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Chapter VIII

PROJECT SHERWOOD:

ORIENTATION LECTURES PRESENTED AT OAK RIDGE NATIONAL LABORATORY

NOVEMBER - DECEMBER 1955

VIII. Stability

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VIII. STABILITY

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Date Issued

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VIII. STABILITY

For the past two years, the central question in the Sherwood business has been the stability of the various proposals which are under development. It is not enough to demonstrate that a specific proposal has a sufficiently long single particle containment, and that a steady state configuration (omitting diffusion and inherent geometrical leaks) exists in which the plasma pressure drops to zero on some fixed surface in the magnetic region. There will always be small fluctuations about this steady state, or equilibrium, solution. It is also necessary that the time behavior of the system be such that these perturbations die away in time or oscillate around the equilibrium solution. In such a case, the equilibrium is said to be stable. If the perturbations grow in time, the equilibrium is unstable.

It is generally found that if instabilities exist in a plasma the amplitude of the perturbations e-fold in a time comparable with the time it takes a sound wave to cross some dimension of the plasma. At thermonuclear temperatures this is of the order of microseconds. Consequently, if instabilities exist they are much more serious than normal loss or diffusion rates. It is imperative that the instabilities be predicted, if they exist, and that methods of overcoming them be devised.

The problem of stability of fluid motions in ordinary hydrodynamics is an exceedingly complicated subject. The situation is perhaps even more difficult when attention is focused on the behavior of an ionized gas in magnetic and electric fields. The interaction of a hydrodynamic fluid with electromagnetic fields forms the new and interesting subject of hydromagnetics.

A fluid of this sort has some interesting and simplifying properties. In all cases of interest the conductivity of the plasma is very large and the time during which the system is to hold together is relatively short. Hence, it is often sufficient to assume that the conductivity is infinite.

<u>Equilibrium Solutions</u>

Before turning to the question of the stability of this hydromagnetic fluid, it is necessary to have an equilibrium state which is to be perturbed. Perhaps the simplest steady-state equations which this fluid must satisfy are the following:

$$\nabla P = \vec{J} \times \vec{B} \tag{8.1}$$

$$\nabla \times \vec{B} = 4\pi \vec{J} \tag{8.2}$$

$$\nabla \cdot \vec{B} = 0 \tag{8.3}$$

where P is the gas pressure, $\vec{J}$ the current density and $\vec{B}$ the magnetic field. These equations already represent a serious compromise with reality. In the first equation, a non-linear term involving the mass velocity of the fluid has been omitted as well as a term representing a force due to a possible charge density in an electric field. However, if attention is confined to equilibria in which there are no mass velocities or electric fields, these equations are almost correct. The most important remaining discrepancy is the use of an isotropic scalar pressure in Eq. (8.1). In actuality, this term should be the divergence of a stress tensor, denoted by $\tilde{T}$. If there are enough collisions during an instability to keep the velocity distribution of the particles isotropic, this is a valid approximation. In practice, this situation is far from true. A later section in this chapter will discuss some rough attempts at theories with a tensor pressure.

Equations (8.1) to (8.3) do not yield a unique solution, even in a given magnetic geometry. This has already been pointed up in Chap. VII, where the study of the steady-state pinch proceeded from a consideration of just these three equations. It was necessary there to choose a specific pressure distribution in order to obtain a solution. In an actual situation, one must include particle sources, diffusion losses, and finite plasma conductivity in order to obtain a unique solution. Kruskal[34] has shown that the complete set of steady-state equations may be expected to yield a unique solution. Nevertheless, it is customary to use only Eqs. (8.1) to (8.3) and assume as simple a pressure distribution as possible in order to study the stability of the resulting equilibrium.

Normal Mode Analysis

The earliest hydromagnetic problems treated in the Sherwood Program were first, an analogy to the Rayleigh instability problem of hydrodynamics and second, the stability of the pinch. These situations were analyzed by Kruskal and Schwarzschild[22] using the normal mode analysis. The starting point of this method is the time-dependant equations of motion of the plasma. These are:

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34. M. D. Kruskal, The Steady State Plasma Equations for the Stellarator Under Diffusion, NYO-7307, PMS-17, (May, 1955).

