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Title: Ellipsoid-Simplex hybrid for hyperspectral anomaly detection

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ELLIPSOID-SIMPLEX HYBRID FOR HYPERSPECTRAL ANOMALY DETECTION

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Abstract: The problem of anomaly detection in hyperspectral imagery is expressed in terms of a minimal volume set in a high-dimensional space that encloses the bulk of the data samples. The venerable RX algorithm employs an ellipsoid for this volume, but endmember methods can be used to create a simplex volume. This talk describes a hybrid ellipsoid-simplex volume and characterizes its performance on hyperspectral imagery by computing a plot of volume versus false alarm rate. This plot provides a generic measure of quality (smaller volumes are better) without requiring the identification of specific anomalies in the data.

ELLIPSOID-SIMPLEX HYBRID FOR HYPERSPECTRAL ANOMALY DETECTION

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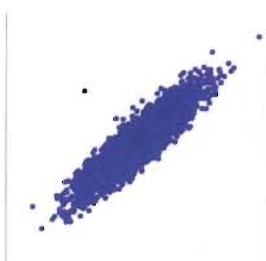
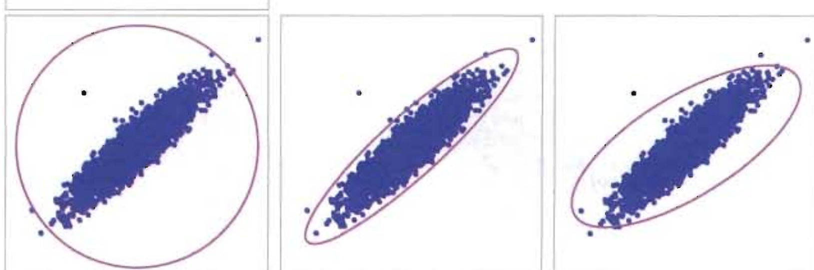
Anomaly detection



- Anomalous points are unlike the others
- Anomalous points are far from others
- Mahalanobis depends on covariance
- Threshold: 99.9% are not anomalous
- Best detector has smallest volume



Family of concentric ellipsoids

- Gaussian distribution: $p(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^d |W|}} \exp\left[-\frac{1}{2} r^2(\mathbf{x})\right]$
- Mahalanobis distance: $r(\mathbf{x}) = \left[(\mathbf{x}-\mu)^T W^{-1} (\mathbf{x}-\mu)\right]^{1/2}$
- Covariance matrix: W
- Volume: $V(r) = \underbrace{\frac{\pi^{d/2}|W|^{1/2}}{\Gamma(1+d/2)}}_{\text{constant actually matters}} r^d$

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Endmember math

- Model data with a k-dimensional simplex
- There are $k+1$ vertices: $\mathbf{e}_0, \mathbf{e}_1, \ldots \mathbf{e}_k$.
- Let $E = [\mathbf{e}_0 \mathbf{e}_1 \cdots \mathbf{e}_k] \in \mathbb{R}^{d\times(k+1)}$
- Point $\mathbf{x}$ is in *subspace* of simplex if $E\mathbf{a} = \mathbf{x}$ for some $\mathbf{a}$ that satisfies sum-to-one constraint: $\mathbf{1}^T\mathbf{a} = 1$; i.e.,

$$\begin{bmatrix} \mathbf{1}^T \\ E \end{bmatrix} \mathbf{a} = \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix}$$

- Point $\mathbf{x}$ is in *interior* of simplex if $\mathbf{a} \succeq 0$
- Interpret $\mathbf{a}$ as vector of *abundances* of endmember materials in pixel $\mathbf{x}$

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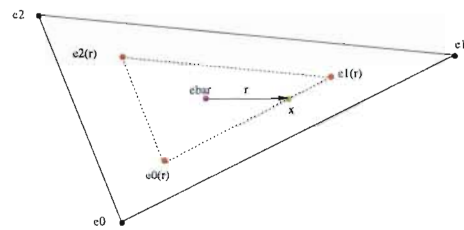
Family of concentric simplices



