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Algebraic Calculation of ERB Dilution, Capture, Retardation, Decay, and Dose

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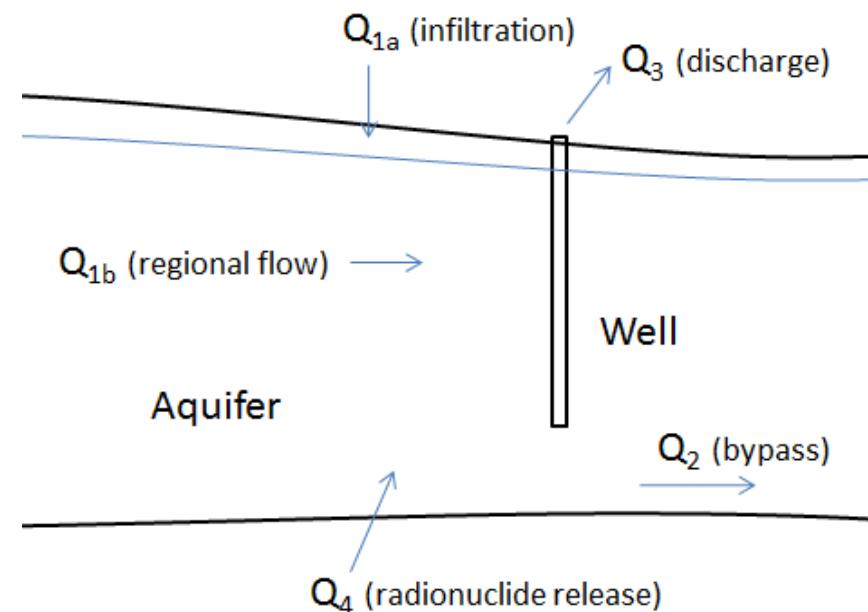
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Outline

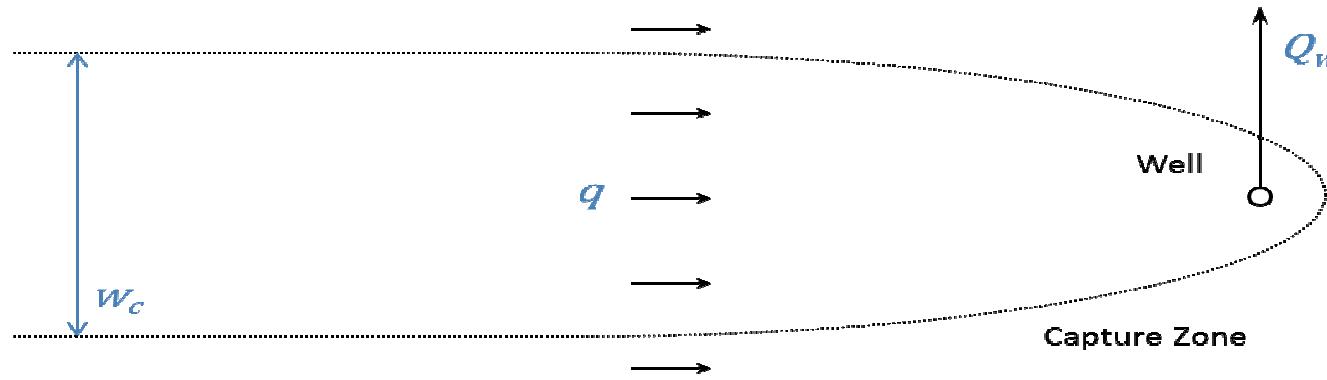
- Review IAEA Example Reference Biosphere (ERB) models
 - Involving groundwater wells (primary exposure pathways)
 - Limitations
 - Must know dilution *a priori*
 - Assume no travel time in aquifer (no sorption, no radioactive decay)
- Show that dilution can be estimated algebraically
 - From the geometries of the plume and well capture zone
- Show that travel time in aquifer can be important
 - Especially for highly sorbed radionuclides
- Present several analytical solutions
 - For different types of releases into a generic aquifer
 - Advective/diffusive
 - Point source/line source/area source