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} + \epsilon\vec{E} - \nabla P + \rho\vec{g} \quad (8.4)$$

$$\nabla \cdot (\rho\vec{v}) = - \frac{\partial\rho}{\partial t} \quad (8.5)$$

$$\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = \frac{1}{\sigma} (c\vec{J} - \epsilon\vec{v}) \quad (8.6)$$

$$\frac{1}{P} \frac{dP}{dt} = \frac{\gamma}{\rho} \frac{d\rho}{dt} \quad (8.7)$$

$$\nabla \times \vec{B} = 4\pi\vec{J} + \frac{1}{c} \frac{\partial\vec{E}}{\partial t} \quad (8.8)$$

$$\nabla \cdot \vec{B} = 0 \quad (8.9)$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial\vec{B}}{\partial t} \quad (8.10)$$

$$\nabla \cdot \vec{E} = 4\pi\epsilon \quad (8.11)$$

The first equation represents the force equation. Note that a scalar pressure has been assumed and that a possible gravitational term has been added. Here ρ is the plasma density, $\vec{g}$ the gravitational acceleration, ϵ the charge density, and $\vec{v}$ the mass velocity of the plasma. The electromagnetic quantities are in mixed Gaussian units and the conductivity σ is in esu. The Eulerian derivative is denoted by d/dt, and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \quad (8.12)$$

The second equation is the mass conservation relation, while the third is the generalized Ohm's law. In most applications σ will be taken to be infinite and the right hand side of this equation set equal to zero. It should be noted that some additional terms, which are usually small, have been omitted in Eq. (8.6). These terms may be found in Spitzer's book.[35] The fourth equation states that the motion is adiabatic. Here γ is the ratio of specific heats of the plasma. This equation implies that heat transfer within the plasma is negligible. If this is not true some more complicated relation must be used. Finally, the last four equations are the familiar Maxwell equations. Note that no distinction need be made between $\vec{B}$ and $\vec{H}$, and $\vec{D}$ and $\vec{E}$, since all currents and charge densities in the medium are treated explicitly. It is assumed in these equations that there are no particle or heat losses from the plasma and no particle or energy sources within it. Otherwise, one must include the appropriate equations.

Equations (8.4) to (8.11) represent a formidable set of relations, particularly since they are non-linear in character. Hence, the first step in treating them is to linearize the equations. This is accomplished by writing each physical variable as the sum of the unperturbed equilibrium value (denoted by a subscript zero) and a small perturbed part, thus for example:

$$\vec{B} = \vec{B_0} + \vec{B_1}, \tag{8.13}$$

and then neglecting all terms of second order or higher in the perturbed variables in the resulting equations.

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35. L. Spitzer, Jr., Physics of Fully Ionized Gases, p. 21, Interscience Publishers, Inc., New York (1956).

Further progress ia made by taking a Fourier transform of the perturbed quantities in time and in as many spatial variables as possible. Thus, for example, in the case of the pinch the unperturbed solutions are functions of the radial distance r, only. Hence, one can write, for example,

$$\vec{B_1} = \vec{B_1}(r)\, e^{\omega t}\, e^{i(m\theta + kz)} \qquad (8.14)$$

where m must be an integer in order that the solutions be single-valued and k may have any real value. The final step consists in solving the set of coupled, homogeneous ordinary differential equations resulting from the substitution of Eq. (8.14) in the linearized relations subject to the proper boundary conditions. The final result is in the form of a single "characteristic" equation which is a function of ω, k, m, and the unperturbed variables. The system is unstable or stable depending on whether or not there exist solutions of this "characteristic" equation with ω having a real positive part.

