LLNL-CONF-612492

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January 23, 2013

Stanford Geothermal Workshop
Stanford, CA, United States
February 11, 2013 through February 13, 2013

PROCEEDINGS, Thirty-Eighth Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, February 11-13, 2013
SGP-TR-198

AN INITIAL VALUE OF INFORMATION (VOI) FRAMEWORK FOR GEOPHYSICAL DATA APPLIED TO THE EXPLORATION OF GEOTHERMAL ENERGY

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ABSTRACT

We present a value of information (VOI) methodology for the geothermal exploration problem. Our work shows how the non-uniqueness of geophysical data affects its value (usefulness) to decision-makers who have to make decisions based on uncertain information. We evaluate the information reliability with geophysical forward-modeling, thereby simulating the physics and limitations associated with the measurement technique. To demonstrate the method, we quantify how reliable magnetotellurics (MT) surveys are at detecting permeable zones containing hot fluid. We vary the reservoir's geological and petro-physical properties, such as reservoir fluid temperatures, effective matrix permeabilities and fluid salinity. There is a non-unique relationship between electrical conductivity (what MT measures) and permeability of reservoir rock and/or the reservoir fluid. Our generic example is used to generate VOI results that estimate the uncertainty associated with reservoir properties inferred from MT measurements.

INTRODUCTION

Value of Information (VOI), from the field of decision analysis, quantifies how relevant and reliable any particular information source is, given a decision with a highly uncertain outcome. VOI can be used to justify the costs of collecting the proposed data. This has been demonstrated in the literature for oil exploration (see review by Bratvold et al, 2009), but we apply it here to the geothermal energy exploration decision. Our work illustrates the implementation of a VOI methodology given the uncertainties of geothermal exploration and considering magnetotelluric (MT) data. Any aspect of this demonstration could be refined to better represent an actual field case or a different source of information (e.g., temperature or heat flow measurements).

Initially we consider a simple exploration decision regarding a possible resource: Should we produce (drill) or not? We assume that the decision outcome only depends on the question "*is it hot and will it flow?"* We assume that porosity is a good proxy for permeability, and that porosity and temperatures can be obtained from the electrical resistivities estimated by inverting MT data.

VOI estimates the possible increase in expected utility by gathering information. In its simplest form, the VOI equation can be expressed as:

$$VOI = V_{with\,information} - V_{prior} \qquad (1)$$

where value *V*, is the metric used to quantify the outcome of a decision; the higher the value, the more "successful" an outcome of a decision is. This paper is organized according to eq. 1. First, V_{prior} will be addressed by describing how the prior uncertainty of the subsurface is represented with multiple realizations of Earth models. Second, we will describe how the value with information ($V_{with\ information}$) can be estimated by simulating the MT response on the prior models. Specifically, we devise a method for estimating MT's reliability to distinguish economic versus uneconomic geothermal reservoirs.

PRIOR MODELS: UNCERTAINTY IN BRINE SALINITY, TEMPERATURE AND MATRIX POROSITY

We define a group of prior models that represent the earth in 3 layers: overburden, reservoir, and basement (Fig. 1). The overburden represents the layer from the surface to 1.5 km depth. The top of the reservoir is at 1.5 km depth is 500 m thick. The basement layer extends from 2 to 250 km depth (which represents 5 skin depths). Figure 1 and subsequent figures plot only the top 10km. The electrical resistivities assigned to the overburden and basement layers are fixed at 100 and 255 ohm-m respectively.

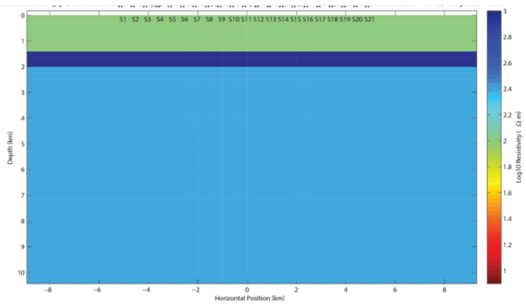

Figure 1: Example of 3 layer electrical resistivity model (overburden, reservoir, & basement). 21 MT Stations are shown on the surface

To capture the uncertainty in reservoir properties, many different electrical resistivities are assigned to the middle layer which represents the potential geothermal reservoir. We utilize laboratory-derived rock physics relationships (Ucok et al, 1980) to describe how the salinity and temperature of the brine affect its electrical resistivity. The prior models represent varying porosity/permeability values of the bulk matrix. We assume Archie's equation to compute the rock's bulk resistivity given porosities and brine resistivity.

