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Data-Based Stochastic Evaluation of Closed-Loop Stability and Performance Metrics

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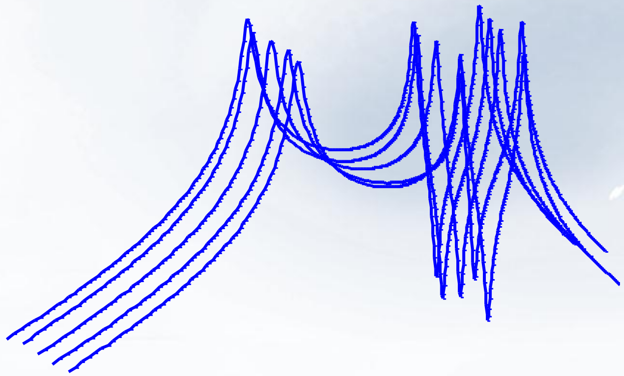
What am I going to talk about?

- Motivation
  - Structures are Random
- Controller Design for Uncertain Dynamical Systems
  - Stochastic data-based approach with no Sys ID
- Estimating the Family of Models
  - Karhunen-Loeve Expansion
  - Kernel Density Estimator
  - Markov Chain Monte Carlo
- Application to Controlled Systems
- Conclusions

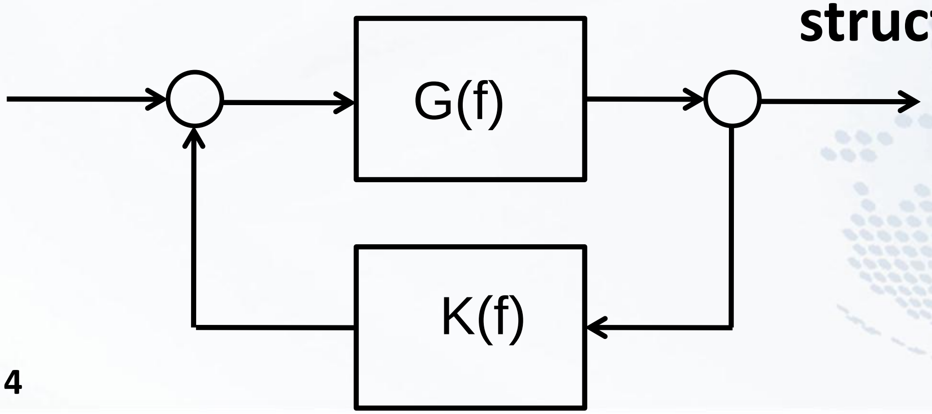
Uncertainty is present in all systems

- **Variability (uncertainty) exists:**
  - **Between nominally identical structures**
  - **Between test trials of the same structure**
  - **In models created to capture the dynamics of structures**
  - **In environmental conditions under which a structure operates**
- **The uncertainty and variability must be considered when analyzing uncertain systems**

Random variability from a controls perspective



- The system to be controlled is stochastic
- Controller is deterministic (and optimized for stability and control of one plant in ensemble of stochastic plants)
- How well does the controller perform on the ensemble of stochastic structures?

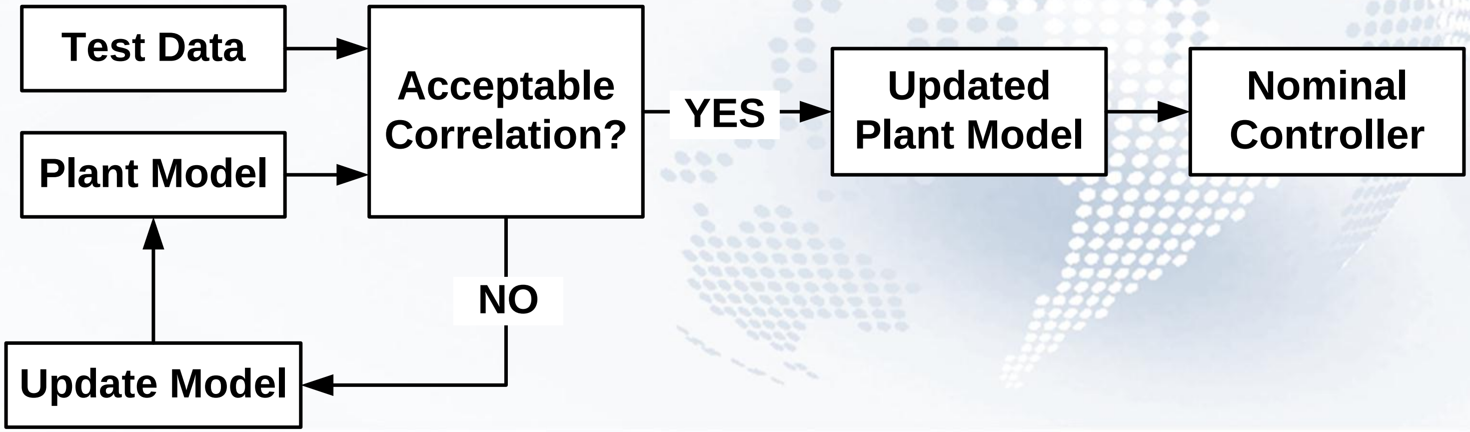


Uncertainty impacts on controller design

- A controller for a dynamical system must take into account the plant uncertainty
  - Will the ensemble of systems be closed-loop stable?
  - Will controlled systems in the ensemble have performance expected from the nominal system?
- Uncertainty is treated with stochastic and deterministic methods
  - Gain and phase margins
  - Deterministic robust control methods (H_∞, QFT)
  - Stochastic robust control, stochastic robustness analysis