The results of Kruskal and Schwarzschild[22] for the case of the ordinary pinch have already been described in Chap. VII. The m = 1 mode, which corresponds to the "kink" perturbation, was found to be unstable for all wavelengths k. The m = 0 mode, which is the "sausage" instability, is also unstable for all wavelengths[24] while the higher modes m ≥ 2 are unstable only for sufficiently small wavelengths. The first problem treated in Ref. 22 was the case of an infinitely conducting fluid supported against gravity by a magnetic field. This equilibrium, in complete analogy to the Rayleigh instability problem, was also found to be unstable.

Kruskal and Tuck[24] then added a magnetic field to the pinch in the longitudinal direction both inside the plasma and out. It was found that this stabilized the short wavelength instabilities. Finally, the recent work by Rosenbluth[25] (using a form of the variational technique to be described in the next section) considered the combined effect of an internal longitudinal field and an external conducting shell. The results indicated that there were indeed regions of complete stability of the pinch but with some strong restrictions on the maximum compression of the pinch and on the maximum value of the external longitudinal field.

A diagram of some of the results given by Rosenbluth is shown in Fig. 8.1.

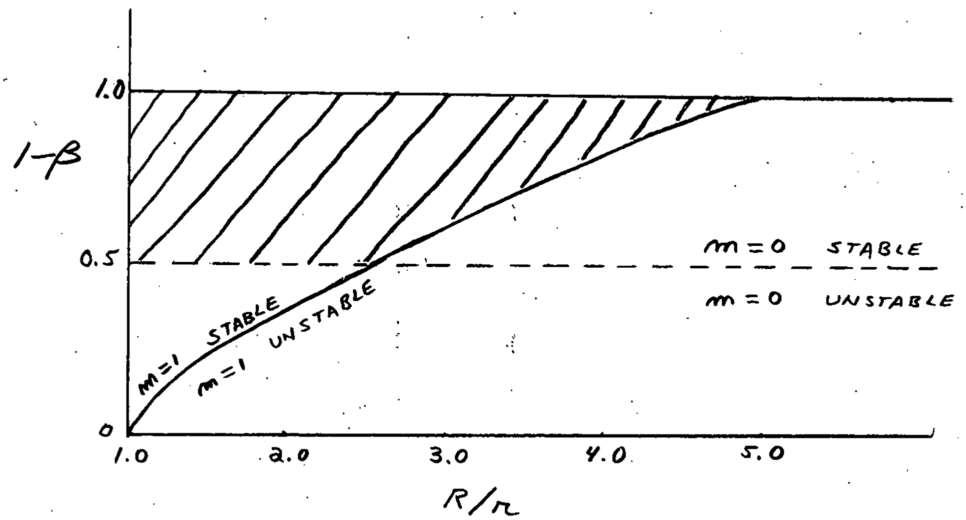

Fig. 8.1. Stability Zone for No External B_z

This result is for the case of no longitudinal field external to the pinch. The quantity β is the ratio of the constant material pressure in the pinch to the external magnetic pressure at the boundary. Hence,

$$\beta = \frac{8\pi P}{B_\theta^2(r)} = 1 - \frac{B_z^2}{B_\theta^2(r)} \qquad (8.15)$$

The external conductor radius is denoted by R and the pinch radius by r. It was found that the m = 0 mode was unstable for any compression if $\beta > 0.5$. In addition, the m = 1 mode is unstable at a given value of β for any compression greater than a number varying between 1 and 5. No stability at all exists for $R/r > 5$. The region of complete stability is indicated by the shaded zone in Fig. 8.1. A similar diagram may be drawn for any other given value of the external B_z field. The general nature of these results is that the zone of stability shrinks to the left of the diagram as the external B_z field increases in value. For example, when B_z^2 external is equal to $\frac{1}{2} B_\theta^2(r)$, there is no stability at all for R/r greater than about 1.85 and the maximum compression for β of 0.5 is about 1.2.