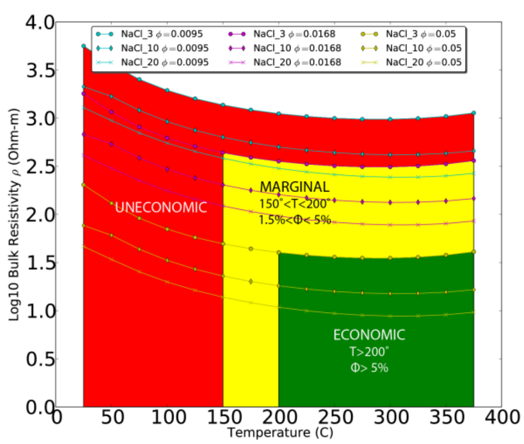

Figure 2: Log10 Bulk Resistivity as a function of porosity of matrix, and salinity and temperature of the brine. Each reservoir resistivity value belongs to 1 of the 3 economic categories: economic (green), marginal (yellow), and uneconomic (red).

Figure 2 shows 3 different salinities (3%, 10%, and 20% concentrations), 3 different porosity values (0.95%, 1.68% and 5%) at brine temperatures ranging 25-375˚C. Using Carmen-Kozeny (Carmen, 1956), these porosity values could correspond to the following permeabilities: 10^{-6}, 10^{-8}, and 10^{-9} Darcy). We generate 135 prior models where each model's reservoir layer is assigned the electrical resistivity value from one resistivity marker in Figure 2. For example, Figure 1 shows a reservoir resistivity of 1000 ohm-m, corresponding to 0.95% porosity, 100˚C and 10% NaCl concentration (according to empirical functions from Ucok et al, 1980).

Each of these models is associated with an economic viability category. Three regions describing the reservoirs' potential profitability are defined: economic (porosity ≥ 5% and temperature ≥ 200˚C), marginally economic (1.68% ≤ porosity < 5% and 150˚≤ temperature < 200˚C) or non-economic (porosity <1.68% and temperature <150˚C) geothermal resources. These regions are shown in green, yellow and red respectively in Figure 2. These represent the *true economic viability category* θ:

$$\theta_i \quad i \ni \{\text{Economic, Marginal, Uneconomic}\} \qquad (2)$$

where each of the prior models belongs to one of these categories. Let us represent each model by

$$\boldsymbol{z}^{(t)}(\theta_i) \quad t = 1, \dots, T \qquad (3)$$

where vector $\boldsymbol{z}$ contains the electrical resistivity, temperature, porosity, and any other relevant properties of the model and t indexes all T=135 prior models. These categories are crude representations of what the value outcome of each model could be. These categories could be replaced by a function that relates the permeability/porosity and temperature to the production potential, but we choose this simplification to demonstrate our methodology.

Figure 3 shows resistivity histograms for all the models in Figure 2. All three categories span the log10 electrical resistivity range 1.3 to 1.6. These overlaps illustrate that electrical resistivity cannot uniquely determine whether the reservoir is "hot and will flow."