Typical controller design

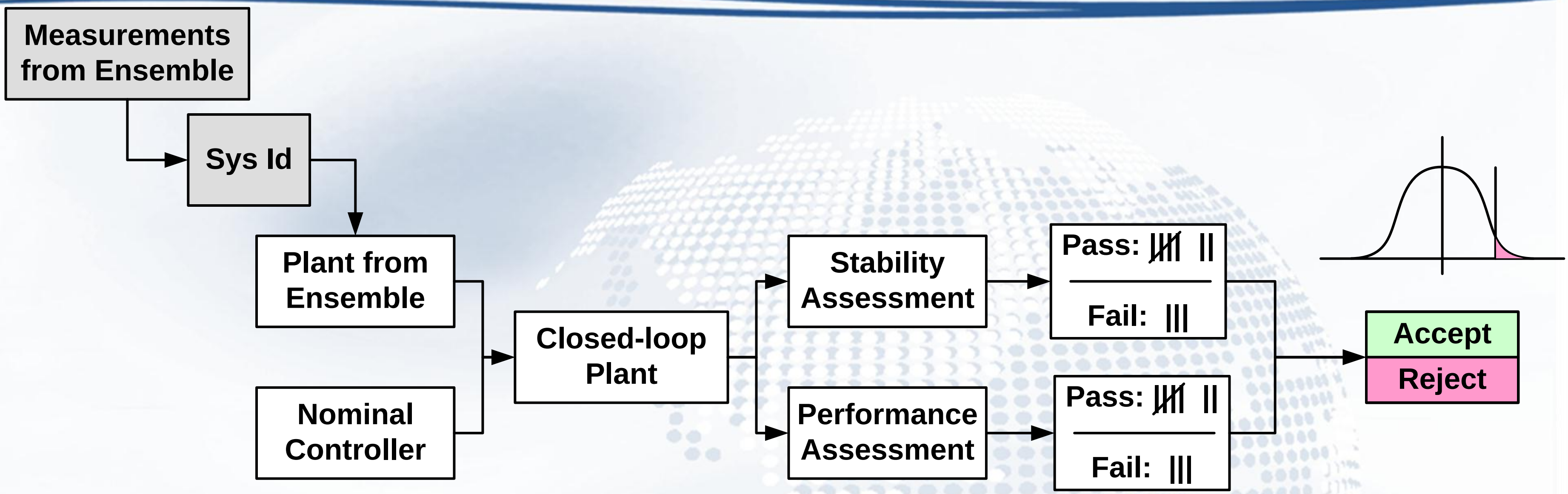
- **A plant controller needs the plant model**
  - Created from 1st principles or identified from test
  - Usually only a "nominal" model from the ensemble
- **Using a plant model in controller design allows for optimization for the nominal system**
- **If the plant order is high, but controller order is limited, the plant must be reduced – this can introduce error**



Stochastic evaluation methods are more flexible than deterministic methods

- **Deterministic methods demand stability for all possible uncertainties**
  - **Hard bounds**
  - **Often conservative**
- **Stochastic methods admit a more realistic, population based, evaluation**
  - **Increased controller flexibility should allow performance improvements**

Stochastic closed-loop performance evaluation



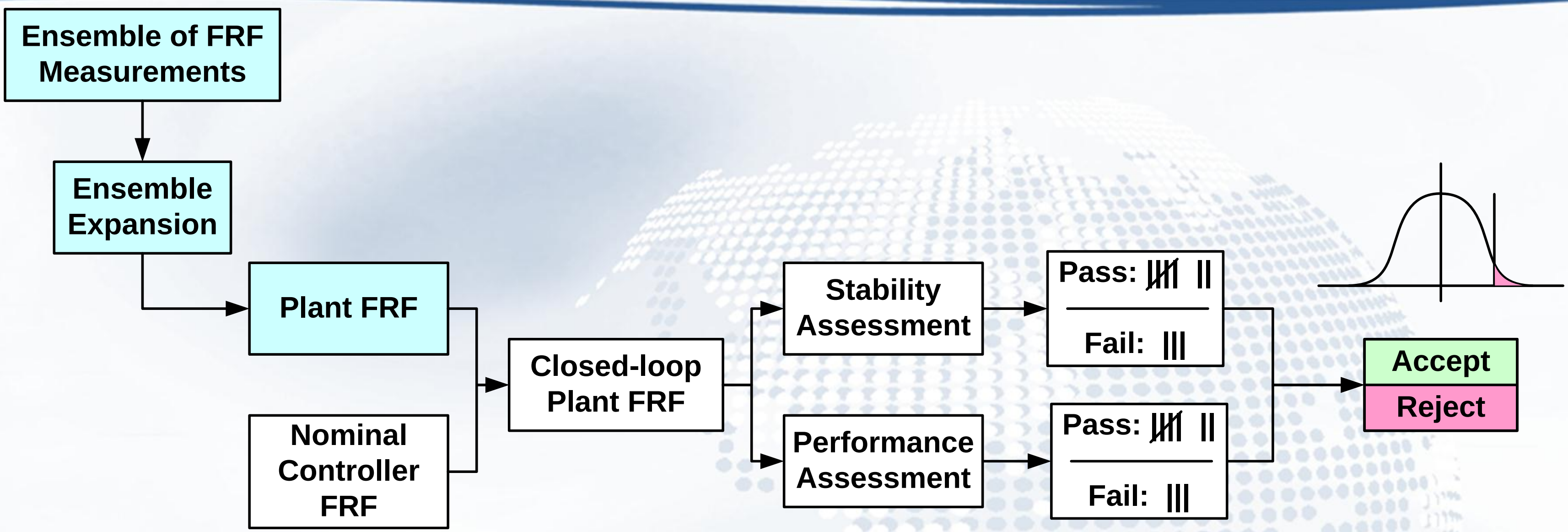
Advantages

- Ensemble measure of stability and performance
  - Stability Margins
  - Performance Cost Functions

Disadvantages

- Requires system identification (difficult)
  - Model error and uncertainty
- Requires a large ensemble

The new method uses measured FRFs to evaluate the closed-loop system



Advantages

- Does not require system identification
  - Avoids Sys ID errors
- Does not require a large ensemble

