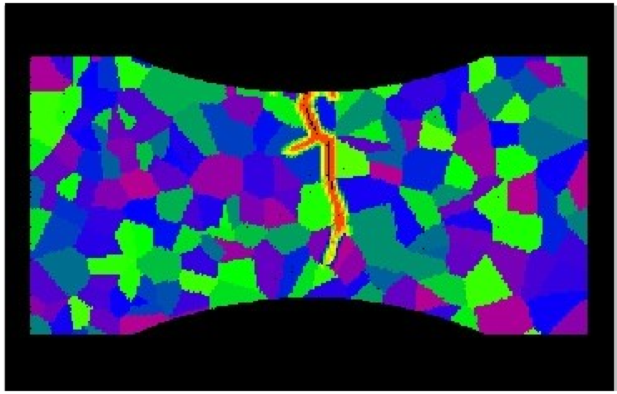

Variable length scale in a peridynamic continuum

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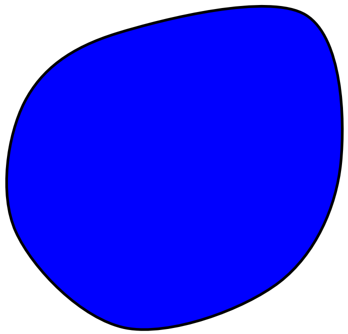
Background

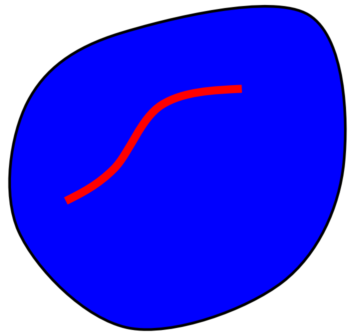
- Long-term goal: local-nonlocal coupling.
- The peridynamic (nonlocal) equations reduce to the Cauchy (local) model when the length scale approaches zero.
- So, let's try to obtain a local-nonlocal coupling method within the peridynamic continuum equations.
  - The method is required to pass a "continuum patch test" (to be described).
  - Seek to reduce artifacts at a local-nonlocal interface.

Purpose of peridynamics

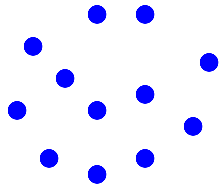
- To unify the mechanics of continuous and discontinuous media within a single, consistent set of equations.



Continuous body



Continuous body
with a defect



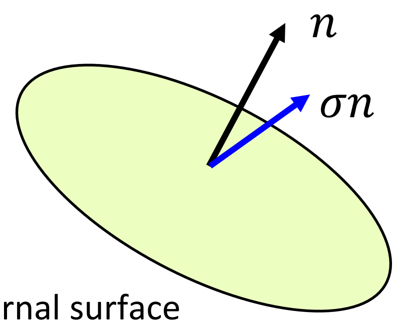
Discrete particles

- Why do this?
  - Avoid coupling dissimilar mathematical systems (A to C).
  - Model complex fracture patterns.
  - Communicate across length scales.

Peridynamics basics:
The nature of internal forces

Standard theory

Stress tensor field
(assumes contact forces and smooth deformation)

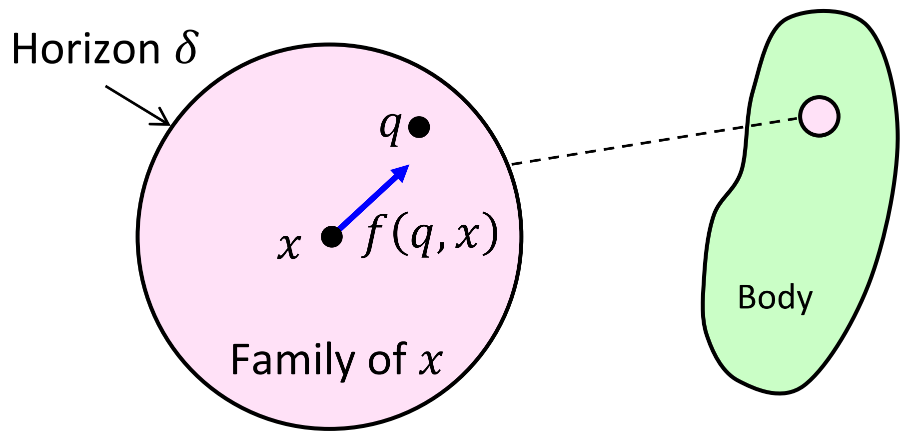


$$\rho \ddot{u}(x,t) = \nabla \cdot \sigma(x,t) + b(x,t)$$

Differentiation of contact forces

Peridynamics

Bond forces within small neighborhoods
(allow discontinuity)



$$\rho \ddot{u}(x,t) = \int_{H_x} f(q,x) dV_q + b(x,t)$$

Summation over bond forces

Peridynamics basics: Force state

- $\mathbf{f}(\mathbf{x}, \mathbf{q})$ has contributions from the material models at both $\mathbf{x}$ and $\mathbf{q}$.

