Final Report

New Grid and Discretization Technologies for Ocean and Ice Simulations

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This is a collaborative project involving Florida State University (Max Gunzburger, PI) and the University of South Carolina (Lili Ju, PI) and this **final report** *is submitted for both grants number DE-FG02-07ER64432 and DE-FG02-07ER64431.*

Todd Ringler *and* ***Steven Price*** *of the Los Alamos National Laboratory have been full partners in several projects associated with the grants to Florida State University and University of South Carolina. In addition, Florida State and South Carolina personnel have collaborated with other scientists at Los Alamos and at the National Center for Atmospheric Research.*

The goals of the grant-related research were as follows.

- *High-quality unstructured, non-uniform Voronoi and Delaunay grids* – develop a new, nonuniform meshing technology, based on the spherical centroidal Voronoi tessellation (SCVT) concept, applicable to regions on the sphere and three-dimensional regions appropriate for atmosphere, ocean, ice sheet, and other climate modeling applications.

- *Improved finite element and finite volume discretization schemes* – develop efficient, accurate, and robust finite volume, finite element, and adaptive methods for ocean and ice-sheet modeling applications; the methods take full advantage of the high-quality SCVT-based grids.

- *Testing and implementation of new technologies* – test the new meshing and discretization technologies on application test-bed problems and incorporate them into production codes.

Towards meeting the goals of the grant, we pursued several projects. Here, we provide brief descriptions of several of these projects and provide sample results obtains. Details can be found in the papers prepared under grant sponsorship.

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1. Variable Resolution Centroidal Voronoi Grids for Climate Simulations

Goal. Develop *a technology for generating high-quality unstructured, variable resolution Voronoi and Delaunay meshes* for use on the whole sphere, in regions of the sphere, and three-dimensional regions appropriate for ocean, atmosphere, ice sheet, and other climate modeling applications.

Approach. Use the *centroidal Voronoi tessellation* (CVT) and the *spherical centroidal Voronoi tessellation* (SCVT) concepts, as well as the corresponding Delaunay tessellations (CVDT and SCVDT) as the bases for mesh generation. These meshes are very easy to generate; furthermore, they can be efficiently generated, e.g., SCVT meshes having $O(10^6)$ nodes (roughly corresponding to a 15 km global resolution) can be generated on a desktop computer in an hour or so.

Accomplishments. Scalable algorithms for the construction of variable resolution CVT and SCVT-based meshes as well as the associated Delaunay CVDT and SCVDT meshes have been developed, implemented, and tested. *The methodology results in superior meshes* in the following respects:

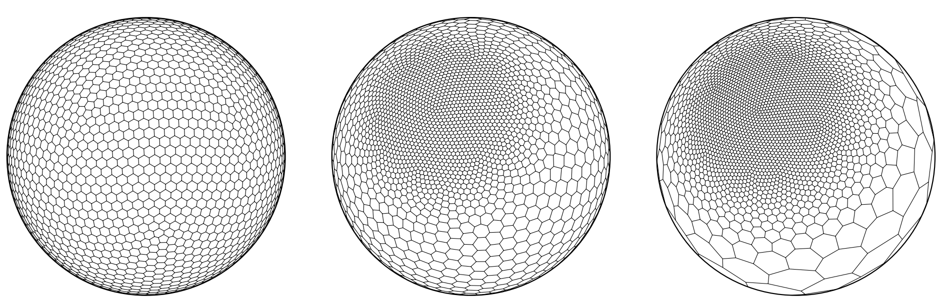
- great flexibility and preciseness are allowed for how grid points are non-uniformly distributed;
- grid cells are very well shaped;
- transitions from coarse to fine grid regions are very smooth;
- meshes having a sufficient resolution for climate modeling applications can be generated very efficiently.

All of these features are very desirable in climate modeling applications. Specifically, we have accomplished the following:

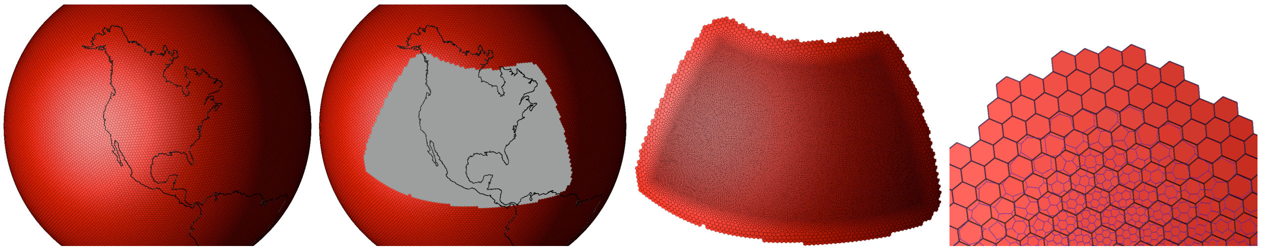
- variable resolution SCVT grids on the whole sphere have been fully developed and tested in regional modeling settings;
- variable resolution CVT and SCVT surface grids for ocean modeling applications have been fully developed and are now been used as a basis for the development of full three-dimensional ocean grid generation software;
- three-dimensional, variable resolution CVT-based prismatic grids have been fully developed and used for finite element ice-sheet modeling.

