SAND2012-2491C

# Solving Large-Scale Inverse Problems in Elasticity Using Sequential Quadratic Programming (SQP)

**Miguel Aguiló**, D. Ridzal, J. Young

Optimization and Uncertainty Quantification
Sandia National Laboratories, Albuquerque, New Mexico
maguilo@sandia.gov

**March 28, 2012**

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.
Research sponsored by the NNSA Advanced Scientific Computing Program under the Computer Science Research Foundation

# Outline

# Introduction

## Motivation

- Direct characterization of material properties is not always possible or desirable
- Nondestructive/noninvasive material characterization is important in many engineering applications, e.g. bio, aero
- Variations in structural response can be induced by
  - Disease; or
  - Damage

## Challenges

- Solution of large-scale inverse problems with models given by partial differential equations (PDEs)
- Characterization of the uncertainties in the model inputs for high-dimensional parameter spaces

Wellington Hospital Cardiac Service

Aloha Airlines Flight 243, 1988

# SQP Summary

- Matrix-free full-space SQP algorithm with trust regions.
- Handles inequality constraints – control *and* state constraints.
- Linear solver accuracy adjusted dynamically, based on SQP progress.
- Approximate Schur preconditioning reuses forward PDE solvers.
- Fully scalable – assuming scalable forward PDE solvers.

# Inverse Estimation of Parameters in Linear Elastodynamics

# Problem Overview

Find the shear modulus $\mu$ and bulk modulus $\kappa$ from measurements $\{\widehat{u}_i\}_{i=1,\ldots,d}$ related to the solution $\{u_i\}_{i,\ldots,d}$ of the boundary value problem

$$\begin{aligned} -\sigma_{ij,j} &= \omega^2 \rho u_i && \text{in} && \Omega, \\ u_i &= 0 && \text{on} && \Gamma_u, \\ \sigma_{ij} n_j &= \tau_i && \text{on} && \Gamma_\tau. \end{aligned}$$

The stress tensor $\sigma_{ij}$ is defined as

$$\sigma_{ij} = \mu C^{\mu}_{ijkl} \epsilon_{kl}(u) + \kappa C^{\kappa}_{ijkl} \epsilon_{kl}(u)$$

$$C^{\mu}_{ijkl} = \delta_{ik}\delta_{jl} + \delta_{il}\delta jk - \frac{2}{3}\delta_{ij}\delta_{kl} \qquad C^{\kappa}_{ijkl} = \delta_{ij}\delta_{kl}$$

and the strain tensor is defined to be the symmetric part of the displacement gradient, as follows,

$$\epsilon_{ij}(u) = \frac{1}{2}(u_{i,j} + u_{j,i}).$$

Here, $\Omega \subseteq \mathbb{R}^d$, $d = \{1, 2, 3\}$ is the computational domain with boundary $\partial\Omega$ and indices $i, j, k$, and $l$ take on values $1, \ldots, d$.

# Full-Space $L^2$ Formulation

## Problem

We consider the following nonlinear programming problem (NLP):

$$\begin{aligned} \underset{(u,\mu,\kappa)\in\mathcal{U}\times\mathcal{G}\times\mathcal{B}}{\text{minimize}} \quad & \left\{ \frac{1}{2}\|u_i - \widehat{u}_i\|^2 + R(\mu) + R(\kappa) \right\} \\ \text{subject to} \quad & -\sigma_{ij,j} = \omega^2 \rho u_i \quad \text{in} \quad \Omega \\ & \sigma_{ij} n_j = \tau_i \quad \text{on} \quad \Gamma_\tau \end{aligned}$$

where $\mathcal{U} = \{u_i : u_i \in H^1(\Omega),\ u_i = 0 \text{ on } \Gamma_u\}$, $\mathcal{G} = \{\mu : \mu \in L^2(\Omega)\}$, $\mathcal{B} = \{\kappa : \kappa \in L^2(\Omega)\}$.

## Lagrangian

$$\mathcal{L}(u_i, \mu, \kappa, \lambda_i) = \frac{1}{2}\|u_i - \widehat{u}_i\|^2 + R(\mu) + R(\kappa) + \langle \lambda_i, -\sigma_{ij,j} - \omega^2 \rho u_i \rangle + \langle \lambda_i, \tau_i - \sigma_{ij} n_j \rangle_{\Gamma_\tau}$$

