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## Multiphase Uranium EOS

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We present the results of an empirically derived multiphase Uranium equation of state. The equation of state includes the orthorhombic, bcc and fluid phases. The effects of phase transitions (Clausius-Clayperon, volume changes, specific heats) are treated self-consistently. We will also present comparisons of the equation of state to electronic structure results.

# Multiphase Uranium EOS

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# Introduction

- Each phase is constructed with its own Helmholtz free energy using conventional modeling techniques.
- The phase boundaries are calculated by matching the Gibbs free energy.
- Each phase represents the minimum of the Gibbs free energy.
- All phases have been constrained to the available data,  $\alpha$ -U in particular.

# Three-term Decomposition of Free Energy

- We express the Helmholtz free energy as:

$$F(V, T) = \phi_0(V) + F_{\text{ion}}(V, T) + F_{\text{el}}(V, T)$$

$\phi_0(V)$  cold curve contribution (Rose analytic form)

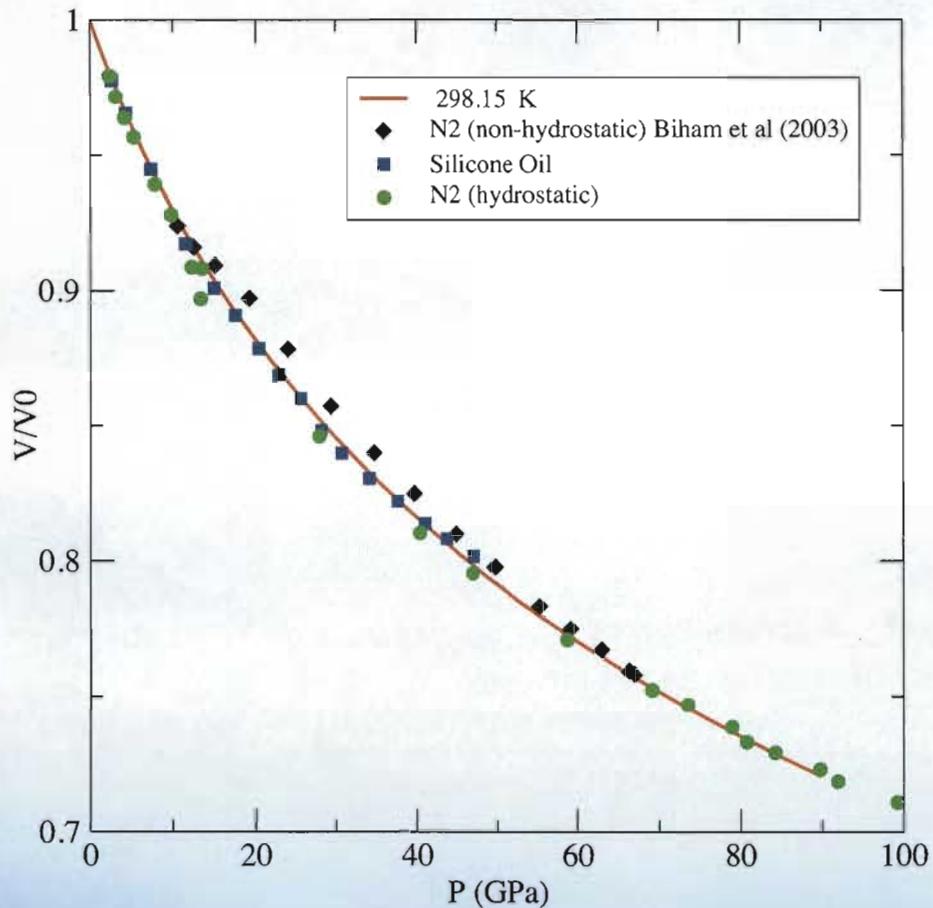
$F_{\text{ion}}(V, T)$  cold + thermal ionic contribution (Debye model with a correction for liquid)

$F_{\text{el}}(V, T)$  thermal electronic contribution (Linear specific heat)

Questions: Is this free energy too simple? Should we include anharmonic, electron-phonon and/or a more complicated electronic part?