Impact on the climate modeling community. SCVT meshes form the basis and are an enabling technology for the MPAS (Model for Prediction Across Scales) multi-resolution climate modeling system currently under joint development by LANL and NCAR. SCVT meshes have also been extensively used by the COSIM group at LANL. Software suites are being developed for parallel SCVT-based grid generation on the whole sphere and on regions of the sphere that can the climate community can use in their ocean, ice, and atmosphere models.

Example: Regionally refined global grids on the sphere



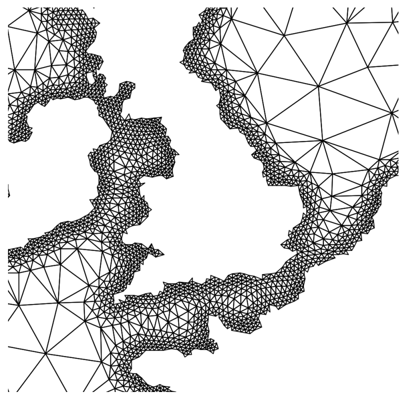
Left to right: quasi-uniform global grid; locally refined grid; severely refined grid. All meshes have the same number of cells so that local refinement causes the coarse grid to get coarser.



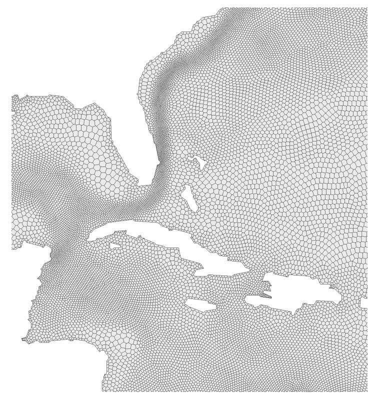
Left to right: global 40,962 node coarse grid; region to be refined; 65,026 node refined grid in the region of interest; near its boundary, the refined grid exactly matches the coarse grid so that smooth grid transitions result. Coarse grid is held fixed during refinement so that local refinement causes an increase in the number of cells.

Example: CVT-based grids of the North Atlantic

CVDT of the North Atlantic

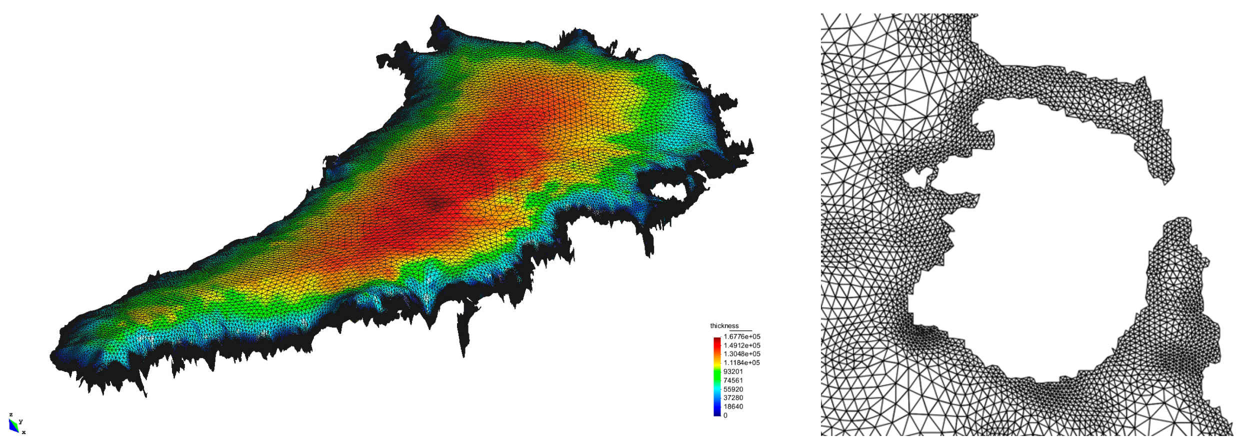


Zoom in of a CVDT



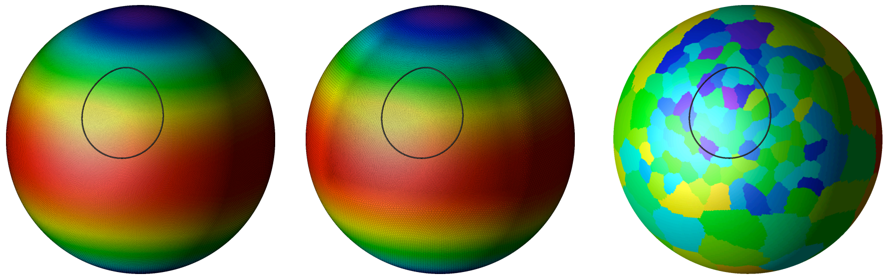
Zoom in of a CVT

Example: Three-dimensional CVT-based prismatic grids of Greenland



Left: a 866,918 node CVDT grid for Greenland 5 km resolution ice-thickness data and a 33,343-node CVDT of the Greenland ice surface. Right: zoom-in of the grid showing the high quality of the CVDT refinement.

