LA-UR- 11-03166

Approved for public release; distribution is unlimited.

Title: Selecting an Optimal Consumption Strategy based on Multiple Reliability Criteria Utilizing a Pareto Frontier

Author(s): Lu Lu
Christine Anderson-Cook

Intended for: Quality and Productivity Research Conference
Roanoke, VA
June 8-10, 2011

Los Alamos NATIONAL LABORATORY EST. 1943

Selecting an Optimal Consumption Strategy based on Multiple Reliability Criteria Utilizing a Pareto Frontier

Abstract

Managers are often faced with difficult decisions about how to balance multiple competing objectives when selecting a best strategy to consume units in their inventory or stockpile. We propose a two-phase decision-making process using a Pareto frontier approach to identify a good consumption strategy for a population of units which age over time, and are to be used in fixed time intervals. The approach selects several good strategies for a subset of representative units in the first phase and then projects the consumption patterns back to the original population and uses them as the starting point to search for a fine-tuned final solution. The Pareto front optimization approach and graphical tools to facilitate improved decision-making are used in both phases of the proposed process. The complete decision-making process is illustrated for a population of single–use nonrepairable units, such as missiles or batteries, while balancing three competing objectives: most consistent reliability for units used across all time intervals, lowest uncertainty to estimate reliability, and highest average reliability.

Selecting an Optimal Consumption Strategy based on Multiple Reliability Criteria Utilizing a Pareto Frontier

Lu Lu and Christine M. Anderson-Cook
Statistical Sciences Group, Los Alamos National Laboratory

Quality and Productivity Research Conference - June 2011

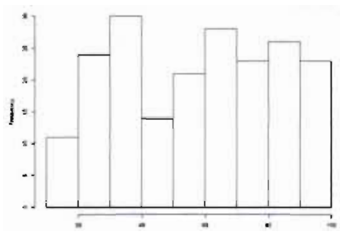
LA-UR **-*****

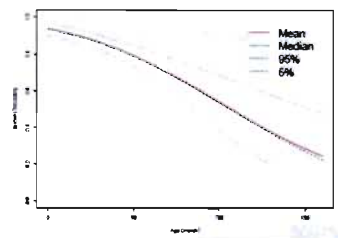
Outline

- Motivating problem - how to choose a “best” strategy when consuming a collection of units while balancing multiple competing objectives based on reliability simultaneously
- Measures for multiple objectives – translating features into quantitative measures
- A two-phase decision-making process utilizing the Pareto frontier approach – select a palette of strategies and provide information on trade-offs between criteria and robustness of strategies to different choices of relative importance of criteria specified by the user to make an informed decision
- Conclusions

Problem Statement

- We have a stockpile of 200 single-use non-repairable units (batteries/flash lights) with known ages that are going to be used at a rate of 50 units every year for four years
- Current strategy: a convenience sample – go to the warehouse and take first ones found
- How can we identify a "best" strategy for using the units?
- Background Information:





Model and Analysis Basics

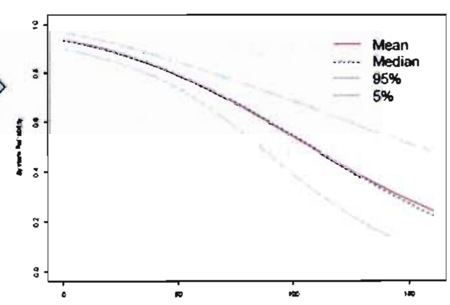
- Sample Data
  - 227 units tested at different ages
  - Response: Pass or Fail for each unit
- Model
  - Probit regression

    $Y_i \sim \text{Bernoulli}(p_i)$

    $p_i = \Phi(\beta_0 + \beta_1 A_i)$
- Analysis
  - Bayesian approach
  - Diffuse priors

    $\beta_0, \beta_1 \sim \text{Uniform}(-1000, 1000)$

Reliability Summary



Multiple Objectives based on Reliability

- p_{ij} = Probability that the j-th unit will work if it is consumed at the end of the i-th year
- The main quantity of interest is the success rate of units consumed at each time interval:

$$R_i = \sum_{j=1}^{50} p_{ij} / 50, i = 1, \cdots, 4$$

- Multiple objectives:
  - ➢ Most consistent success rate over all time intervals
  - ➢ Least uncertainty associated with estimation
  - ➢ Highest average success rate