It is of interest to note that the stabilizing tendency of the B_z field is entirely dependant upon its being embedded in the plasma. A recent calculation by R. J. Mackin and A. Simon (unpublished) considered the case of a linear coaxial cylinder of plasma with longitudinal fields existant in the hollow center of the cylinder, in the plasma itself and external to the cylinder. It was found that the trapped magnetic field in the hollow center of the pinch did not contribute to stability (in fact, it had no effect at all on the m = 0 instabilities) and that this function is entirely performed by the longitudinal field in the plasma itself.

The Variational Method

The method of normal mode analysis was historically the first to be applied to Sherwood stability problems. In particular, it was and is a good method for analyzing simple geometries, such as the pinch, in which one can solve the resulting differential equations to obtain the eigenmodes. The method is considerably less flexible when more complicated geometries are considered.

Interest in more complicated geometries was aroused by Edward Teller at the 1954 Princeton meeting when he expressed doubts that any of the systems we were dealing with were stable. He likened containment of the plasma by magnetic fields to containment of a gas by a large number of rubber bands, which would be highly unstable, and illustrated his remarks by the following example. Let the magnetic field be excluded from the plasma and let the system be cylindrically symmetric with a bulge as shown in Fig. 8.2. The dotted

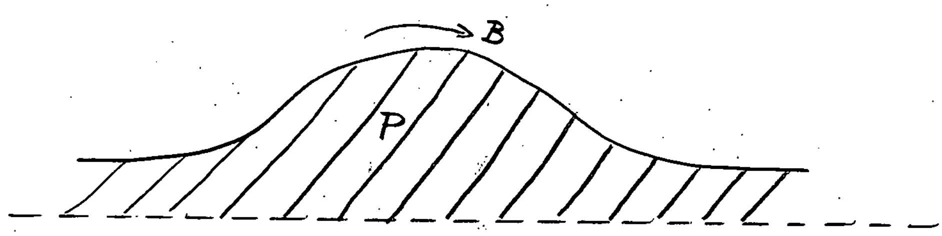

Fig. 8.2. Magnetically Confined Bulge

line is the axis of symmetry of the system and the trace of the surface in the plane of the paper is a line of magnetic flux. Assume that a small ripple occurs on the surface, and that this ripple occurs all along its included flux lines. Thus a cross section of the plasma at any axial point has the form shown in Fig. 8.3.

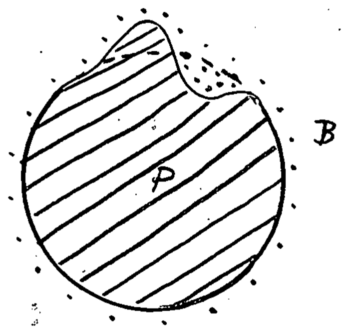

Fig. 8.3. Surface Ripple.

Assume that the ripple preserves the volume of the plasma. In that case the plasma pressure and hence the plasma energy is unchanged. Assume further that the flux lines which were in the shaded region above the dotted line are now moved into the newly available volume in the trough of the ripple and under the dotted line, and that the remainder of the magnetic field is undisturbed. Owing to the curvature of the bulge, the area in the shaded region above the dotted line must be somewhat smaller than the area in the trough below the dotted line if the total volumes are to be equal. Hence, the magnetic field strength is reduced somewhat in the trough and the total magnetic field energy is reduced. The total potential energy of the system has decreased as a result of this ripple and so the system is unstable to this perturbation.

It was shown almost immediately that Teller's particular argument was wrong, but that his intuitive idea was correct. The only error in his argument is the extension of the ripple all along the included lines of magnetic flux. The regions of reverse curvature at the left and right of the bulge in Fig. 8.2 have a reverse effect on the potential energy change and actually overcome the instability produced by the central section of positive curvature. The system is unstable to little "flutes" or ripples which do not extend the whole length along a line of force but terminate before the curvature changes sign.

Perhaps another way to see the instability of the plasma is by an argument due to Conrad Longmire. If the flutes are very thin they leave the rest of the magnetic field undisturbed moving only a little flux and keeping the plasma pressure constant. Since the field decreases outward because of its curvature the same gas pressure meets a lower magnetic pressure at the top of the very thin flute and the flute continues to grow.

