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Chapter VII

PROJECT SHERWOOD:

ORIENTATION LECTURES PRESENTED AT OAK RIDGE NATIONAL LABORATORY

NOVEMBER - DECEMBER 1955

VII. Pinch Devices

OAK RIDGE NATIONAL LABORATORY · U.S. ATOMIC ENERGY COMMISSION

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VII. PINCH DEVICES

Albert Simon

Date Issued
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VII. PINCH DEVICES

The devices described in the previous two chapters have at least one element in common in that they attempt to confine a plasma by use of externally generated magnetic fields. A third method for the confinement of a plasma differs from those described above in that an internal magnetic field, produced by currents induced in the plasma, is used. Owing to the magnetic attraction of parallel currents, there will be a tendency for the discharge in the plasma to contract under the action of its self-magnetic fields. This phenomenon is called the pinch effect.

The pinch effect was first suggested in a paper by W. H. Bennett[19] in 1934. The effect was rediscovered and treated in detail by L. Tonks[20] in 1939. In 1951 J. Tuck at Los Alamos proposed that the pinch effect be utilized for the achievement of a controlled thermonuclear reactor. The result of this suggestion was the establishment of a Sherwood project at Los Alamos under the direction of J. Tuck and which is concerned with the development of pinch devices. Pinch studies have also been underway at Berkeley since 1955. Recent classified discussions with the British have revealed that their thermonuclear effort is based upon exploitation of the pinch effect. Finally, the information released by the Russians to date is concerned entirely with experimental studies on the pinch effect. The Russians did imply however that they are investigating other schemes.

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19. W. H. Bennett, Magnetically Self-Focussing Streams, Phys. Rev. 45, 890 (1934).
20. L. Tonks, Theory of Magnetic Effects in the Plasma of an Arc, Phys. Rev. 56, 360 (1939).

The Steady State Pinch

It is instructive to derive the relations between the pressure, magnetic field, and current in a steady state pinch. In the absence of electric or external fields, the steady state force equation for a plasma takes the form given in Eq. (2.15).

$$\nabla P = \vec{J} \times \vec{H} \tag{7.1}$$

Combining this expression with the steady state Maxwell equation,

$$\text{curl } \vec{H} = 4\pi \vec{J}, \tag{7.2}$$

one obtains the expression given in Eq. (2.17). This is

$$\nabla\left(P + \frac{H^2}{8\pi}\right) = \frac{1}{4\pi} (\vec{H} \cdot \nabla)\vec{H}. \tag{7.3}$$

This expression may now be applied to the specific geometry of an infinite linear pinch. Consider an infinite cylindrical column of plasma as shown in Fig. 7.1. The plasma is entirely confined within a cylindrical surface of

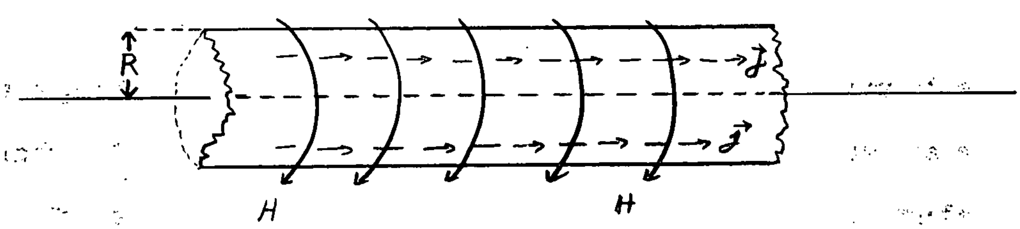

Fig. 7.1. The Linear Pinch

radius R. Hence the plasma is zero outside of this radius. The current flows entirely in the axial, or z, direction. The self magnetic field $\vec{H}$ is entirely in the θ-direction. By symmetry, all three quantities can be functions of the radial distance only. Assuming this geometry, Eq. (7.3) takes the form

$$\frac{d}{dr}\left(P + \frac{H^2}{8\pi}\right) = -\frac{H^2}{4\pi r}, \tag{7.4}$$

since

$$\frac{d}{d\theta}\vec{\theta} = -\vec{r} \tag{7.5}$$

where $\vec{\theta}$ and $\vec{r}$ are unit vectors in the indicated directions. Equation (7.4) may be rewritten as

$$\frac{dP}{dr} = -\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2H^2}{8\pi}\right), \tag{7.6}$$

which may be integrated immediately. The result is

$$\frac{H^2(r)}{8\pi} = -\frac{1}{r^2}\int_0^r r^2 \frac{dP}{dr} \cdot dr \tag{7.7}$$

since H and dP/dr must vanish at the origin. Similarly, Eq. (7.2) takes the form

$$\mathcal{J}(r) = \frac{1}{4\pi r}\frac{d}{dr}(rH) \tag{7.8}$$

In the framework of these equations, we are free to choose P(r) as any even function of the radius which vanishes for $r > R$. The corresponding magnetic field and current are then determined by Eqs. (7.7) and (7.8). In practice, of course, the actual pressure distribution would be determined by a balance between diffusion losses and sources of fresh plasma. This complication will be ignored, and some simple distributions assumed, in order to illustrate the nature of the results.