Example: Application of regionally refined grids to PDE solutions



Global (left) and regionally refined (middle) grids, both having 82,547 nodes; the partitioning of the grid for good load balance (right). The results for day 15 using the uniform and regionally refined grids are indistinguishable. Regional refinement allows for the employment of parameterizations that are not applicable for coarse grids.

2. High-Accuracy, Scalable Finite Element Model for Three-Dimensional Stokes Ice-Sheet Modeling

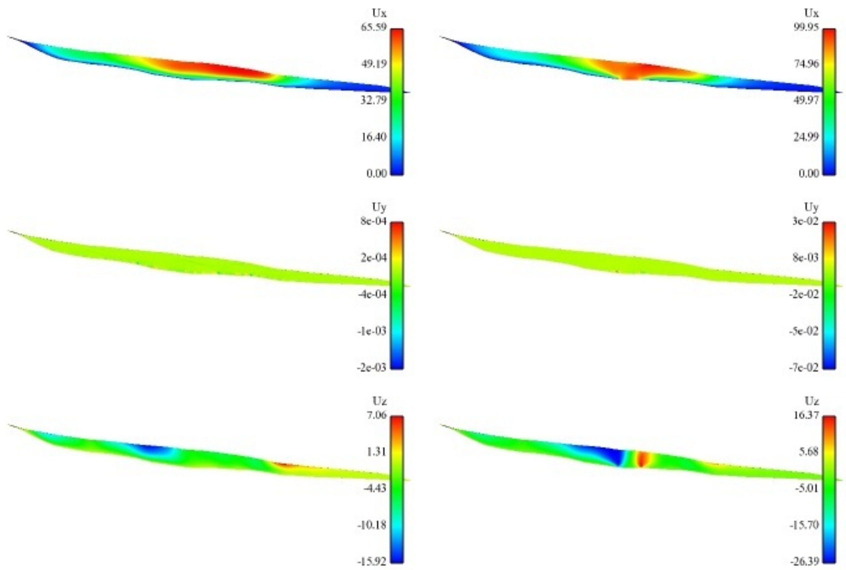
Goal. Develop an *accurate and efficient computational model for ice-sheet dynamics, temperature, and evolution* based on the three-dimensional Stokes dynamical model coupled to an energy equation for the temperature and to a surface evolution equation.

Approach. In order to obtain the grid resolution needed to produce predictions in an accurate and efficient manner, we use *finite element discretizations based on the three-dimensional, prismatic, variable resolution CVDT meshes* of the ice sheet discussed in Section 1. Highly scalable *parallel solvers* are also used to enhance efficiency.

Accomplishments. We have created a highly efficient and accurate capability for modeling ice sheets based on a fully coupled, three dimensional model for the dynamics, temperature, and height evolution of ice sheets based on the Stokes equation dynamics model and using variable resolution three-dimensional CVDT grids and parallel solvers. The governing equations are formulated as a three-dimensional nonlinear Stokes system for ice-sheet dynamics, an advection-diffusive energy equation for the temperature evolution, and a mass-conservation equation for the ice thickness. Our computational ice-sheet model features a new parallel finite element implementation using variable resolution layered tetrahedral meshes. Discretization is based on the high-accuracy Taylor-Hood element pair (quadratic elements for velocity and linear elements for pressure) that is stable without any need for penalization or stabilization. Both no-sliding and sliding boundary conditions at the ice-bedrock boundary can be applied. In particular, we proposed and implemented an efficient and effective approach for handling the Rayleigh friction boundary condition through a rotated coordinate system at each velocity node on the sliding boundary. In addition, effective solvers for the linear system using preconditioning techniques for the saddle-point system such as Algebraic Multigrid (AMD) were discussed and implemented. The parallel libraries HYPRE (http://acts.nersc.gov/hypre/) and PETSC (http://www.mcs.anl.gov/petsc/) in Trillinos were used in our implementation. Through analytical solution tests (see Section 3) and the well-known ISMIP-HOM ice-sheet benchmark experiments, we demonstrated that our finite element nonlinear Stokes model performs well compared to other published and established Stokes models. The parallel solver was also shown to be efficient, robust, and scalable.