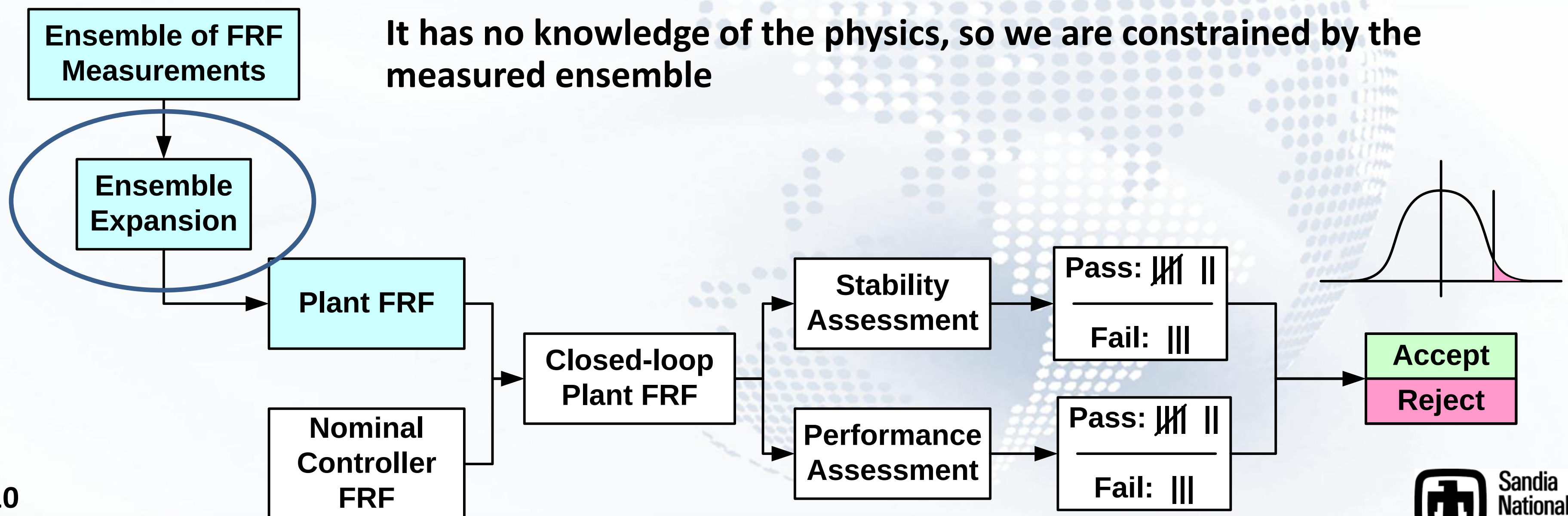
Disadvantages

- Synthetic plants are “clones”
  - True population representation is unknown
- Underlying source of variability is unknown

Expanding an ensemble

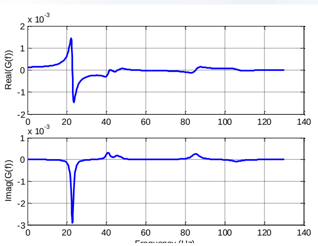
- **Problem: We rarely have a large ensemble from with which to work**
- **Solution: Synthetically expand test ensemble in FRF space using Karhunen-Loeve expansion (KLE)**
  - **Note that the expanded ensemble contains only linear combinations of the measured ensemble**

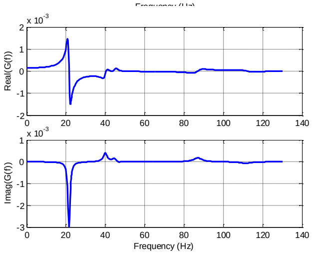
It has no knowledge of the physics, so we are constrained by the measured ensemble



Karhunen-Loeve Expansion

$$x(f) = \mu_X(f) + \sum v_k(f) \times w_k^{1/2} \times u_k$$





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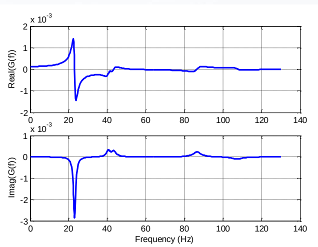

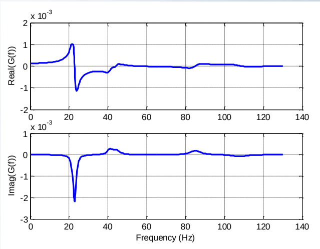

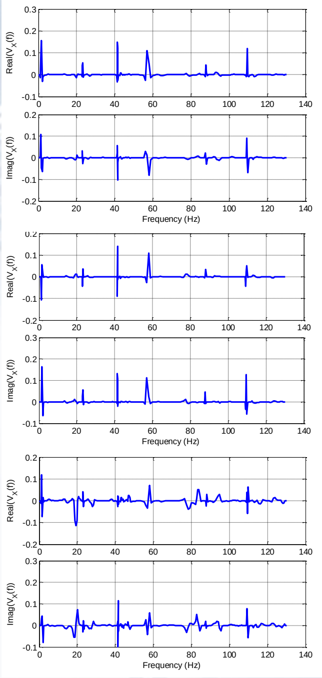

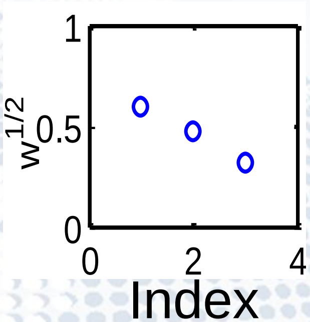

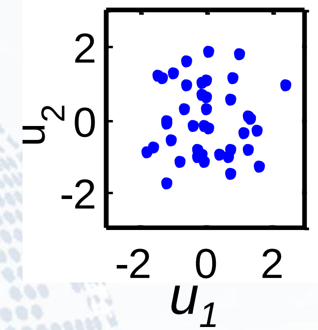

The first three can be approximated using measured realizations.

The u_k are zero-mean, unit variance, uncorrelated.

The KL expansion models a random process and its realizations as a mean function plus a product of shape functions, amplitude functions, and randomizing factors.