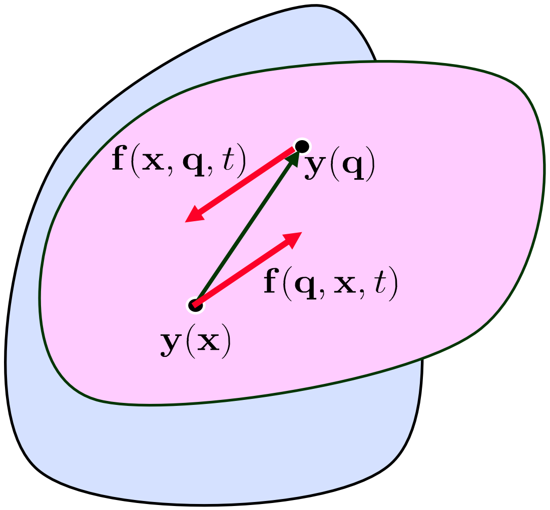
$$\mathbf{f}(\mathbf{x}, \mathbf{q}) = \mathbf{t}(\mathbf{x}, \mathbf{q}) - \mathbf{t}(\mathbf{q}, \mathbf{x})$$

$$\mathbf{t}(\mathbf{x}, \mathbf{q}) = \underline{\mathbf{T}}[\mathbf{x}]\langle \mathbf{q} - \mathbf{x} \rangle, \qquad \mathbf{t}(\mathbf{x}, \mathbf{q}) = \underline{\mathbf{T}}[\mathbf{q}]\langle \mathbf{x} - \mathbf{q} \rangle$$

- $\underline{\mathbf{T}}[\mathbf{x}]$ is the *force state*: maps bonds onto bond force densities. It is found from the constitutive model:

$$\underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$$

where $\hat{\underline{\mathbf{T}}}$ maps the deformation state to the force state.



Peridynamic vs. local equations

State notation: $\underline{\text{State}}\langle\text{bond}\rangle = \text{vector}$

Relation	*Peridynamic theory*	*Standard theory*
Kinematics	$\underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \frac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \int_{\mathcal{H}} \Big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\Big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative)	$\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient)
First law	$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} + q + r$	$\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$

$$\underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle \cdot \dot{\underline{\mathbf{Y}}}\langle\boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$

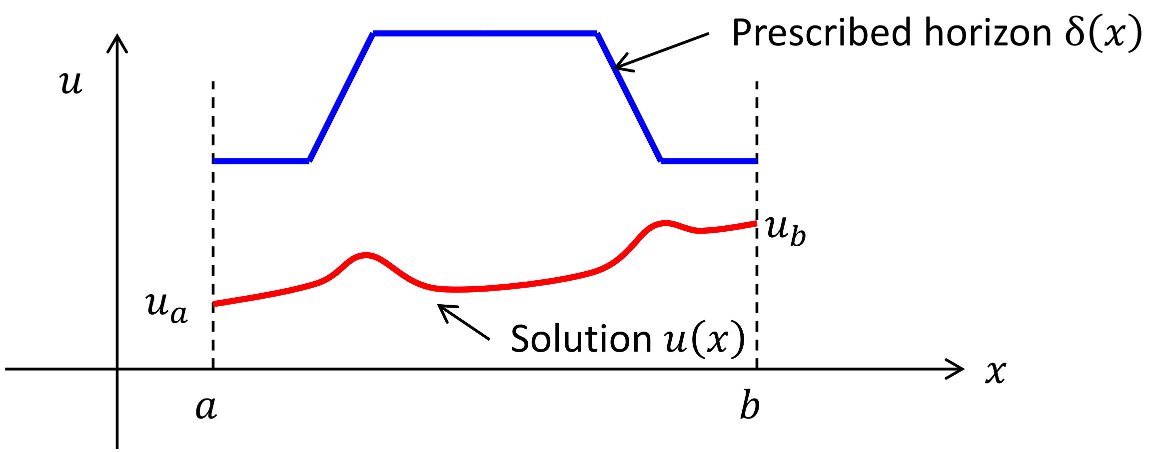
Mixed blessing of nonlocality

- Nonlocality is necessary to achieve the goals of peridynamics , but it entails some practical difficulties.
- Example: nonuniform horizon in a bar with “homogeneous bulk properties”.
- We know how to scale a material model so the Young’s modulus is independent of horizon.

$$T_\delta\langle\xi\rangle = \delta^{-2} Z\langle\xi/\delta\rangle$$

where Z is a reference force state that depends only on strain.