Since the instability problem was serious for almost all geometries of interest a general theory seemed desirable. A very powerful technique is available by use of variational methods and this is known as the δW formalism. Consider a displacement perturbation $\vec{\xi}(\vec{r})$ of the material of the plasma. Imagine pincers from outside displacing every element of plasma through a distance $\vec{\xi}(\vec{r})$. Since the matter is nearly infinitely conducting the lines of force are frozen in the plasma. From their varying density one can calculate the new field strength at the end of the displacement. In addition, knowing the varying density of the plasma and assuming adiabatic compression allows one to calculate the change in gas pressure. These two results allow a calculation of the change in total potential energy of the system.

Let the total potential energy be denoted by W. Then,

$$W = \int \left(\frac{B^2}{8\pi} + \frac{P}{\gamma - 1} \right) d\tau \tag{8.16}$$

where the integration is over all space. It can be shown that the change in W due to a displacement $\vec{\xi}(\vec{r})$ is

$$\delta W = \frac{1}{8\pi} \int \left\{ \left[\nabla \times (\vec{\xi} \times \vec{B}) \right]^2 + \vec{\mathcal{J}} \cdot \vec{\xi} \times \left[\nabla \times (\vec{\xi} \times \vec{B}) \right] + 4\pi\gamma P (\nabla \cdot \vec{\xi})^2 + 4\pi (\vec{\xi} \cdot \nabla P)(\nabla \cdot \vec{\xi}) \right\} d\tau \tag{8.17}$$

where B and P are equilibrium values. If δW is positive external work must be done to carry out the displacement and the system is stable to this $\vec{\xi}(\vec{r})$. If δW is negative the system is unstable.

It may be shown that the ratio

$$\lambda^2 = - \frac{\delta W}{\frac{1}{2} \int \rho \xi^2 \, d\tau}, \tag{8.18}$$

where ρ is the equilibrium density, is stationary (i.e., maximum, minimum, or saddle point) with regard to the possible displacement functions $\vec{\xi}(\vec{r})$ whenever $\vec{\xi}(r)$ corresponds to an eigenmode of the system. In such cases, the time behavior of the displacement is

$$\vec{\xi}(\vec{r},t) = \vec{\xi}(r)\, e^{\omega t} \tag{8.19}$$

and

$$\lambda^2 = \omega^2$$

Hence, if a displacement is found which makes δW negative, one is assured that there exists at least one unstable eigenmode of the system with an eigenvalue ω^2 which is

$$\omega^2 \geq \lambda^2, \tag{8.20}$$

where λ^2 is the ratio given by Eq. (8.18) for this displacement. Hence, a minimum value can be found for the blowup rate. The precise value can only be found by actually determining the stationary displacements for the system. It is important to note that the existence of an instability can be detected simply by finding any $\vec{\xi}(\vec{r})$ which makes δW negative and that is not necessary to obtain the stationary values unless information on ω^2 and the shape of the eigenmodes is required.

Using this variational method[36] the following results were found. Consider any cylindrically symmetric equilibrium system in which B_θ is zero so that the lines of force lie in planes which include the axis. One of these planes is shown in Fig. 8.4 and illustrates the case of a bulge in the field lines. To each line of flux in this diagram there corresponds a magnetic