# Alpha Phase Uranium

## Room Temperature Comparison to Select DAC Data



### Imposed Model Constraints/Assumptions

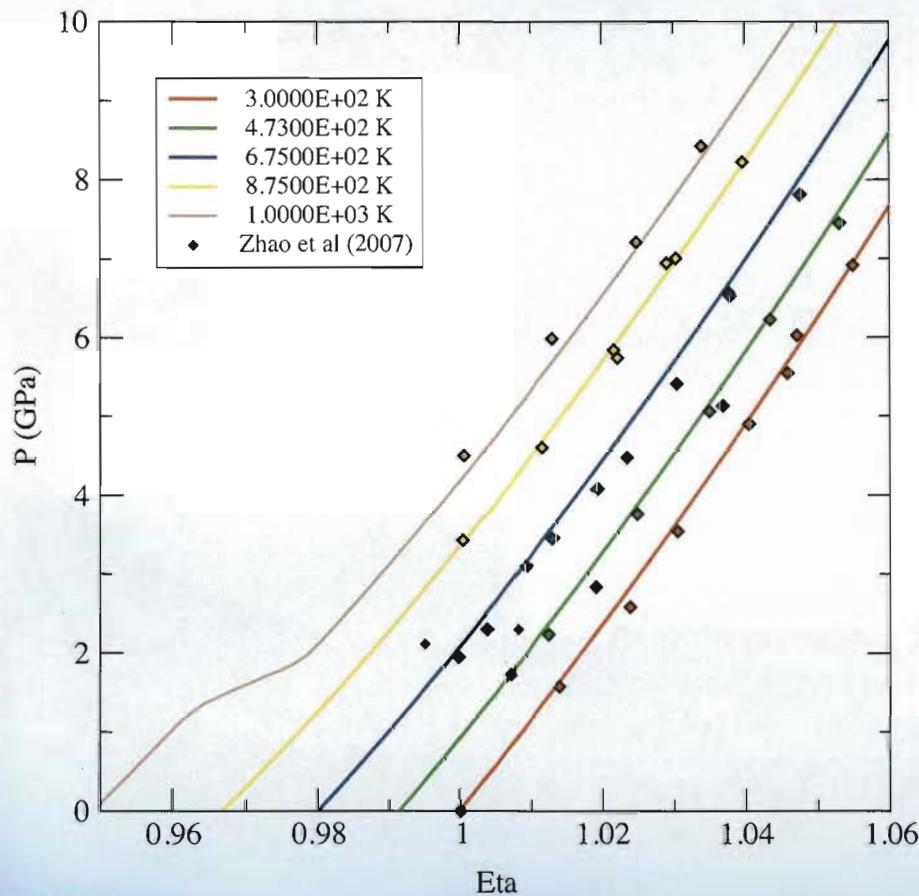
- $B_s$ ,  $dB_s/dP$
- $\rho_{ref}$

Analytic cold curve is adjusted to match data

- $B_s = 114.5$  GPa
- $B' = 5.7$
- $B_T = 111.0$  GPa
- $\rho_{ref} = 19.0429$  g/cc