First, consider a simple parabolic pressure variation which vanishes smoothly at the boundary,

$$P(r) = \frac{P}{R^2} (R^2 - r^2) \qquad (7.9)$$

where P is the pressure at the axis of the pinch. By Eqs. (7.7) and (7.8), one finds

$$\begin{aligned} H &= \sqrt{4\pi P}\, \frac{r}{R} && r \leq R \\ &= \sqrt{4\pi P}\, \frac{R}{r} && r \geq R, \end{aligned} \qquad (7.10)$$

and

$$\begin{aligned} \mathcal{J} &= \frac{\sqrt{4\pi P}}{2\pi R} && r \leq R \\ \mathcal{J} &= 0 && r \geq 0. \end{aligned}$$

The constant value of the current density is not a general property but instead is a peculiarity of the specific pressure distribution which was assumed.

Another possible distribution is a constant pressure P which falls discontinuously to zero at the boundary. In this case, H vanishes in the interior of the plasma and has the value

$$H = \sqrt{8\pi P}\ \frac{R}{r} \qquad r \geq R \tag{7.11}$$

in the exterior region. The current is entirely a surface current in this case and has the magnitude per unit length as follows:

$$\mathcal{J}_{\text{surface}} = \sqrt{\frac{P}{2\pi}}\ . \tag{7.12}$$

The results obtained above may be used to obtain an estimate of the pinch currents which are required to confine a plasma having the typical thermonuclear properties, i.e., a temperature of about 10 kev and a particle density of about 10^{15}. The resulting pressure is about 16 atmospheres, or 16×10^6 dynes/cm^2. By Eq. (7.10), the corresponding magnetic field at the plasma surface is

$$H \cong 14 \text{ kg},$$

and the current density in the plasma is

$$\mathcal{J} \cong \frac{4}{\sqrt{\pi}}\ \frac{10^3}{R} \ \text{emu/cm}^2. \tag{7.13}$$

The total current through the plasma is then,

$$J \cong 4\sqrt{\pi}\ R\ 10^4 \text{ amps} \tag{7.14}$$

The minimum radius of a pinch should certainly be large compared to the Larmor radius of an ion in the magnetic field near the plasma surface. A 10 kev deuteron has a radius of 1.8 cm in a 14 kg field. Hence $R > 1.8$ cm and $J > 1.3 \times 10^5$ amp. It is clear that currents in the neighborhood of 10^5 amp or larger will be needed for a pinch device. Needless to say, a practical pinch would be set up in a toroidal geometry so as to avoid the end losses of a linear system.

Dynamics of the Pinch

The previous discussion is highly academic in that it deals only with the static relations within a steady state pinch under the assumption of constant current. It is even more academic when one realizes that no steady state pinch has yet been accomplished and that all observations so far have been on the transient behavior of a pinch for times of the order of 1 millisecond or less. A much more interesting (and difficult) problem is the behavior of a pinch in time from the initial application of a driving electric potential. M. Rosenbluth has investigated this problem in detail[21] and his results are summarized in the next section.

Consider a finite conducting cylinder of radius $\bar{R}$ and length ℓ filled with a fully ionized plasma. If a potential difference V is applied across the tube in the axial direction, a current will begin to flow in this direction. Rosenbluth assumes that the plasma is infinitely conducting and hence that this current flows only on the outer surface of the plasma. As a

21. M. Rosenbluth, Infinite Conductivity Theory of the Pinch, LA-1850 (Sept., 1954).

result of this current, a magnetic field is formed outside the plasma in the θ-direction. The combined effect of the crossed electric and magnetic fields at the surface of the plasma results in each particle being forced to move in the inward radial direction. Ths situation is shown in Fig. 7.2. Here R_o

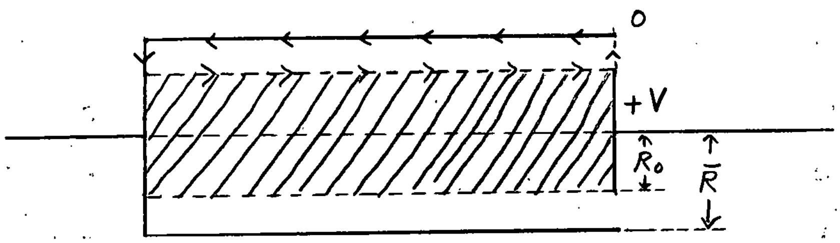

Fig. 7.2. Dynamics of the Pinch

is the radius of the instantaneous plasma surface indicated by the dotted line.