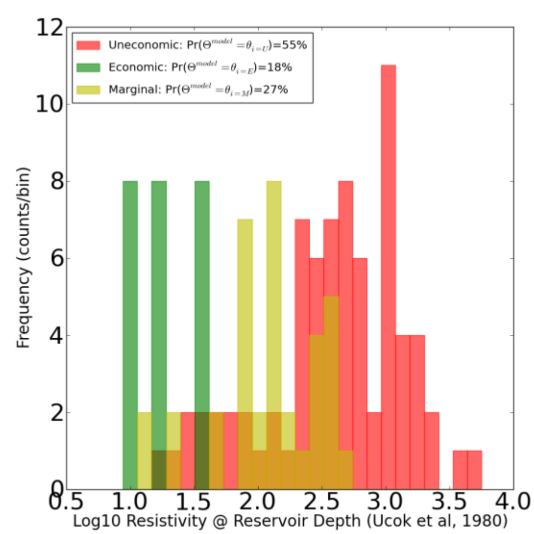

Figure 3: Histogram of log10 electrical resistivity of the reservoir layer (all curve markers in Figure 2).

We will now describe how each prior model is linked to possible economic outcomes. This will be summarized in the quantity V_{prior}, which translates our prior uncertainty (or current state of information) into an expected (or average) outcome from our decision.

V_{PRIOR}: THE BEST DECISION OPTION GIVEN PRIOR UNCERTAINTY

Decision analysis concepts are often described in terms of lotteries and prizes (Pratt et al, 1995). By choosing to drill or not, a decision maker is choosing whether or not to participate in a lottery with certain perceived chances of winning a prize (drilling into a profitable reservoir); however, this lottery also involves the chances of losing money (drilling into an uneconomic reservoir). By utilizing V_{prior}, a decision-maker can logically determine when one should participate in this lottery given both the prior uncertainties and possible gains and losses.

The value metric allows for comparison between outcomes from different decision alternatives, which can be represented by function g_a.

$$v_a^{(t)}(\theta_i) = g_a\big(\boldsymbol{z}(\Theta = \theta_i)^{(t)}\big) \qquad (4)$$
$$a = 1,2 \quad i = E, M, U \quad t = 1,..T$$

We assume only 2 possible alternatives (a = 1 or 2): drill/produce the reservoir or do nothing. Table 1 defines the 6 possible outcomes, which is a result of these 2 decision alternatives and the 3 possible reservoir categories. The columns represent the decision alternatives (a=1 and a=2) and the rows the different subsurface categories (Θ_E, Θ_M, Θ_U).

Table 1: Table of nominal value outcomes for the 2 possible decision options (columns) and 3 possible economic viability categories of the unknown subsurface (rows).

Decision option→ ↓Reservoir Category	a = 1 (drill)	a = 2 (do nothing)
θ_i= economic (E)	\$200	\$0
θ_i= marginal (M)	\$50	\$0
θ_i= uneconomic (U)	\$0 to -\$500	\$0

Table 1 values do not reflect realistic gains (payout when you produce an economic reservoir) or losses (loss on investment when you drill an uneconomic reservoir). Instead, VOI results will be presented as a function of the ratio of the loss to gain (loss range shown in Table 1); for example, \$200 means that there is a possible profit (revenue – cost) of \$200 if an economic reservoir is produced. Although not logical, the range of "losses" includes a gain (positive outcome). This is simply for demonstration purposes so that the behavior of the VOI quantities can be visualized. Realistic gains and losses for a particular field site can be easily substituted in Table 1 and into the methodology.

If a continuous function were established to link each prior model to its value outcome, the average outcome for models within a category will need to be calculated for both possible alternatives (a=1 and a=2)

$$v_a(\theta_i) = \frac{1}{T_i}\sum_{t=1}^{T_i} v_a^{(t)}(\theta_i) \qquad (5)$$
$$a = 1,2, i = E, M, U$$

where T_i represents the total number of models belonging to each of the three categories.

All the necessary quantities have been introduced to calculate V_{prior}.

$$V_{prior} = \max_a\left(\sum_{i=1}^{3} Pr(\Theta = \theta_i)v_a(\theta_i)\right) \qquad (6)$$
$$a = 1,2$$

In words, V_{prior} quantifies the best the decision-makers can do with the current uncertainty (no MT data has been collected), which are reflected in the prior probabilities $Pr(\Theta = \theta_i)$. V_{prior} identifies which decision alternative gives *on average* the best outcome (done through the $\max_a$). When considering a specific location for geothermal exploration, these prior probabilities should come from a geologist and/or other experts with knowledge of the geologic structure and history. For now, we use the number of models in each of the three categories depicted in Figure 3, such that $Pr(\Theta = \theta_E) = 18\%$, $Pr(\Theta = \theta_M) = 27\%$ and $Pr(\Theta = \theta_U) = 55\%$.