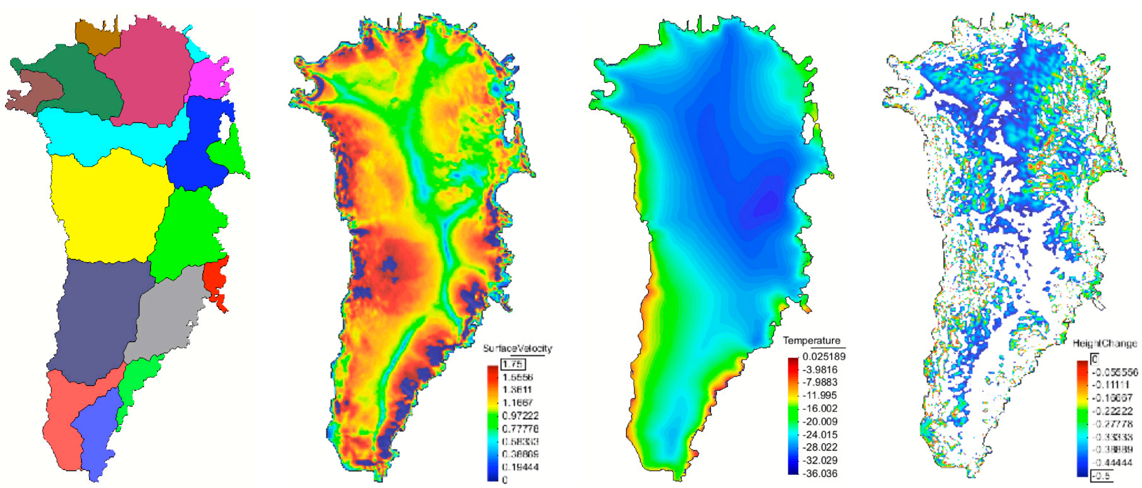
Impact on the climate modeling community. We have developed a software suite for high-fidelity, three-dimensional ice-sheet modeling that would be openly available to the climate community. In addition, this software is currently being interfaced with and thus made part of the MPAS climate system model currently being developed by LANL and NCAR and will thus be available to the climate community through that avenue.

Example: ISMIP-HOM ice-sheet benchmark experiments



Cross-sectional view of simulation results for ISMIP-HOM Experiment E. Left: ice fixed to the basal boundary; right: with a sliding zone at the basal boundary. From top to bottom: the components horizontal in plane and out of plane velocity components and the vertical velocity component.

Example: Greenland ice sheet



Simulation results for Greenland after one year. Left to right: partition into 16 sub-regions using the METIS package for partitioning; surface velocity; surface temperature; height change.

3. Manufactured Solution for Ice-Sheet Computational Model Verification

Goal. Develop an analytic solution for the nonlinear Stokes equation mathematical model for ice sheets that can be used to verify the accuracy of results obtained from computational models for those equations.

Approach. We synthesized an exact solution of three-dimensional, isothermal, nonlinear Stokes mathematical model for flows in glaciers and ice sheets equations using the method of manufactured solutions. The solution construction procedure starts with kinematic boundary conditions and is mainly based on the solution of a first-order partial differential equation for the ice velocity that satisfies the incompressibility condition. The manufactured solutions depend on the geometry of the ice-sheet, basal sliding parameters, and ice softness. Initial conditions are taken from the periodic geometry of a standard problem of the ISMIP-HOM benchmark tests. The upper surface is altered through the manufactured solution procedure to generate an analytic solution for the time-dependent flow problem.

Accomplishments. We took great pains to make sure that our manufactured solution contains many of the features commonly seen in ice sheets so that the use of such solutions provides a realist verification test for computational ice sheet models. We used our manufactured solution to verify our parallel, high-order accurate, finite element Stokes ice-sheet model. Simulation results from the computational model show excellent, high-order accurate convergence of the computational model results to the manufactured exact solution.

Impact on the climate modeling community. Manufactured solution technique is often used for the verification of computational models in many fields. The manufactured solution for the three-dimensional Stokes ice-sheet model that we have developed can be used by all groups developing such models to verify their models. Furthermore, the manufactured solution we have developed provides an excellent setting for comparing the fidelity of different computational models so that it provides a valuable tool for assessing the relative usefulness of those models. Furthermore, because code verification is often a necessarily step for the validation of mathematical models, the manufactured solution we have developed provides an avenue for model validation as well.

Example: Verification of the 3D Stokes-Ice Sheet Model

Mesh	DOF	Velocity error	Convergence rate	Pressure error	Convergence rate
$20\times20\times5$	56,184	26.7	–	19.1	–
$40\times40\times10$	424,364	4.00	2.74	6.70	1.51
$80\times80\times20$	3,296,724	0.316	3.66	0.182	1.88

The manufactured solution is used to verify our three-dimensional finite element ice-sheet model. Errors are measure with respect to the L^2 norm. High-order convergence rates are achieved.

4. Unified Grids for Multi-Physics Modeling

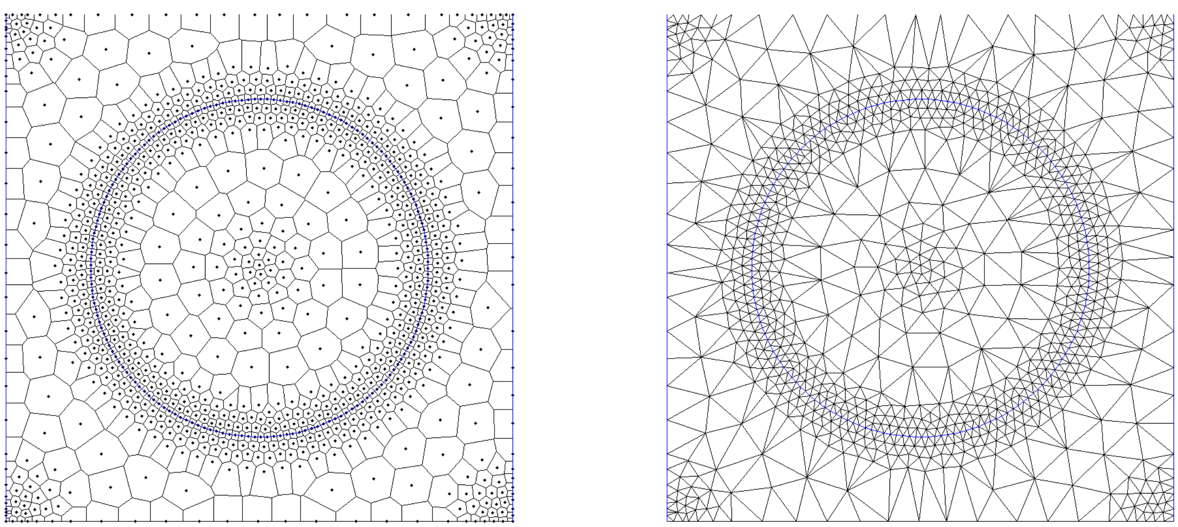
Goal. Develop a capability for constructing high-quality, variable resolution grids that match at the interface between domains for multi-domain, multi-physics problems.