## First-Order Necessary Optimality Conditions

$$F(u_i, \mu, \kappa, \lambda_i) = \left\{ \begin{array}{c} (u_i - \widehat{u}_i) - (C_{ijkl}\epsilon_{kl}(\lambda))_{,j} - \omega^2 \rho u_i \quad \text{in} \quad \Omega \\ \nabla_\mu R(\mu) + \epsilon_{ij}(\lambda) C^{\mu}_{ijkl} \epsilon_{kl}(u) \\ \nabla_\kappa R(\kappa) + \epsilon_{ij}(\lambda) C^{\kappa}_{ijkl} \epsilon_{kl}(u) \\ -(C_{ijkl}\epsilon_{kl}(u))_{,j} - \omega^2 \rho u_i \quad \text{in} \quad \Omega; \ \tau_i - \sigma_{ij} n_j \quad \text{on} \quad \Gamma_\tau \end{array} \right\} = 0$$

# Full-Space $L^2$ Formulation

## Hessian of the Lagrangian

$$F'(u_i, \mu, \kappa, \lambda_i)(\delta u_i, \delta\mu, \delta\kappa, \delta\lambda_i) = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{E} & \mathbf{F} & 0 & \mathbf{G} \\ \mathbf{H} & 0 & \mathbf{I} & \mathbf{J} \\ \mathbf{K} & \mathbf{L} & \mathbf{M} & 0 \end{bmatrix}$$

## Jacobian

$$\mathbf{D} = -(C_{ijkl}\epsilon_{kl}(\widehat{\delta\lambda}))_{,j} - \omega^2\rho\,\widehat{\delta\lambda_i} \qquad \mathbf{G} = \epsilon_{ij}(\widehat{\delta\lambda})C^{\mu}_{ijkl}\epsilon_{kl}(u) \qquad \mathbf{J} = \epsilon_{ij}(\widehat{\delta\lambda})C^{\kappa}_{ijkl}\epsilon_{kl}(u)$$

$$\mathbf{K} = -(C_{ijkl}\epsilon_{kl}(\widehat{\delta u}))_{,j} - \omega^2\rho\,\widehat{\delta u_i} \qquad \mathbf{L} = -(\widehat{\delta\mu}C^{\mu}_{ijkl}\epsilon_{kl}(u))_{,j} \qquad \mathbf{M} = -(\widehat{\delta\kappa}C^{\kappa}_{ijkl}\epsilon_{kl}(u))_{,j}$$

# Full-Space $L^2$ Formulation

## Hessian of the Lagrangian

$$F'(u_i,\mu,\kappa,\lambda_i)(\delta u_i,\delta\mu,\delta\kappa,\delta\lambda_i)=\begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{E} & \mathbf{F} & 0 & \mathbf{G} \\ \mathbf{H} & 0 & \mathbf{I} & \mathbf{J} \\ \mathbf{K} & \mathbf{L} & \mathbf{M} & 0 \end{bmatrix}$$

## Hessian Operator

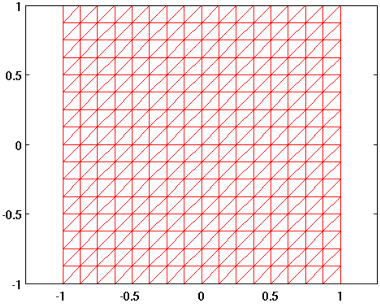
$$\mathbf{A}=\beta\,\widehat{\delta u_i} \qquad \mathbf{B}=-(\widehat{\delta\mu}C^{\mu}_{ijkl}\epsilon_{kl}(\lambda))_{,j} \qquad \mathbf{C}=-(\widehat{\delta\kappa}C^{\kappa}_{ijkl}\epsilon_{kl}(\lambda))_{,j}$$

$$\mathbf{E}=\epsilon_{ij}(\lambda)C^{\mu}_{ijkl}\epsilon_{kl}(\widehat{\delta u}) \qquad \mathbf{F}=\nabla^2_{\mu}R(\mu)\widehat{\delta\mu}$$

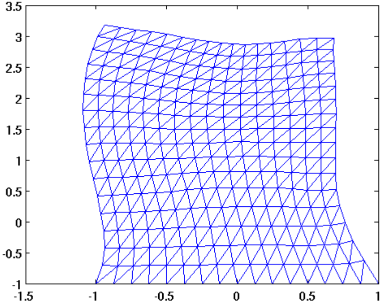
$$\mathbf{H}=\epsilon_{ij}(\lambda)C^{\kappa}_{ijkl}\epsilon_{kl}(\widehat{\delta u}) \qquad \mathbf{I}=\nabla^2_{\kappa}R(\kappa)\widehat{\delta\kappa}$$