# Alpha Phase Uranium

## Comparison to Static Compression Diffraction Data



Imposed Model Constraints/Assumptions

- $B_s, dB_s/dP$
- $\rho_{ref}$

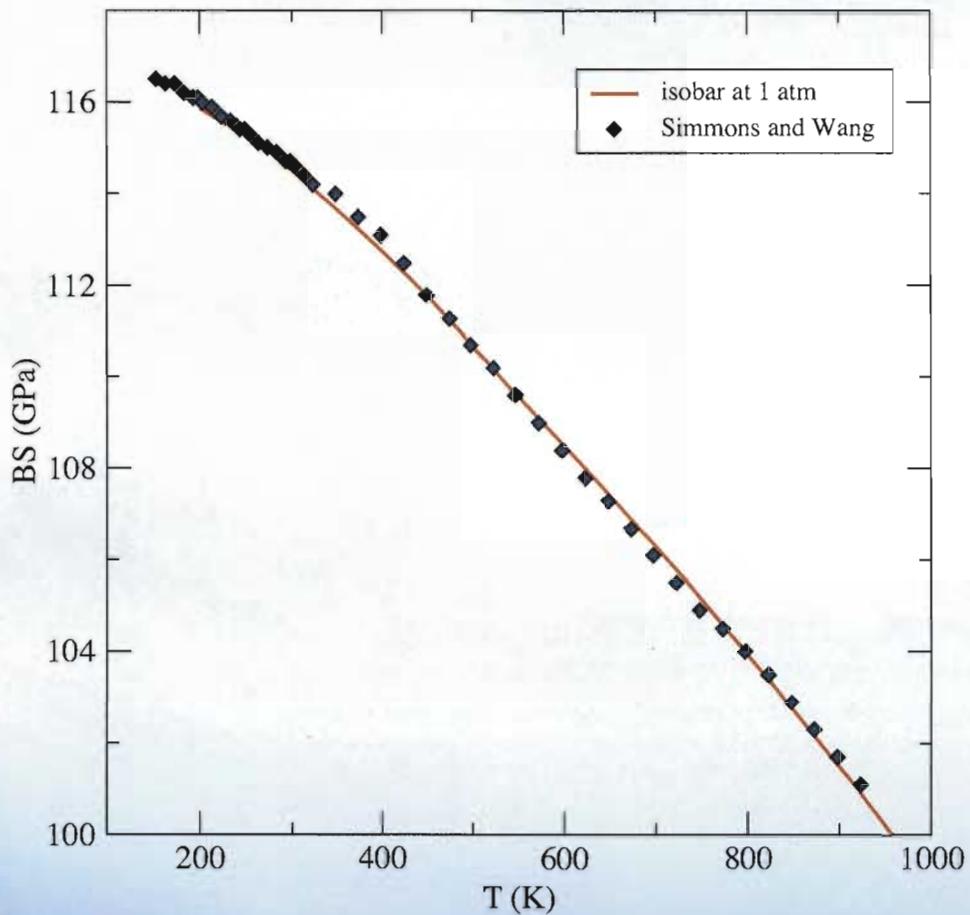
Validates choice for the following

$$\gamma = \frac{\alpha B_s V}{C_p} = 2.16$$

$\frac{d\gamma}{d \ln \rho}$  is small

# Alpha Phase Uranium

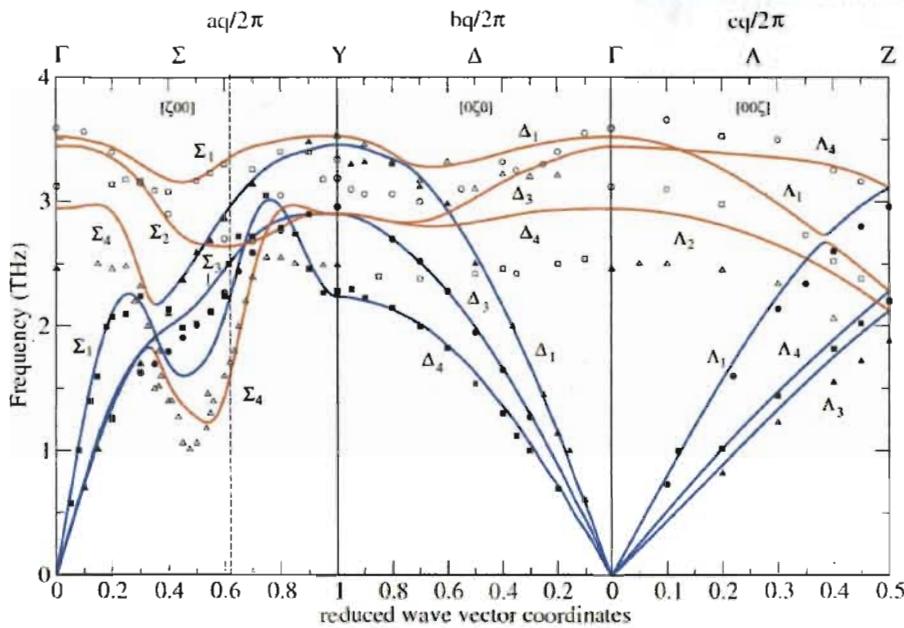
## Comparison to Thermal Dependence of Bulk Moduli Data



### Imposed Model Constraints/Assumptions

- $B_s, dB_s/dP$