The green line of Figure 4 graphs the resulting V_{prior}, given the values of the possible gains/losses shown in Table 1 and the prior models of Figure 3. The x-axis of V_{prior} represents the ratio of the loss versus the gain, such that the losses become less as you move to the right. A ratio = 0 indicates an unrealistic but ideal situation: the potential for loss is 0 regardless of the potential for gains. A ratio of -1.0 indicates that one could loose one dollar for every dollar that could be gained. Returning to the lottery example, when V_{prior} is 0, the decision-maker should "not participate in the lottery" (i.e. don't drill) given the current state of information. V_{prior}=0 tells the decision-maker that the decision alternative to "do nothing" will yield the higher outcome on average. V_{prior}=0 reflects the potential for large losses when you "participate in the lottery" or drill to produce a geothermal reservoir. The decision-maker would only be wise to participate in the lottery when $V_{prior} > 0$, which for this particular set-up is when the absolute value of the loss is 0.5 or less of the possible gain.

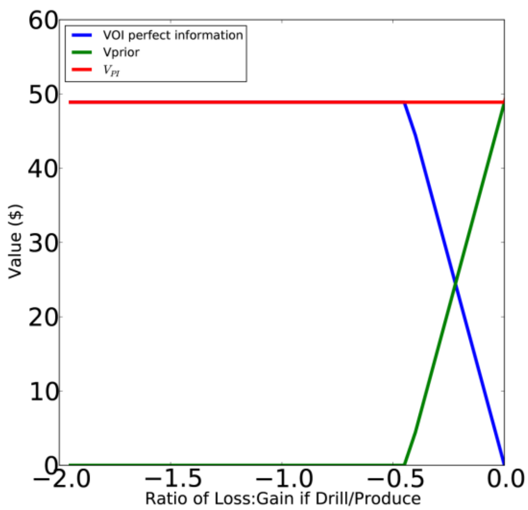

Figure 4: Graph depicting Vprior (green), Value with Perfect information (V_{PI}- red), and Value of Perfect Information (VOI_{PI} -blue)

VALUE OF PERFECT INFORMATION (VOI_{PI}): UPPER BOUND ON THE INFORMATION'S VALUE

Also shown in Figure 4 are the quantities of value *with* perfect information (V_{PI}, red curve) and value *of* perfect information (VOI_{PI}, blue curve). The value of perfect information can be calculated using Equation 1, by substituting in V_{PI} for the value with information ($V_{with\ information}$). VOI_{PI} assumes that an information source exists that will always identify the correct economic viability category θ_i without errors. Like V_{prior}, V_{PI} only depends on the prior uncertainty and potential gains/losses of the problem.

$$V_{PI} = \sum_{i=1}^{3} Pr(\Theta = \theta_i)\left(\max_{a} v_a(\theta_i)\right) \tag{7}$$

Here, we see that for each reservoir category θ_i, we can choose the best decision alternative *a* (this is reflected in $\max_{a}$ being performed before the average). V_{PI} is plotted in red in Figure 4, which shows that it is invariant to the loss-gain ratio. This is because if we know the reservoir is uneconomic, we will always choose not to participate in the lottery; therefore, we remove the chance of loss by collecting perfect information. The blue line of Figure 4 plots the resulting VOI_{PI}, which increases as V_{prior} decreases (Equation 1). With our current state of information, we would not enter the lottery when the potential losses were too high relative to the gains. But with a flawless information source to allow us to avoid these losses, we may choose to participate in the lottery. Since it assumes error-free information, the VOI_{PI} quantity will give an upper bound on what we could expect for any information source. Now we consider imperfect MT data and we estimate its reliability when distinguishing between the three possible geothermal categories θ_i.