ERB Models

- Example Reference Biospheres (ERBs)
 - Developed in *"Reference Biospheres" for Solid Radioactive Waste Disposal*, IAEA-BIOMASS-6, IAEA (2003)
- ERB 1A and ERB 1B
 - calculate dose rate ($H_{E,i}$) resulting from the concentration of radionuclide i in a well ($C_{3,i}$)
 - $$H_{E,i} = C_{3,i} * I * dcf_i$$
 - I = individual consumption rate
 - dcf_i = ingestion dose coefficient
- For ERB 1B, dilution in the aquifer and well should not be arbitrarily chosen
 - Can vary from 1 to 10^6 or more
 - Dose rate is highly sensitive to the chosen dilution



Capture Zone

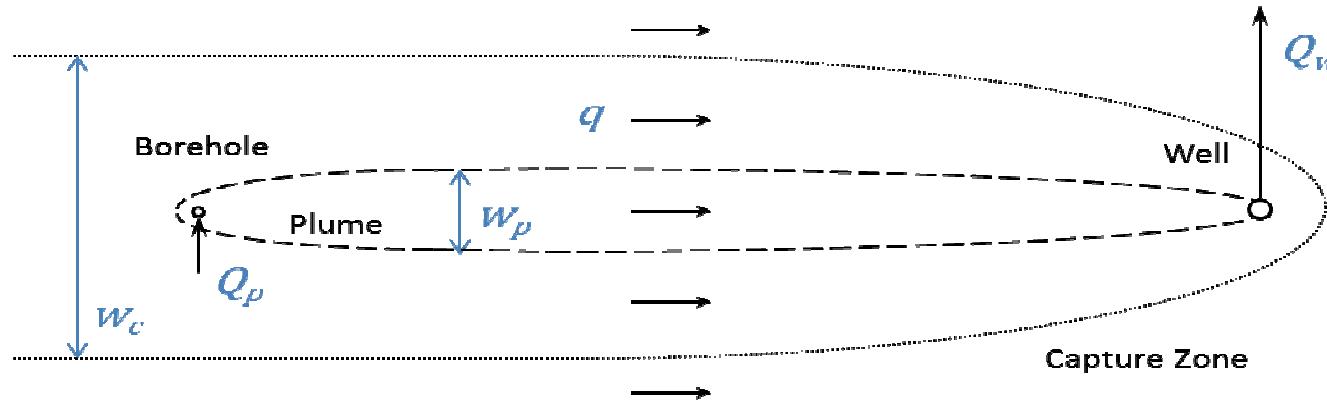


- Analytical solution for generic model aquifer
 - Model aquifer conditions
 - Uniform saturated thickness b [m]
 - Uniform regional specific discharge (darcy velocity) q [m yr^{-1}]
 - Fully penetrating, fully screened well
 - Width of the well capture zone w [m] is

$$w = \frac{Q_w}{bq} - \frac{Q_w}{\pi bq} \tan^{-1} \frac{w}{2x}$$
 (Javandel and Tsang 1986, *Ground Water*, 35(5) 842-847)
 - Maximum capture zone width w_c [m] (as distance x gets large) is