- $\rho_{ref}$
- $\Gamma$  – linear specific heat coefficient

# Alpha Phase Uranium Phonons and Thermodynamics

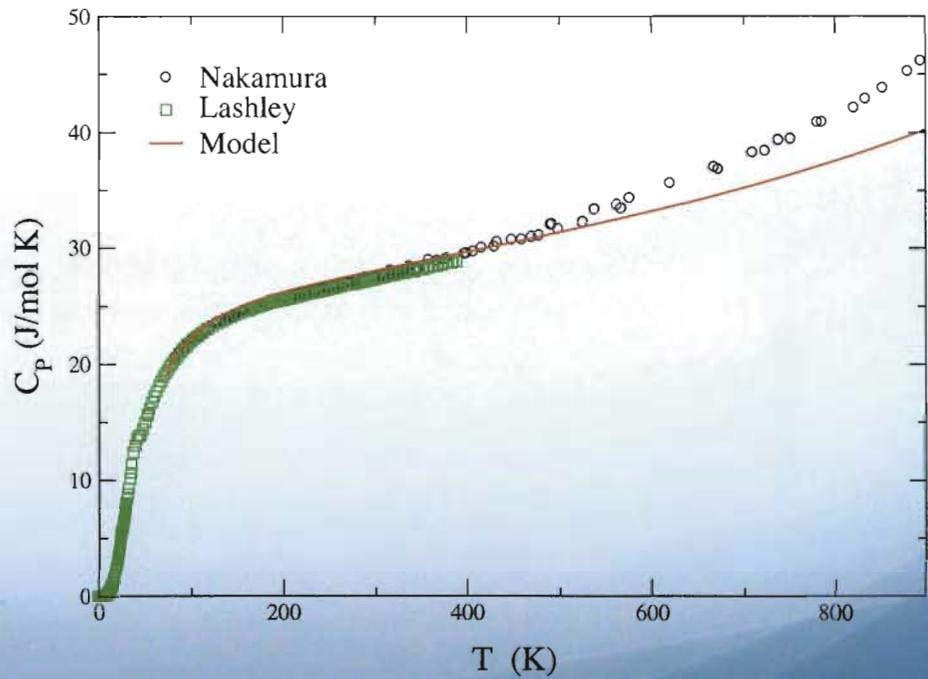


Empirical model based on specific heat, entropy.

Debye T,  $\theta=173$  K.

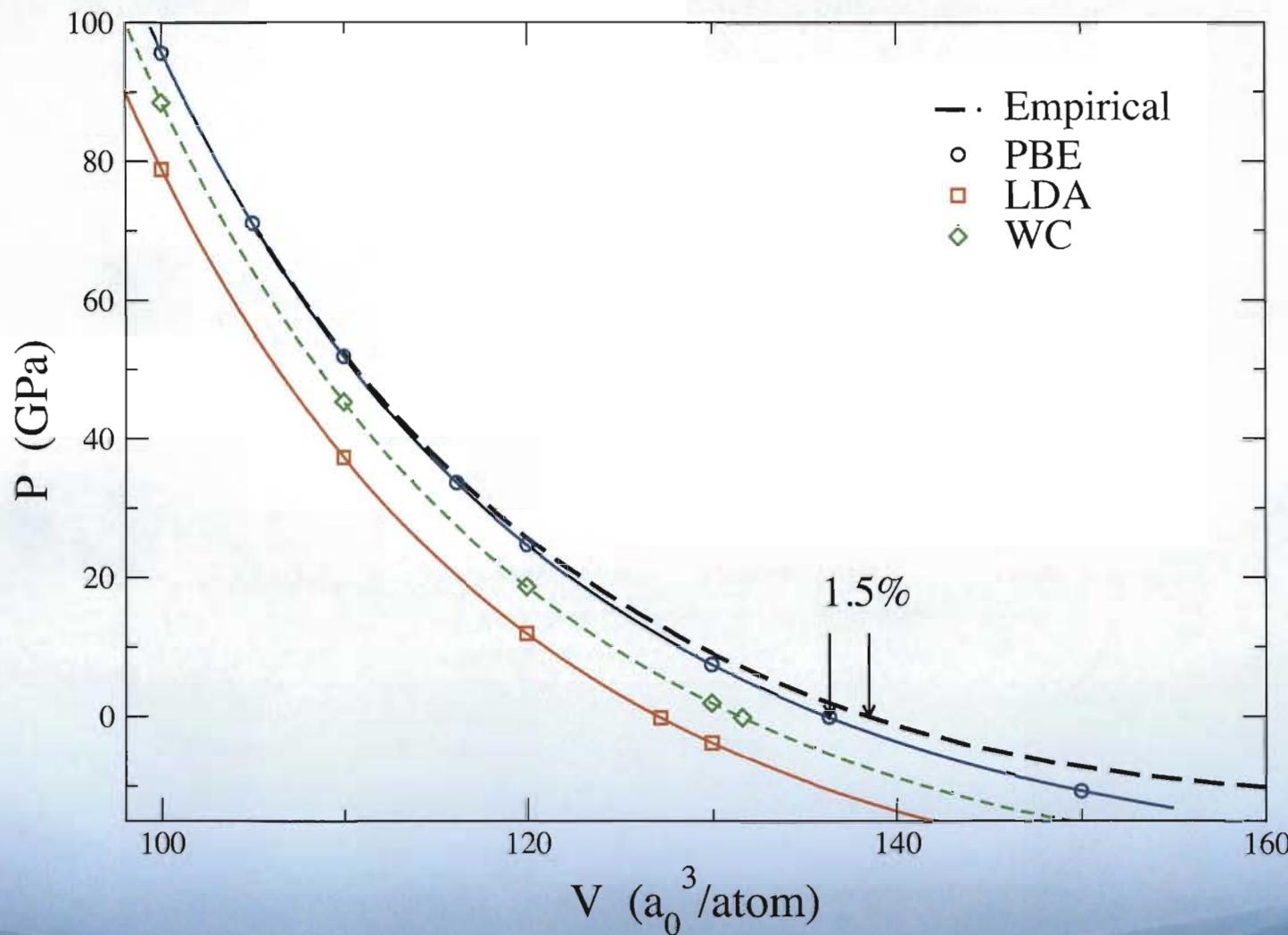
DFT calculation of  $\alpha$ -U phonon dispersion by J. Bouchet, PRB 77 2008, generally agrees well with experiment.