Karhunen-Loeve Expansion

- **The matrix form of the KL expansion is**

$$X_{rp} \cong \boldsymbol{v}\boldsymbol{w}^{1/2}\boldsymbol{U} + \boldsymbol{\mu}_x$$

- **The expansion is a straightforward SVD**
- **For MIMO FRFs, we vectorize the FRF matrix and stack the real and imaginary parts**
- **Once realizations can be generated from the random vector *U*, realizations of the random process x_{rp} can be generated**
- **To generate realizations from *U* we need to express its joint probability density function (PDF)**

Markov Chain Monte Carlo

- We wish to draw samples from a source with the joint PDF created with the KDE
- A generated vector is accepted or rejected as a new realization based on its likelihood of occurring (calculated with Kernel Density Estimator)
- When enough randomizing vectors have been accepted, they can be turned back into FRF realizations using the original decomposition:

$$x_{gen} = \mu_x + VWu^*_{gen}$$

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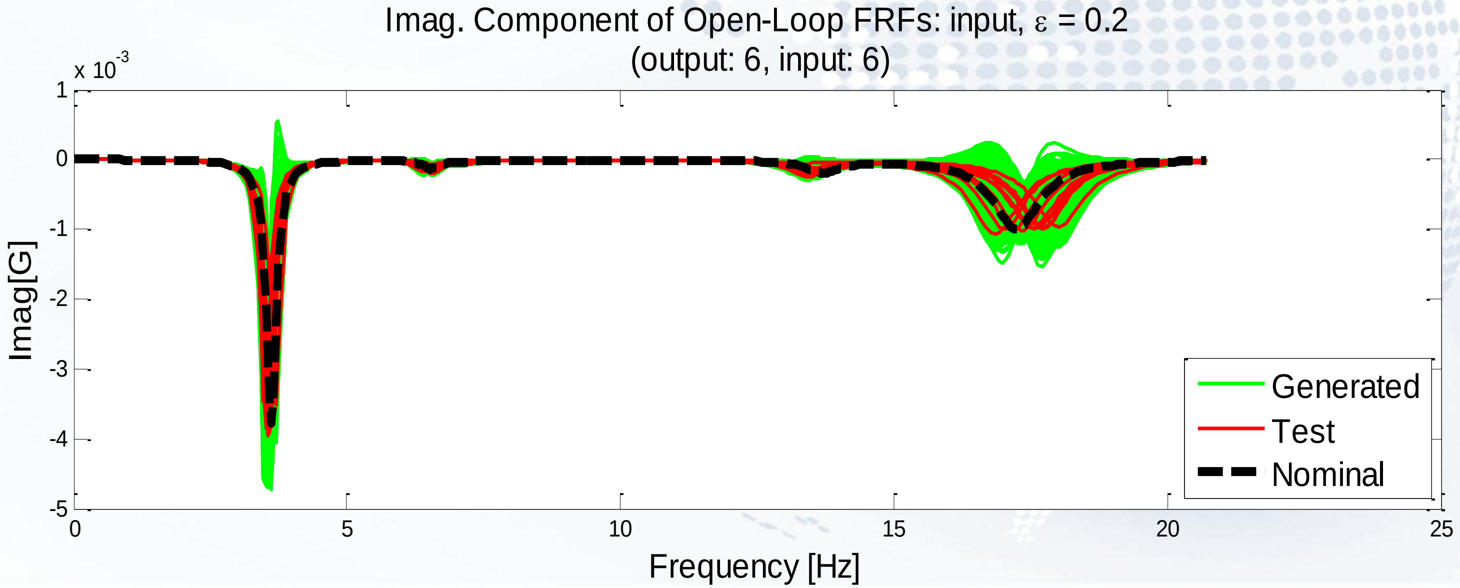
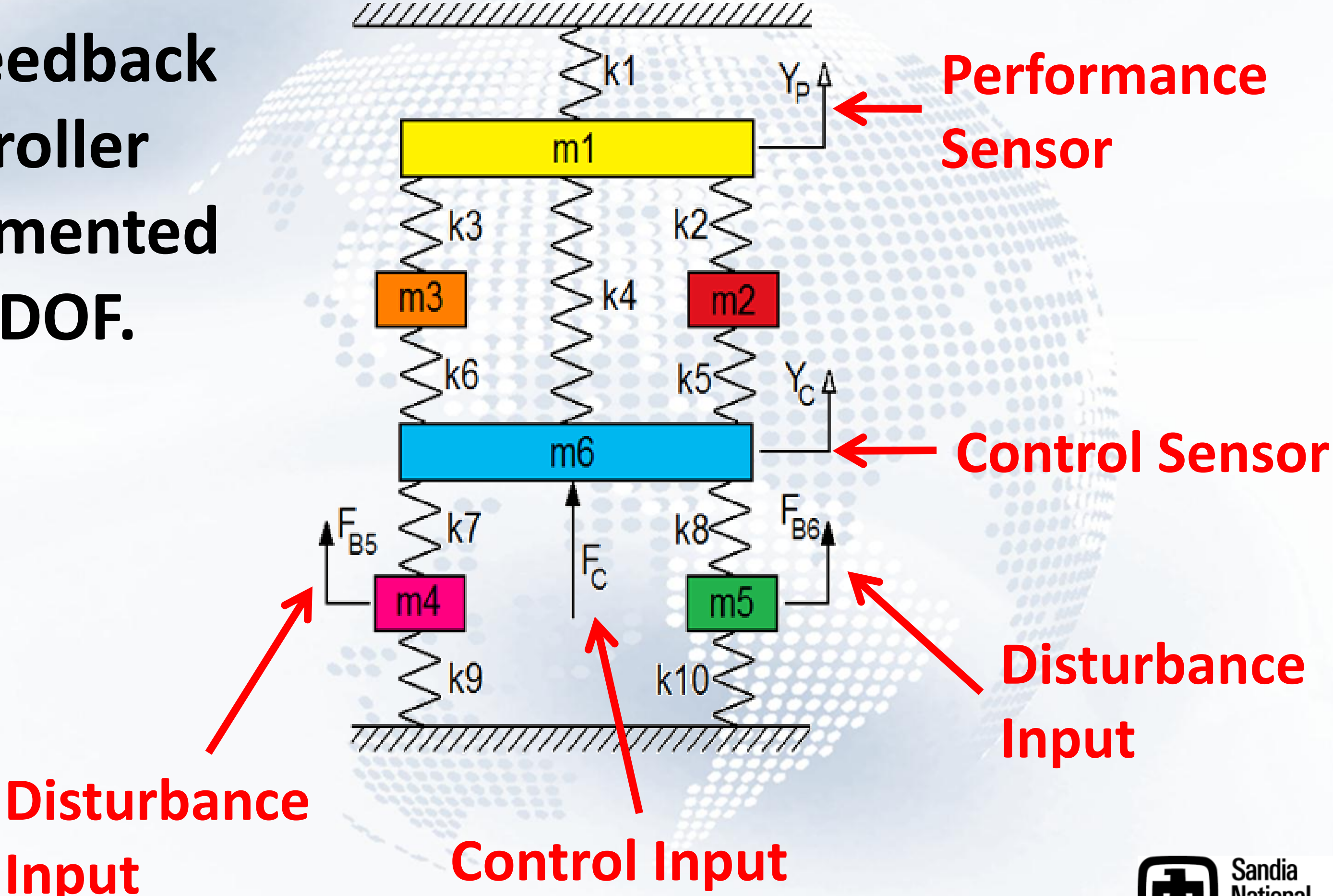
The KLE method sometimes gives non-physical results