Since the plasma is infinitely conducting, the electric field in the moving frame of the plasma must vanish. This implies the following relation between the surface electric field in the plasma, E_s, and the radial velocity $\dot{R}_o$ of the plasma:

$$\vec{E}_s + \frac{\vec{v} \times \vec{H}}{c} = E_s + \frac{\dot{R}_o H}{c} = 0.$$

Hence

$$E_s = - \frac{\dot{R}_o H}{c} \qquad (7.15)$$

Next, it should be noted that the surface electric field is not simply given by $-V/\ell$ due to the fact that the moving surface and changing magnetic field in the exterior of the plasma produces a magnetic induction with associated electric fields. This induced field may be obtained by taking a line integral of E around the path indicated by the arrows in Fig. 7.2. The path legs in the conducting wall give no contribution since the tangential field must vanish. The total integral is

$$\oint \vec{E}\cdot d\vec{\ell} = \ell E_s + V. \tag{7.16}$$

Now by Maxwell's equation

$$\text{curl } \vec{E} = -\frac{1}{c}\frac{\partial H}{\partial t}$$

Hence

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{1}{c}\int \frac{\partial H}{\partial t}\, ds \tag{7.17}$$

where the integral on the right is over the area enclosed by the path of integration. The magnetic field in the region outside the plasma has the usual value

$$H = \frac{2I}{r} \qquad r \gtrsim R_o \tag{7.18}$$

where I is the total current through the plasma. Substituting Eqs. (7.18) and (7.16) in Eq. (7.17), one obtains

$$\ell E_s + V = -\frac{\ell}{c}\int_{R_o}^{\bar{R}} 2\dot{I}\,\frac{dr}{r} \tag{7.19}$$

Hence, the surface electric field at the plasma is

$$E_s = -\frac{V}{\ell} - \frac{2\dot{I}}{c} \ln\left(\frac{\bar{R}}{R_o}\right). \tag{7.20}$$

Substitute for E_s by using Eq. (7.15). The result is

$$-\frac{V}{\ell} = -\frac{\dot{R}_o H}{c} + \frac{2\dot{I}}{c} \ln\left(\frac{\bar{R}}{R_o}\right).$$

Finally, by use of Eq. (7.18), this may be written as

$$E_o = \frac{\partial}{\partial t}\left[\frac{2I}{c} \ln\left(\frac{\bar{R}}{R_o}\right)\right] \tag{7.21}$$

where $E_o = -V/\ell$ is the applied electric field. Equation (7.21) is a purely inductive relation between the current, plasma radius R_o and applied voltage. The relation must necessarily be inductive since no dissipative forces have been introduced. Equation (7.21) may be rewritten as

$$I \ln \frac{\bar{R}}{R_o} = \frac{c}{2}\int_0^t E_o dt. \tag{7.22}$$

Further progress in detailing the transient behavior of the pinch may be made only by assuming some model for the hydrodynamics of the plasma under compression. The link between the inside and outside is the requirement that the magnetic pressure balance the surface gas pressure. Thus,

$$P_s = \frac{H_o^2}{8\pi} = \frac{I^2}{2\pi R_o^2} \tag{7.23}$$

Several hydrodynamic models have been considered by Rosenbluth. The simplest one is the snow-plow model which assumes that all material which is swept up by the magnetic piston is piled up in a very thin layer at the boundary and moves with it. In this case, the momentum equation for the surface becomes

$$\frac{d}{dt}\left(M\dot{R}_o\right) = -2\pi R_o P_s \qquad (7.24)$$

where M is the mass per unit length swept up by the snow-plow. Now

$$M = \pi(\bar{R}^2 - R_o^2)\rho_o \qquad (7.25)$$

where ρ_o is the initial gas density. A final relation may be obtained by substituting Eqs. (7.15), (7.23), and (7.22) in Eq. (7.24). The result is

$$\frac{d}{dt}\left[(\bar{R}^2 - R_o^2)\dot{R}_o\right] = -\frac{c^2\left[\int_0^t E_o dt\right]^2}{4\pi\rho_o R_o\left[\ln \frac{\bar{R}}{R_o}\right]^2} \qquad (7.26)$$

This equation may be reduced to dimensionless form by the following substitutions (for the case of E_o a constant in time):

$$\eta = \frac{R_o}{\bar{R}} \qquad \tau = \frac{t\sqrt[4]{\frac{c^2E_o^2}{4\pi\rho_o}}}{\bar{R}} \qquad (7.27)$$

and the resultant equation may be solved numerically.

The results show a current and radius which tend smoothly to zero in time. This is to be expected since the snow-plot model makes no provision for the effects of such things as finite conductivity and back-shocks from the center. More complex hydrodynamic models show a behavior in which an initial compression is followed by an outward bounce followed by another compression etc. The specific results depend, of course, upon the time behavior of the applied voltage and the model assumed. A more general relation is expressed, however, by the results of Eq. (7.27). This scaling law shows that the velocity of compression of the plasma surface is of the order of magnitude as follows:

$$v_r = \sqrt[4]{\frac{c^2 E_o^2}{4\pi\rho_o}} \tag{7.28}$$

It should be noted before leaving this subject that the Rosenbluth theory above is often referred to as the M-theory (M standing for motor), and that the same results have been obtained by the Russians. It should also be pointed out that Rosenbluth has studied the structure of the surface layer by considering individual particle orbits. He shows that the magnetic and electric fields drop to zero in a distance Δ of the order of

$$\Delta \cong \sqrt{\frac{mc^2}{8\pi n e^2}} \tag{7.29}$$

Here n is the particle density and m is the electron mass. This thickness is quite small, of the order of magnitude 1 mm thick, in cases of interest.