- Simplex delineated by endmembers: $\mathbf{e}_0$, $\mathbf{e}_1$, and $\mathbf{e}_2$.
- Centroid at $\overline{\mathbf{e}}$.
- "Radius" r associated with point $\mathbf{x}$ in subspace of simplex
  - Defined so $\mathbf{x}$ is just inside the simplex of $\mathbf{e}_0(r)$, $\mathbf{e}_1(r)$, and $\mathbf{e}_2(r)$,
  - where $\mathbf{e}_i(r) = (1-r)\overline{\mathbf{e}} + r\mathbf{e}_i$.
- Radius given by formula $r(\mathbf{x}) = 1 - (k+1)\min[\mathbf{a}(\mathbf{x})]$
  - note: $\mathbf{a}(\mathbf{x}) \succeq 0$ implies $r(\mathbf{x}) \le 1$

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Volume of simplex

- Define $\widehat{E} = [\mathbf{e}_1 - \mathbf{e}_0, \mathbf{e}_2 - \mathbf{e}_0, \ldots, \mathbf{e}_k - \mathbf{e}_0] \in \mathbb{R}^{d\times k}$.
- Volume: $V = |\widehat{E}^T\widehat{E}|^{1/2}/k!$
- For simplex of "radius" r, volume is:

$$V(r) = \underbrace{\frac{|\widehat{E}^T\widehat{E}|^{1/2}}{k!}}_{\text{constant}} r^k$$

- Because we were careful about the constant pre-factors, we can compare ellipsoid-based anomaly detection to simplex-based anomaly detection: the smaller the volume, the better the detector.

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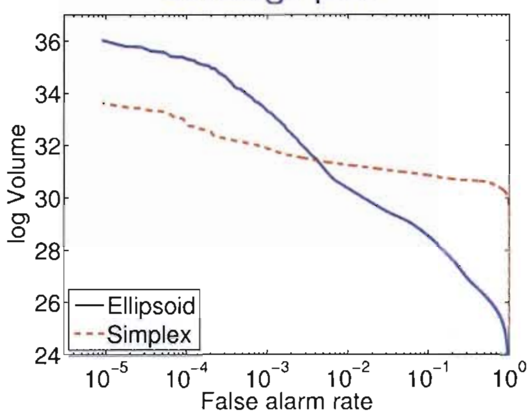
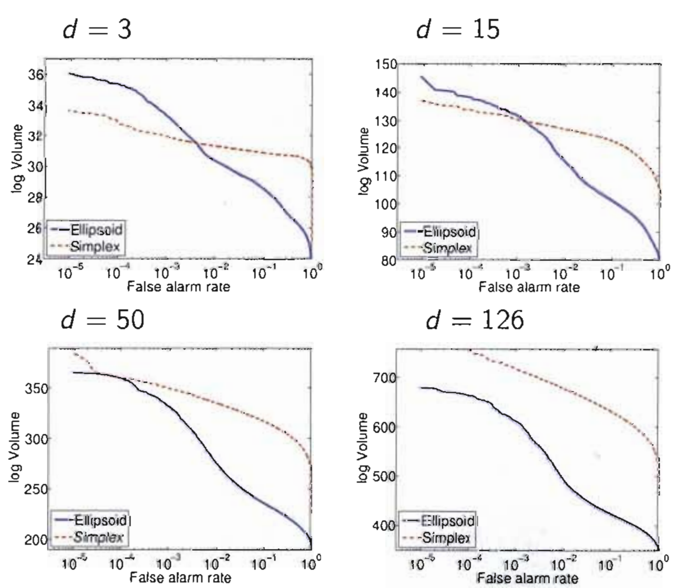
Coverage plot



- To cover fraction $1-\alpha$ of data requires volume V (*i.e.*, to achieve a false alarm rate of α)
- Plot volume V versus false alarm rate α
- At very low false alarm rates, simplex is better (*i.e.*, smaller) than ellipsoid

Coverage plot (cont'd)



Ellipsoid-simplex hybrid

While hyperspectral data is not typically Gaussian, there is a sense (*e.g.*, see [Adler-Golden 2009, Bajorski 2008, Theiler et al 2005]) that it can be Gaussian in "some directions," particularly the directions with lower variance. This intuition suggests a hybrid model for characterizing hyperspectral data: simplex-like in the large-variance directions and ellipsoid-like in the low-variance directions.

What does an ellipsoid-simplex hybrid look like?

- In 2-d
  - pure ellipse, pure triangle
  - hybrid: rectangle
- In 3-d
  - pure ellipsoid, pure tetrahedron
  - hybrid: cylindrical ellipse (elliptical cylinder?)
  - hybrid: triangular prism
- In d dimensions
  - First k dimensions are modeled as $k+1$ endpoint simplex
  - Remaining $d-k$ dimensions are modelled with covariance matrix

Ellipsoid-simplex hybrid: how do you make one?