Measures for Multiple Objectives

- $\hat{R}_{ik}^B = \sum_{j=1}^{50} \hat{p}_{ijk}^B / 50$ is the estimate of R_i using the *k*-th draw of parameters from MCMC simulation
- Bayesian estimate of R_i and its associated uncertainty

$$\hat{R}_i^B = \sum_{k=1}^{M} \hat{R}_{ik}^B / M, \quad s(\hat{R}_i^B) = \sqrt{\sum_{k=1}^{M} (\hat{R}_{ik}^B - \hat{R}_i^B)^2 / (M-1)}$$

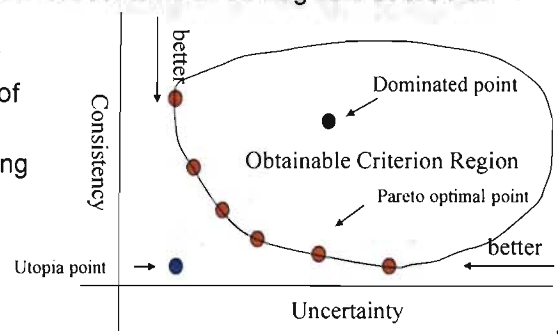
➢ Average over time: $\bar{\hat{R}}^B = \sum_{i=1}^{4} \hat{R}_i^B / 4$

➢ Consistency over time: $\sqrt{\sum_{i=1}^{4} (\hat{R}_i^B - \bar{\hat{R}}^B)^2 / 3}$

➢ Overall uncertainty: $\sum_{i=1}^{4} s(\hat{R}_i^B)$

Pareto Frontier Approach – Pareto Optimization

- One strategy *Pareto dominate* another strategy if all its criteria values are no worse than the other's and at least one of its criteria values is strictly better than the other's
- A strategy is *Pareto optimal* if and only if no other strategy dominates it; a Pareto set contains all strategies that are Pareto optimal
- The *Pareto frontier* consists of the set of points of criteria values corresponding to the set of Pareto optimal strategies



UNCLASSIFIED

Utopia Point Approach

- If the front is comprised of a large number of points, if must be reduced to a manageable set of points for further decision-making
- The *Utopia point approach* ranks Pareto optimal strategies according to their proximity to the *Utopia point* (an 'ideal' situation which is optimal for all criteria) based on a chosen metric and choose the strategies that are closest to "ideal"
- A fine mesh of all possible weight combinations for different criteria are evaluated to study the impact of relative importance of individual criteria specified by the user on which strategy is selected based on the given weights
- A smaller set of strategies are selected which are optimal for at least one of possible weights

Pareto Front vs. Desirability Function

- We use the weighted absolute distance metric which is equivalent to the additive desirability function
- The desirability method requires repeated optimizations for each set of weights but the Pareto approach conducts the Pareto optimization search once and then allow flexible exploration of different weights and metrics with little extra computational effort
- A rich set of graphical summaries can be used to facilitate final decision-making by revealing the trade-offs between criteria and exploring the robustness of different strategies to different weight choices and the sensitivity of final solutions to different metrics and scaling schemes (will be illustrated later)

Pareto Optimization Search Algorithm

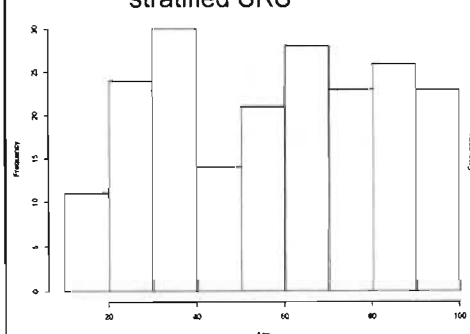
- Step 1: Pareto search based on a single random start
  - Randomly generate a usage strategy as a starting point
  - Randomly select two units from two different time intervals and create a new strategy by swapping the two units
  - Replace the current "best" strategies by the new one if the new one is strictly better
  - A Pareto front and the Pareto set of strategies are populated along the searching process
  - The search stops when there is no update of current "best" strategy for a pre-specified number of swaps
- Step 2: Repeat Step 1 for multiple random starts and combine results to obtain a cumulated Pareto front and a set of Pareto optimal strategies