- **The KLE doesn't know about the structure…**
  - It simply combines basis vectors in random linear combinations which can give FRFs that aren't physical
- **A screening method is described in another paper at this conference**



A spring-mass example

- **A positive position feedback (PPF) controller was implemented at the 6th DOF.**



Stability metric aims to assure Nyquist stability

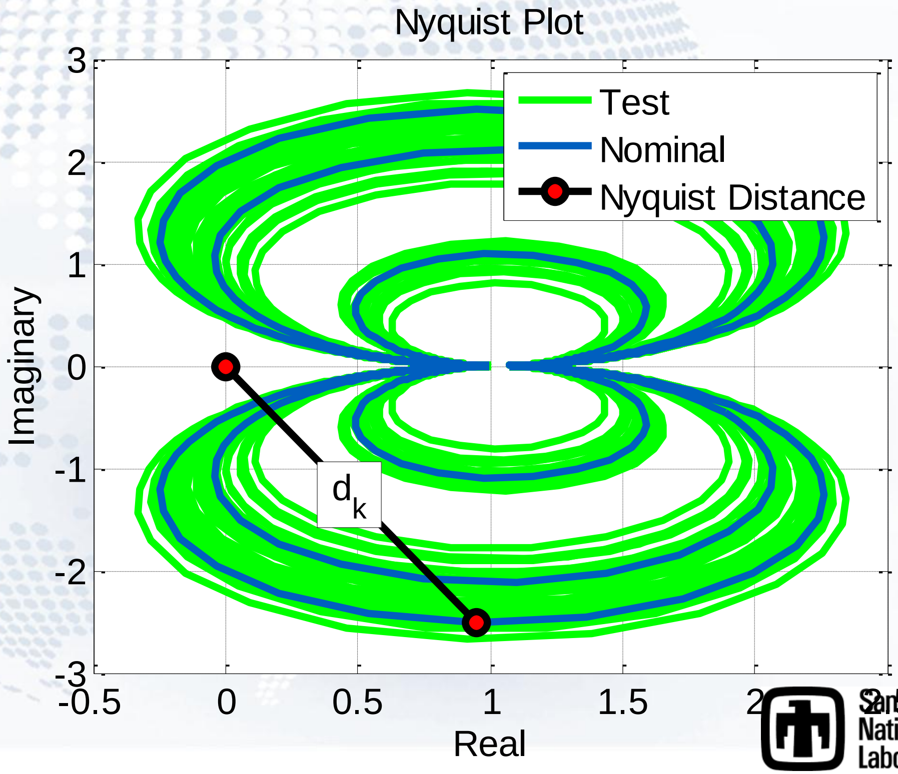
- **This stability metric penalizes curves which are near the critical point**
  - **Maximizes stability**

$$J_s = \frac{1}{\pi}\sum_{k=1}^{n_f} \frac{1}{d_k^{\,2}} \Delta\omega$$

$$d_k = det\left(\boldsymbol{I} + \boldsymbol{G}_k \boldsymbol{Q}_k\right)$$

Open-Loop Plant

Controller

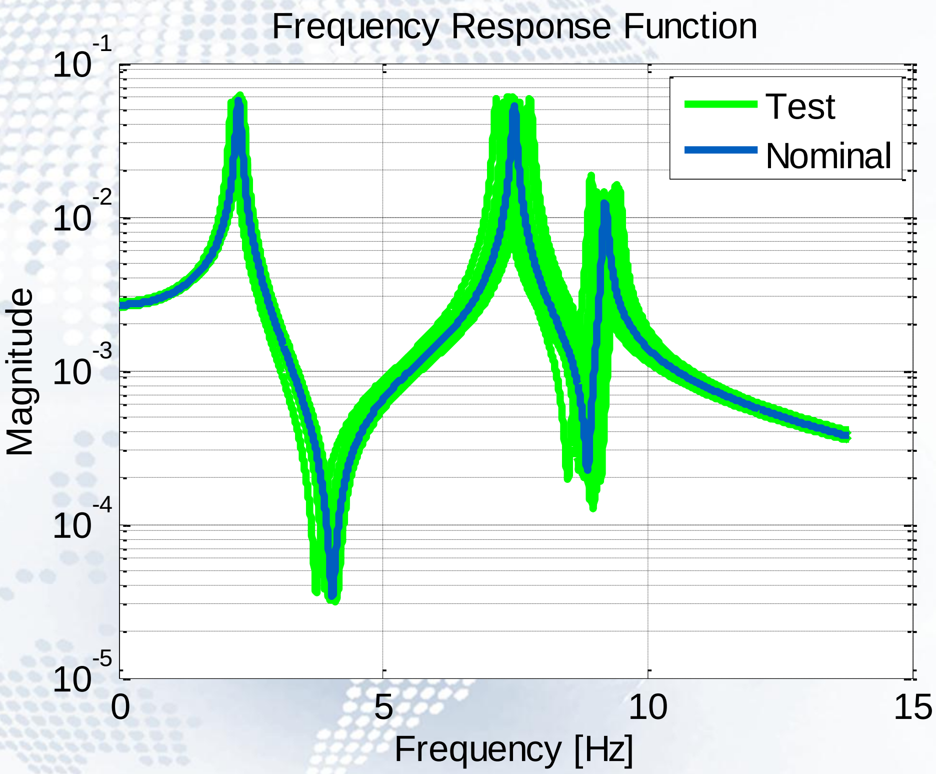


Performance metric aims to minimize response magnitude

- **This performance metric is proportional to the area underneath the FRF magnitude curve**

$$J_p = \frac{1}{\pi}\sum_{k=1}^{n_f} tr[\boldsymbol{H}_k \boldsymbol{H}_k{}^H]\Delta\omega$$