Gives effective Debye T,  $\theta_2=168$  K.



# Cold Pressure – Empirical vs. DFT

DFT provided by Carl Greeff

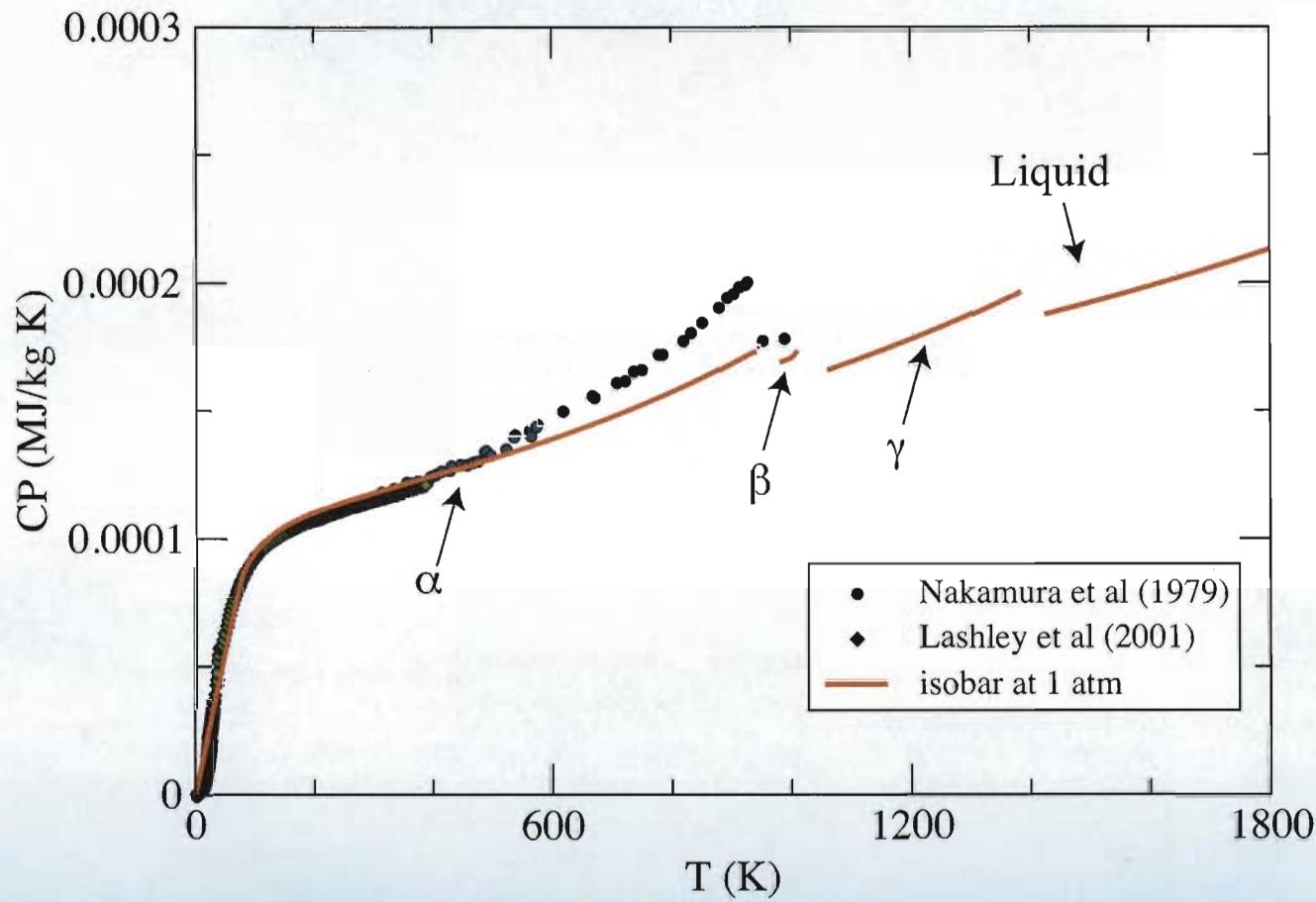


# Alpha Phase Uranium

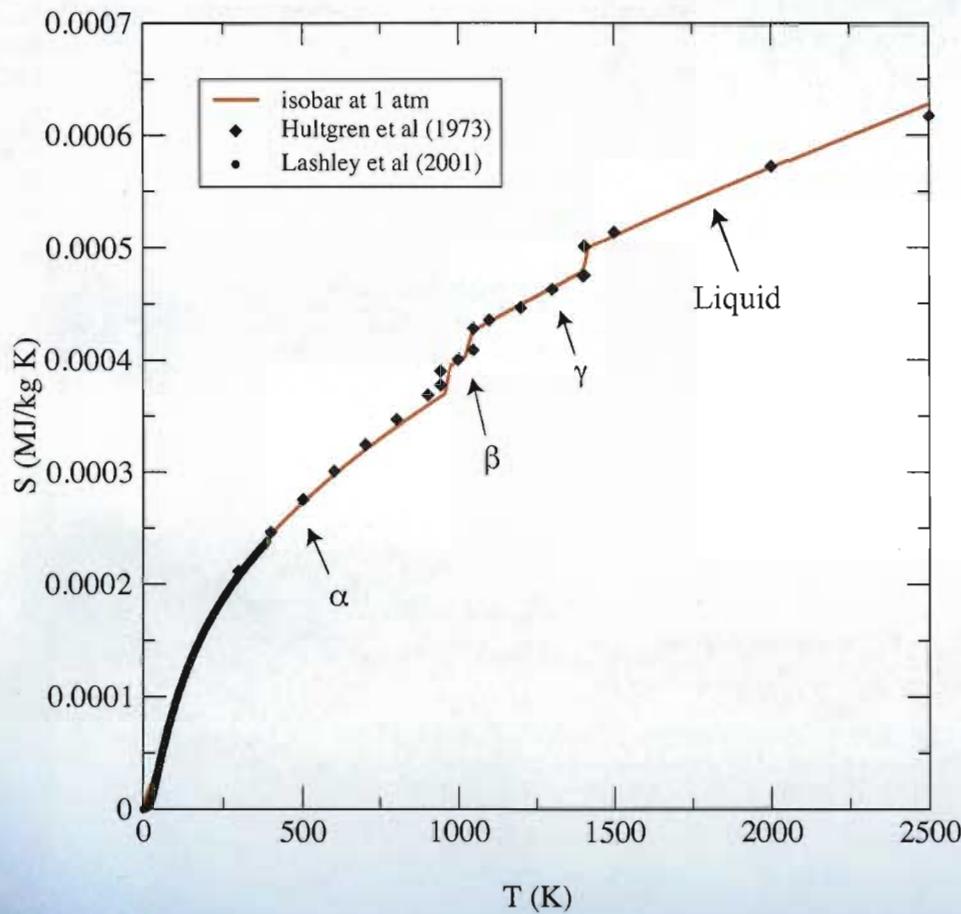
- Alpha Phase
  - Appears to match data
  - Agrees well with DFT
- Next Steps
  - Constrain other phases to available data and bind the phases together using phase boundary data
  - Phase transitions are treated self-consistently (Clausius-Clayperon, volume changes, specific heats)

# Multiphase Uranium

## Comparison to Specific Heat Data



# Multiphase Uranium Comparison to Entropy Data



## Imposed Model Constraints/Assumptions

Relative values of:

- $\theta_{\text{Debye}}$
- $\gamma$

The ratios of the various  $\theta_{\text{Debye}}$  for each phase are set to match the data.

Entropy difference between the phase are:

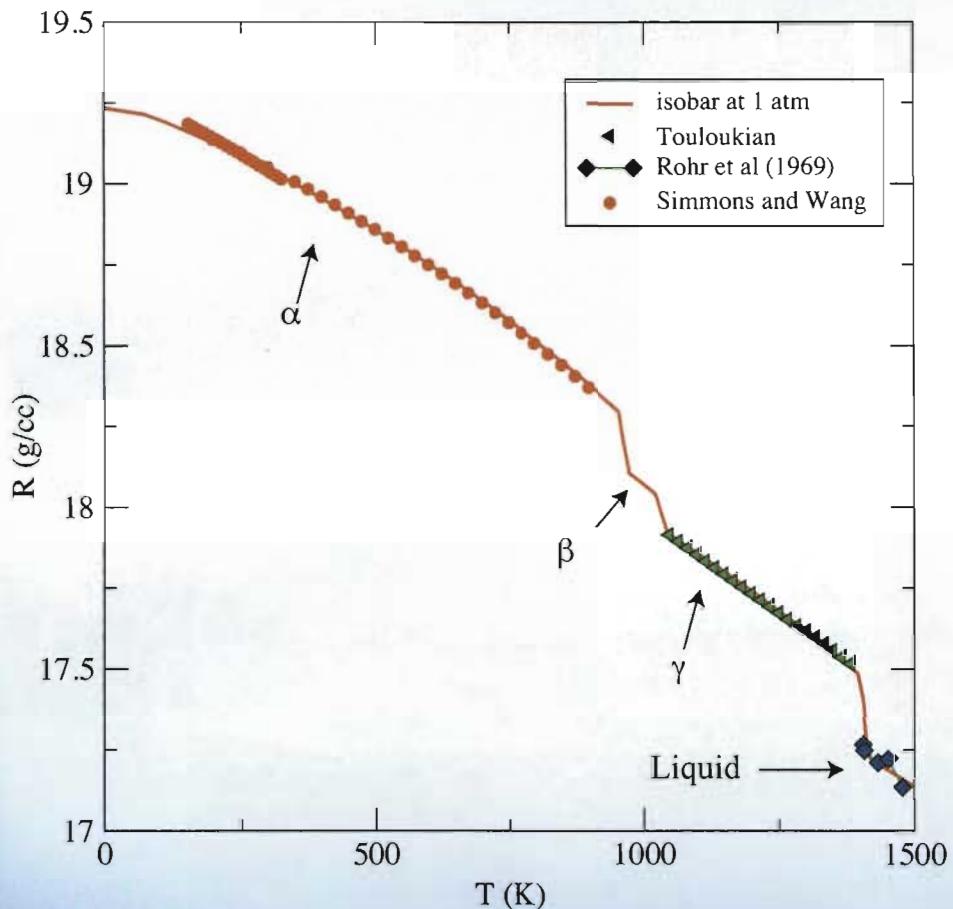
$$\alpha - \beta \quad (\Delta S)_P \approx 0.617 \text{ k}_B/\text{atom}$$