# Results

**Undeformed Grid**

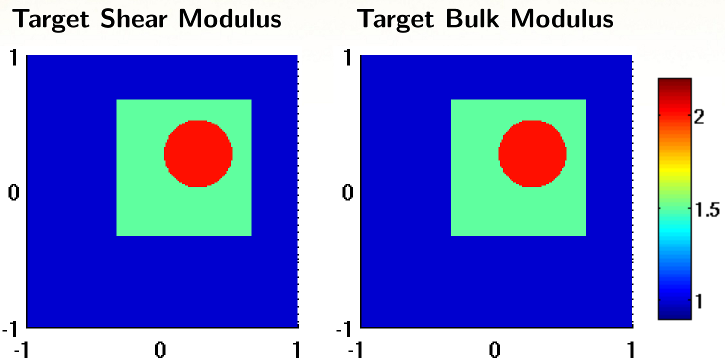



**Deformed Grid**



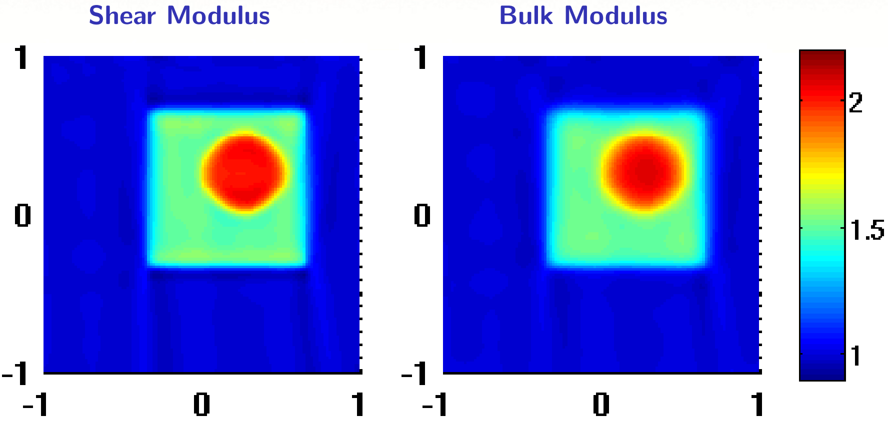

- Our domain $D$ is $[-1,1]^2$, the grid is uniform triangular, $128 \times 128$
- Data is sampled on a finer grid $256 \times 256$
- We solve the PDE using Galerkin FEM with a piecewise linear basis
- As the initial guess, we use constant $\mu = 1$ *Pa* and $\kappa = 1$ *Pa*
- Total number of unknowns is equivalent $99,330$

# Results

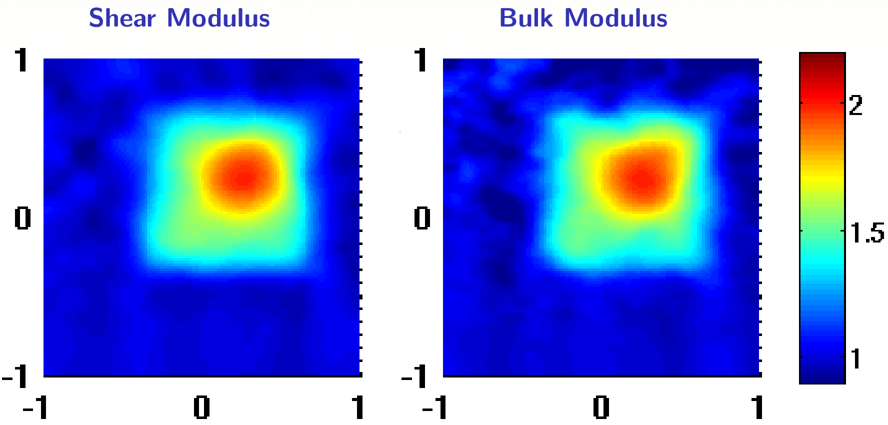



- Our domain $D$ is $[-1,1]^2$, the grid is uniform triangular, $128 \times 128$
- Data is sampled on a finer grid $256 \times 256$
- We solve the PDE using Galerkin FEM with a piecewise linear basis
- As the initial guess, we use constant $\mu = 1\ Pa$ and $\kappa = 1\ Pa$
- Total number of unknowns is equivalent $99,330$

