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## Testing a Alternative Singles Rate Dead Time Correction Algorithm for use in Neutron Multiplicity Analysis

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### ABSTRACT

Neutron multiplicity analysis based on multiplicity shift register (MSR) logic applied to the pulse train in hardware or to list mode data is an established non-destructive assay technique for the quantification of spontaneously fissile materials. To obtain accurate results requires the data to be corrected for dead time losses. Dead time also influences the observed variance. Dead time correction algorithms are being evaluated at Los Alamos National Laboratory (LANL) as part of a strategic R&D program to enable correlated neutron measurements to be applied with confidence to ever more demanding scenarios, for example; reflective or highly multiplying items. One can treat dead time as part of the detector characteristics in a forward calculation. The most common assay solution or inversion involves first correcting the observed rates and adopting the simple algebraic point-model) rather than running complex forward calculations iteratively.

In examining the present day expressions for the dead time corrected singles, doubles and triples rates derived from MSR histograms according to the widely deployed dead time correction scheme developed by Dytlewski, we find that implicit within them is an alternative form of the singles dead time correction. The correction is assay item specific in that it derives from the dead time perturbed histograms themselves rather than being a function only of the observed singles rate. This means that the simplistic approximate correction for the trigger (singles) rate employed by Dytlewski may be replaced so that an internally self-consistent formalism covering all three multiplicity rates is obtained. A correction factor that depends on the concentration of correlated events present on the pulse train, as encoded in the histograms is to be expected and deals with the in-burst dead time losses. In this work we present the alternative singles correction factor.

Dytlewski is not the only deadtime correction scheme in common use and we examine other approximations and important results to shed light on the bigger problem.

### INTRODUCTION

At high counting rates the correction for dead time losses can contribute a significant uncertainty to plutonium assay by passive neutron coincidence counting. Even at low rates biases creep in

because current dead time correction methods do not account for ‘within burst’ losses. Re-examining dead time treatment to obtain a self consistent formulation therefore forces one to reconsider the foundations of multiplicity analysis and push back the state of the art.

To obtain precise results in a reasonable time neutron multiplicity counters have been developed with exceptionally high efficiency ( $>0.65$  counts per neutron emerging from the item) which is necessary since the probability of observing a multiplet of order  $n$  depends on the efficiency raised to the  $n$ -th power. In tandem the trend has been to design for exceptionally short effective  $1/e$ -dieaway time ( $<25\mu\text{s}$  for polyethylene moderated  $^3\text{He}$  proportional counter based systems) which minimizes the accidental (chance pile-up) coincidence rate which must be subtracted and otherwise severely limits the statistical precision on the higher moments. Together these two achievements have allowed the envelope of measurement problems to be expanded considerably and mean that for highly multiplying metallic items, as one example, the instantaneous counting rate within a fission chain is extremely high and also that the pulse trains are densely correlated and can no longer be approximated as random for the purposes of applying reliable dead time corrections. This work is part of a larger effort directed at understanding and providing practical solution to this challenge [1]. In this work we review the standard approaches and examine the theoretical underpinnings.

## REVIEW OF COMMON METHODS

In Passive Neutron Coincidence Counting (PNCC) the most widely applied means of making corrections for dead time losses to the Singles (Totals or gross) and Doubles (Reals or pairs) rates is to use empirical relationships. The idea of using exponential factors in terms of effective dead time parameters chosen to correct characterization test data was established long ago in the safeguards arena [2] and has served the community extremely well. The form of the Dead Time Correction Factors (DTCFs) is as follows:

$$CF_D = e^{\delta_R \cdot S_m} = e^{(a+b \cdot S_m) \cdot S_m}$$

$$CF_S = e^{\delta_T \cdot S_m} = e^{\frac{1}{4}(a+b \cdot S_m) \cdot S_m} = CF_D^{1/4}$$

The Reals DT parameter,  $\delta_R$ , was initially measured by plotting the normalized logarithm of the net (i.e. Accidentals corrected) Reals rate as a function of Totals rate varied by adding Am/Li sources which boost the random (non time correlated) neutron rate. This approach naturally suggests parameterising  $\delta_R$  in terms of low order polynomial of  $S_m$ , the observed or measured singles rate (i.e. not DT corrected), to accommodate deviations from linearity. The two free parameters  $a$  and  $b$  may also be obtained by counting  $^{252}\text{Cf}$  sources of various strengths and requiring the ratio of DTC doubles to the DTC singles,  $D_c/S_c$ , to be constant as it should because the fissioning system is constant.