$$\beta - \gamma \quad (\Delta S)_P \approx 0.514 \text{ k}_B/\text{atom}$$

$$\gamma - \text{Liquid} \quad (\Delta S)_P \approx 0.526 \text{ k}_B/\text{atom}$$

# Multiphase Uranium

## Comparison to Density vs. Temperature Data



### Imposed Model Constraints/Assumptions

Relative values of:

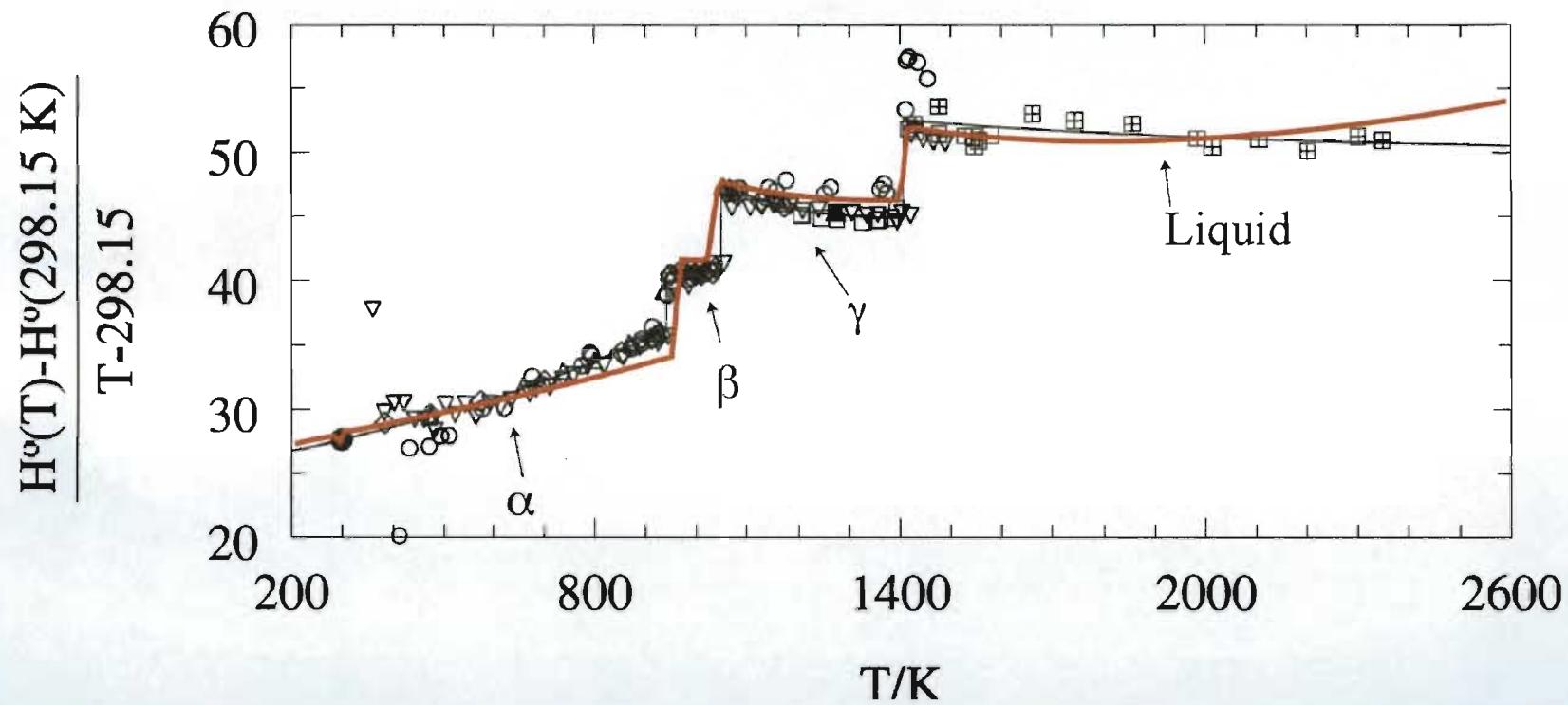
- $\rho_{ref}$
- $\gamma$

The densities for each phase are set to match the data.