# Computed Parameters, No Noise



# Computed Parameters, 3% Noise



# SQP Performance

| k | f(x_k) | \|\| c(x_k) \|\| | \|\|L_x(x_k,lam_k)\|\| | Delta_k | \|\| n_k \|\| | \|\| t_k \|\| | itcg |
|---|---|---|---|---|---|---|---|
| 0 | 6.93090e-01 | 3.062110e-13 | 1.781733e-02 | 1.00e+02 | | | |
| 1 | 8.10553e-03 | 7.771920e-03 | 5.954813e-04 | 5.33e+02 | 2.88e-10 | 7.61e+01 | 72 |
| 2 | 4.42933e-03 | 3.788758e-03 | 1.870175e-04 | 1.21e+02 | 4.45e+00 | 1.66e+01 | 3 |
| 3 | 3.33968e-03 | 1.020809e-02 | 3.779170e-05 | 1.06e+02 | 7.87e-01 | 1.51e+01 | 6 |
| 4 | 3.18639e-03 | 1.049432e-02 | 1.377018e-05 | 1.32e+01 | 5.20e-01 | 1.32e+01 | 8 |
| 5 | 3.13484e-03 | 1.069195e-02 | 5.134040e-06 | 1.32e+01 | 2.57e-01 | 1.32e+01 | 11 |
| 6 | 3.12505e-03 | 4.355073e-03 | 2.525812e-06 | 1.29e+01 | 2.16e-01 | 6.43e+00 | 12 |
| 7 | 3.12184e-03 | 3.640399e-03 | 1.741322e-06 | 4.56e+00 | 5.90e-02 | 4.56e+00 | 14 |
| 8 | 3.12074e-03 | 1.006678e-03 | 1.064661e-06 | 1.60e+01 | 2.54e-02 | 2.28e+00 | 14 |
| 9 | 3.12026e-03 | 8.401915e-04 | 8.417355e-07 | 2.35e+00 | 6.01e-03 | 2.35e+00 | 16 |
| 10 | 3.12010e-03 | 3.957708e-04 | 5.254084e-07 | 2.35e+00 | 3.75e-03 | 1.18e+00 | 16 |
| 11 | 3.12004e-03 | 1.138563e-04 | 2.018372e-07 | 3.83e+00 | 9.26e-04 | 5.47e-01 | 16 |
| 12 | 3.12002e-03 | 9.428050e-05 | 1.820050e-07 | 4.35e-01 | 3.12e-04 | 4.35e-01 | 16 |
| 13 | 3.12001e-03 | 1.194862e-04 | 1.865811e-07 | 4.35e-01 | 2.27e-04 | 4.35e-01 | 16 |
| 14 | 3.12000e-03 | 3.342200e-05 | 7.061537e-08 | 1.52e+00 | 2.84e-04 | 2.17e-01 | 16 |
| 15 | 3.11999e-03 | 3.390794e-05 | 8.867013e-08 | 2.13e-01 | 8.19e-05 | 2.13e-01 | 16 |
| 16 | 3.11999e-03 | 4.356250e-05 | 7.217764e-08 | 2.13e-01 | 8.81e-05 | 2.13e-01 | 17 |
| 17 | 3.11999e-03 | 4.831992e-05 | 7.877364e-08 | 2.13e-01 | 1.03e-04 | 2.13e-01 | 17 |
| 18 | 3.11999e-03 | 5.779310e-05 | 1.907613e-08 | 2.13e-01 | 1.29e-04 | 2.13e-01 | 21 |
| 19 | 3.11999e-03 | 5.475901e-06 | 1.825626e-10 | 4.24e-01 | 1.45e-04 | 6.06e-02 | 37 |
| 20 | 3.11999e-03 | 4.199292e-10 | 6.559367e-12 | 4.24e-01 | 1.33e-05 | 8.24e-04 | 42 |

- We observed **quadratic convergence** in all numerical studies

# Inverse Estimation of Parameters in Nonlinear Elastostatics

**Problem Overview**

**NLP Problem**

**Results**

# Problem Overview

Find the shear modulus $\mu$ and bulk modulus $\kappa$ from measurements $\{\widehat{u}_i\}_{i=1,\ldots,d}$ related to the solution $\{u_i\}_{i,\ldots,d}$ of the boundary value problem

$$\begin{aligned} -(F_{iK}S_{KL})_{,L} &= 0 && \text{in} && \Omega_0, \\ u_i &= 0 && \text{on} && \Gamma_u^0, \\ (F_{iK}S_{KL})n_L^0 &= \tau_i^0 && \text{on} && \Gamma_\tau^0. \end{aligned}$$

For a Saint Venant-Kirchhoff Material, the second Piola-Kirchhoff stress tensor $S_{KL}$ is defined as

$$S_{KL} = \mu C^{\mu}_{IJKL} E_{IJ} + \kappa C^{\kappa}_{IJKL} E_{IJ}$$

$$C^{\mu}_{IJKL} = \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK} - (2/3)\delta_{IJ}\delta_{KL} \qquad C^{\kappa}_{IJKL} = \delta_{IJ}\delta_{KL}$$

The deformation gradient $F_{iK}$ and Green-Lagrangian strain tensor $E_{KL}$ are respectively defined as

$$F_{iK} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_K^0} + \delta_{iK}\right) \quad E_{IJ} = \frac{1}{2}\left(F_{iK}F_{iL} + \delta_{KL}\right).$$

Here, $\Omega_0 \subseteq \mathbb{R}^d,\ d = \{1,2,3\}$ is the computational domain on the reference configuration with boundary $\partial\Omega_0$ and indices $i,I,J,K$ and $L$ take on values $1,\ldots,d$.