The Kink Instability

As has already been indicated, it has been found experimentally that all pinch discharges up to the present time are extremely unstable. In typical cases, the pinch may be formed for a few microseconds but then the discharge breaks up and fills the tube in a comparable time. The observation of this instability was no surprise since a theoretical prediction[22] of the effect was available quite early in the game. A physical picture of the effect is shown in Fig. 7.3.

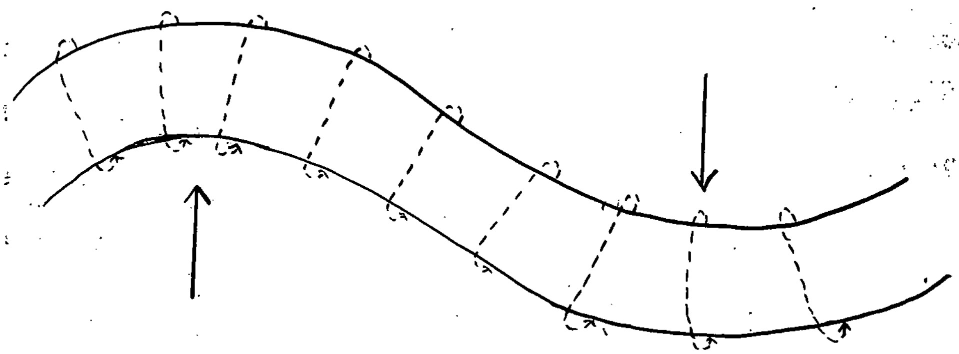

Fig. 7.3. The Kink Instability

What is shown here is the effect of a lateral perturbation or "kinking" on a cylinder of plasma. The dotted lines represent the lines of force of the self-magnetic field due to the pinch current. As a result of the kinking, the lines of force are brought closer together on the inside of the bend and are farther apart on the outside of the bend. The resultant magnetic pressure is greater on the inside and a net force acts in such a direction as to increase the bend. Thus, once a slight kink develops, it will grow in size until the discharge breaks up and the plasma fills the tube.

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22. M. Kruskal and M. Schwartzschild, Some Instabilities of a Fully Ionized Plasma, Proc. Roy. Soc. A233, 348 (1954).

The amplitude of the kink increases exponentially, with an e-folding time approximately equal to the time required for a sound wave to travel a distance equal to the wave-length of the perturbation or equal to the geometric mean of the wave-length and the pinch radius, whichever is the larger. The result is an extremely fast breakup. For example, at a temperature of 10 ev the speed of sound in deuterium is about 3×10^6 cm/sec. Hence, a wavelength of 1 cm will e-fold in less than a microsecond. A discussion of some of the observed behaviors of unstable pinches will be found in the last section of this chapter.

There is a second instability of the pinch which is of interest. This is the so-called "Sausage" instability. The unstable deformation in this case corresponds to a necking-down or constriction of the plasma as shown in Fig. 7.4. This instability tends to grow even more rapidly than the kink instability. Although there is no direct experimental proof of this instability, there is some indirect evidence of its existence.

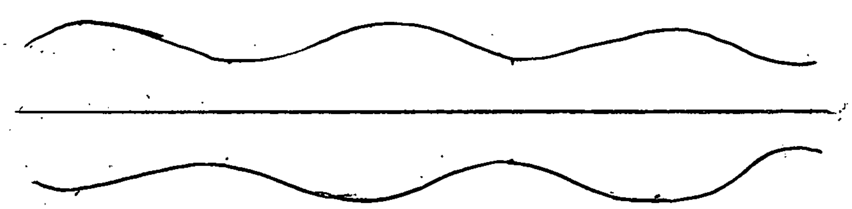

Fig. 7.4. Sausage Instability

Stabilization of the Pinch

It may be possible to live with an unstable pinch. A discussion of this possibility will be found in the next section. However, it is clearly desirable to attempt to find ways to stabilize the system. Attempts to achieve this end have been uniformly unsuccessful until very recently when a combination of theoretical and experimental observations have led to a sharp upsurge of optimism.

It has been known for some time[23] that the long wave-length kink instabilities could be eliminated by encasing the pinch within a conducting shell. Furthermore, the beneficial action of a longitudinal magnetic field (in the axial direction) upon the short wave length instabilities has been pointed out by Kruskal and Tuck.[24] However, low power experiments at Los Alamos gave no indication of improvement resulting from longitudinal field. Recently Rosenbluth[25] has investigated the combined action of longitudinal fields and a conducting shell in detail and has found a region of complete stability. It appears that comparatively modest longitudinal fields will suffice. However, it is important that the net compression of the plasma be small and that little or no longitudinal field remain outside of the plasma after pinching. This last feature may make achievement of the stabilized pinch quite difficult since the internal longitudinal field tends to leak out of the plasma quite rapidly at low temperatures.

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23. J. L. Tuck, Conference on Thermonuclear Reactions, WASH-146, p. 51 (April 1953).
24. M. Kruskal and J. L. Tuck, Instability of a Pinched Fluid with a Longitudinal Magnetic Field, LA-1716 (Nov. 1953).
25. M. Rosenbluth, Stability of the Pinch, LA-2030 (April 1956).