- But when you use this to model equilibrium of a bar with variable horizon, you get a “wrong” result:

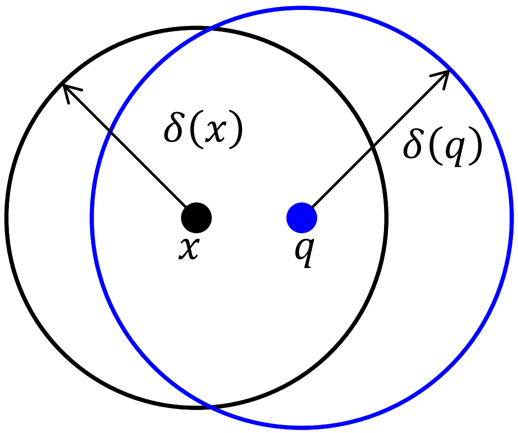


Origin of artifacts

- The peridynamic force density operator $L(x)$ involves the force state not only at x but also the force states at all points within the horizon.

$$0 = L(x) + b, \qquad L(x) = \int_{-\infty}^{\infty} \{T_{\delta(x)}[x]\langle q - x\rangle - T_{\delta(q)}[q]\langle x - q\rangle\} dq$$

so simply scaling the material model at x is not sufficient.



“Patch test” requirement for a coupling method

- In a deformation of the form

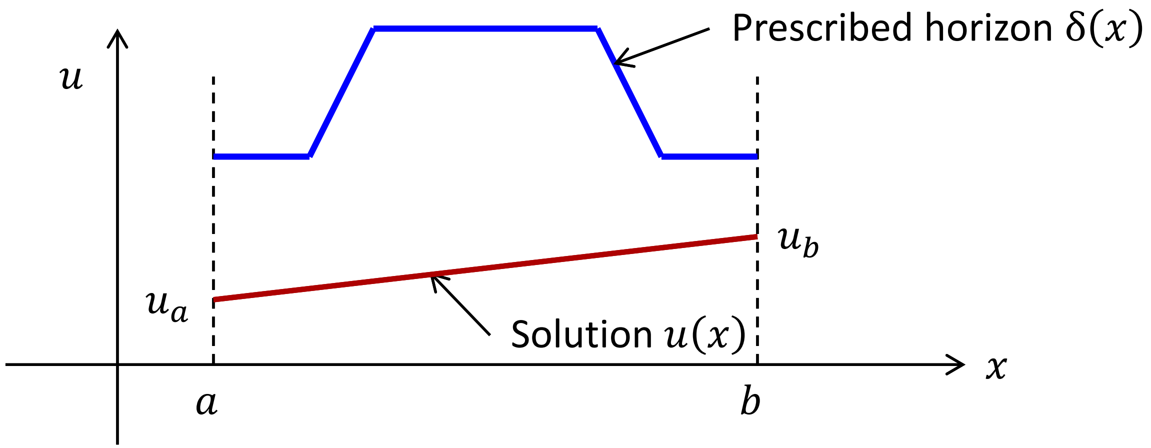
$$u(x) = a + Hx$$

where H is a constant and the material model is of the form

$$T[x]\langle\xi\rangle = \delta^{-2}(x) Z\langle\xi/\delta(x)\rangle$$

where $\delta(x)$ is a prescribed function and Z is a state that depends only on H, we require

$$L(x) = 0 \quad \text{for all } x.$$



Peridynamic stress tensor

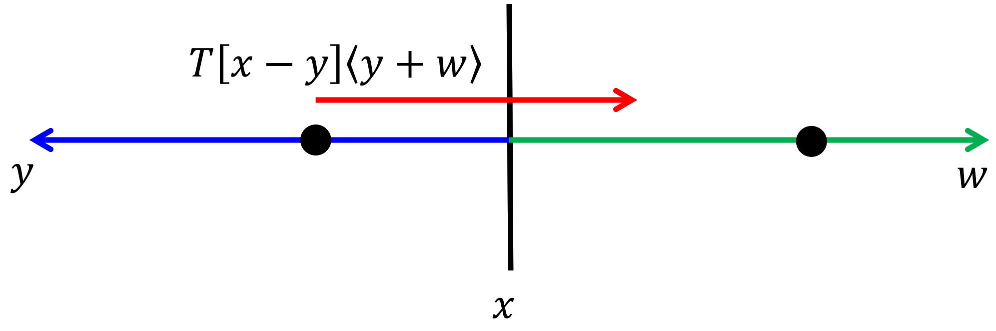
- Define the peridynamic stress tensor field by

$$v(x) = \int_0^\infty \int_0^\infty \{T[x-y]\langle y+w\rangle - T[x+y]\langle -y-w\rangle\}\, dy\, dz$$

- Identity:

$$\frac{dv}{dx} = \int_{-\infty}^\infty \{T[x]\langle q-x\rangle - T[q]\langle x-q\rangle\} dq$$

- $v(x)$ is the force per unit area carried by all the bonds that cross x.