# Full-Space $L^2$ Formulation

## Problem

We consider the following NLP:

$$\begin{aligned} \underset{(u,\mu,\kappa)\in\mathcal{U}\times\mathcal{G}\times\mathcal{B}}{\text{minimize}} \quad & \left\{ \frac{1}{2}\|u_i - \widehat{u}_i\|^2 + R(\mu) + R(\kappa) \right\} \\ \text{subject to} \quad & -(F_{iK}S_{KL})_{,L} = 0 \quad \text{in} \quad \Omega_0 \\ & (F_{iK}S_{KL})n_L^0 = \tau_i^0 \quad \text{on} \quad \Gamma_\tau^0 \end{aligned}$$

## Lagrangian

$$\mathcal{L}(u_i,\mu,\kappa,\lambda_i) = \frac{1}{2}\|u_i-\widehat{u}_i\|^2+R(\mu)+R(\kappa)+\langle\lambda_i,-(F_{iK}S_{KL})_{,L}\rangle+\langle\lambda_i,\tau_i^0-(F_{iK}S_{KL})n_L^0\rangle_{\Gamma_\tau^0}$$

## First-Order Necessary Optimality Conditions

$$F(u_i,\mu,\kappa,\lambda_i) =$$

$$\left\{ \begin{array}{c} \beta\,(u_i - \widehat{u}_i) - (\lambda_{i,K}S_{KL})_{,L} - (F_{iK}(\mathbf{C}_{IJKL}\,(F_{jK}\lambda_{j,L} + \lambda_{j,K}F_{jL})))_{,L} \quad \text{in} \quad \Omega_0 \\ \nabla_\mu R(\mu) + \lambda_{i,L}F_{i,K}C^{\mu}_{IJKL}E_{IJ} \\ \nabla_\kappa R(\kappa) + \lambda_{i,L}F_{i,K}C^{\kappa}_{IJKL}E_{IJ} \\ -(F_{iK}S_{KL})_{,L} \quad \text{in} \quad \Omega_0 \;;\; \tau_i^0 - (F_{iK}S_{KL})n_L^0 \quad \text{on} \quad \Gamma_\tau^0 \end{array} \right\} = 0$$

# Full-Space $L^2$ Formulation

## Hessian of the Lagrangian

$$F'(u_i, \mu, \kappa, \lambda_i)(\delta u_i, \delta\mu, \delta\kappa, \delta\lambda_i) = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{E} & \mathbf{F} & 0 & \mathbf{G} \\ \mathbf{H} & 0 & \mathbf{I} & \mathbf{J} \\ \mathbf{K} & \mathbf{L} & \mathbf{M} & 0 \end{bmatrix}$$

## Jacobian

$$\mathbf{D} = -(F_{iK}C_{IJKL}(F_{jK}\widehat{\delta\lambda}_{j,L} + \widehat{\delta\lambda}_{j,K}F_{jL}))_{,L} - (\widehat{\delta\lambda}_{i,K}S_{KL})_{,L}$$

$$\mathbf{G} = \widehat{\delta\lambda}_{iL}F_{iK}C^{\mu}_{IJKL}E_{IJ}$$

$$\mathbf{J} = \widehat{\delta\lambda}_{iL}F_{iK}C^{\kappa}_{IJKL}E_{IJ}$$

$$\mathbf{K} = -(F_{iK}C_{IJKL}(F_{jK}\widehat{\delta u}_{j,L} + \widehat{\delta u}_{j,K}F_{jL}))_{,L} - (\widehat{\delta u}_{i,K}S_{KL})_{,L}$$

$$\mathbf{L} = -(\widehat{\delta\mu}F_{iK}C^{\mu}_{IJKL}E_{IJ})_{,L}$$

$$\mathbf{M} = -(\widehat{\delta\kappa}F_{iK}C^{\kappa}_{IJKL}E_{IJ})_{,L}$$

# Full-Space $L^2$ Formulation

## Hessian of the Lagrangian

$$F'(u_i, \mu, \kappa, \lambda_i)(\delta u_i, \delta\mu, \delta\kappa, \delta\lambda_i) = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{E} & \mathbf{F} & 0 & \mathbf{G} \\ \mathbf{H} & 0 & \mathbf{I} & \mathbf{J} \\ \mathbf{K} & \mathbf{L} & \mathbf{M} & 0 \end{bmatrix}$$