A Two-phase Decision-making Process

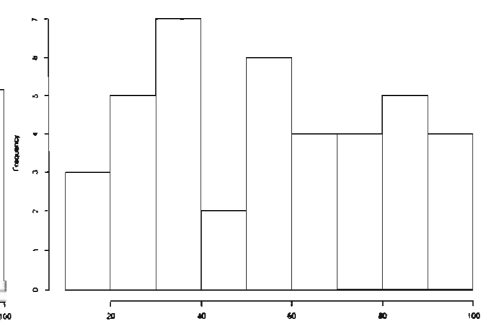
- **Phase I**:
  - ➢ Select a subset of representative units from the population
  - ➢ Select a set of optimal strategies using the Pareto frontier approach
  - ➢ Project each of the selected strategies from Phase I to the original population
- **Phase II**:
  - ➢ Use the projected strategies as the starting points for the Pareto optimization search for the original population to obtain fine-tuned solutions

Phase I – Selection of a Subset of Units

- Select a subset of 40 units as a representative sample of the original population using simple random sampling (SRS) or stratified SRS



Original Population

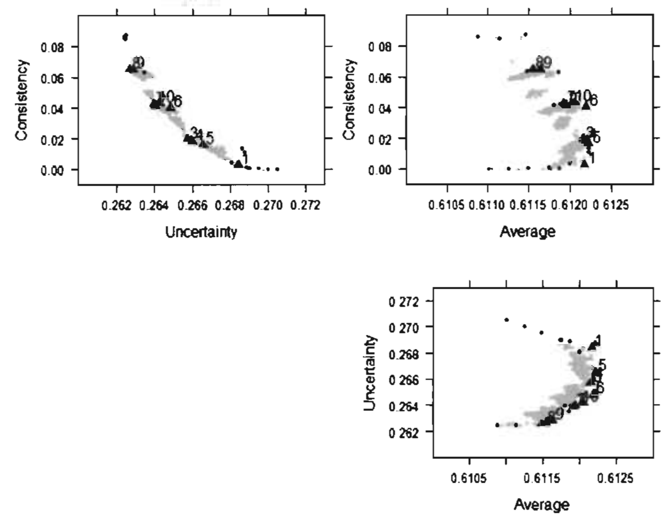


A representative sample

UNCLASSIFIED

Phase I – Pareto Optimization for the Subset

- Pareto front for the subset of units



Los Alam
NATIONAL LABORA
Operated by the Los

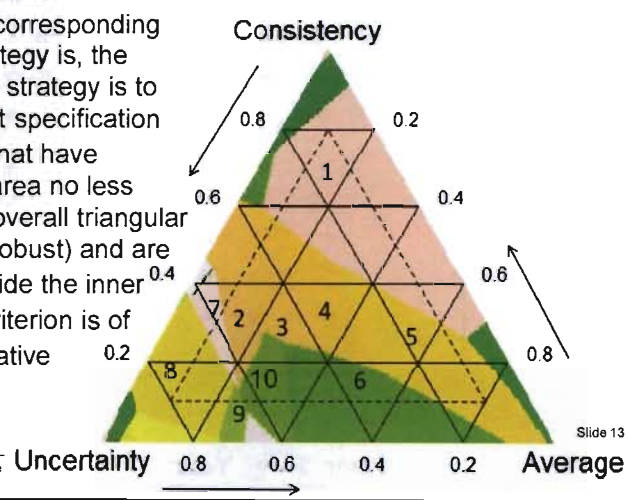
Slide 12

NNSA

UNCLASSIFIED

Phase I – Graphical Summaries

- The **mixture plot** (Cornell, 2002) showing the weight distribution for strategies selected using the Utopia point approach
  - The bigger the corresponding region for a strategy is, the more robust the strategy is to uncertain weight specification
  - The strategies that have corresponding area no less than 1% of the overall triangular area (not least robust) and are not entirely outside the inner triangle (each criterion is of at least 10% relative importance)