This doubles DTCF has a specific functional form which one would like to justify at a more fundamental level. Also we would like to understand the assumed factor of  $\frac{1}{4}$  applied to  $\delta_R$  to get  $\delta_T$  as this is generally not questioned.

In multiplicity counting, where the triples rate is extracted from a shift-register histogram, the most widely used DTC approaches are rooted in the work of Dytlewski [3]. Although Dytlewski shows how, under certain simplifying assumptions, how to derive DTCs for doubles and triples from the histograms given the effective extending dead time,  $d$ , of the system he does not explicitly address the singles rate loss. Instead he assumes a simple first order form, namely:

$$CF_S \sim e^{d \cdot S_m}$$

There are a number of reasons to seek an alternative DT correction formulation for singles counting. For one, the classical empirical form is at least suggestive, through experience, that inclusion of a higher order term would be a beneficial whereas Dytlewski *ad hoc* suggestion uses only a linear term in the exponent. Also we'd ideally like to develop a consistent treatment between traditional PNCC and conventional multiplicity analysis approaches. The doubles and triples rates are derived from signal (single or event) triggered histograms and so the ratio of rates appearing in quantitative assay are independent of the trigger losses BUT it is important to remember that the absolute mass assay is directly dependent on the singles rate correction so it behooves us to get it right.

There is another short coming however and that is for a random neutron source where the DTC doubles and triples rates are zero ( $D_c=0$ ,  $T_c=0$ ) for an ideal counter with a fixed extending deadtime these simple forms in terms only of the measured singles rate do not reproduce the known theoretical result. For an ideal counting system described by a constant extending deadtime of duration  $d$  per event, it is well known [4] that for a pure Poisson emission neutron-source (and hence pulse train) the deadtime corrected singles event rate  $S_c$  is obtained from  $S_m$ , the singles event rate uncorrected for deadtime, as follows:

$$S_c = S_m \cdot e^{d \cdot S_c}$$

This form differs from that suggested by Dytlewski in that  $S_c$  replaces  $S_m$  in the exponent. If the correction factor is not too severe we can replace  $S_c$  in the exponent by  $S_m$  and obtain an approximate form:

$$S_c = S_m \cdot e^{d \cdot S_m} \approx S_m \cdot \left( 1 + d \cdot S_m + \frac{(d \cdot S_m)^2}{2!} + \dots \right) \text{ in the limit } (d \cdot S_m) \ll 1$$

although we shall present a much better polynomial approximation later.

Because the form of the exact expression is transcendental a convenient general iterative solution for applications is to form a nest of exponential functions. For the range of practical applications of interest here the singles dead time correction factor,  $CF_s$ , is not expected to exceed about 1.2214 (say, which would correspond to  $d = 100$  ns and  $S_c = 2$  MHz which would yield what would be considered by an instrument designer to be a somewhat high  $d \cdot S_c$  product) and so the order of the nesting to ensure adequate convergence is not high, seven is the value we have used successfully over a number of years, which yields essentially exact numerical results (better than 0.00023% deviation over the range quoted). So, for practical applications we may write for the ideal random source and extending deadtime case:

$$CF_s(\text{random}) = \frac{S_c}{S_m} = \exp(x \cdot \exp(x \cdot \exp(x \cdot \exp(x \cdot \exp(x \cdot \exp(x \cdot \exp(x))))))))$$

where  $x = d \cdot S_m$

From the exact solution for the random source case we are also able to write an exact Taylor series expansion. This follows from well known results for the Lambert W-function for real-valued W on the main (unique valued) branch. That is we may write:

$$\begin{aligned} CF_s &= \exp(d \cdot S_c) \\ &= \exp\left(x + x^2 + \frac{3}{2} \cdot x^3 + \frac{8}{3} \cdot x^4 + \frac{125}{24} x^5 + \frac{54}{5} \cdot x^6 + \frac{16807}{720} \cdot x^7 + \dots \right. \\ &\quad \left. + \frac{n^{n-1}}{n!} \cdot x^n + \dots\right) \end{aligned}$$