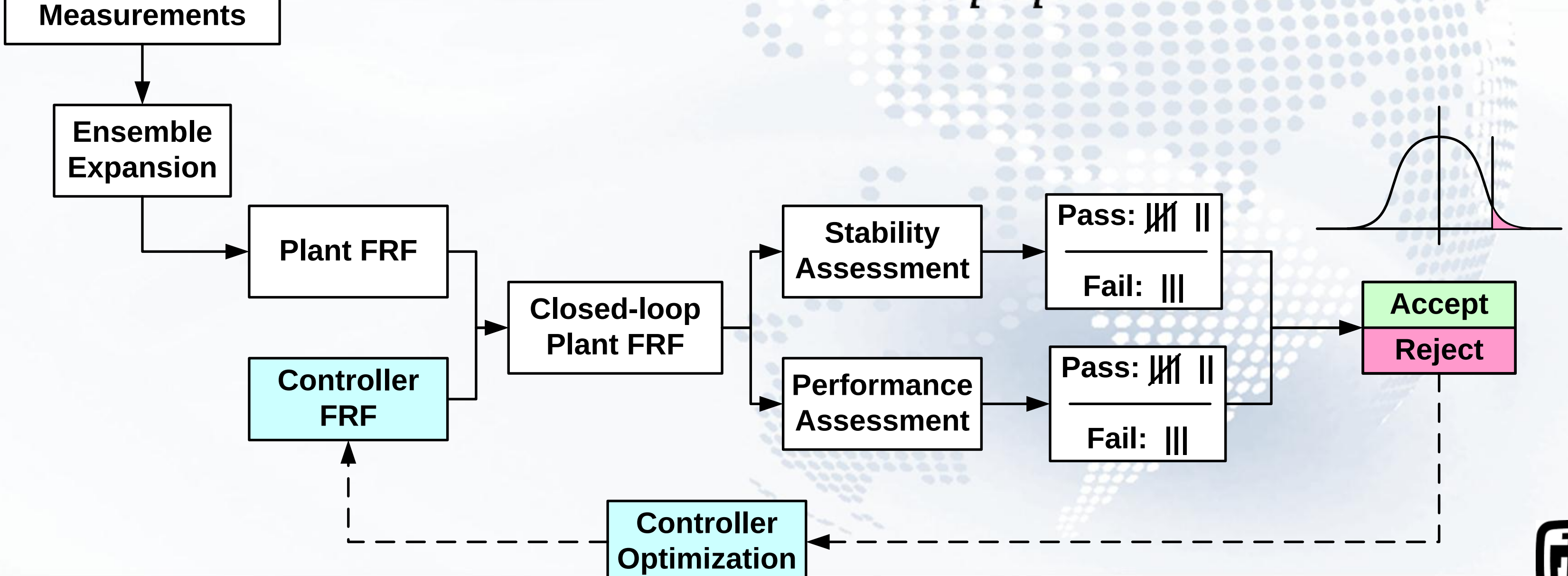
$$\boldsymbol{H}_k = [\boldsymbol{I} + \boldsymbol{G}_k \boldsymbol{Q}_k]^{-1}\boldsymbol{G}_k$$



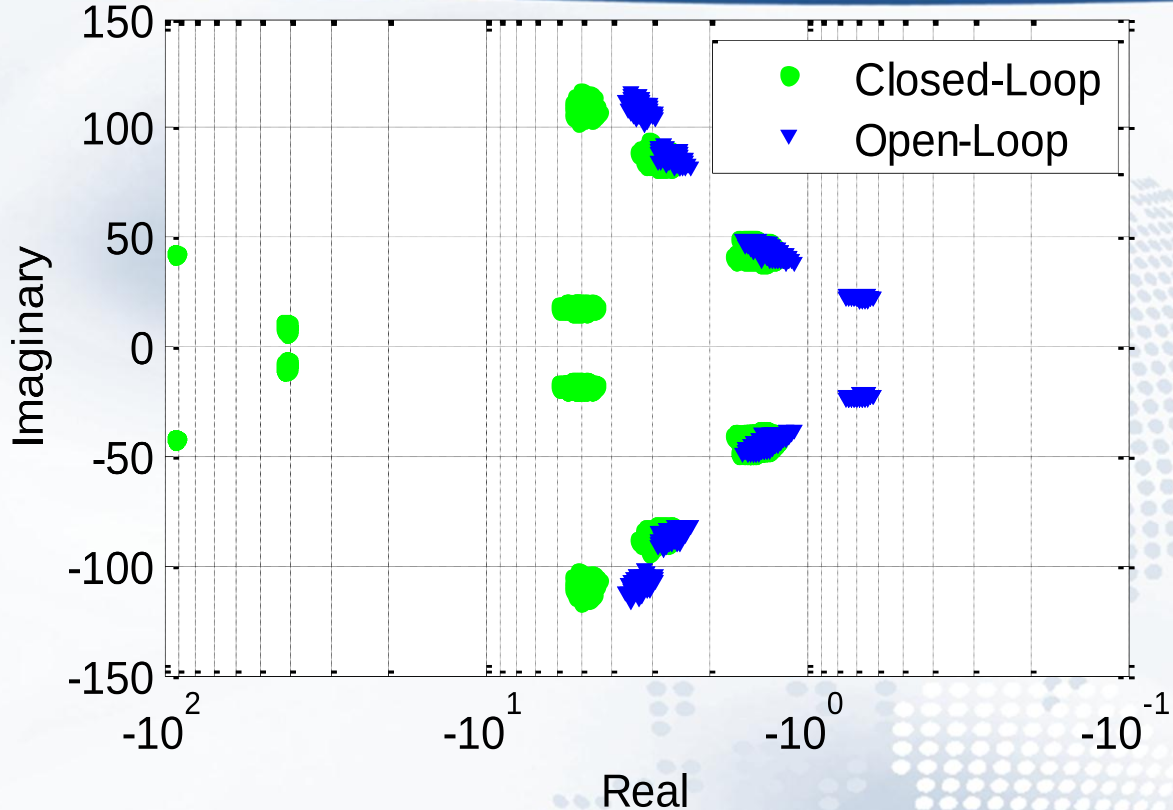
A combined cost function is created by combining metrics