Peridynamic stress tensor: special case

- Under our assumption that

$$T[x]\langle\xi\rangle = \delta^{-2}(x) Z\langle\xi/\delta(x)\rangle$$

one finds directly that

$$\upsilon(x) = \int_{-\infty}^{\infty} \xi T\langle\xi\rangle \, d\xi = \int_{-\infty}^{\infty} \xi Z\langle\xi\rangle \, d\xi$$

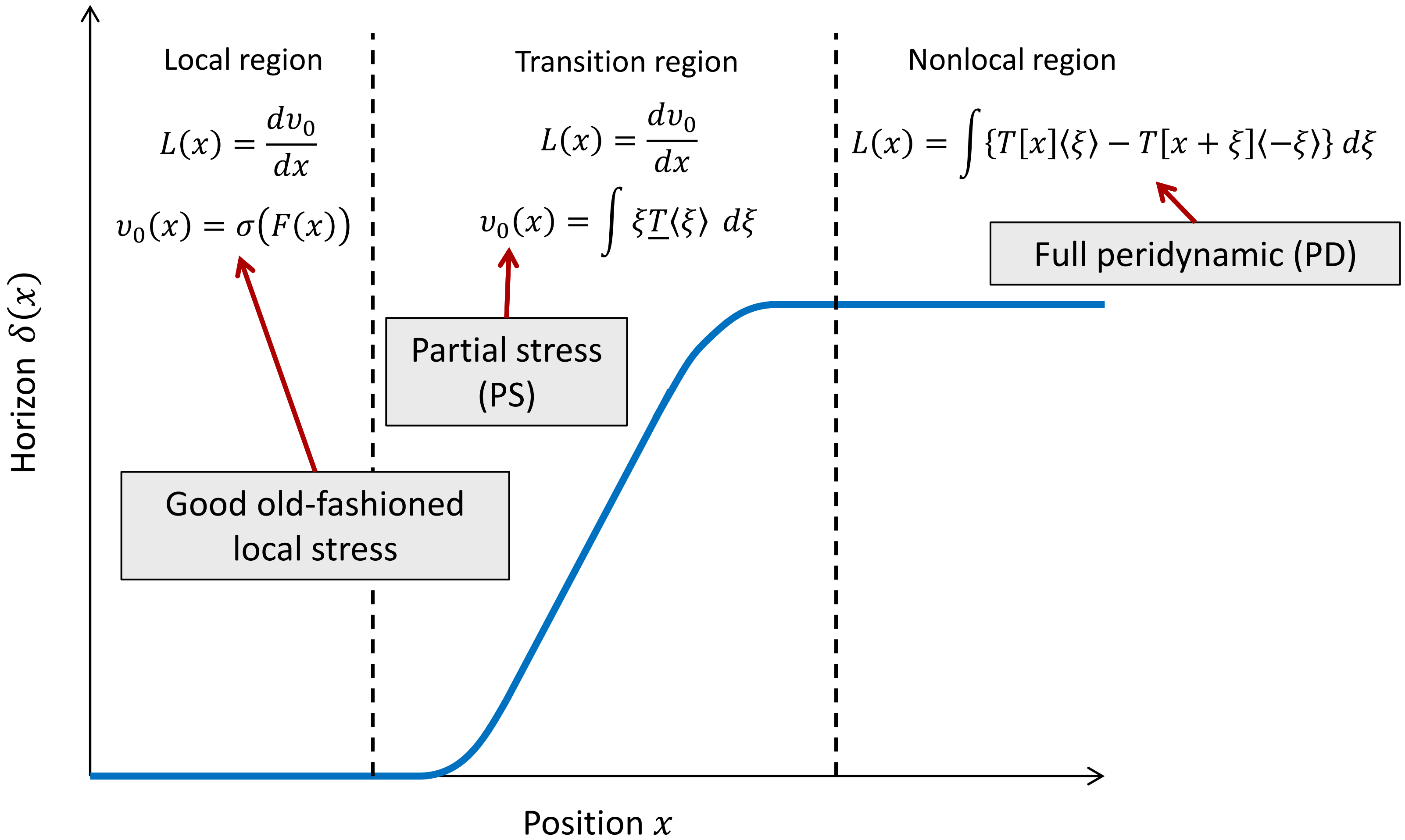
independent of x, so $d\upsilon/dx = 0$.