This provides an alternative means to numerical iteration for the exact evaluation in the case of a truly random pulse train subject to DT that obeys the mathematics of ideal extending behavior. Convergence of the series is not especially fast for finite values of  $x$  however, as can be seen by the trend in the value of the coefficients. However, for an operational range of practical concern ( $x < 0.25$ , say) including terms up to the tenth power agreement between the Taylor series expansion and the seven nested exponential form is better than about 0.032% at the top end of the range and this should not limit assay accuracy. However, we have elected to use the nested exponential form in our work. Note that if we compare the functions in the exponents directly we may write the corrected rate in terms of a (better) polynomial in terms of the measured rate as follows:

$$S_c = S_m \cdot \left[1 + x + \frac{3}{2} \cdot x^2 + \frac{8}{3} \cdot x^3 + \frac{125}{24} x^4 + \dots + \frac{(n+1)^n}{(n+1)!} \cdot x^n + \dots\right]$$

It is both interesting and necessary to consider the consequence of when the DT is of the extending type but sampled from a distribution rather than being fixed for all detector pulses. We shall consider a simple single channel counting system, for example a cluster of  $^3\text{He}$  gas filled proportional counters serviced by a single amplifier-discriminator module where the

variation in DT may be conceptually attributed to the distribution of the width of the analog voltage signals feeding the discriminator threshold. We may expect the DT to have a complex distribution of values between some minimum value  $\check{d}$  and some maximum value  $\hat{d}$  governed by the convolution of the internal detector response and the action of the electronics. The pulse shapes from the  $^3\text{He}$  gas filled proportional counters exhibit a wide variation. The shaper in the amplifier and whether unipolar or bipolar circuits are used also plays it part. Recall that in the paralyzable (extending) model of dead time losses for an event to be recorded it must arrive at least a dead time after the pulse preceding it so that the observed singles rate is a consequence of pairs of detection events spaced by at least a dead time apart. The inter-pulse separation, restricting our discussion for simplicity to the case of a Poisson source, is governed by the interval distribution normalized [4] over all positive time,  $t$ :

$$p(t).dt = S_c \cdot e^{-S_c \cdot t} \cdot dt$$

Let us define the probability of observing a DT of duration  $\tau$  after a given event by  $p(\tau).d\tau$ . Then the probability of intervals  $> \tau$ , which is also the probability that an event will be observed, is then obtained from the following combination of probabilities:

$$\int_{\check{d}}^{\hat{d}} p(\tau) \left[ \int_{\tau}^{\infty} p(t).dt \right] \cdot d\tau$$

To proceed we need the distribution of DT values. For illustrative purposes only let us pick a simple rectangular distribution, namely:

$$p(\tau).d\tau = \frac{d\tau}{\hat{d} - \check{d}}$$

Given this distribution we may define the mean DT according to  $\bar{d} = \frac{1}{2} \cdot (\hat{d} + \check{d})$  and the half spread given by  $\Delta = \frac{1}{2} \cdot (\hat{d} - \check{d})$ . Evaluating the integral using standard forms we obtain upon rearranging the DT loss factor as follows:

$$\begin{aligned} \frac{1}{CF_S} &= e^{-\bar{d} \cdot S_c} \cdot \frac{1}{S_c \cdot \Delta} \cdot \frac{[e^{S_c \cdot \Delta} - e^{-S_c \cdot \Delta}]}{2} = e^{-\bar{d} \cdot S_c} \cdot \frac{\sinh(S_c \cdot \Delta)}{S_c \cdot \Delta} \\ &= e^{-\bar{d} \cdot S_c} \cdot \left[ 1 + \frac{(S_c \cdot \Delta)^2}{3!} + \frac{(S_c \cdot \Delta)^4}{5!} + \frac{(S_c \cdot \Delta)^6}{7!} + \frac{(S_c \cdot \Delta)^8}{9!} + \dots \right] \end{aligned}$$