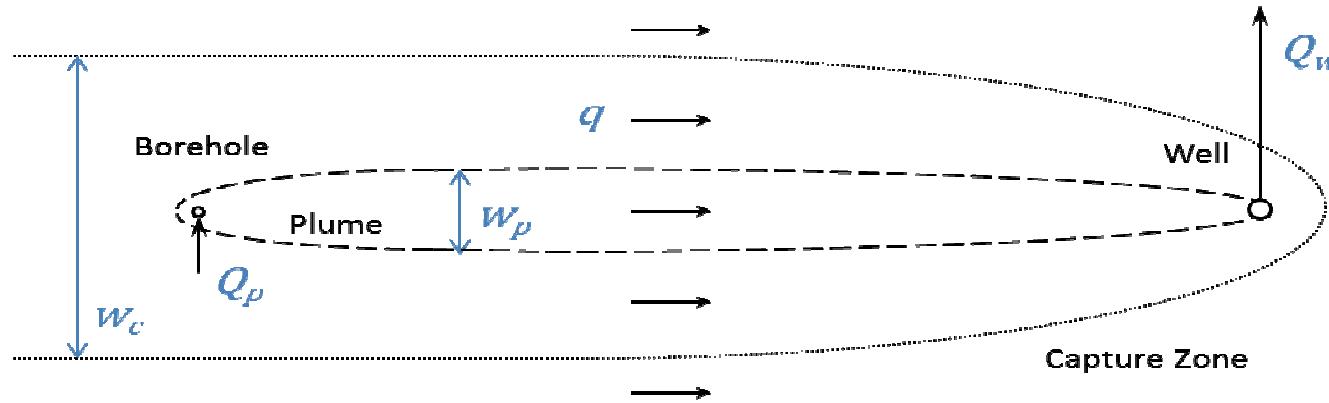
$$w_c = \frac{Q_w}{bq}$$

Plume Width and Dilution



- Advective release from a fully penetrating vertical borehole (VBA) (e.g., abandoned unsealed exploratory borehole)
 - Assume well is on same streamline as borehole
 - Assume large Peclet number in aquifer (negligible dispersion)
 - Negligible transverse dispersion maximizes capture at the well
 - Negligible longitudinal dispersion maximizes peak concentrations at well
 - Maximum plume width w_p [m] is:
$$w_p = \frac{Q_p}{bq}$$
 - Dilution:
$$D_{p,w} = \frac{w_c}{w_p} = \frac{Q_w}{Q_p} \quad \text{if } Q_w < Q_p$$

Plume Capture



- Fraction of plume captured, vertical borehole advective (VBA)

$$F_{vba} = \begin{cases} 1 & \text{if } Q_p \leq Q_w \\ \frac{w_c}{w_p} = \frac{Q_w}{Q_p} & \text{otherwise} \end{cases}$$

- Travel time in aquifer for radionuclide i

$$T_i = \frac{\ln R_{f,i}}{q}$$

L = distance btn borehole and well

n = porosity

$R_{f,i}$ = retardation factor

- Concentration at well of radionuclide i released from borehole at time $t - T_i$

$$C_{w,i}(t) = \frac{c_{p,i}(t-T_i)e^{-\lambda T_i}}{D_f(t-T_i)}$$

λ = decay constant

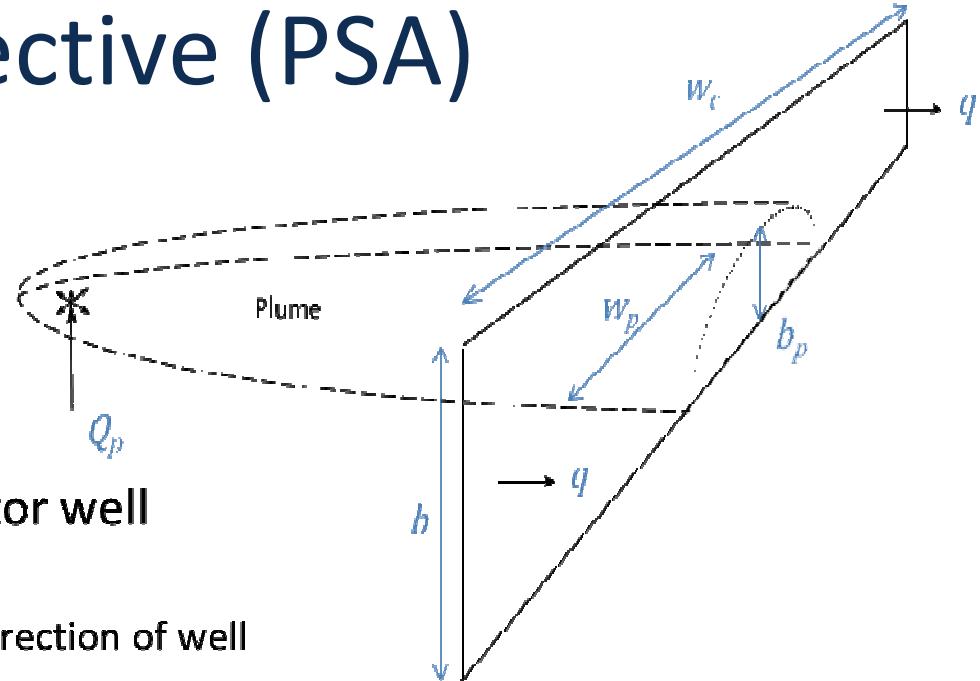
Point Source Advective (PSA)

- Point source release

- Advective (PSA)
- Top or bottom of aquifer
- E.g., from unsealed borehole, shaft, fracture zone, etc.