Approach. We use CVT and SCVT gridding strategies to generate variable resolution grids, where refinement is effected to both resolve the interface between domains (as well as outer boundaries) and in interior regions of interest in each domain. The matching of the grids on the two sides of an interface is effected by projecting, at every step of the iterative method for the construction of the CVT or SCVT grids, Voronoi generators near the interface to the nearest point on the interface.

Accomplishments. Grid generation for multiple-domain multiple-physics problems in which the single-physics components are applied on disjoint abutting domains is usually done separately in each domain, thus resulting in grids that do not match along the interfaces between the domains. This happens, e.g., in cases for which legacy codes are used. Mismatching can result in the two grids have overlaps and gaps and have grid points that do not match along the interface between the domains. Using very simple settings, we illustrated the possible difficulties that can arise when using non-matching grids. Then, using the approach we describe above, we developed the capability to construct grids that match perfectly at the interface between the two domains; users can fix some or all points along the interface, or leave the algorithm to select all those points. The capability we developed allows for grid refinement both to resolve boundaries, including interfaces, and also to refine in regions of interest. We illustrated the matching-grid construction algorithm through its application to simple examples as well as to multi-domain gridding on the globe.

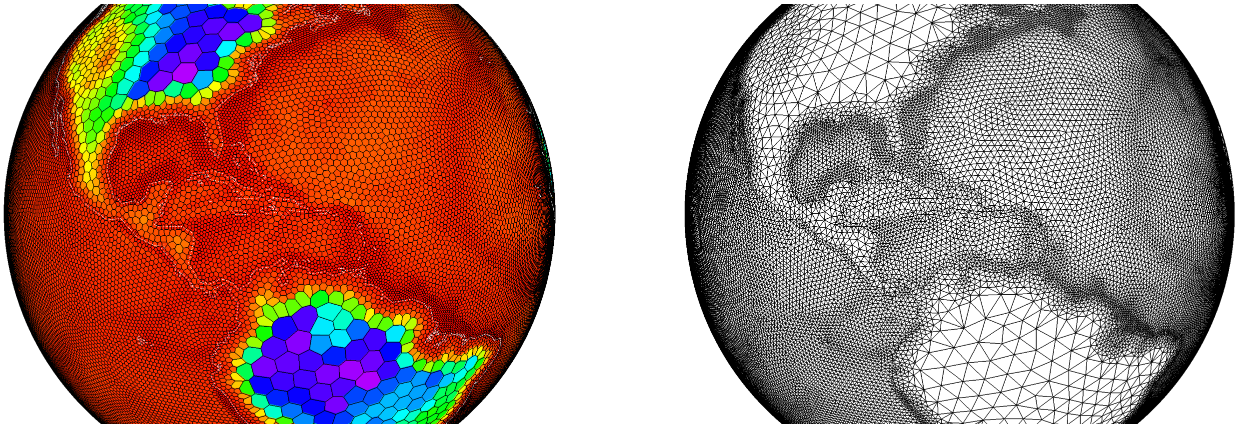
Impact on the climate modeling community. Climate system models consist of multiple components (ocean, land ice, atmosphere, etc.) which are applied on disjoint but abutting domains. It is along the interfaces between the domains that coupling occurs between components. Transferring information between the components is compromised by the use of grids that do not match along the interfaces because, e.g., interpolation is needed and there may be grid gaps or overlaps due to the mismatching. Having grids that perfectly match along interfaces along with having the ability to refine the grid to resolve the interfaces (and in regions of interest) could be of substantial benefit for component-coupling strategies in climate system models.

Example: A simple two-domain problem.



Matching grids for two domains with a circular interface. The Voronoi generators (left) lie on the interface whereas the triangle edges of the corresponding Delauney are aligned with the interface. The grids are individually refined to both better resolve the interface (the circle) and at regions of *interest (the center of the circle and the corners of the square).*

Example: Matching ocean-land grids.



A portion of a variable resolution matching ocean-land grid focused; Voronoi mesh with cells color coded according to the areas of the cells (left) and the corresponding Delauney mesh (right).

Other Project Participants

Additional personnel have been supported by the grants at Florida State University and the University of South Carolina.