We see that for this special case the form of the DT loss is the same as for the fixed DT scenario but in terms of the mean DT and modified by the function in terms of the variable  $S_c \cdot \Delta$ . Since the half spread value  $\Delta$  is likely to be of the same order as the mean value  $\bar{d}$  for  $^3\text{He}$  proportional counters we see from the expansion in the [ ] brackets that the modifier is fortuitously of second order smallness meaning that even though in real life the dead time is not fixed we are justified in using the ideal functional form – at least over some operational range to achieve adequate

correction for many purposes. For illustration consider a single proportional counter connected to a amplifier-discriminator operating within the vicinity of the existing demonstrated limit of performance for a single amplifier-discriminator. Thus, let us assume  $S_c = 100$  kHz and further let us take notional values of  $2\mu s$  for  $\bar{d}$  and  $1\mu s$  for  $\Delta$ . The value of  $1/CF_s$  then takes on the value of  $0.819.[1 + 0.006666 + 0.000013 + \dots]$ . Thus the bias introduced in this somewhat extreme example of using the mean DT value is only about 0.7% and by picking an effective DT value (a little higher than true) better agreement over a narrow range of interest can be expected. We now have an appreciation of why the results of simple DT loss theory, but with fixed *effective* parameters to take the place of what are in fact physical parameters that follow some distribution have been found to work adequately provided the DT losses are modest.

The simple discussion presented here needs to be extended, of course, to the case of correlated pulse trains and once we step outside first order approximation we lack solid guidance which is why our new studies are needed. Theoretical, experimental and simulation methods may all be gainfully employed in the evaluation. Space prohibits discussion here.

Returning to the classical empirical form

$$CF_s \approx e^{\frac{1}{4}\delta.S_m} = e^{\frac{1}{4}(a+b.S_m).S_m}$$

where we emphasize that the effective Doubles deadtime parameter  $\delta$  is distinct from  $d$ . If we equate terms in this approximate form with those of equal order in the exact expansion for the random (Poisson) source case we find from writing:

$$CF_s \approx \exp(x + x^2 + \dots) \approx \exp\left(\frac{1}{4} \cdot (a + b.S_m).S_m\right)$$

where as previously  $x = d.S_m$  that we can relate  $b$  to  $a$  and both to  $d$  and is so doing reduce the classic empirical form to a single parameter form. Explicitly we obtain:

$$\frac{a}{4} = d, \text{ and } \frac{b}{4} = d^2$$

or alternatively these relations for the real DTC model parameters in terms of the paralyzable DT can be re-expressed as:

$$a = 4.d, \text{ and } b = \frac{a^2}{4}$$

With this exact pick of parameters the approximate correction factor formula using this particular definition of  $\delta$  will, however, for a mathematically ideal counter, will always underestimate the exact value because of the neglect of the higher order terms in the exponent. Of course over a limited range the values of  $a$  and  $b$  can be chosen empirically so that the bias between the true value and the approximate estimate is reduced. Bearing in mind that a closer fit is needed at the

higher end of the counting rate range where the correction differs from unity the most one can choose to emphasis agreement in this region since the magnitude of the absolute deviations in the extreme  $x \ll 1$  region is not important.

To illustrate the accuracy of the exact series expansion consider the case at the upper end of our posited counting range namely  $d.S_c = 0.2$  which corresponds to  $x \approx 0.1634$  and a true singles deadtime correction factor of about 1.2214. If we include only the first two terms in the exponent (which is roughly equivalent to the  $\frac{1}{4}\delta.S_m$  approximation discussed above) the correction factor is underestimated by about 0.94% ( $= 100 \cdot (1 - \frac{\text{Approximation}}{\text{Exact}})$ ). With the inclusion of each additional higher order term the magnitude of the underestimation decreases becoming successively 0.29%, 0.094%, 0.032% and so on. The convergence and agreement is worse for higher  $x$  values (which is typically beyond our range of concern). The extent to which the deviations identified matter in practice is of course not just a consequence of formal mathematical analysis since in the real world the behavior of the system will not obey the simple extending DT theory perfectly and for the pulse trains of interest to PNCC and multiplicity analysis the pulse train is not random. But for completeness it is useful to understand the underlying mathematics presented. For example using  $b = \frac{a^2}{4}$  is a useful starting point for empirical iterative work.