36. E. Frieman et al., Stability Criteria, Conference on Controlled Thermo-Nuclear Reactions, TID-7503 (Feb., 1956).

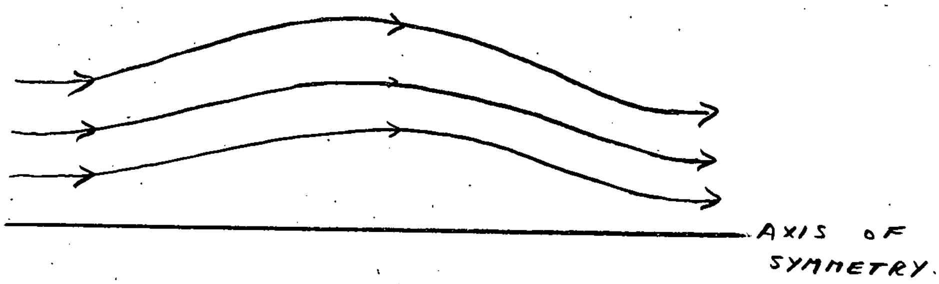

Fig. 8.4. Magnetic Surfaces

surface which is generated by rotating the line of flux about the axis. Denote the flux contained between this surface and the axis by ψ. The quantity ψ is a natural coordinate of the problem. Let P be the value of the pressure on the magnetic surface, V the volume contained inside this surface and let P' and V' be the derivatives of these quantities with respect to ψ. Then for systems in which the gas pressure is low (small β) and also for a number of large β cases the system is stable or unstable as M"V"/M'V' is positive or negative,

$$\frac{M''V''}{M'V'} \gtreqless 0 \begin{cases} \text{stability} \\ \text{neutral} \\ \text{unstable} \end{cases} , \tag{8.21}$$

where

$$\frac{M''}{M'} = \frac{V''}{V'} + \frac{P'}{\gamma P} . \tag{8.22}$$

It may be shown that a system in which the magnetic lines are concave toward the axis (which is the case illustrated in Fig. 8.4) have a negative value of V"/V'. Hence, stability is possible only if M"/M' is positive.

By Eq. (8.22) this implies that the gas pressure increases outward and this is compatable with a confined plasma only if there is a finite pressure drop at the boundary. However, any such system with a finite jump in pressure at its boundary is unstable to surface perturbations, e.g., the "flute" instabilities. Thus it may be expected that the "bulge" regions of the Stellarator as well as the central regions of the Mirror Machine will be unstable and some stabilizing mechanism must be sought. It also seems clear that the devices with reverse curvature of the magnetic lines, such as the Picket Fence and Cusp devices, should be inherently stable.

Stabilization of the Stellarator

L. Spitzer has suggested that an external magnetic field transverse to the main B_z field of the Stellarator would tend to bind the lines of force and thus stabilize the system. This suggestion has resulted in the investigation[37] by variational techniques of a number of problems involving transverse fields superimposed on the main B_z field. In most of these problems, transverse fields with helical symmetry have been used. It has been found that the stabilizing effect of the transverse field is due to a non-uniform twist of the field lines. Thus if F' represents the twist or rotational transform angle of the flux on a magnetic surface, the beneficial effect is due to the existence of a non-zero F''. The result of these calculations leads to the speculation that the general form for δW in such systems is

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37. E. Frieman, Recent Results on Stability, Conference on Controlled Thermonuclear Reactions, TID-7520 (Sept., 1956).

$$\delta W \sim M''(V'' - P'L') + (F'')^2$$

where L' is the weighted average of B^{-2} over a flux tube. Thus one can achieve greater stability with systems whose F" is large.

This stabilizing action may be viewed in another way. At the interface the external lines of force make an angle with the internal lines of force. If a flute tried to follow the external lines of force it would wrinkle the internal lines. In the same way a flute following the internal lines would wrinkle the external ones. If the change in angle (F") is large enough the situation becomes stable to flute instabilities.

Research is continuing on methods of stabilizing the Stellarator. It appears that modest transverse fields will stabilize systems with small β, but not those with $\beta = 1$. Some consideration is being given to eliminating the figure-eight entirely and using a helical torus in its place.

Similar considerations may also apply to the Mirror Machine. In addition, attention is being paid to possible beneficial effects resulting from terminating the magnetic lines beyond the mirror on a metal plate (thus tying down the lines) and to the possible effect of conduction along the field lines smearing out the electric fields which accompany instabilities.