Recent experiments on linear pinches at Los Alamos and Berkeley have indicated that longitudinal fields seem to suppress the short wave length instabilities, as predicted. It should be emphasized that the theoretical calculations so far are valid only for a linear pinch and have not yet been extended to a toroidal configuration.

Suppose, for the moment, that the theoretical prediction may be trusted and that there is indeed a region of complete stability of the pinch. There are still several major problems which would have to be overcome before a successful device could be operated. One of these is the tendency for the magnetic field to leak out of the plasma and into the vacuum. The rate at which this occurs may be estimated quite easily. By Maxwell's equations and Ohm's law,

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad (7.31)$$

$$\nabla \times \vec{H} = \frac{4\pi \vec{I}}{c} \qquad (7.32)$$

$$\nabla \cdot \vec{H} = 0 \qquad (7.33)$$

$$\vec{I} = \sigma \vec{E}, \qquad (7.34)$$

where the displacement current has been neglected and where σ, the conductivity of the plasma, is in esu. Take the curl of Eq. (7.32) and substitute from Eqs. (7.34) and (7.31). The result is

$$\nabla \times (\nabla \times \vec{H}) = -\frac{4\pi\sigma}{c^2}\frac{\partial \vec{H}}{\partial t},$$

which may be rewritten, by use of Eq. (7.33) as,

$$\nabla^2 \vec{H} = \frac{4\pi\sigma}{c^2} \frac{\partial \vec{H}}{\partial t} \tag{7.35}$$

If the magnetic field falls off spatially in the conductor with a characteristic length L, its e-folding time τ, may be written as,

$$\tau = \frac{4\pi\sigma L^2}{c^2} \tag{7.36}$$

The conductivity of a fully ionized plasma is readily estimated. Under the action of an electric field E, an electron has an acceleration eE/m for a time of the order of λ/v, where λ is the mean free path. Hence the average acquired velocity is

$$\bar{v} = \frac{eE\lambda}{mv}$$

and the current, in esu, is:

$$\mathcal{J} = ne\bar{v}$$

$$= \frac{ne^2\lambda}{mv} E$$

where n is the electron density. The resultant conductivity, in esu, is

$$\sigma \cong \frac{ne^2\lambda}{mv} = \frac{e^2}{mv\sigma_c}$$

where σ_c is the electron-ion coulomb cross sections. By Eqs. (2.9) and (2.10), this expression may be written as

$$\sigma \cong \frac{mv^3}{80\pi e^2}$$

$$= \frac{(3kT)^{3/2}}{80\pi \sqrt{m}\, e^2} \tag{7.37}$$

which has the numerical value

$$\sigma \cong 6 \times 10^{12} (kT)^{3/2} \tag{7.38}$$

with kT in ev. Hence, by Eq. (7.36)

$$\tau \cong 0.84 \times 10^{-7} (kT)^{3/2} L^2 \text{ sec} \tag{7.39}$$

At a working temperature of 10 kev, with a pinch which is a few centimeters in radius, it seems clear that the leak time can be of the order of seconds, which should be adequate. However, the same pinch at a temperature of 1 ev would have a leak time of the order of microseconds. Hence, in order to avoid instabilities, it will be necessary to heat the plasma very rapidly so as to bring it up to a temperature at which the stabilizing longitudinal field can be held for a reasonable period.

The second major problem is the heating of the pinch. It has just been demonstrated that this process must be accomplished very quickly at first. One of the most natural ways to heat a plasma is by compression. Compression in turn, is an automatic consequence of the method used to establish the pinch. As was shown in the section on dynamics of the pinch, the application of an electric field across the plasma results in a compression of the pinch with a velocity given by Eq. (7.28). Since the surface acts as a magnetic piston, every particle which strikes it is reflected with an increase in velocity equal to twice the surface velocity. The resultant increase in energy is then proportional to the mass of the particle which is reflected. Hence, this method is most efficient for heating of the ions, which is a desirable property. Furthermore, it may not be too difficult to apply electric fields which result in surface compression velocities of the order of 10^8 cm/sec. This would imply that every ion struck by the magnetic piston is accelerated up to thermonuclear energies.

Unfortunately, Rosenbluth's stability studies have also shown that there is a maximum compression of the pinch which can be tolerated before instability sets in again. The criterion may be expressed in terms of the ratio of the radius of the external conductor to the radius of the pinch. This ratio may not be larger than 5 for the case of a pinch having negligible gas pressure, (i.e., where the internal pressure is mainly due to the magnetic pressure of the longitudinal field). For a more reasonable case, in which the gas pressure is comparable to that of the longitudinal field, the maximum ratio is more like 2 or 3. This restriction implies that any large scale heating by means of plasma compression must be so programmed that the radius of the pinch is not below the critical limit for times which are of the order of instability time or longer. This is very likely a serious constraint on this type of heating.