## Hessian Operator

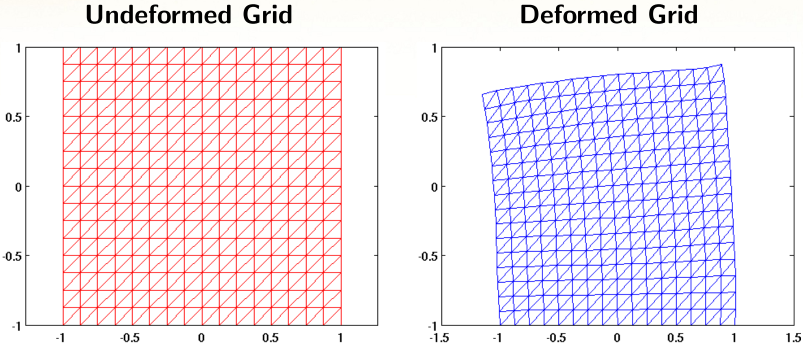
$$\mathbf{A} = \widehat{\delta u}_i - (\lambda_{i,K} C_{IJKL}(F_{jK}\widehat{\delta u}_{j,L} + \widehat{\delta u}_{j,K} F_{jL}))_{,L} - (\widehat{\delta u}_{i,K} C_{IJKL}(F_{jK}\lambda_{j,L} + \lambda_{j,K} F_{jL}))_{,L}$$
$$-(F_{iK} C_{IJKL}(\widehat{\delta u}_{j,K}\,\lambda_{j,L} + \lambda_{j,K}\,\widehat{\delta u}_{j,L}))_{,L}$$

$$\mathbf{B} = -(\widehat{\delta\mu}\, F_{i,K} C^{\mu}_{IJKL}(F_{jK}\lambda_{j,L} + \lambda_{j,K} F_{jL}))_{,L} \qquad \mathbf{C} = -(\widehat{\delta\kappa}\, F_{i,K} C^{\kappa}_{IJKL}(F_{jK}\lambda_{j,L} + \lambda_{j,K} F_{jL}))_{,L}$$
$$-(\widehat{\delta\mu}\,\lambda_{i,K} C^{\mu}_{IJKL} E_{IJ})_{,L} \qquad -(\widehat{\delta\kappa}\,\lambda_{i,K} C^{\kappa}_{IJKL} E_{IJ})_{,L}$$

$$\mathbf{E} = \lambda_{i,L} F_{i,K} C^{\mu}_{IJKL}(F_{jK}\widehat{\delta u}_{j,L} + \widehat{\delta u}_{j,K} F_{jL}) + \lambda_{i,L}\widehat{\delta u}_{i,K} C^{\mu}_{IJKL} E_{IJ} \qquad \mathbf{F} = \nabla^2_{\mu} R(\mu)$$

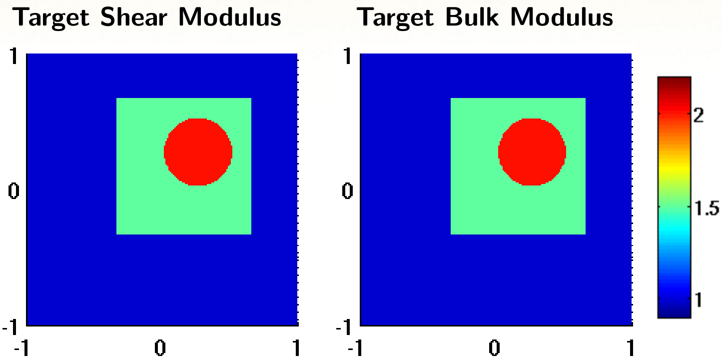
$$\mathbf{H} = \lambda_{i,L} F_{i,K} C^{\kappa}_{IJKL}(F_{jK}\widehat{\delta u}_{j,L} + \widehat{\delta u}_{j,K} F_{jL}) + \lambda_{i,L}\widehat{\delta u}_{i,K} C^{\kappa}_{IJKL} E_{IJ} \qquad \mathbf{I} = \nabla^2_{\kappa} R(\kappa)$$