We may take this approximation as means to estimate the dead time correction bias in the reals rate for assay item 2 relative to a reference case item 1 due to uncertainty in the dead time parameter,  $a$ . Assuming the dominant reals dead time loss can be corrected by the factor (adopting shorthand notation for convenience):

$$CF = \exp[(a + b.S).S] \approx \exp\left[\left(a + \frac{a^2}{4}.S\right).S\right]$$

We may write the relative correction factor as follows:

$$\psi = \frac{CF_2}{CF_1} = \exp \left[ a.(S_2 - S_1) + \frac{a^2}{4}.(S_2^2 - S_1^2) \right]$$

The fractional deviation in the relative correction factor due to a change in the value of the dead time parameter  $a$  may then be expressed as:

$$\frac{\psi_2 - \psi_1}{\psi_1} = \frac{\psi_2}{\psi_1} - 1 = \exp \left[ (a_2 - a_1).(S_2 - S_1) \cdot \left\{ 1 + \frac{1}{4} \cdot (a_2 + a_1).(S_2 + S_1) \right\} \right] - 1$$

If we take  $a_1$  as our best estimate of the dead time parameter and set  $a_2 = a_1 + \sigma_{a_1}$ , that is to the value of  $a_1$  incremented by the experimental uncertainty at the one standard deviation level,  $\sigma_{a_1}$ ,

then the given expression gives an estimate of the bias in the real rate for item 2 relative to item 1 due solely to the dead time parameter uncertainty. In an assay based on a linear real calibration curve this will propagate directly about the reference case. This type of analysis has value when for example comparing the real rate from two notionally similar fresh fuel assemblies measured with an active collar containing an Am/Li source to induce fission in the LEU. If one assembly contains poison rods or has had rods removed we are interested in detecting the change relative to the base case as accurately as possible and quantifying the associated dead time loss uncertainty. We see immediately that when  $a$  is well determined ( $a_2 \sim a_1$ ) and the total rates are also similar ( $S_2 \sim S_1$ ) the relative real dead time correction bias will be small. This can be quantified using actual values on a case by case basis. However, for illustrative purposes, suppose we are operating in the somewhat extreme regime of  $a \cdot S \sim 0.25$  corresponding to a real DT correction factor of about  $\exp(a \cdot S + \frac{1}{4} \cdot (a \cdot S)^2) \approx 1.304$ . If the DT parameter  $a$  is determined to about 5% meaning  $(a_2 - a_1) \sim 0.05 \cdot a_1$  and the difference in rates  $(S_2 - S_1) \sim 0.1 \cdot S_1$  and we make the approximation  $\frac{1}{4} \cdot (a_2 + a_1) \cdot (S_2 + S_1) \sim a_1 \cdot S_1$  then we see by inspection that in our expression for  $(\psi_2/\psi_1 - 1)$  the mantissa in the exponential takes on a value of about  $0.05 \times 0.1 \times 1.25$  times or 0.63% of the value used to make the full DT correction. This corresponds to about a  $(100 \times 0.0063 \times 0.266) = 0.17\%$  potential bias in this worked and somewhat contrived example. In practice active collar measurements on fresh fuel is a low counting rate application and does not tax the counting system. But for measurements of spent nuclear fuel for pin diversion the situation could well be quite different in future applications. Additionally we recognize that when we determine the effective dead time parameters  $a$  and  $b$  experimentally the limitations of the dead time correction model cancel out to some extent, and further the parameters are dependent on the dynamic range picked to determine them. How much this might affect the estimates of systematic bias is unclear and would require a case by case assessment.

Another simple yet important result is note worthy of mentioning. It is to do with the benefit of distributing the count rate between a number,  $n$ , of counting chains in order to reduce the overall dead time losses. For simplicity consider a system in which the true count rate  $S_c$  is obtained from a given array of  $^3\text{He}$  gas filled proportional counters serviced by an ideal single amplifier-discriminator. Assume for illustration a fixed extending DT,  $d$ , and a Poisson source. The relation between the true or correct singles rate and the observed or measured rate,  $S_m$ :

$$S_m = S_c \cdot e^{-d \cdot S_c}$$

Suppose that the array can be divided into a number  $n$  of equivalent counting chains each with its own amplifier-discriminator which is the dominant manifestation of DT. In this case the observed rate for the system is:

$$S_m = \sum_n \left( \frac{S_c}{n} \right) \cdot e^{-d \cdot \left( \frac{S_c}{n} \right)} = S_c \cdot e^{-\left( \frac{d}{n} \right) \cdot S_c}$$

Written this way we see that the effective DT parameter for the distributed system is just the DT of a single element divided by the number of equivalent elements in the system, that is to say the effective DT is for the system is  $(d/n)$ . In practice detectors may be ganged together in various numbers to try and get a near even count rate loading given that the efficiency per  $^3\text{He}$  tube varies from inner ring to outer ring or in the case of a non cylindrical moderator location in general. But since we are dealing with small integers the matching is often imperfect. Furthermore, some items, such as inhomogeneous waste and scrap do not give a symmetrical output and this can be emphasized if the item is rotated during the measurement because the rates in each detector bank will not be steady. These effects and the consequence of combining unmatched counting banks on correlated DT losses needs to be examined further both theoretically and experimentally and via empirical numerical simulation.