Some Miscellaneous Results

There has been a great deal of research on the problem of stability in the Sherwood Project. It would not be possible to describe all the work in detail in a set of survey lectures, such as these. Instead a few selected topics will be briefly described in the remainder of this chapter.

Heating Instability in the Stellarator. The confining field of the Stellarator is a longitudinal B_z field which undergoes a rotational transform in a single revolution around the device. If a heating current is passed through the Stellarator, this current produces a B_θ field which tends to alter the twist, or rotational transform, of the Stellarator field. M. Kruskal[38] has shown that there is a limiting value of this current above which instability sets in.

Actually, there are two limits depending on which direction the current is going. One may attribute the instability to the removal of the rotational transform by the θ-field of the current. In one direction the rotational twist is removed. In the other direction it is pushed up to 360 deg. The critical limits are proportional to B_z/L where L is the Stellarator length. These differing critical currents have actually been observed.

Rayleigh's Principle and Rotation. The success of the variational method described earlier depends upon the existance of a Rayleigh Principle for the equilibrium system. One way of stating this is that the Hamiltonian of the system shall be separable into a kinetic energy term and a potential energy term with no cross term. Another way of stating this is to require that there be no velocity dependant forces in the perturbed equations. If a Rayleigh principle exists for the system, one is assured that the square of the eigenvalues, ω^2, is real and hence that each eigenmode is pure oscillatory

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38. M. Kruskal, Large Scale Plasma Instability in the Stellarator, PM-S-12, NYO-6045 (April, 1954).

or purely exponential in its time behavior. If a Rayleigh principle does not exist, the eigenvalues are complex and the simple variational methods of Eqs. (8.17) to (8.20) are not useful.

Kruskal has shown that a Rayleigh Principle exists for a hydromagnetic fluid which has no electric fields or mass velocity in its equilibrium state. This requirement has been true for all cases considered by the Princeton group. There is, however, at least one case of interest in which mass velocity does exist in the steady state. H. Snyder[39] has suggested that the kink instability of the pinch might be overcome by imparting a mechanical rotation to the pinched fluid about its axis of symmetry. This suggestion is being investigated at Oak Ridge.

Owing to the fact that variational techniques cannot be used, the problem has been attacked by a numerical scheme. The method is to make use of a high speed digital computer (the ORACLE) to actually integrate the perturbed equations in time. The initial equilibrium is perturbed and the subsequent behavior is watched. If unstable modes exist they should become dominant in time. The results[40] which have been obtained so far are somewhat obscured because of the effects of inherent numerical instabilities produced by the use of a finite difference scheme and by roundoff errors. However, there is

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39. H. Snyder, Effect of Rotational Motion on Plasma Stability, Conference on Thermonuclear Reactions, WASH-289, p. 351 (June, 1955).

40. F. M. Rankin and A. Simon, ORACLE Calculations of Stability, Conference on Controlled Thermonuclear Reactions, TID-7503 (Feb., 1956).

indication that rotation does not produce stability. It does appear to have a stabilizing influence on the low k (or long wavelength) modes.

Tensor Pressure. Perhaps the most serious assumption in the variational method is the assumption that there are enough collisions during an instability to keep the velocity distribution of the particles isotropic and the pressure a scalar. Rosenbluth has considered one aspect of this problem in his paper on stability of the pinch.[25] He assumes a non-isotropic distribution of particle velocities in the equilibrium state. One then uses the adiabatic invariants of the motion to calculate the effect of a perturbation on the orbit of a single particle, and then sums over all orbits to obtain the result. The result reduces to that obtained in the magnetohydrodynamic approximation when the velocity distribution is isotropic. In the more general case, the results depend on ($P_1 - P_3$) where P_1 is the pressure in the direction of the field lines and P_3 the pressure at right angles to the field lines. If $P_3 > P_1$, the pinch is less stable than in the isotropic case. More general considerations along these lines have been given by Brueckner, Chew, Goldberger, Longmire, Low, and Watson.[41]

41. Series of Lectures on Physics of Ionized Gases, LA-2055 (Oct. 1956).