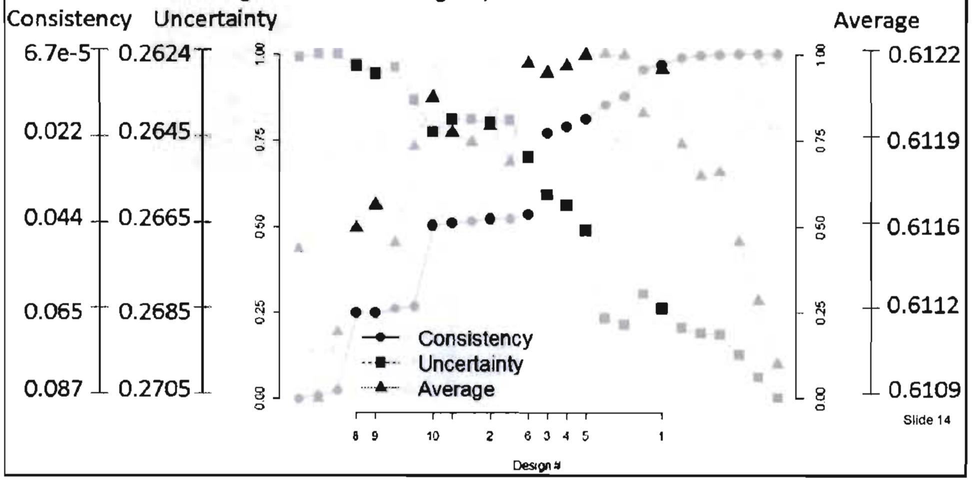
Los Alamos
NATIONAL LABORATORY
Operated by the Los Alamos National Security, LLC for the

Slide 13

UNCLASSIFIED

Phase I – Graphical Summaries (cont'd)

- The trade-offs between criteria among selected strategies
  - Scale the criteria values to between 0 (worst) and 1 (best among selected strategies)

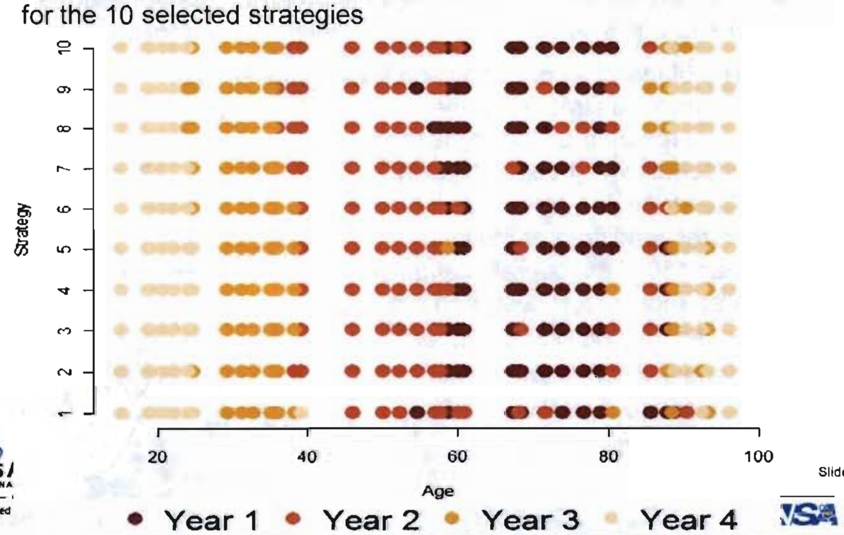


Slide 14

UNCLASSIFIED

Phase I – Graphical Summaries (cont'd)

- Current age of units with different colors for different time intervals for the 10 selected strategies

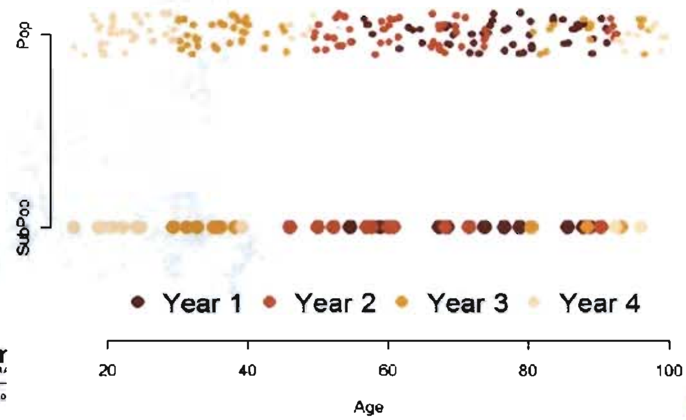


Slide 15

UNCLASSIFIED

Phase I – Project Back to Original Population

- Project selected consumption patterns to the original population
  - sort the subset and the original population by age
  - Match every unit in the subset to every five units in the original population by the relative age position in their population