# Results



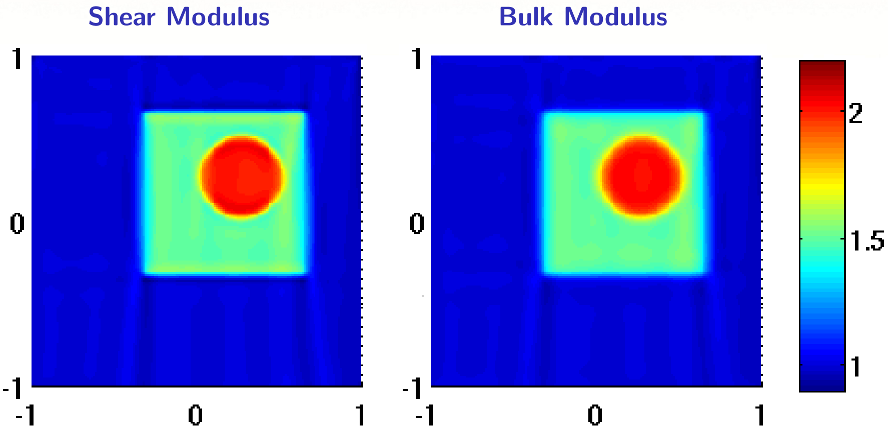

- Our domain $D$ is $[-1,1]^2$, the grid is uniform triangular, $200 \times 200$
- Data is sampled on a finer grid $400 \times 400$
- We solve the PDE using Galerkin FEM with a piecewise linear basis
- As the initial guess, we use constant $\mu = 1$ *Pa* and $\kappa = 1$ *Pa*
- Total number of unknowns is equivalent $161,202$

# Results

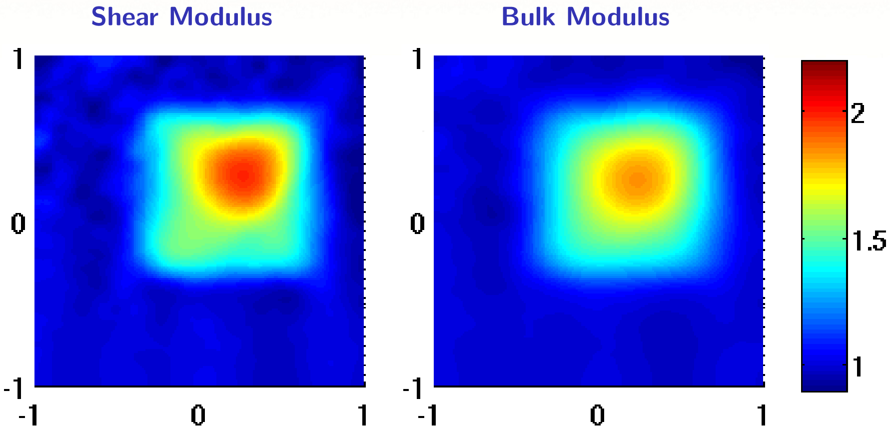



- Our domain $D$ is $[-1, 1]^2$, the grid is uniform triangular, $200 \times 200$
- Data is sampled on a finer grid $400 \times 400$
- We solve the PDE using Galerkin FEM with a piecewise linear basis
- As the initial guess, we use constant $\mu = 1\ Pa$ and $\kappa = 1\ Pa$
- Total number of unknowns is equivalent $161,202$