Density percent difference between the phase are:

$\alpha - \beta$	0.92 % @ 941 K
$\beta - \gamma$	0.59 % @ 1051 K
$\gamma - \text{Liquid}$	1.36 % @ 1408 K

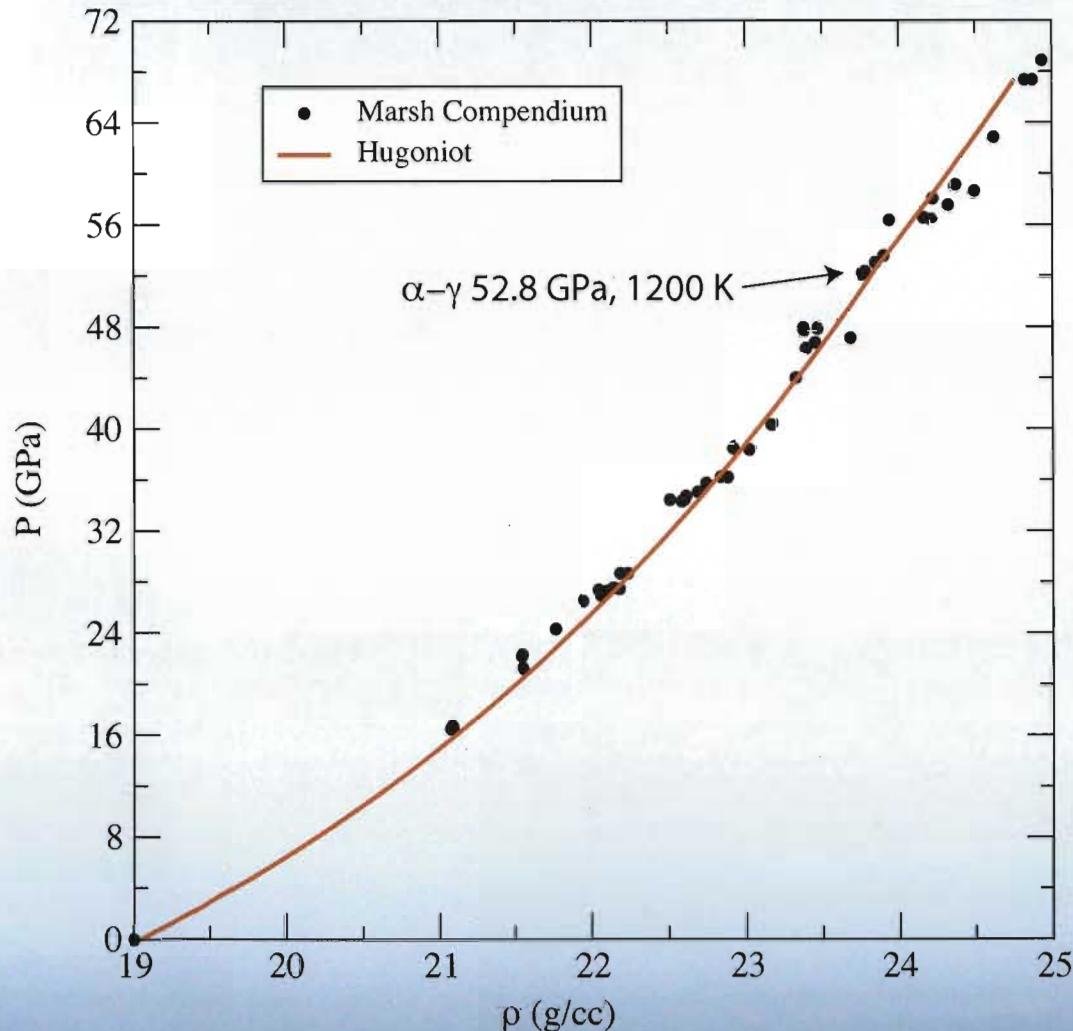
# Multiphase Uranium Comparison to Enthalpy Data



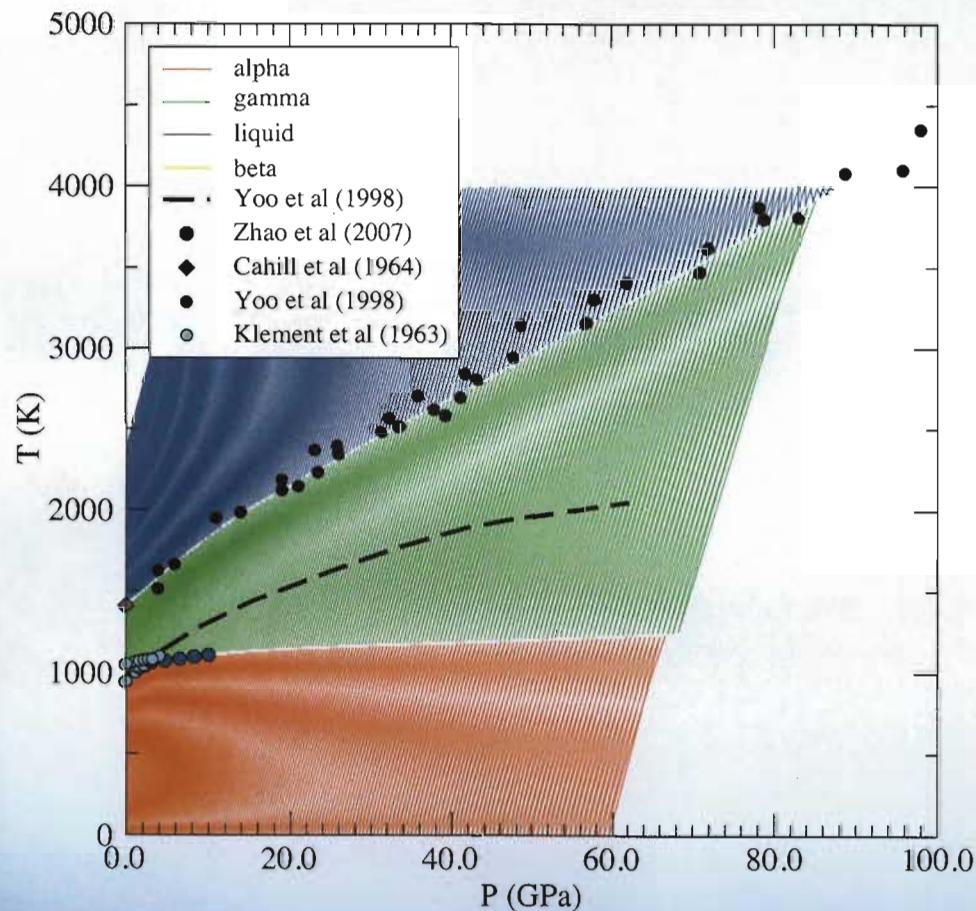
R. J. M. Konings and O. Benes, J. Phys. Chem. Ref. Data, Vol. 39, No. 4, 2010