Of course, there is always the ohmic heating resulting from the finite conductivity of the plasma. As has already been discussed in Chap. 5, this type of heating is not very effective above 100 ev because of the decreased resistivity of the plasma. The most likely prospect is some kind of shock heating. One possible scheme is to create a shock by the sudden application of a large pinch field. Another possibility, suggested by S. Colgate, is to produce shock heating by the use of "collapse" techniques. In this case, the shock is again produced by a magnetic piston; but the magnetic field is an external longitudinal field produced by a solenoidal winding. The main difficulty with these shock schemes is the necessity for the magnetic pressure to rise in a time which is shorter than the sound speed across the diameter of the tube. For a tube of a few centimeters in radius, this implies rise times of the order of 10^{-8} sec. This transit time is too rapid for presently known condensers with large energy storages.

An additional difficulty of the "collapse" scheme is the necessity for programming the external longitudinal "collapse" field to zero intensity within the e-folding time for instabilities to grow. As was pointed out earlier, Rosenbluth has shown that any appreciable external longitudinal field sharply reduces the region of stability of the pinch. Hence the "collapse" field must be reduced sharply within the time required for a sound wave to travel around the circumference of the torus (instability to long wave lengths).

One final factor should be mentioned in this discussion of pinch instability. Recent experimental observations both in Britain and in this country indicate that the pinch tends to break into a corkscrew type of instability shortly after it is formed. It is believed that this behavior is in agreement with theory, since the compressions in these experiments were well beyond the Rosenbluth limit. In that case, long wave length instabilities of a helical type are predicted. The observed direction of the helix is also in agreement with theory. The formation of a helical instability may possibly have a partially stabilizing tendency inherent in it. As a result of the helical shape, the pinch current itself now tends to produce a longitudinal magnetic field in the plasma, as well as the original B_θ field. This field is in such a direction as to reinforce the original longitudinal field. Hence, a substantial increase in containment time may result from this mechanism.

Economics of the Pinch

As will be shown below, the economics of the stabilized pinch are very favorable compared to those of the Stellarator or Mirror machine. The chief reason for this is the highly efficient way in which the magnetic field is produced. Not only is the plasma a much better conductor than copper, at a temperature of 10 kev, but the peak magnetic field occurs at the plasma surface, where it is needed, rather than in the coils. These factors permit larger particle densities, smaller physical dimensions and smaller input energies for the system. If the pinch cannot be stabilized, the situation is much less favorable. This possibility is considered at the end of this section.

The pinch device is necessarily a pulsed gadget. This follows from the fact that the geometry is necessarily toroidal in order to eliminate end losses. In this case, the applied voltage must be obtained by inductive action, which implies pulse operation. Furthermore, as has been mentioned, stability will only persist until a fraction of the internal longitudinal field of the plasma has leaked out into the vacuum. This places a practical upper limit on the duration of a pulse. Assume now that a toroidal stabilized pinch has been established. By some sort of shock heating, the temperature has been raised very quickly at the beginning of operation to thermonuclear temperatures. The pinch is now in a steady state which will persist for a time τ limited by field diffusion.

The input energy per unit length to the system during the pulse consists of four parts. One is the energy in the pinch field. If the external field at the plasma surface is denoted by B_θ, this contribution is

$$E_\theta = \int_r^R \frac{B^2}{8\pi} \, 2\pi r' dr',$$

where r is the pinch radius and R the radius of the external conductor. Now

$$B = B_\theta \frac{r}{r'} \tag{7.41}$$

and thus

$$E_\theta = \pi r^2 \frac{B_\theta^2}{8\pi} \cdot 2 \ln \frac{R}{r}. \tag{7.42}$$

The second contribution is the energy required initially to heat the gas up to working temperature.

This contribution is

$$E_G = \pi r^2 NkT \tag{7.43}$$

$$= \pi r^2 P$$

where N is the particle density and P the final gas pressure. The third contribution is the energy in the longitudinal, or B_z, field. This is

$$E_z = \pi r^2 \frac{B_z^2}{8\pi} \tag{7.44}$$

But, by the pressure balance condition,

$$\frac{B_z^2}{8\pi} + P = \frac{B_\theta^2}{8\pi} \tag{7.45}$$

at the plasma surface. Hence the three energy contributions above can be written as

$$E_\theta + E_G + E_z = \pi r^2 \frac{B_\theta^2}{8\pi} \left(1 + 2 \ln \frac{R}{r}\right) . \tag{7.46}$$

The final energy contribution is from the ohmic heating of the plasma during the duration of the pulse. If a uniform current density $\mathcal{J}$ is assumed in the plasma, this energy input is

$$E_R = \pi r^2 \frac{\mathcal{J}^2}{\sigma} t \tag{7.47}$$

where σ is the plasma conductivity and t is the duration of the pulse. Now the pulse duration is limited by the time required for the B_z field to diffuse a distance of the order of the pinch radius. By Eq. (7.36) this time is

$$t \cong \frac{4\pi\sigma r^2}{c^2} . \tag{7.48}$$

Hence, substituting in Eq. (7.47),

$$E_R = \pi r^2 \frac{4\pi j^2 r^2}{c^2} .$$

But the pinch field is related to the current density by the relation:

$$B_\theta = \frac{2I}{cr} = \frac{2\pi r j}{c} \tag{7.49}$$

Substituting this above,

$$E_R \cong \pi r^2 \frac{B_\theta^{\,2}}{\pi} . \tag{7.50}$$

Combining Eqs. (7.46) and (7.50), the net energy input per unit length to the pinch becomes:

$$E_{IN} \cong \pi r^2 \frac{B_\theta^{\,2}}{8\pi} \left(9 + 2 \ln \frac{R}{r} \right) \tag{7.51}$$

It should be noted that the energy input in the form of ohmic heating is about 8 times larger than the thermal energy content of the gas.