## EXTENSIONS

When correlations are present on the pulse train we might expect the singles DTCF to not only depend on the true (DTC) singles rate but also to be governed in part by the degree of correlation. This is because the higher the proportion of time correlated events on the pulse train the greater will be the fraction of events with shorter inter-pulse separation and hence the greater the chance of dead time (pileup) loss. Another way to think about this is to consider what we refer to as “within-burst” DT losses. Imagine a weak source of correlated neutrons – for example a spontaneous fission source that decays with the emissions of bursts of neutrons. In the limit that the burst rate tends to zero, that is  $d \cdot S_c \rightarrow 0$  we still expect DT losses because the neutron bursts are detected over a short period of time commensurate with the characteristic lifetime of neutrons in the system. A DTCF that depends solely on  $S_c$  and not on the item specific density distribution (inter-pulse spacing distribution), that is to say the multiplicity histograms or put another way the  $(D_c/f_d)/S_c$ -ratio (where  $f_d$  is the PNCC coincidence gate utilization factor) must only be approximate even within the theoretical construct of the rest of the DT correction formalism (developed for doubles and triples). It follows therefore that to fit into the Dytlewski DTC scheme an approach which is dependent on the accidentals or  $B_i$ -histogram would seem ‘natural’. Recognizing the deficiency and ultimately the empirical nature of the Dytlewski approach provision is made for additional factors (of the type  $\sim 1 + c \cdot S_m$  or  $e^{c \cdot S_m}$ ), that effectively increase the DTCF for the trigger rate for doubles and triples.

In work recently presented elsewhere [5] we have developed an item specific singles DTC derived from the accidentals histogram using Dytlewski’s assumptions and  $\alpha$ -functions. By item

specific we mean the correction will be specific to the degree of correlation and not just the totals rate. On simulated test data for an AWCC (JCC-51) detector we demonstrated this form worked extremely well. The form of the correction is:

$$CF_S = \frac{S_c}{S_m} = \frac{\left( \frac{\sum_{i=1}^{\infty} \alpha_i \cdot B_i}{\sum_{i=0}^{\infty} B_i} \right)}{S_m \cdot T_g}$$

where  $T_g$  is the coincidence gate width.

We have also revisited what we consider to be seminal but overlooked work of Mattes and Hass [6] to obtain DTC easily implementable expressions for a detector with 1/e-decay time  $\tau$  of:

$$S_m = S_c \cdot \left( 1 - \frac{D_c/f_d}{S_c} \cdot \frac{d}{\tau} \right) \cdot \exp \left[ -S_c \cdot d \cdot \left( 1 - \frac{1}{2} \cdot \frac{D_c/f_d}{S_c} \cdot \frac{d}{\tau} \right) + O\left(\left[\frac{d}{\tau}\right]^4\right) \right]$$

$$D_m = D_c \cdot \left( 1 - 2 \cdot S_c \cdot d \cdot \left[ 1 - \frac{D_c/f_d}{S_c} \cdot \frac{d}{\tau} \cdot \left( 1 + \frac{1}{2} \cdot \frac{\theta_2}{\theta_1} \right) \right] \right) \cdot \exp \left[ -2 \cdot S_c \cdot d \cdot \left( 1 - \frac{D_c/f_d}{S_c} \cdot \frac{d}{\tau} \right) + O\left(\left[\frac{d}{\tau}\right]^4\right) \right]$$

We reserve detailed discussion and application of this result for another time but point out it goes a long way to justifying the success of the classical empirical DTC expressions at modest rate while not being a complete solution to the problem.

## CONCLUSION

We have reviewed some important dead time correction theory for neutron coincidence counting applications which as far as we are aware has been applied empirically by the NDA community but never explicitly explained to the user.

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