Janet Peterson (Professor, FSU)
In addition to co-advising Geoffrey Wolmendorff, a graduate student (see below), Professor Peterson's work is focused on the fourth project described above.

Huai Zhang (Research Associate Professor, USC)
Huai Zhang worked with Lili Ju (USC), Max Gunzburger (FSU), Todd Ringler (LANL), and Steve Price (LANL) on the first and second projects described above.

Vani Cheruvu (Postdoctoral Associate, FSU)
Vani Chevuru was advised by Max Gunzburger (FSU) and collaborated with Lili Ju (USC) and Todd Ringler (LANL) on the first project described above.

Tao Cui (Visiting Scholar, FSU)
Tao Cui worked with Lili Ju (USC) and Max Gunzburger (FSU) on the second project described above.

Doug Peterson (Graduate Student, FSU)
Doug Peterson was advised by Max Gunzburger (FSU) and collaborated with Todd Ringler (LANL) and Mark Peterson (LANL). He completed a M.S. degree in August 2009 and then a Ph.D. degree in August 2011. His work was directed at testing the relative merits of different grids within the POP and HYPOP ocean models.

Geoffrey Wolmendorff (Graduate Student, FSU)
Geoffrey Wolmendorff was advised by Max Gunzburger (FSU) and Janet Peterson (FSU) and collaborated with Todd Ringler (LANL). He completed a M.S. degree in August 2008 and and then a Ph.D. degree in August 2011. He worked on the first project described above.

Wei Leng (Visiting Graduate Student, FSU)
Wei Leng worked with Lili Ju (USC), Max Gunzburger (FSU), and Steve Price (LANL) on the

second and third projects described above.

Li Tian (Graduate Student, USC)

Li Tian was advised by Lili Ju (USC) and received a Ph.D. degree in August 2009. His work on a priori and a posteriori error estimates for finite element/volume methods for PDEs on surfaces was directly relevant to the first and second projects described above.

Xiao Xiao (Graduate Student, USC)

Xiao Xiao was advised by Lili Ju (USC). She completed a M.S. degree in May 2010 and and then a Ph.D. degree in May 2012. Her work is related to the first and second projects described above.

Jing Liu (Graduate Student, USC)

Jing Liu was a graduate student advised by Lili Ju (USC). He completed a M.S. degree in Mathematics in August 2010. He worked on the first project described above.

Papers Prepared Under Grant Sponsorship

- T. RINGLER, L. JU, AND M. GUNZBURGER, *A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*, Ocean Dynamics, Vol. 58, pp. 475-498, 2008.
- H. NGUYEN, J. BURKARDT, M. GUNZBURGER, L. JU AND Y. SAKA, *Constrained CVT meshes and a comparison of triangular mesh generators*, Computational Geometry: Theory and Applications, Vol. 42, pp. 1-19, 2009.
- L. JU AND Q. DU, *A finite volume method on general surfaces and its error estimates*, Journal of Mathematical Analysis and Applications, Vol. 352, pp. 645-668, 2009.
- Q. DU, L. JU AND L. TIAN, Analysis of a mixed finite-volume discretization for fourth-order equations on general surfaces, *IMA Journal of Numerical Analysis*, Vol. **29**, pp. 376-403, 2009.
- L. JU ,W. WU, AND W.-D. ZHAO, *Adaptive finite volume methods for steady convection-diffusion equations with mesh optimization*, Discontinuous and Continuous Dynamical Systems B, Vol. 11, pp. 669-690, 2009.
- L. JU, L. TIAN, AND D. WANG, *A posteriori error estimates for finite volume approximations of elliptic equations on general surfaces*, Computer Methods in Applied Mechanics and Engineering, Vol. 198, pp. 716-726, 2009.
- H. NGUYEN, M. GUNZBURGER, L. JU, AND J. BURKARDT, *Adaptive anisotropic meshing for steady convection-dominated problems*, Computer Methods in Applied Mechanics and Engineering, Vol. 198, pp. 2964-2981, 2009.
- Q. DU, M. GUNZBURGER, AND L. JU, *Advances in studies and applications of centroidal Voronoi tessellations*, Numerical Mathematics: Theory, Methods and Applications, Vol. 3, pp. 119-142, 2010.
- L. JU, T. RINGLER, AND M. GUNZBURGER, *Voronoi diagrams and their application to climate and global modeling*, Lecture Notes in Computational Science and Engineering – Numerical Techniques for Global Atmospheric Models, Vol. 80, pp. 313-342, Ed. by P. Lauritzen, C. Jablonowski, M. Taylor, R. Nair, Springer, 2011.