The energy production per unit length of the pinch during the duration of the heating pulse is given by the usual relation,

$$E_{OUT} = n_D n_T \, (\overline{\sigma v})_{D-T} \, \bar{E} \, \pi r^2 \tau \tag{7.52}$$

where $\bar{E}$ is the energy produced in a D-T reaction. Now by the usual definition of the quantity β,

$$P = \beta \frac{B_\theta^2}{8\pi}$$

and

$$P = (n_e + n_D + n_T)\, kT.$$

If a 50-50 D-T mixture is assumed,

$$n_D = n_T = \frac{\beta}{4} \frac{B_\theta^2}{8\pi} \frac{1}{kT} \qquad (7.53)$$

Combining Eqs. (7.53), (7.52), (7.51), and (7.48),

$$\frac{E_{OUT}}{E_{IN}} = \frac{\beta \ B_\theta^2 (\overline{\sigma v})_{D-T} \ \bar{E} \ \sigma r^2}{32 \ c^2 (kT)^2 \left[9 + 2 \ln \frac{R}{r} \right]} \qquad (7.54)$$

If it is assumed that this ratio must be of the order of 3 or larger in order to have excess energy to sell, one obtains a minimum condition on the pinch radius.

$$r^2 \geq \frac{96 \ c^2 (kT)^2 \left[9 + 2 \ln \frac{R}{r} \right]}{\beta^2 \ B_\theta^2 (\overline{\sigma v})_{D-T} \ \bar{E} \ \sigma} . \qquad (7.55)$$

Since σ is given by Eq. (7.38) it is clear that

$$r^2 \sim \frac{\sqrt{kT}}{(\overline{\sigma v})_{D-T}} \frac{1}{B_\theta^2} \qquad (7.56)$$

The temperature dependant term varies slowly with temperature and has a minimum in the neighborhood of 100 Kev. It is clear that the minimum pinch radius varies inversely as the pinch field for fixed temperature.

Rosenbluth has shown that the pinch will be unstable to the "sausage" type instability if $\beta > 0.5$. Hence let us choose this limiting value as the operating condition. In addition, a compression of greater than 2.5 is also unstable for this value of β. Hence, R/r will be assumed to be equal to 2.5. Numerical values can now be inserted in Eq. (7.55), where it will be assumed that the reaction energy is 10 Mev and the value of σ is given by Eq. (7.38). Thus

$$r^2 \geq 10^{-7} \cdot \frac{\sqrt{kT}}{(\overline{\sigma v})_{D-T} \, B_\theta^2} \tag{7.57}$$

where kT is in ev. If attention is focused on the D-T reaction at kT = 10 kev, then $(\overline{\sigma v}) \doteq 10^{-16}$ by Table 2.1 and

$$r^2 \geq \frac{10^{11}}{B_\theta^2} . \tag{7.58}$$

If comparison is to be made with the previous economic considerations for the Stellarator and Mirror device, the surface pinch field should be chosen to be 30 kg. In this case, the minimum pinch radius is 10.5 cm which is considerably smaller than the results given in Eq. (5.64) and Eq. (6.26). The two principle reasons for this advantage over the other devices are the larger conductivity of a plasma at 10 Kev compared to the conductivity of copper and the more efficient geometrical usage of the magnetic field.

The situation is even more favorable than indicated by this comparison. A plasma surface field of about 30 kg implies a maximum field strength in the coil windings of about 50 kg for the case of the Stellarator and Mirror device. This limiting value, in turn, is set by considerations of coil strength and fabrication difficulties. In the case of the pinch, the maximum field occurs at the pinch itself and falls off to R/r (= 2.5) of its value at the coils which make up the conducting wall. Hence, if the same strength limit, 50 kg, is chosen at the conducting wall, the maximum pinch surface field becomes 125 kg. Inserting this value in Eq. (7.58) yields

$$r \gtrsim 2.5 \text{ cm.} \qquad (7.59)$$

This radius, and corresponding field strength, will be used to illustrate the properties of a pinch device.

The total input energy per unit length is obtained by inserting the proper numerical values in Eq. (7.51). This is

$$E_{IN} = 1.3 \times 10^4 \text{ joules/cm.} \qquad (7.60)$$

The duration of the pinch is found by use of Eqs. (7.48) and (7.38). This result is

$$\tau = 0.52 \text{ sec.} \qquad (7.61)$$

Hence, the input power per unit length during the pulse is

$$P = \frac{1.3 \times 10^4}{0.52} = 25 \text{ kw/cm} \qquad (7.62)$$

$$= 2.5 \text{ Mw/meter.}$$

The thermal power developed is 3 times this value. However, the salable power should be of the order of the input power.