SIMULATING POSSIBLE MT DATA COLLECTION, NOISE AND INVERSION

VOI is used to determine whether particular data is worth purchasing and thus, it must be calculated before the intended data is collected. To estimate the reliability of the data to reveal the principal uncertainty to the decision (θ_i for our example), one must either use a suite of calibrated field data or use synthetic models and forward modeling to predict the data. We perform the later and utilize the prior models which represent our current uncertainty.

The workflow to estimate the VOI of MT can be described in 7 steps.

1. The MT response is forward modeled for each prior model.
2. 5% random Gaussian noise is added to each response.
3. Geophysical inversion is performed for each noisy data set; one inverted electrical resistivity model is obtained for every prior model.
4. For each inversion result, automatic interpretation is used to locate the reservoir boundaries and select its resistivity ρ.
5. The Data Likelihood/Reliability is calculated by comparing the reservoir's inverted and interpreted electrical resistivity (ρ) to its prior model's original economic viability θ_i.
6. The Information Posterior is calculated: use Bayes rule to estimate the probability of any given economic category θ_i given an inverted electrical resistivity (ρ).
7. Calculate the value of imperfect information (VOI_{II}) using the Information Posterior.

For steps 1 and 3 we utilize a 2D MT code based on Constable, et al. (1987). For Step 4, a gradient algorithm locates the 2 greatest vertical resistivity changes and uses these to define the top and bottom of the reservoir layer. It selects all the resistivities that are between the top and bottom since they all define the inferred reservoir layer.

Figure 1 (true model) differs from Figure 5 (inverted/recovered model) for several reasons. First, MT induces horizontal currents in the subsurface at frequencies between 0.01 to 100 Hz (Trainor-Guitton & Hoversten, 2011). Although these currents have deep penetration, they are insensitive to thin resistors (such as the low porosity and low temperature reservoir shown in Figure 1). Second, the MT problem is usually an underdetermined problem: there are fewer data points than resistivity parameters that need to be resolved. The inversion process smoothes the resistivity structures (through regularization) in order to mitigate problems caused by the underdetermined condition. Therefore, the recovered models will not perfectly match the prior model from which the data came.

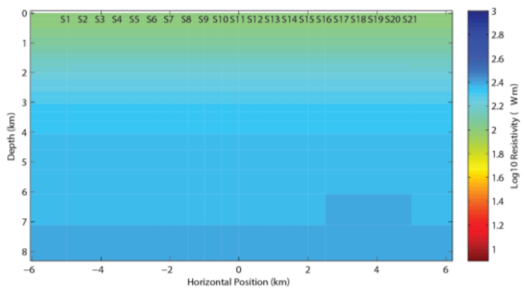

Figure 5: Inversion result of Figure 1. Porosity=0.95%, temperature=100°C and salinity=10%.

The top and bottom of Figure 6 plots the result of Steps 5 (the data likelihood) and 6 (the information posterior) respectively. Both plot the log10 interpreted electrical resistivity on the x-axis. The data likelihood (which is also the reliability) is expressed as:

$$Pr\left(P = \rho_j \middle| \Theta = \theta_i\right) \qquad (8)$$
$$i = \{E, M, U\}\ \mathrm{j} = 1, \ldots, \mathrm{J}$$

where j indexes the bins of the electrical resistivity.

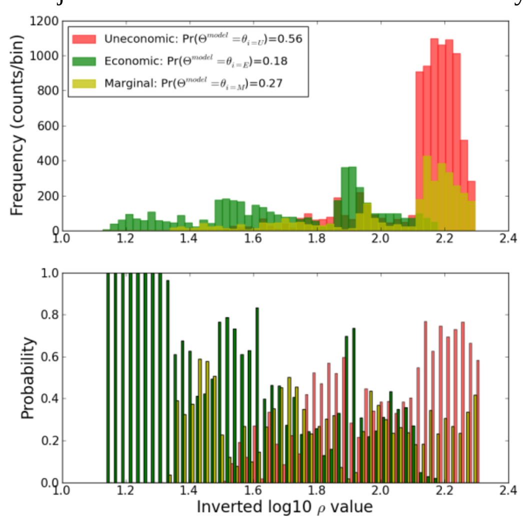

Figure 6: Data Likelihood/Reliability (top). Information Posterior (bottom). X-axis is the inverted reservoir electrical resistivity (log10).