- **Each metric is weighted by its importance in the final result**
- **This combined metric could be used in controller optimization**

$$J = \alpha_p J_p + \alpha_s J_s$$

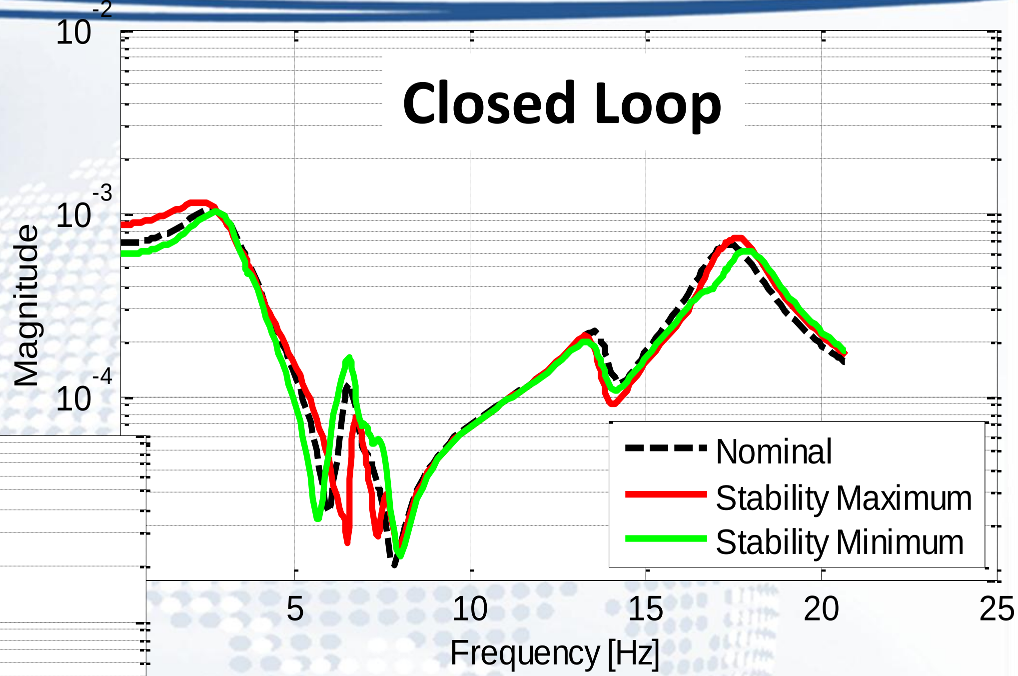


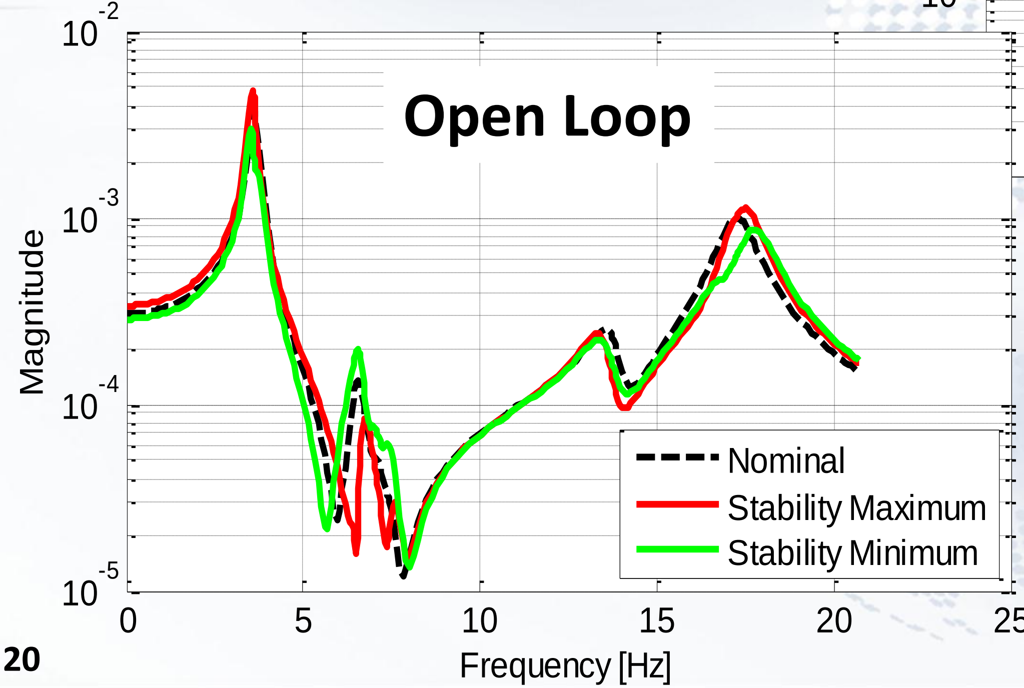
Root-Loci



The loci of eigenvalues for the open-loop and closed-loop systems of randomly generated plants