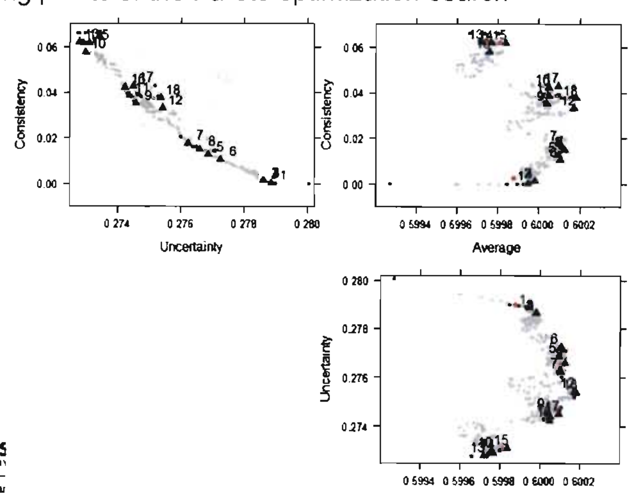


Slide 16

UNCLASSIFIED

Phase II: Pareto Search for Fine-tuned Solution

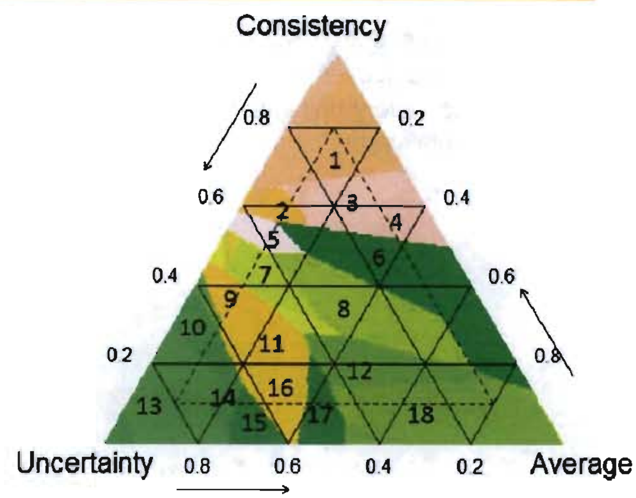
- The Pareto front obtained using the projected strategies as the starting points of the Pareto optimization search