- Idea: within a coupling region in which δ is changing, compute the force density from

$$L(x) = \frac{d\upsilon_0}{dx}(x), \qquad \upsilon_0(x) = \int_{-\infty}^{\infty} \xi T\langle\xi\rangle \, d\xi$$

- Here, $T(x)$ is determined from whatever the deformation happens to be near x.
- Clearly this L passes the "patch test."
- υ_0 is called the ***partial stress*** field.

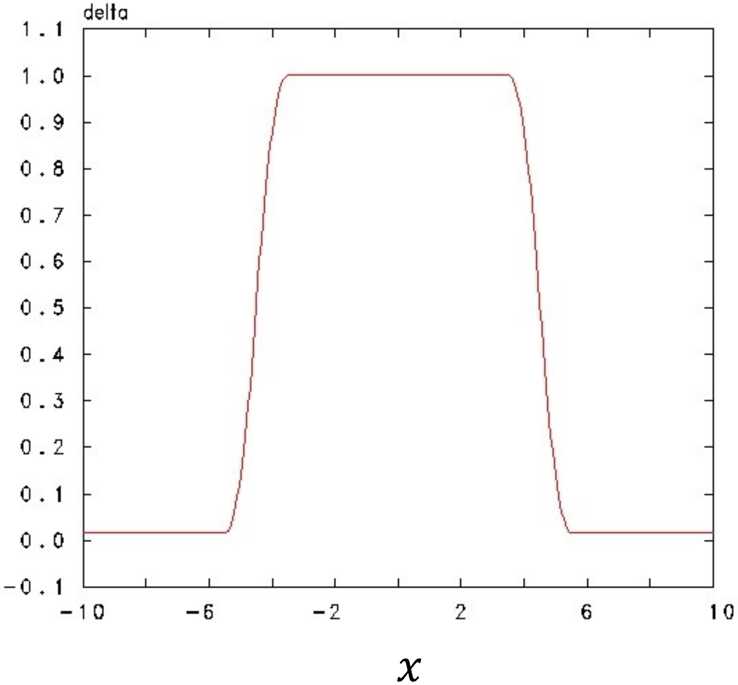
Local-nonlocal coupling idea



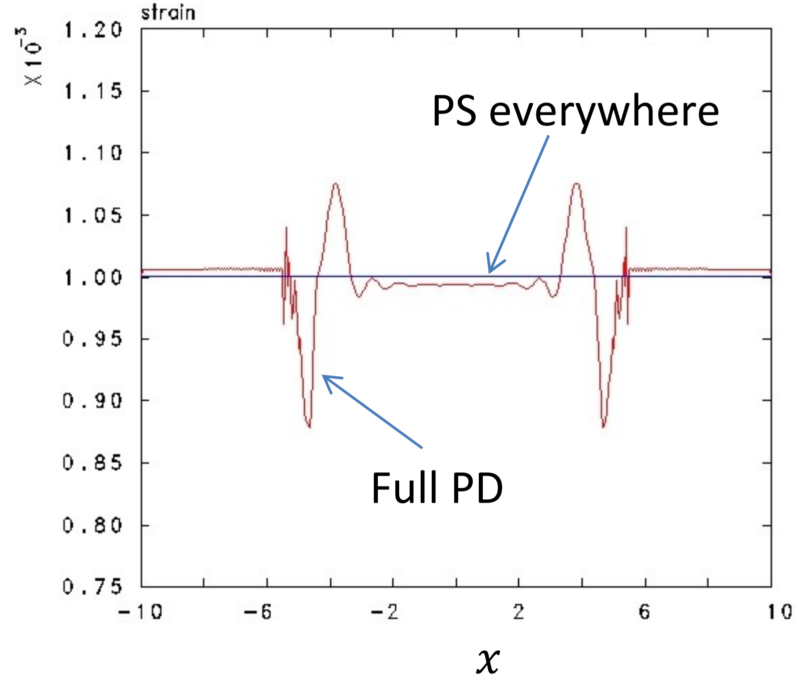
Continuum patch test results

- Full PD shows artifacts, as expected.