- Same generic aquifer and receptor well

- Constant saturated thickness
- Constant regional flow velocity in direction of well
- Well is fully penetrating, fully screened, and more than $3w$ down gradient
- Vertical anisotropy (a) important for point source release



$$a = \left(\frac{K_h}{K_v} \right)^{\frac{1}{2}}$$

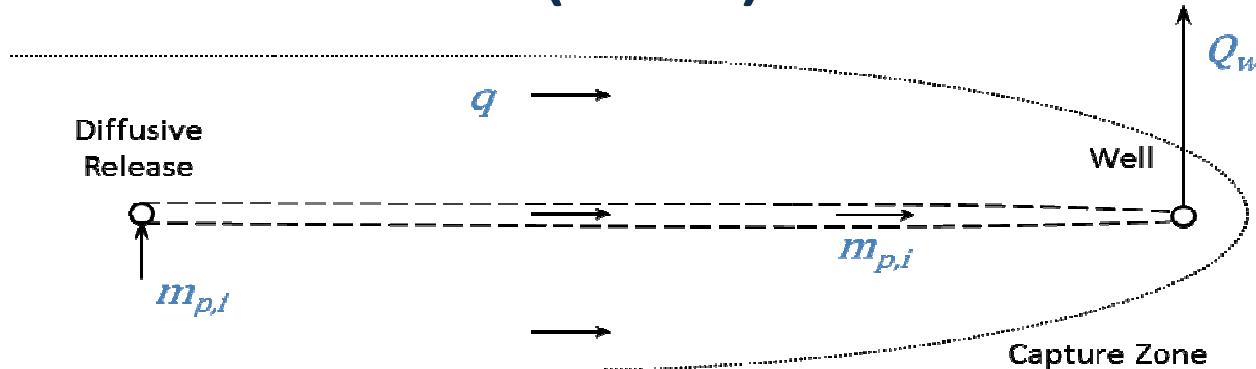
Plume cross-section $w_p = 2 \left(\frac{2Q_p a}{\pi q} \right)^{\frac{1}{2}}$ $b_p = \left(\frac{2Q_p}{\pi q a} \right)^{\frac{1}{2}}$ semi-ellipse, assuming $b_p < b$

- Dilution

$$D_{f,psa} = \frac{w_c b}{A_{psa}} \quad \text{where} \quad A_{psa} = \frac{\pi}{4} w_p b_p \quad \text{if } w_p \leq w_c$$

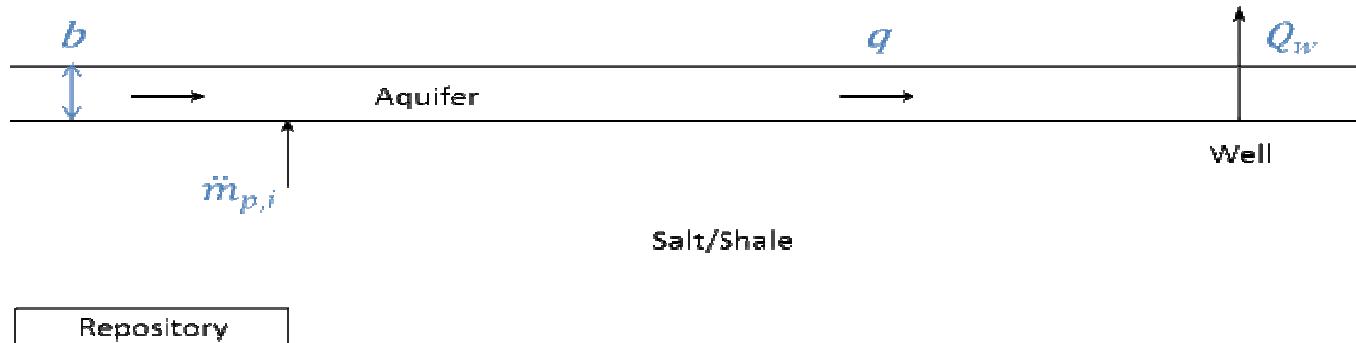
$$= 2 \int_0^{\frac{w_c}{2}} \left(\frac{b_p}{\frac{1}{\pi} w_p} \sqrt{\left(\frac{1}{2} w_p \right)^2 - y^2} \right) dy \quad \text{otherwise}$$