Slide 17

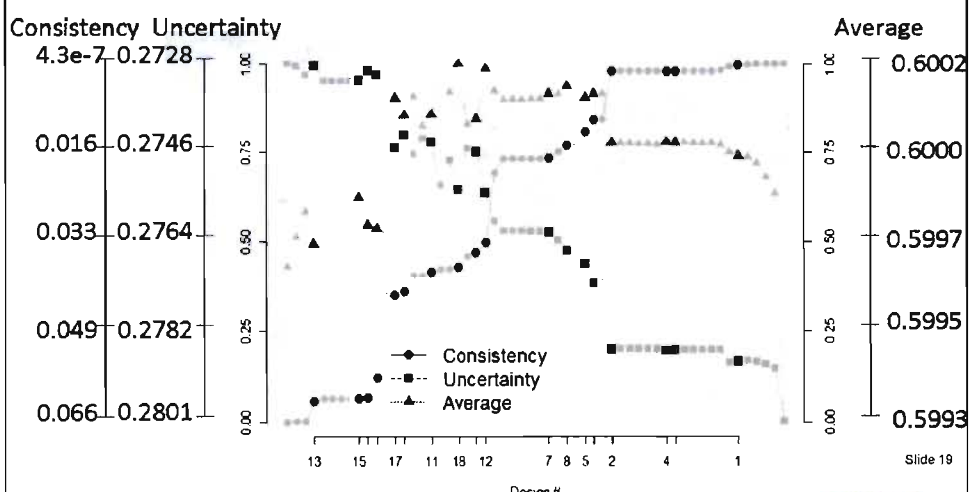
Phase II – Graphical Summaries

- The Mixture plot



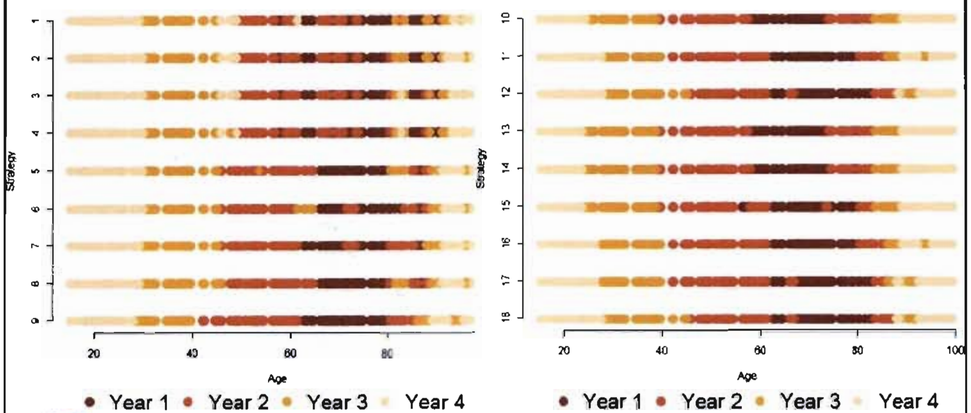
Phase II – Graphical Summaries (cont'd)

- Trade-off plot by using to (0,1) scale based on selected strategies

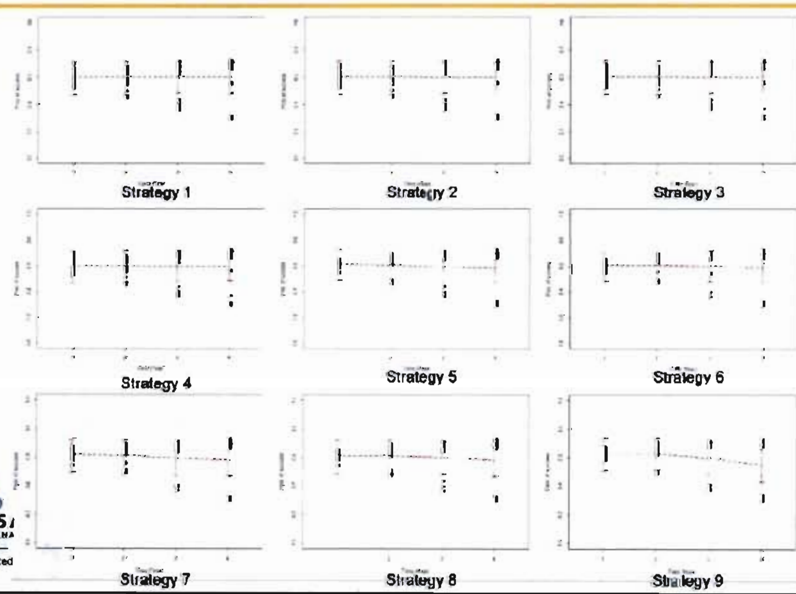


Phase II – Graphical Summaries (cont'd)

- Current age of units with different colors for being used at different time intervals

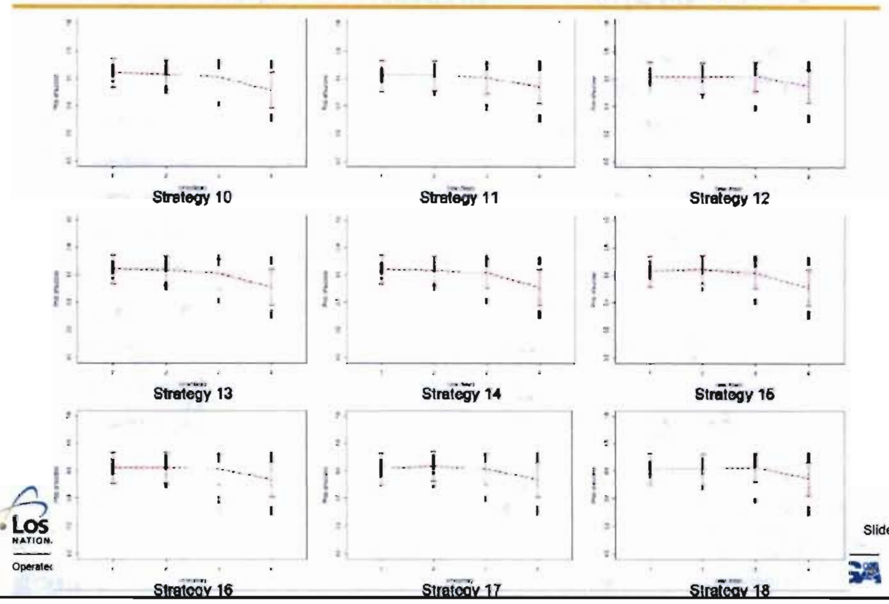


Phase II – Graphical Summaries (cont'd)