Horizon

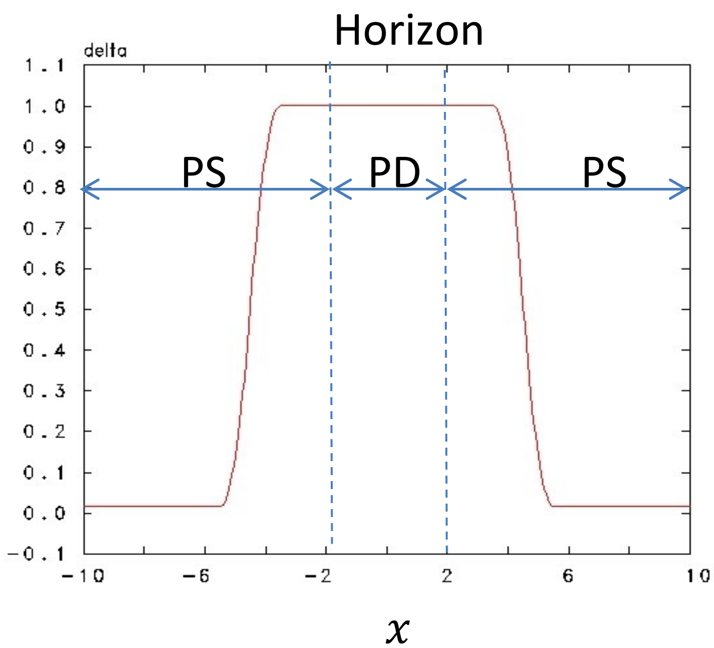


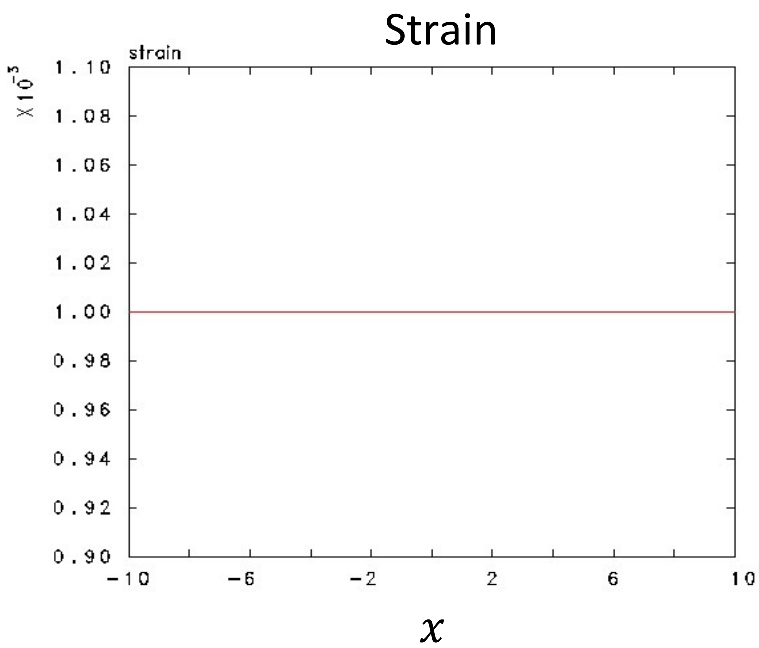
Strain



Continuum patch test with coupling

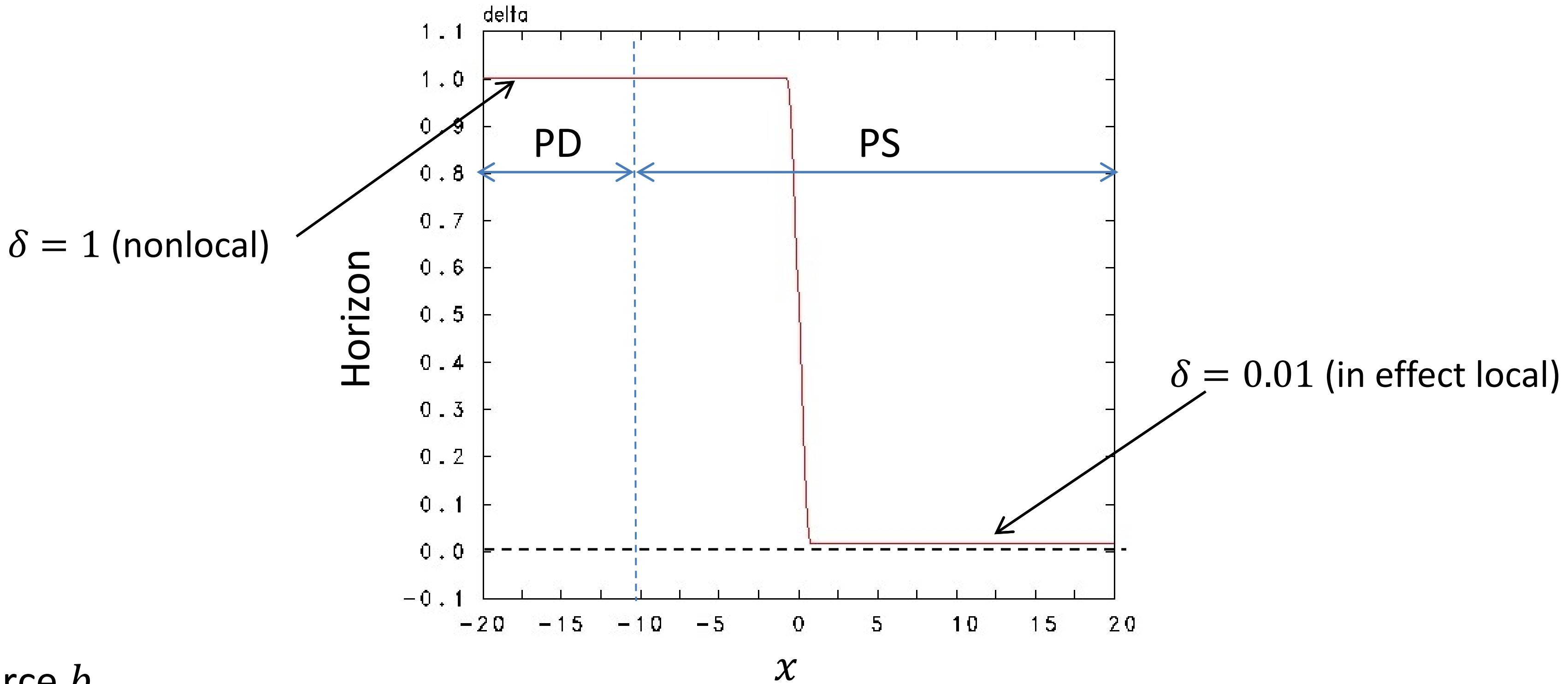
- No artifacts.