# Computed Parameters, No Noise



# Computed Parameters, 3% Noise



# SQP Performance

| k | f(x_k) | \|\| c(x_k) \|\| | \|\|L_x(x_k,lam_k)\|\| | Delta_k | \|\| n_k \|\| | \|\| t_k \|\| | itcg |
|---|---|---|---|---|---|---|---|
| 0 | 1.21121e-02 | 1.418450e-13 | 1.501655e-03 | 1.00e+02 | | | |
| 1 | 9.18257e-05 | 8.198400e-03 | 8.177939e-06 | 1.75e+02 | 7.65e-11 | 2.50e+01 | 3 |
| 2 | 1.00124e-05 | 1.281381e-03 | 1.438794e-06 | 2.45e+02 | 4.52e+00 | 3.47e+01 | 4 |
| 3 | 3.00828e-06 | 6.600252e-04 | 3.330121e-07 | 2.45e+02 | 8.05e-01 | 1.85e+01 | 6 |
| 4 | 1.79307e-06 | 1.086107e-03 | 2.098207e-07 | 1.03e+01 | 8.05e-02 | 1.03e+01 | 6 |
| 5 | 9.72900e-07 | 1.804024e-04 | 2.018572e-07 | 7.20e+01 | 5.72e-02 | 1.03e+01 | 6 |
| 6 | 1.76429e-08 | 9.444912e-04 | 6.134319e-09 | 7.20e+01 | 2.24e-02 | 2.91e+01 | 21 |
| 7 | 2.34142e-08 | 1.549139e-05 | 2.226323e-08 | 7.20e+01 | 8.81e-02 | 8.96e-01 | 5 |
| 8 | 9.14361e-09 | 8.921297e-05 | 2.967412e-09 | 6.59e+00 | 1.38e-04 | 6.59e+00 | 19 |
| 9 | 6.09451e-09 | 4.709693e-05 | 1.808598e-09 | 6.59e+00 | 6.44e-03 | 5.02e+00 | 23 |
| 10 | 5.17726e-09 | 3.097234e-05 | 6.411598e-10 | 6.59e+00 | 2.41e-03 | 4.14e+00 | 31 |
| 11 | 5.11002e-09 | 4.721732e-07 | 3.776935e-10 | 6.59e+00 | 2.41e-03 | 4.34e-01 | 17 |
| 12 | 4.97759e-09 | 6.359808e-06 | 6.916146e-11 | 3.29e+00 | 2.98e-05 | 3.29e+00 | 62 |
| 13 | 4.97340e-09 | 1.623117e-07 | 1.259521e-11 | 3.86e+00 | 6.39e-05 | 5.51e-01 | 76 |
| 14 | 4.97334e-09 | 3.230133e-09 | 1.555454e-12 | 3.86e+00 | 3.33e-06 | 5.25e-02 | 66 |
| 15 | 4.97334e-09 | 1.433939e-10 | 6.650761e-13 | 2.37e-02 | 7.79e-08 | 1.18e-02 | 58 |

# Further Advances

## Precise Embedding of Uncertainty into the Inverse Problem Formulation

To directly incorporate the notion of uncertainty into the mathematical formulation of the inverse problem we require

- **Research:** Advances in algorithmic research such as new optimization algorithms, parallel solvers, and numerical methods for stochastic computation
- **Numerical Tools:** Use of the best available numerical tools for a rigorous implementation of the advances in algorithmic research

# Conclusions

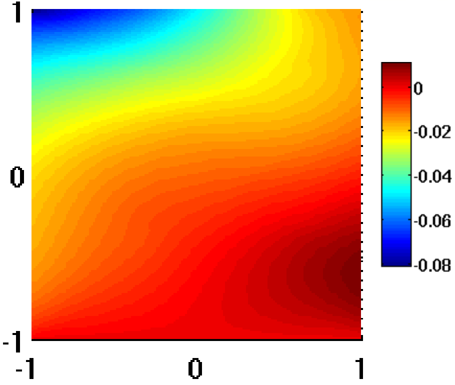
For large-scale PDE-constrained inverse problems:

- We can successfully apply the SQP to relevant **large-scale** inverse problems in solid mechanics
- We can successfully apply the SQP to large-scale inverse problems in nonlinear elasticity **without** using continuation methods
- Second-order optimization algorithms can achieve **quadratic** convergence rates [1]
- **Advances** in algorithmic research are required to accomplish a precise embedding of stochastic discretizations into our mathematical models

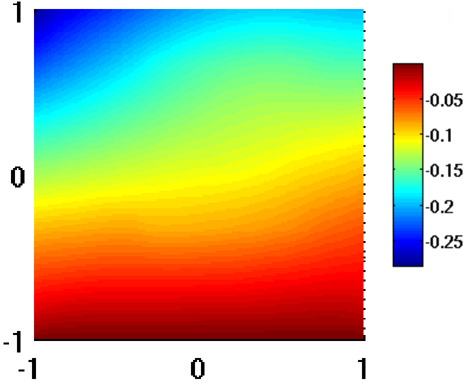
**1 D. Ridzal, M. Aguiló, and M. Heinkenschloss "Numerical study of a matrix-free trust-region SQP method for equality constrained optimization," Sandia Technical Report SAND2011-9346, December 2011**

# Computed Displacement Field, No Noise

$U_1$ **Field**

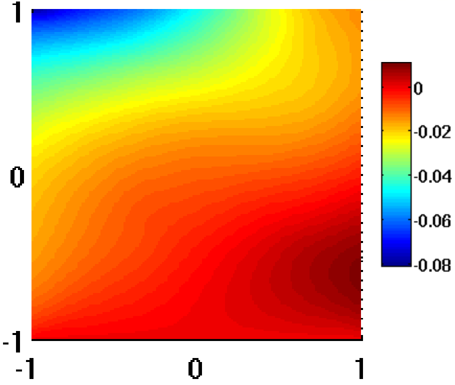



$U_2$ **Field**



# Computed Displacement Field, 3% Noise

$U_1$ **Field**




$U_2$ **Field**