- Q. Du, L. Ju, and L. Tian, *Finite element approximation of the Cahn-Hilliard equations on surfaces*, Computer Methods in Applied Mechanics and Engineering, Vol. 200, pp. 2458-2470, 2011.
- H. Zhang, L. Ju, M. Gunzburger, T. Ringler, and S. Price, *Coupled models and parallel simulations for three-dimensional full-Stokes ice sheet modeling*, Numerical Mathematics: Theory, Methods and Applications, Vol. 4, pp. 359-381, 2011.
- T. Ringler, D. Jacobsen, M. Gunzburger, L. Ju, M. Duda, and W. Skamarock, *Exploring a multi-resolution modeling approach within the shallow-water equations*, Monthly Weather Review, Vol. 139, pp. 3348-3368, 2011.
- L. Ju, L. Tian, X. Xiao, and W.-D. Zhao, *Covolume-upwind finite volume approximations for linear elliptic partial differential equations*, Journal of Computational Physics, Vol. 231, pp. 6097-6120, 2012.
- W. Leng, L. Ju, M. Gunzburger, S. Price, and T. Ringler, *A parallel high-order accurate finite element nonlinear Stokes ice sheet model and benchmark experiments*, Journal of Geophysical Research, Vol. 117, F01001 (24 pages), 2012.
- W. Leng, L. Ju, M. Gunzburger, and S. Price, *Manufactured solutions and the numerical verification of three-dimensional Stokes ice sheet models*, Cryosphere, Vol. 7, pp. 19-29, 2013.
- Q. Chen, M. Gunzburger, and M. Perego, *Well-posedness results for a nonlinear Stokes problem arising in glaciology*, submitted to SIAM Journal on Mathematical Analysis.
- G. Womerldorff, J. Peterson, M. Gunzburger, and T. Ringler, *Unified matching grids for multidomain multiphysics simulations*, submitted to SIAM Journal on Scientific Computing.

Talks Delivered

- *"New grid and discretization technologies for ocean and ice simulations"* (Poster), Climate Change Prediction Program Meeting, Indianapolis, September 2007. (by M. Gunzburger et. al.)
- *"Adaptive finite volume methods for steady convection-diffusion equations"*, Computational Nano Seminar, University of South Carolina, November 2007. (by L. Ju)
- *"Grids for ocean and ice modeling"*, Graduate Student Seminar, Florida State University, January 2008. (by M. Gunzburger)
- *"Analysis of a mixed finite volume discretization for fourth-order equations on general surfaces"*, SIAM Southeastern Atlantic Section Annual Meeting, Orlando, March 2008. (by. L. Ju)
- *"Adaptive finite element method based on conforming centroidal Voronoi Delaunay triangulations"*, Colloquium, Department of Mathematics, Xiangtan University, China, May 2008. (by. L. Ju)

- *“Grids and discretization schemes for ocean and ice modeling”*, The sixth International Conference on Scientific Computing and Applications, Pusan National University, Busan, Korea, June 2008. (by. M. Gunzburger)
- *“Adaptive finite element method based on conforming centroidal Voronoi Delaunay triangulations”*, Colloquium, Xiangtan University, China, May 2008. (by L. Ju)
- *“A posteriori error estimates for finite-volume approximation of elliptic equations on general surfaces”*, SIAM Annual Meeting, San Diego, July 2008. (by L. Tian, graduate student)
- *“A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations”*, Colloquium, Florida International University, January 2009. (by M. Gunzburger)
- *“A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations”*, Colloquium, George Mason University, January 2009. (by M. Gunzburger)
- *“A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations”*, Conference on Energy, Wind and Water: Algorithms for Simulation, Optimization and Control, Auckland, New Zealand, February 2009. (by M. Gunzburger)
- *“A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations”*, Colloquium, University of North Carolina at Charlotte, February 2009. (by M. Gunzburger et. al)
- *“A multi-resolution method for climate system modeling”*, SIAM Conference on Computational Science and Engineering, Miami, March 2009. (by L. Ju et. al)
- *“Analysis of a mixed finite-volume discretization of fourth-order equations on general surfaces”*, SIAM Conference on Computational Science and Engineering, Miami, March 2009. (by L. Tian, graduate student)
- *“High-order methods for ocean modeling”*, SIAM Conference on Computational Science and Engineering, Miami, March 2009. (by V. Cheruvu, postdoctoral associate)
- *“A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations,* Colloquium, Oxford University, United Kingdom, March 2009. (by M. Gunzburger)
- *“Adaptive finite volume methods for steady convection-diffusion equations with mesh optimization”*, The 33th SIAM Southeastern Atlantic Section Annual Meeting, Columbia, SC, April 2009. (by L. Ju)
- *“Analysis of a mixed finite-volume discretization of fourth-order equations on general surfaces”*, The 33th SIAM Southeastern Atlantic Section Annual Meeting, Columbia, SC, April 2009. (by L. Tian, graduate student)
- *“New grid and discretization technologies for ocean and ice simulations”* (Poster), Climate Change Prediction Program Meeting, Bethesda, April 2009. (by M. Gunzburger et. al.)
- *“A parallel solver for three-dimensional full-Stokes ice sheet modeling”* (Poster), Climate Change Prediction Program Meeting, Bethesda, MD, April 2009. (by L. Ju et. al.)