$u = 0$ $\qquad$ $u = 0.02$

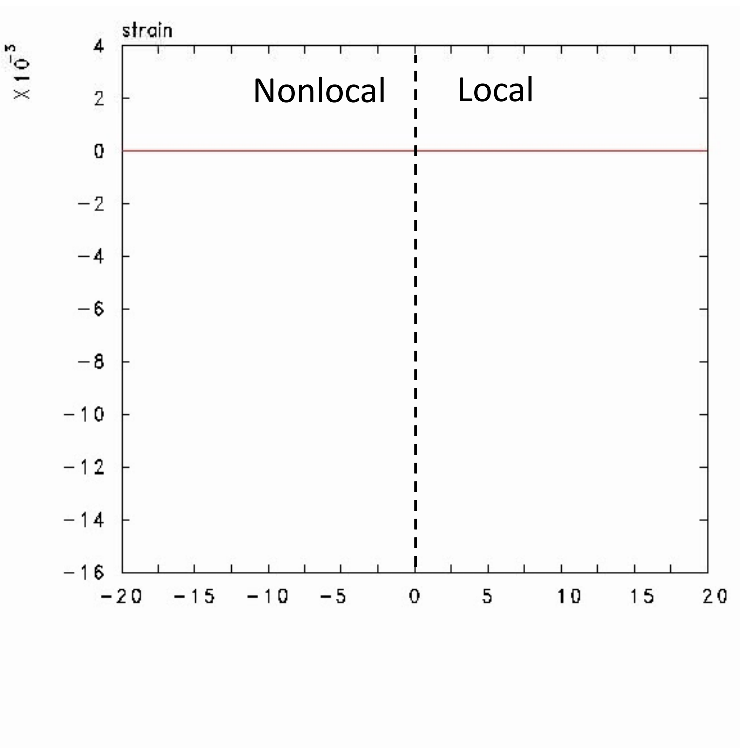
Pulse propagation test problem



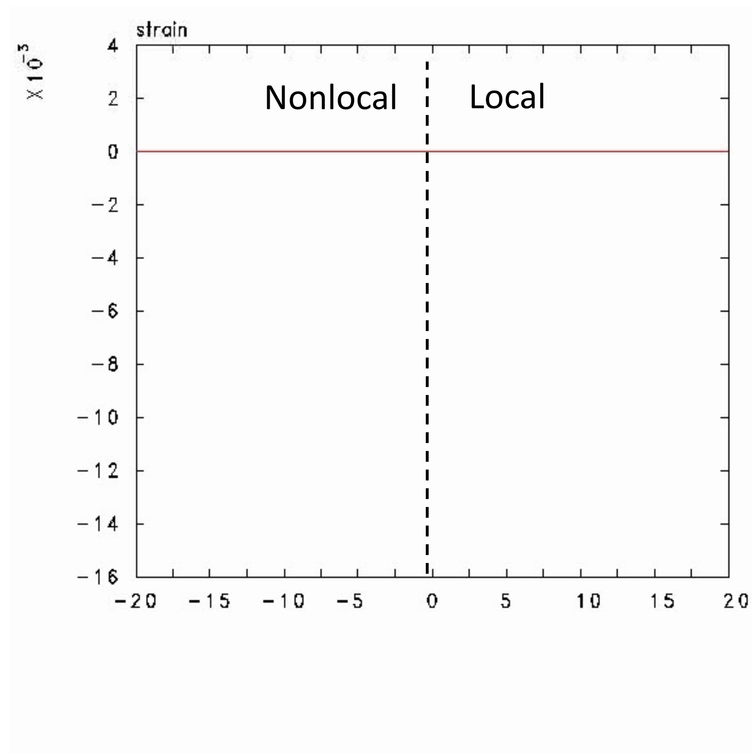
Pulse propagation test results

- Movies of strain field evolution

Full PD everywhere



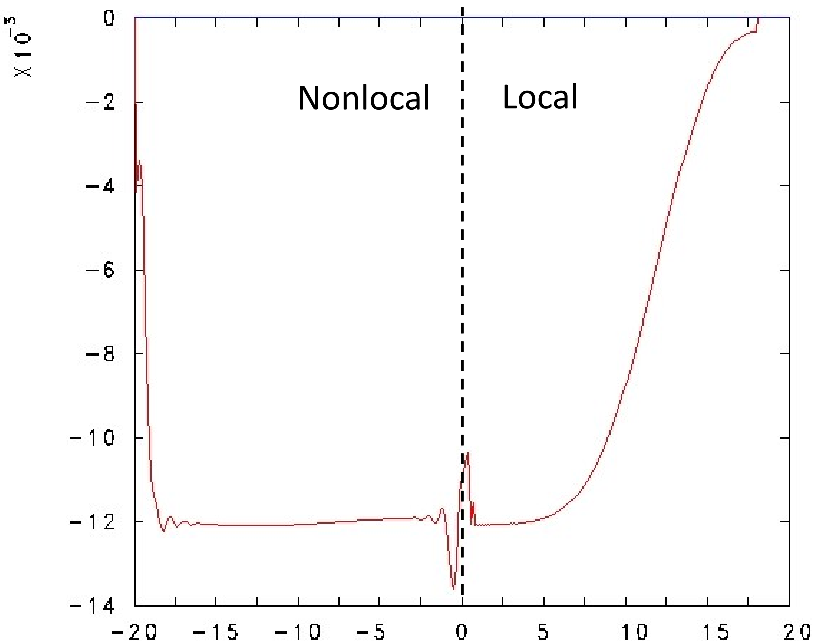
Coupled PD-PS



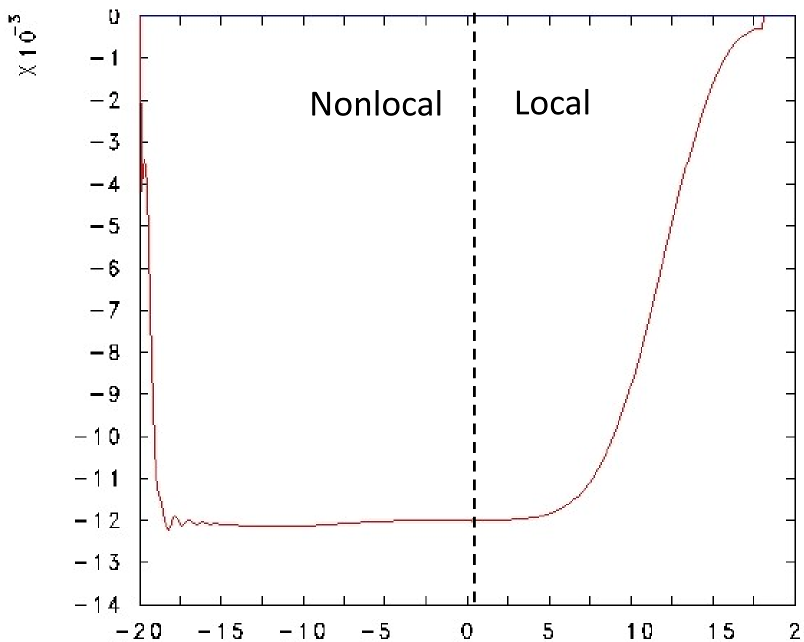
Pulse propagation test results

- No artifacts in the coupled model the local-nonlocal transition.

Full PD everywhere



Coupled PD-PS



Discussion

- The partial stress approach may provide a means for local-nonlocal coupling within the continuum equations.
- Similar to Virtual Internal Bond method (Gao & Klein, JMPS, 1998).
- PS is inconsistent from an energy minimization point of view.
  - Not suitable for a full-blown theory of mechanics (as PD is).
  - Minimizing the total energy with a nonlocal material model results in full PD expression for momentum balance, not PS.
  - Not yet clear what implications this may have in practice.
  - Use full PD for crack progression.