# Multiphase Uranium

## Comparison to Shock Compression Data



# Multiphase Uranium Phase Diagram Compared to Data



Imposed Model Constraints/Assumptions

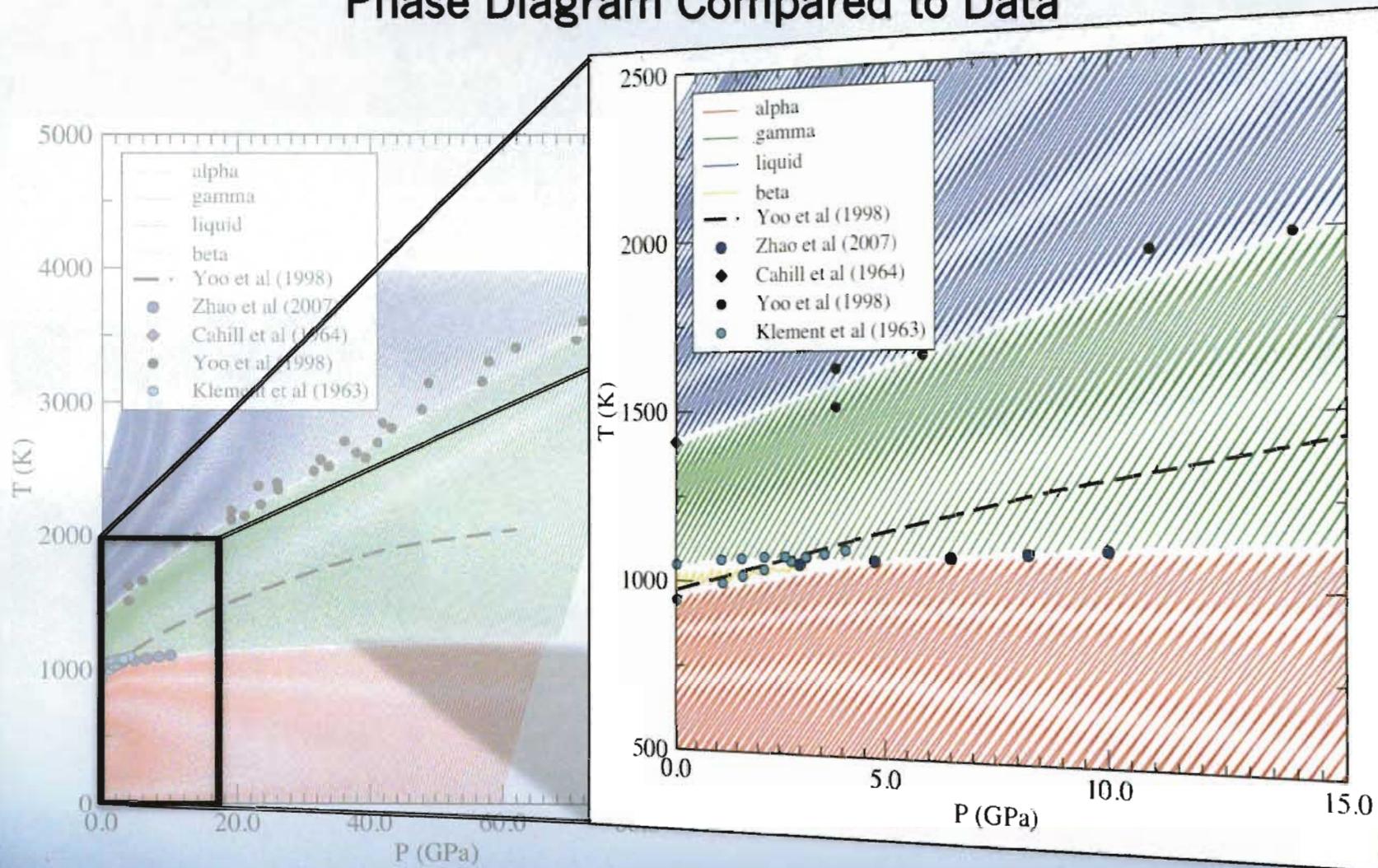
Relative values of:

- $E_0$  – internal energy
- $dB_s/dP$

Constrained to:

$$\frac{dT}{dP} = \frac{\Delta V}{\Delta S}$$

# Multiphase Uranium Phase Diagram Compared to Data



# Conclusions

- We have an excellent thermodynamically consistent representation of the four phases of Uranium.
- More experiments are needed to sample the phases at higher temperature and pressure.
- Would including the anharmonic, electron-phonon, and/or a more complicated electronic contribution to the free energy give us the correct  $C_P$  behavior?