- *"A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations"*, Colloquium, Tulane University, April 2009. (by M. Gunzburger.)
- *"Adaptive finite element method based on conforming centroidal Voronoi Delaunay triangulations"*, Seminar, School of Mathematics and System Sciences, Shandong University, China, June 2009 (by L. Ju)
- *"A parallel solver for three dimensional full-Stokes ice sheet modeling"*, Aubrey Lecture Series, Department of Earth and Ocean Sciences, University of South Carolina, September 2009 (by L. Ju)
- *"Over-relaxation Lloyd method for computing centroidal Voronoi tessellations"*, Cha-Cha Days Workshop 2009, Orlando, FL, October 2009 (by X. Xiao, graduate sdtudent)
- *"Over-relaxation Lloyd method for computing centroidal Voronoi tessellations"*, SIAM Student Conference 2010, Blacksburg, VA, 2010 (by X. Xiao, graduate student)
- *"A parallel solver for three dimensional full-Stokes ice sheet modeling"*, 2010 Spring AMS Southeastern Section Meeting, Lexington, KY, 2010 (by L. Ju)
- *"A parallel solver for three dimensional full-Stokes ice sheet modeling"*, SIAM Southeastern Atlantic Section Annual Meeting 2010, Raleigh, NC, 2010 (by L. Ju)
- *"Adaptive finite element method based on conforming centroidal Voronoi Delaunay triangulations"*, Applied Mathematics Seminar, Department of Mathematics, Ohio State University, 2010 (by L. Ju)
- *"Centroidal Voronoi tessellations: algorithms and applications"*, Frontiers of Scientific Computing Distinguished Lecture Series, Louisiana State University, MD, 2010 (by M. Gunzburger)
- *"Centroidal Voronoi tessellations: algorithms and applications"*, Colloquium, Massachusetts Institute of Techonology, 2010 (by M. Gunzburger)
- *"Greenland ice-sheet simulations via parallel three-dimesnional full- Stokes modeling"* (Poster), DOE Climate Change Modeling Program Science Team Meeting 2010, Gaithersburg, MD, April 2010 (by L. Ju et. al.)
- *"Variable resolution centroidal Voronoi grids for climate simulations"* (Poster), DOE Climate Change Modeling Program Science Team Meeting 2010, Gaithersburg, MD, April 2010 (by M. Gunzburger et. al.)
- *"Centroidal Voronoi tessellations: algorithms and applications"*, Colloquium, Emory University, 2010 (by M. Gunzburger)
- *"Coupled models and parallel simulations for three-dimensional full-Stokes ice sheet modeling"*, International Workshop on Scientific Computing and Nonlinear Partial Differential Equations, Jiuzhaigou, Sichuan Province, China, July, 2010 (by L. Ju)
- *"Covolume-upwind finite volume approximations for linear elliptic partial differential equations"*, SIAM Student Conference, Clemson, SC, February 2011 (by X. Xiao)

- *"A parallel high-order accurate finite element full-Stokes ice-sheet model with validation using benchmark experiments"*, SIAM Southeastern Atlantic Section Annual Meeting 2011, Charlotte, NC, March 2011 (by L. Ju)
- *"A parallel high-order accurate finite element full-Stokes ice-sheet model and benchmark experiments"*, 2011 Spring AMS Western Section Meeting, Las Vegas, NV, April 2011 (By L. Ju)
- *"A parallel high-order accurate finite element nonlinear Stokes ice sheet model and benchmark experiments"*, Applied and Computational Mathematics Seminar, School of Mathematics, Georgia Institute of Technology, April 2011 (by L. Ju)
- *"A parallel high-order accurate finite element nonlinear Stokes ice sheet model and benchmark experiments"*, Seminar, School of Mathematics, Shandong University, China, June 2011 (by L. Ju)
- *"A parallel high-order accurate finite element nonlinear Stokes ice-sheet model and benchmark experiments"*, International Conference on Applied Mathematics and Interdisciplinary Research, Tianjin, China, June 2011 (by L. Ju)
- *"A parallel high-order accurate finite element nonlinear Stokes ice sheet model and benchmark experiments"*, Seminar, Laboratory of Computational Geodynamics, Graduate University of Chinese Academy of Sciences, June 2011 (by L. Ju)
- *"Covolume-upwind finite volume approximations for linear elliptic partial differential equations"*, SIAM Southeastern Atlantic Section Annual Meeting 2012, Huntsville, AL, March 2012 (by L. Ju)
- *"A parallel high-order accurate finite element full-Stokes ice-sheet model with validation using benchmark experiments"*, International Congress on Industrial and Applied Mathematics 2011, Vancouver, Canada, July 2011 (by L. Ju)
- *"Covolume-upwind finite volume approximations for linear elliptic partial differential equations"*, International Conference on Scientific Computing and Applications 2012, Las Vegas, NV, April 2012 (by L. Ju)
- *"The science of ice sheets: The mathematical modeling and computational simulation of ice flows"*, University of Oxford, United Kingdom, 2012 (by M. Gunzburger)
- *"A parallel high-order accurate finite element nonlinear Stokes ice sheet model and benchmark experiments"*, Seminar, School of Mathematics, Capital Normal University, China, June 2012 (by L. Ju)
- *"The science of ice sheets: The mathematical modeling and computational simulation of ice flows"*, Rostchild Lecture, Newton Institute for Mathematical Sciences, Cambridge University, United Kingdom, 2012 (by M. Gunzburger)
- *"Adaptive finite element method based on conforming centroidal Voronoi Delaunay triangulations"*, Seminar, School of Mathematical Sciences, Beihang University, China, July 2012 (by L. Ju)