- Find $k+1$ endmembers: $E = [\mathbf{e}_0 \mathbf{e}_1 \cdots \mathbf{e}_k]$
  - Some algorithms ask you to project to a k dimensional subspace; some do not.
- Let $\mathbf{x}_s$ correspond to the projection of $\mathbf{x}$ to the k dimensional simplex subspace:
  - $\mathbf{x}_s = \mathbf{e}_0 + \widehat{E}\widehat{E}^{\#}(\mathbf{x} - \mathbf{e}_0)$
- Let $\mathbf{x}_e = \mathbf{x} - \mathbf{x}_s$ be the (off-plane) residual
- Compute covariance $W = \langle \mathbf{x}_e \, \mathbf{x}_e^T \rangle$
- For $\mathbf{x}_s$ and $\mathbf{x}_e$, can compute simplex radius r_s and Mahalanobis radius r_e.
  - r_s indicates in-plane anomalousness
  - r_e indicates off-plane anomalousness
- How to combine them into a single measure?

Ellipsoid-simplex hybrid (cont'd)

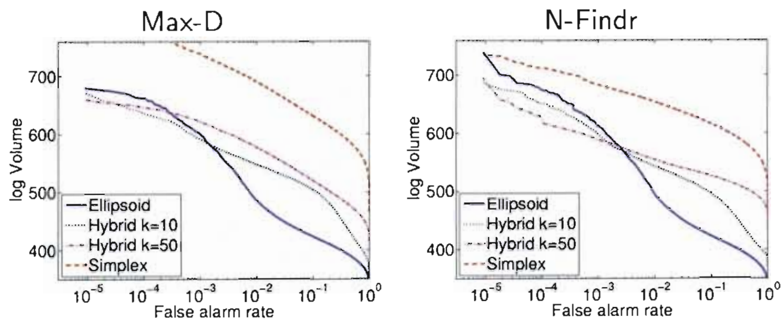
  - r_s indicates in-plane anomalousness
  - r_e indicates off-plane anomalousness
- How to combine them into a single measure?
- Detector fusion: $r = \max(r_s, r_e)$
  - Volume is product of simplex volume and ellipsoid volume

$$V(r) = \frac{\pi^{(d-k)/2}|W|^{1/2}}{\Gamma(1+(d-k)/2)} \frac{|\widehat{E}^T\widehat{E}|^{1/2}}{k!} r^d$$

- Variant: $r = \max(\beta r_s, r_e)$
  - Accounts for different "scale" of r_s and r_e
  - Chose β so that median(βr_s) = median(r_e)
  - Volume formula is modified:

$$V(r) = \frac{\pi^{(d-k)/2}|W|^{1/2}}{\Gamma(1+(d-k)/2)} \frac{|\widehat{E}^T\widehat{E}|^{1/2}}{k!} \beta^{-k} r^d$$

Coverage plots for Ellipsoid-Simplex hybrids



- Full $d = 126$ hyperspectral datacube
- Ellipsoids best characterize the "core" of the data ($\alpha > 0.01$)
- Hybrids are better at characterizing the "periphery"
- N-Findr is better than Max-D (by this metric)

Recap

- Simplex-based anomaly detection
  - Better coverage at smaller volume than ellipsoid-based (RX) anomaly detection
  - Better performance *only* at low dimensions
- Ellipsoid-simplex hybrid for anomaly detection
  - Simplex for k high-variance dimensions,
  - Ellipsoid for $d - k$ low-variance dimensions
- Emphasis on volume as a measure of anomaly detector performance
  - Do you have a better anomaly detector?
    Volume versus false alarm rate provides a way to compare
  - And perhaps, a way to compare endmember algorithms

To Do

- Consider modified fusion of r_s and r_e;
  e.g., find better ways to choose β
- Use coverage plots to compare endmember finding algorithms
- Develop coverage plots as a metric for use in *local* (segmented, or moving-window) anomaly detection schemes
- Characterize/visualize distinction between ellipoid-dominated (off-plane) and simplex-dominated (in-plane) anomalies.
- Develop simplex-based model for $p(\mathbf{x})$ for hyperspectral data
  - *e.g.*, ellipsoid → Gaussian; simplex → ??
  - Use this $p(\mathbf{x})$ for target detection, change detection, etc.

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