As expected most of the low inverted resistivities (log10 ρ < 1.6) come from the economic geothermal category (green). However, in Figure 6 some economic models have interpreted reservoir resistivity as high as 2.2. When we compare this to Figure 3, we see that none of economic models have log10 resistivities higher than ~1.6. This represents some errors that have been introduced by the noisy MT, and by the inversion and interpretation processes.

The information posterior (bottom of Figure 6) is the chronological reverse of the data likelihood (eq. 7):

$$Pr\left(\Theta = \theta_i \middle| P = \rho_j\right) \qquad (9)$$
$$i = \{E, M, U\}\ j = 1, \ldots J$$

where now we have probability of any of the economic viability categories occurring given a particular bin value for the inverted ρ. This is the resistivity's "message" regarding economic viabilty. Notice that very low resistivity values (log10 ρ < 1.4) show a 100% probability of being associated with an economically viable reservoir. The resistivity message becomes quite ambiguous between 1.6 and 1.8 because the probabilities for the 3 categories are roughly equivalent. Previously, it was noted that the range of 1.3 to 1.6 in the prior models includes all three categories (Figure 3).

The smoothing effect of the inversion is noted in the range change of log10 resistivity values recovered versus those in the original prior models. Figure 3 (the values used in the prior models) show a range of 0.94 to 3.75 whereas Figure 6 shows that the recovered (inverted and interpreted) values only range from 1.0 to 2.3.

The information posterior is the form actually used to calculate the value *with* imperfect information V_{II}.

$$V_{II} = \sum_{j=1}^{J} \Pr\left(P = \rho_j\right) \ldots \qquad (9)$$
$$\left\{\max_a \left[\sum_{i=1}^{3} Pr\left(\Theta = \theta_i \middle| P = \rho_j\right) v_a(\theta_i)\right]\right\}$$

Here, the posterior accounts for how often one may incorrectly infer a subsurface category given the inverted electrical resistivity. The posterior is used to weigh the averaged outcome of each alternative and category combination $v_a(\theta_i)$. Since the decision is made after resistivity data has been collected, the best alternative ($\max_a$) is chosen given the interpreted category. Lastly, V_{II} is weighted by the marginal probability $\Pr(P = \rho_j)$, how often any of the particular inverted resistivities occur relative to other resistivity bins.

Figure 7 plots both the value with imperfect information (V_{II}) and the value of imperfect information (VOI_{II}) along with the previously seen V_{prior} and $VOI_{perfect}$. The value of imperfect information is calculated using Equation 1 where now the V_{II} is used in place of the generic term of $V_{with\ information}$. As expected, VOI_{II} (cyan) is lower than

$VOI_{perfect}$ (blue) at all loss-gain ratios. This demonstrates how the highest value outcome will not be realized because of the imperfectness of the data that can mislead the decision maker about the economic viability of the reservoir.

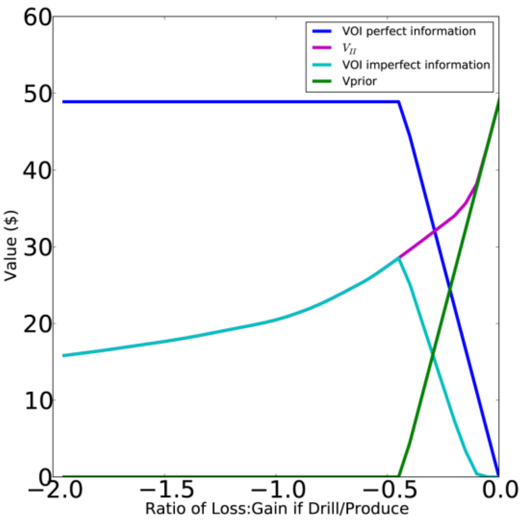

Figure 7: Value with imperfect information (magenta) and Value of imperfect information (cyan). $VOI_{perfect}$ (blue) and V_{prior} (green) also plotted for comparison (from Figure 4).