Diffusive Release (PSD)



- Point source diffusive (PSD) release (e.g., from sealed borehole or shaft)
- Same generic aquifer and fully-penetrating, fully-screened receptor well
 - Narrow plume width, so no bypassing and no minimum distance requirement
- Mass flux of radionuclide i : $m_{p,i}(t)$ [g yr $^{-1}$]
- Dilution occurs, but no dilution factor can be calculated
 - A diffusive flux is not volume-based
- Travel time, retardation (same): $T_i = \frac{\ln R_{f,i}}{q}$
- Concentration at well : $C_{w,i}(t) = \frac{m_{p,i}(t-T_i)e^{-\lambda T_i}}{Q_w}$
- Dose at well (same): $H_{E,i}(t) = C_{w,i}(t) * I * dcf_i$

Broad Diffusive (BD) Release



- Broad diffusive (BD) release (e.g., from low permeability formation)
- Same generic aquifer and fully-penetrating, fully-screened receptor well
- Plume width assumed to be larger than capture zone
- Mass flux to bottom (or top) of aquifer: $\ddot{m}_{p,i}(t)$ [g yr⁻¹ m⁻¹]
 - Units are mass flux per unit lateral width of aquifer perpendicular to flow
- No lateral dilution, only dilution from vertical mixing
 - Again, no dilution factor can be calculated
- Travel time, retardation in aquifer, calculated as before
- Concentration at well :
 - Not a function of well discharge
$$C_{w,i}(t) = \frac{\ddot{m}_{p,i}(t-T_i)e^{-\lambda T_i}}{qb}$$

Example Applications

- Example applications presented in paper
 - Vertical borehole advective (VBA)
 - Variable release rate
 - Point source advective (PSA)
 - Bypassing (plume width sometimes greater than capture zone)
 - Point source diffusive (PSD)
 - Broad diffusive (BD)
- Constants for each example
 - Aquifer and well properties
 - $q = 8 \text{ m yr}^{-1}$, $b = 15 \text{ m}$, $a^2 = 10$, $n = 0.4$, $L = 5 \text{ km}$, $Q_w = 10,000 \text{ m}^3 \text{ yr}^{-1}$
 - Travel time in aquifer, $T_i = 1,000 \text{ yr}$ (consistent with a K_d of 0.76 mL g^{-1})
 - Radionuclide half life = 1,000 yr
 - Implies 50% of radionuclide mass will decay during travel time

VBA Example

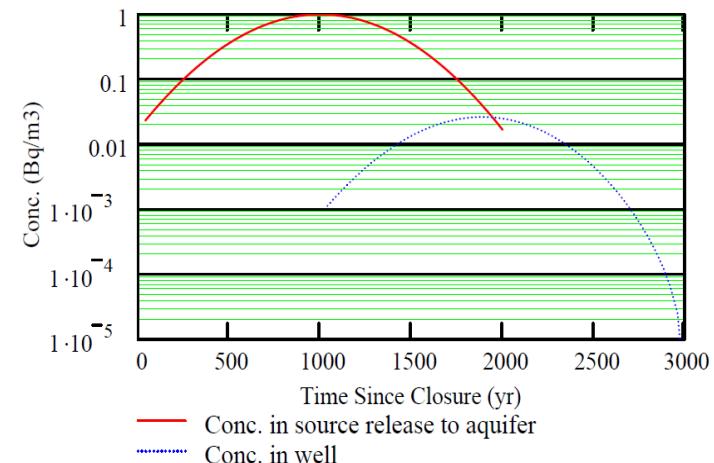
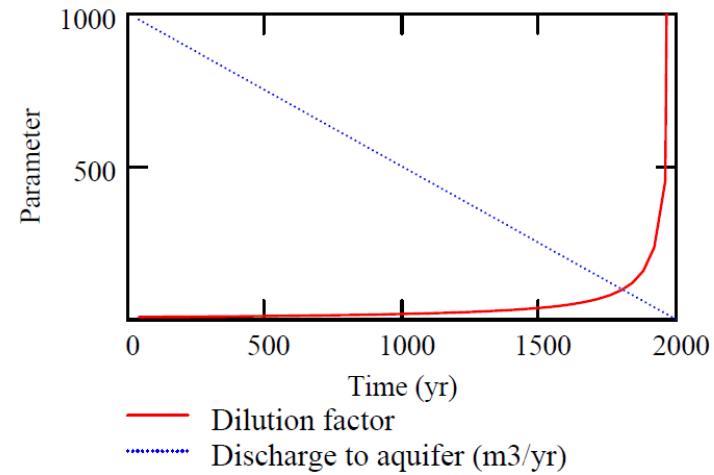
- Vertical borehole advective (VBA)