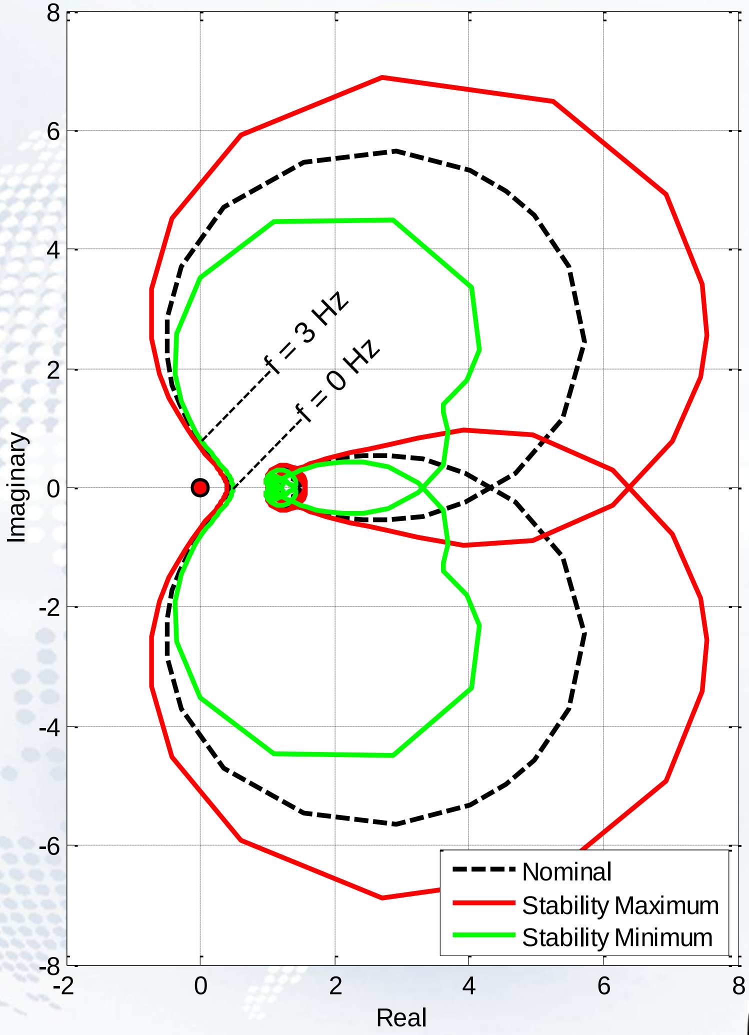
FRF Magnitude



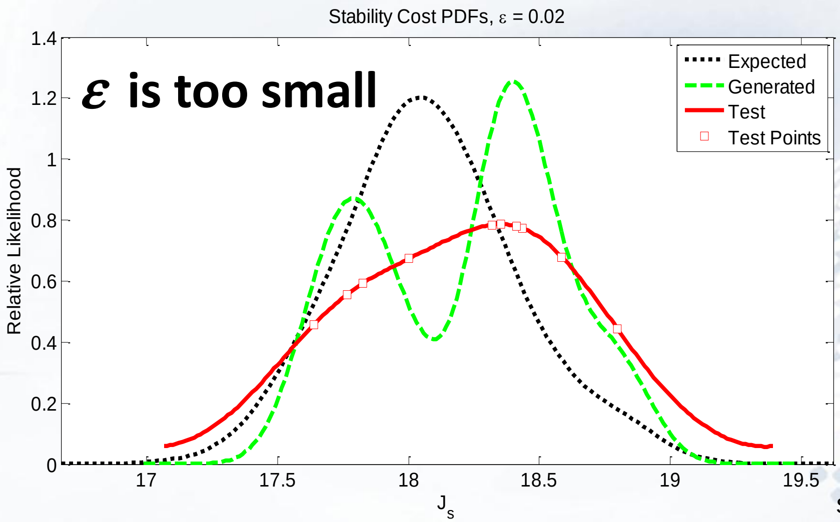


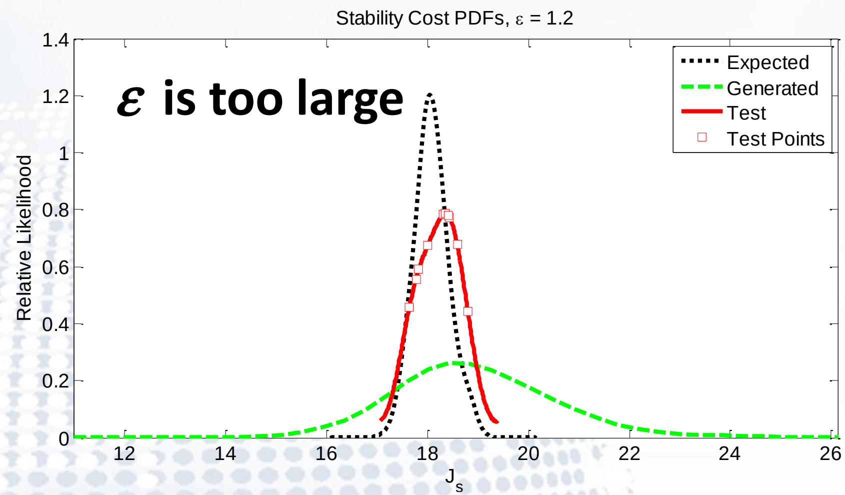
Nyquist Contours

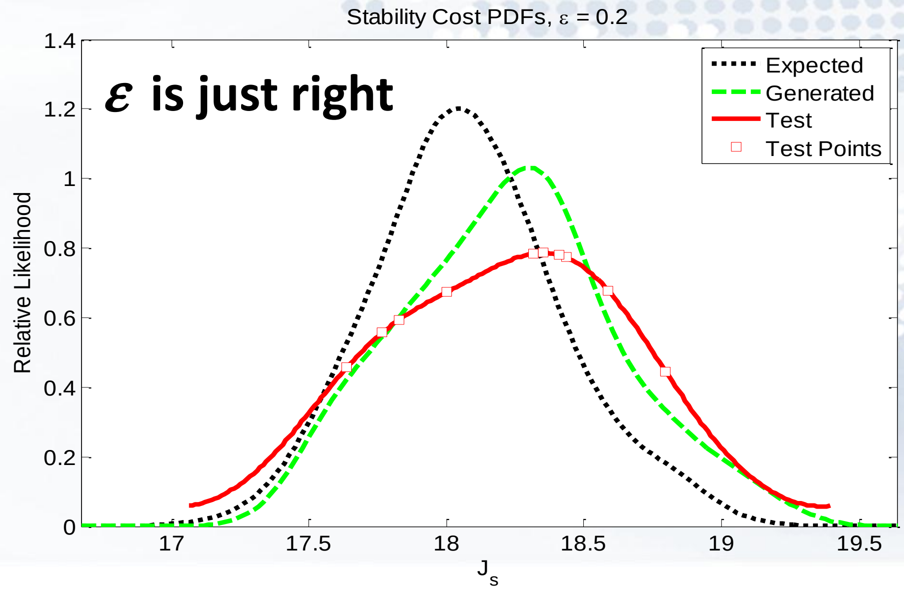
- **Nyquist contours show range of closed-loop performance**
- **System variations were relatively small so the differences are small**



Poor distributions result when ε is not chosen carefully







Conclusions

- KLE does a reasonable job expanding the test ensemble, but it does create some non-physical FRFs
  - This effect seems compounded by closely spaced modes
- The synthetic realizations yield the expected probability density functions from the test data
- Choosing a good value for the smoothing parameter is essential to getting good results
- Certain problems are inherent to the use of small data sets in estimating PDFs
  - A single outlier can skew the estimated distributions

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Acknowledgements

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