From a loss-gain ratio of 0 to -0.5, the value of imperfect information (cyan) steeply increases due to the V_{prior} (green) sharply decreasing. Once V_{prior}=0, $VOI_{imperfect}$ is determined solely by V_{II} (magenta).

COMPLEXITY ADDED TO PRIOR MODELS: THIN CONDUCTOR ABOVE RESERVOIR

In an effort to add more realism to our example, we introduce more subsurface complexity to our prior models by adding a thin conductor above the reservoir layer. Figure 8 shows the same model depicted in Figure 1 (porosity = 0.95%, temperature=100C, salinity= 10% NaCl concentration) but now immediately above the reservoir layer is 300m thick, 10 ohm-m layer. This layer could represent hydrothermal alteration. Cumming (2009) describes how low resistivity structures are "consistently found over geothermal reservoirs" but can also be indicative of sediments related to volcanic valleys. Either way, a thin conductor will likely make the MT's "message" more ambiguous regarding the economic viability of the reservoir because the MT measurements will have less sensitivity to the reservoir. In other words, the conductor will degrade the reliability of the interpreted electrical resistivity ρ to identify θ_i.

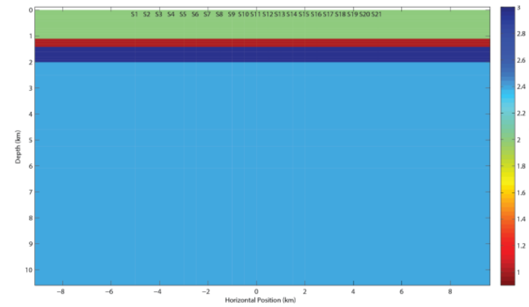

Figure 8: Prior Model same as Figure 1 but with a 300m 10 ohm-m layer above "reservoir layer"

The 10 ohm-m layer is added to all prior models. The first step of VOI is to evaluate the V_{prior} utilizing the prior uncertainties and value outcomes from the decisions alternatives. Although we have added complexity to the prior models, we will keep the prior uncertainties of the different economic categories the same as demonstrated before (ie $Pr(\Theta = \theta_E) = 18\%$, $Pr(\Theta = \theta_M) = 27\%$ and $Pr(\Theta = \theta_U) = 55\%$.). Specifically, the reservoir layer, which determines the outcome for the considered decision, remains the same all the prior models. And the decision alternatives will also be kept the same (Table 1). Therefore, V_{prior} and $VOI_{perfect}$ of Figure 4 still apply to this example.

VOI_{II} WITH PRIOR MODELS WITH THIN CONDUCTOR

The same workflow (Steps 1-7 above) is applied to simulate possible MT data collection, inversion and interpretation. Figure 9 shows the inversion result for the model in Figure 8. Figure 9 differs significantly from Figure 5 due to the addition of the conductor.

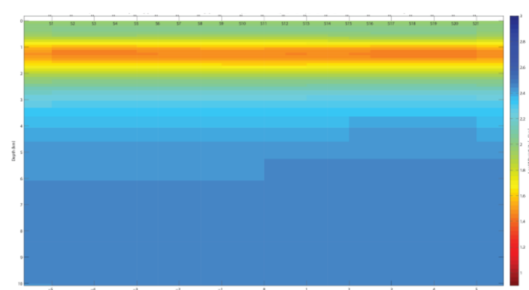

Figure 9: Inversion result noisy data from the Figure 8 model. Porosity=0.95%, temperature=100 and salinity=10.

The data likelihood (reliability) and the information posterior are calculated in the same way as described previously. The result is shown in Figure 10. A visible gap is seen at resistivity values between 1.6 and 1.8. This resistivity range is the yellow value in the Figure 9 inversion, which the automatic interpretation algorithm interprets as the reservoir layer *boundaries*. Therefore, this value is not ever interpreted as the reservoir layer resistivity.