- Variable release rate, $Q_p(t)$
- Constant well discharge, Q_w

- Dilution factor changes with t

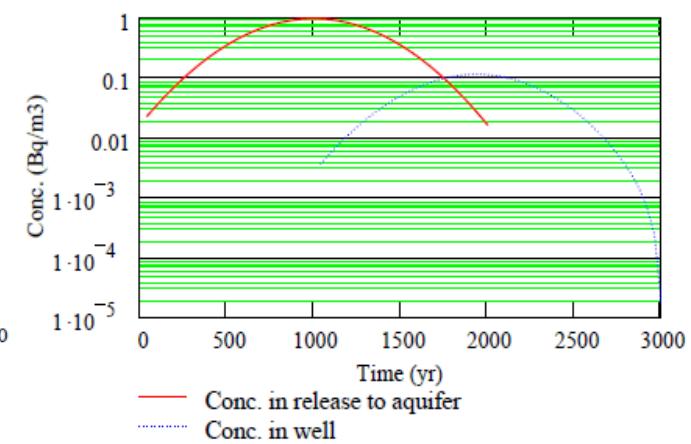
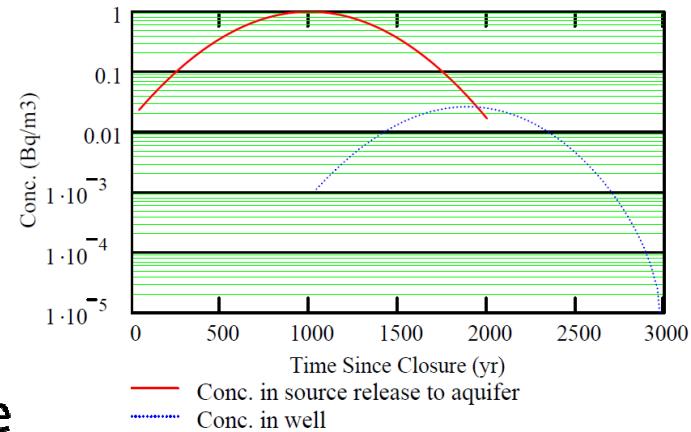
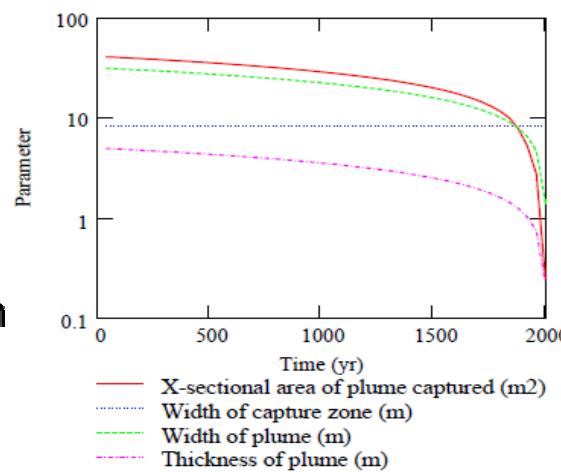
$$D_{f,vba}(t) = \frac{w_c}{w_p(t)} = \frac{Q_w}{Q_p(t)} \quad \text{for } Q_p(t) \leq Q_w$$

- As $Q_p(t)$ goes from 1,000 to $1 \text{ m}^3 \text{ yr}^{-1}$
 $D_{f,vba}(t)$ goes from 10 to 10,000
- Concentration at well
 - Peak arrives at well 1,000 yr after peak release
 - Peak concentration at well is 40-fold lower than peak concentration released
 - 20-fold decrease due to dilution
 - 2-fold decrease due to decay