UNCLASSIFIED

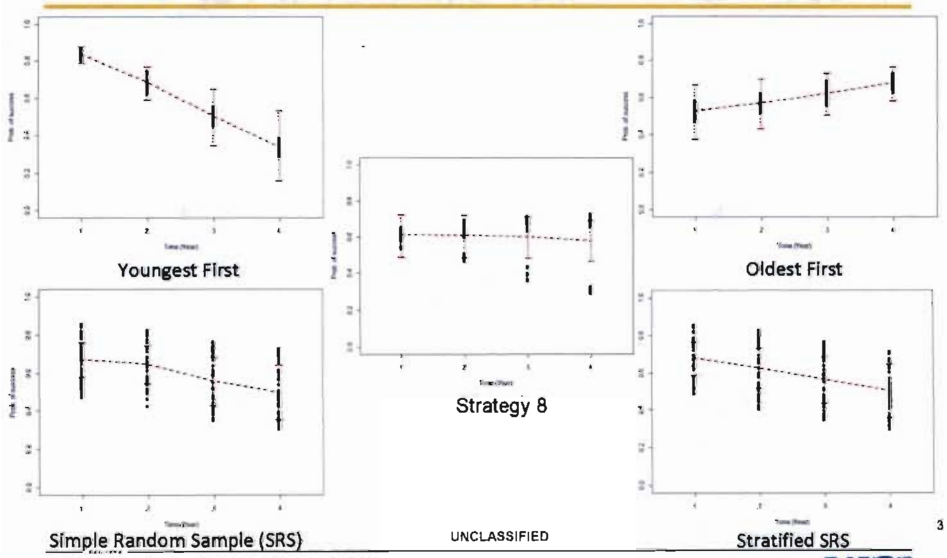
Phase II – Graphical Summaries (cont'd)



Slide 22

UNCLASSIFIED

Strategically Selected vs. Typical Strategies



UNCLASSIFIED

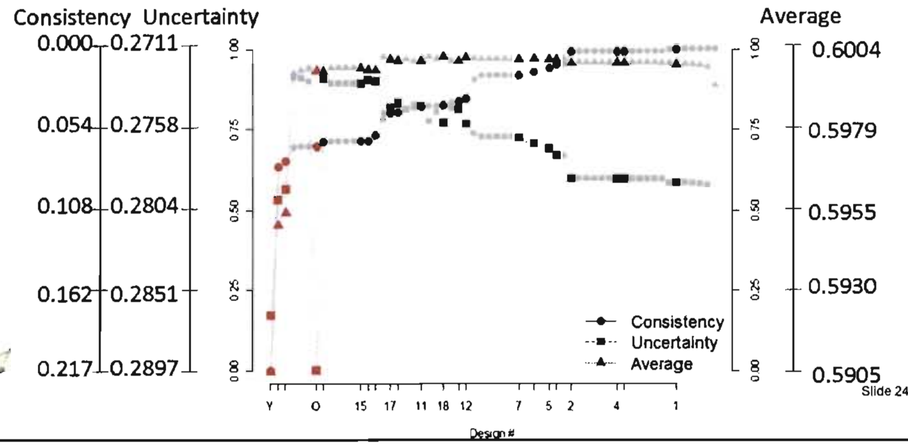
3

Operated by the Los Alamos National Security, LLC for the DOE/NNSA

NNSA

Strategically Selected vs. Typical Strategies

- Trade-off plot by using (0,1) scale based on all possible strategies
  - ➢ Scaling can have substantial impact on graphical summary
  - ➢ Strategically selected strategies consistently and significantly outperform the typical strategies



Conclusions

- Once Pareto front is selected, various weighting, scaling, and metric schemes can be explored easily while desirability approach conducts separate optimizations for each scheme
- Pareto optimization search can be time-consuming for large population; hence a two-phase process is developed to improve the search efficiency
- Graphical summaries are very useful for providing rich information to facilitate an informed decision-making
- Strategically chosen consumption strategies substantially outperform some typical or convenience strategies
- The projected strategies based on Phase I search results can be also be good choices when a decision has to be made in a short time frame