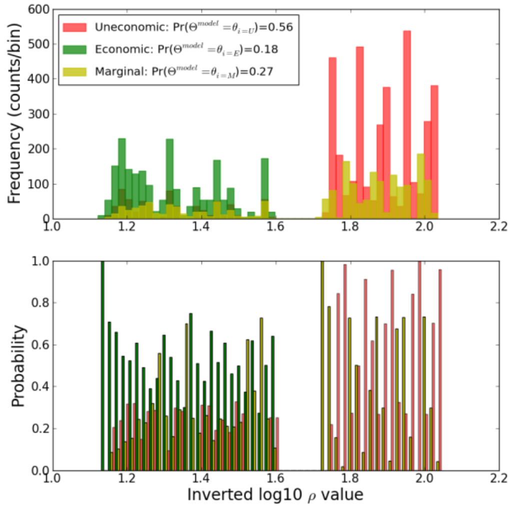

Figure 10: Data Likelihood (top) and Information Posterior (bottom) utilizing prior models that include a 300m 10 ohm-m conductor above the reservoir layer.

Another feature noticeable in Figure 10 is that now some uneconomic models have very low resistivity, this is different from the results in Fig. 6. Before, the lowest log10 interpreted electrical resistivity was 1.5. Figure 10 shows that the thin conductor has caused uneconomic models to have interpreted log10 electrical resistivities of 1.2. Figure 9 demonstrates how this misinterpretation occurs since the MT inversion cannot delineate the 4 distinct layers present in Figure 8.

We then utilize the posterior of Figure 10 into Eq. 9 which results in the V_{II} (magenta) and ultimately VOI_{II} (cyan) shown in Figure 11. The largest difference between Figure 7 (VOI when no conductor exists) and Figure 11 is how V_{II} decreases more sharply with increasing loss (left on the x-axis). This reflects how more misinterpretations occur when a thin conductor is introduced to all prior models.

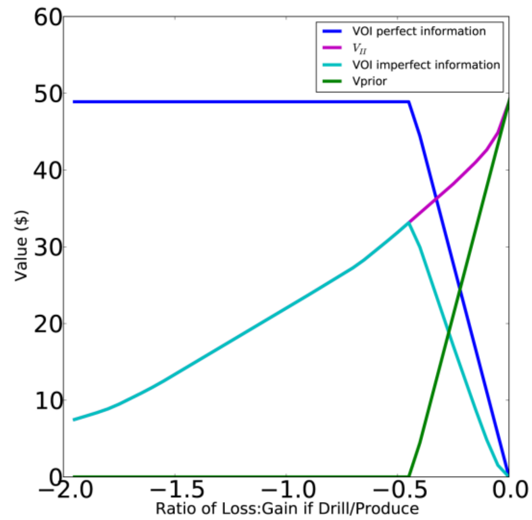

Figure 11: V_{II} and VOI_{II} utilizing prior models that include thin conductor above reservoir layer. V_{prior} and $VOI_{perfect}$ are the same from Figure 4 since the value outcomes and prior uncertainties have not changed.

CONCLUSIONS AND FUTURE WORK

We have presented a general VOI method to determine if MT data should be purchased. Our work shows how the non-uniqueness of geophysical data affects its value (usefulness) to decision-makers who have to make decisions based on uncertain information. This method can be applied to real-world geothermal exploration situations to evaluate the relevancy and reliability of MT to assess the principal uncertainty associated with the production decision. Two examples were presented that demonstrate the sensitivity of VOI_{II} to the reliability of the information, specifically the types of prior models used to forward simulate the MT data.

Future work will include the spatial uncertainty of the reservoir properties, such as thickness, depth or spatial extent of a possible reservoir to be evaluated. Depth is of high importance in geothermal exploration since it is linked to the cost of producing the reservoir. The VOI results from this new work will be independent of what we presented here because we will be evaluating how well MT can assess the thicknesses and spatial extent of the reservoir, not how well it can determine if a reservoir "*is hot and will it flow*."

This research was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. DE-AC52-07NA27344. LLNL-CONF-612492

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