PSA Examples

- Point source advective (PSA) example 1
 - Same $Q_p(t)$ and Q_w as VBA example
 - Same dilution factor, $D_f(t)$
 - Width of plume remains within capture zone
 - Same concentrations at well
- PSA example 2 with reduced well discharge
 - Same as PSA example 1, but Q_w fixed at $1,000 \text{ m}^3 \text{ yr}^{-1}$ instead of $10,000 \text{ m}^3 \text{ yr}^{-1}$
 - Width of plume exceeds width of capture zone for first $\sim 1,800 \text{ yr}$
 - Much of plume bypasses well
 - Dilution of peak is much lower
 - Peak concentration is much higher



Diffusive Release Examples

■ Point source diffusive (PSD) example

- The release concentration is represented by normalizing the mass flux release by the well discharge rate, called hereafter the activity released per dilution volume: $C_{p,i}(t) = \frac{m_{p,i}(t)}{Q_w}$

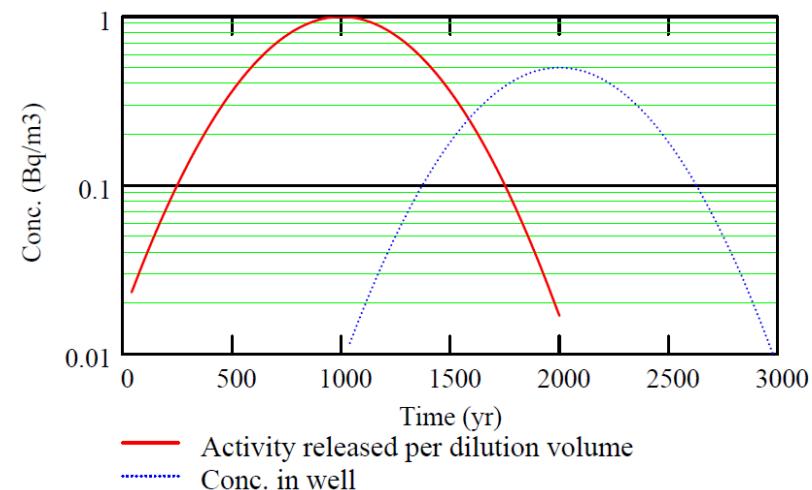
- Reduced concentrations at the well are due only to decay

■ Broad diffusive (BD) example

- The activity released per dilution volume for the BD example is:

$$C_{p,i}(t) = \frac{\ddot{m}_{p,i}(t)}{qb}$$

- Dilution is built into the source breakthrough curve (as in PSD example)
 - Less dilution is expected for the BD model versus the PSD model because PSD dilution is due to vertical and lateral mixing; only vertical mixing for BD model
- The same breakthrough curves at the well are obtained because



Model Summary

- Simple algebraic analytical solutions
- Dilution factors
 - Calculated from inputs
 - Ensures consistency
 - Vary over time when volumetric fluxes change
- Bypassing included
 - When plume width exceeds capture zone
- Travel time and decay included
 - Especially important for highly sorbed Rn's

TABLE I. Factors Affecting Dose Rate Calculations in the Aquifer Release Models

Parameter	VBA	PSA	PSD	BD
Pump rate, $Q_w(t)$	✓	✓	✓	
Release rate, $Q_p(t)$	✓	✓		
Diffusive flux, $m_{p,i}(t)$			✓	
Diffusive flux, $\dot{m}_{p,i}(t)$				✓
Saturated thickness, b		✓		✓
Specific discharge, q	✓	✓		✓
Capture zone width, w_c	✓	✓		
Dilution factor, $D_f(t)$	✓	✓		
Anisotropy, a		✓		
Fraction captured, F		✓		
Well distance, L	✓	✓	✓	✓
Retardation factor, $R_{f,i}$	✓	✓	✓	✓
Decay rate, λ	✓	✓	✓	✓
Porosity, n	✓	✓	✓	✓
Travel time, T_i	✓	✓	✓	✓
Indiv. consump. rate, I	✓	✓	✓	✓
Ingest. dose coeff., dcf_i	✓	✓	✓	✓

Next Steps

- Consider additional release scenarios
 - E.g., release from multiple fractures
- Include in-growth
- Sensitivity analyses
 - Identify important parameters for various release scenarios and radionuclides