So far nothing has been said about the total length of the torus. A large value of the ratio of the major axis of the torus to the minor radius is desirable in one way since problems concerned with centering of the pinch are reduced in this case. On the other hand, a large ratio of major to minor radius would require a larger total input energy and would mean a larger value of the inductance for the system. A low inductance is desirable in order that the steep initial current rise necessary for shock heating be possible. Assume that this ratio is chosen to be 10. In this case, the total energy input required of the condenser bank is

$$E_{IN} \cong 2 \times 10^6 \text{ joules},$$

and the input power is

$$P \cong 4 \text{ Mw},$$

which is also the order of magnitude of the salable power.

It is interesting to note that the total input energy is inversely proportional to the magnitude of the pinch field. This may be recognized by the fact that the input energy is

$$E_{IN} \sim r^3 H_\theta^2$$

where a fixed ratio of major to minor radius has been assumed. By Eq. (7.56), it is clear that

$$E_{IN} \sim \frac{1}{H_\theta} \tag{7.63}$$

On the other hand, since the pulse duration is proportional to r^2, by Eq. (7.48), the power delivered during the pulse is

$$P \sim H_\theta . \tag{7.64}$$

If the pinch cannot be stabilized, there is still a small but finite possibility of extracting useful energy from such a device. The energy production would occur only during the first compression of the pinch under an impressed voltage. After this first compression, kink instabilities would break it up very rapidly. Since only a short time exists for the reaction, very little nuclear burnup would occur unless the particle density becomes very large in the compression. This in turn implies very large driving fields.

If the plasma drives in with a constant velocity $\dot{R}$, the energy given each particle upon being swept up by the field is $M\dot{R}^2$ which becomes the effective temperature of the gas. Hence the input energy on compression is

$$E_{IN} \sim M\dot{R}^2 \, n\pi R^2 = \pi R^2 \rho T$$

where ρ is the initial density and T the effective temperature. The energy output is

$$E_{OUT} \sim (C\rho)^2 \; \frac{\pi R^2}{C} \; (\overline{\sigma v})_{D-T} \, \tau$$

where C is the plasma compression and τ the e-folding time for instabilities. Now τ is proportional to the pinch radius divided by the speed of sound or

$\tau \sim R/(CT)^{1/2}$. Hence

$$\frac{E_{OUT}}{E_{IN}} \sim C^{1/2} \rho R \frac{(\overline{\sigma v})}{T^{3/2}} \qquad (7.65)$$

and if ρ, R, and T can be made large enough, it may be possible to produce more energy than is put in. Estimates by Tuck give results of the following order of magnitude:

$$I \cong 5 \times 10^8 \text{ amp}$$

$$R \cong 5 \text{ to } 10 \text{ meter}$$

$$kT \cong 100 \text{ kev}$$

$$E_{IN} \cong 10^9 \text{ joules.}$$

These are very large numbers, indeed, and it seems clear that this approach is a last-ditch affair. Incidentally, one advantage of this method is the fact that a torus is unnecessary. Since the time scale is so short, the ends of the tube do not affect the interior. Hence a linear pinch may be used which makes unnecessary an inductive discharge.

Other Geometries

From time to time, many geometries other than those described in this and the previous two chapters have been proposed. Most of these have perished because of some obvious flaw. However, there are at least two general classes of alternative geometries which still persist and in which some research (mainly theoretical) still continues. These two general classes are the Picket Fence (or Cusp) device and devices based on ion or electron streams.

The chief, and perhaps only, advantage of these two schemes is their apparent inherent stability. It has already been shown in this chapter that the pinch, up to the time of Rosenbluth, was a very unstable system. In addition, general considerations of stability, to be described in the next chapter, had made it seem plausable that both the Stellarator and Mirror device were also inherently unstable. It was in answer to this depressing situation that the Picket Fence was invented by J. Tuck. The ion stream proposed is quite ancient[19] but was revived for the same reason by W. Bennett and others. A brief description of these proposals follows.

Picket Fence. Intuitive arguments by E. Teller suggested that a situation in which plasmas were confined by magnetic fields wrapped around them were unstable. The inherent tendency seemed to be one in which the plasma slipped out between the field lines and the field lines snapped in like rubber bands. On the other hand, the same intuitive arguments suggested that a situation in which escape of plasma tended to stretch the magnetic lines would be stable. A geometry which has this property is the Picket Fence.[26] A sketch of the geometry is shown in Fig. 7.5.

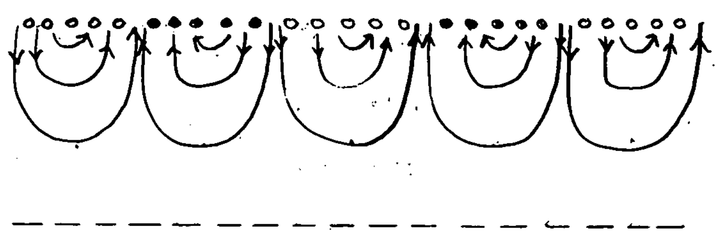

Fig. 7.5. The Picket Fence

26. J. L. Tuck, Picket Fence, Conference on Thermonuclear Reactions, WASH-184, p. 77 (January, 1955).

The open circles represent windings with current out of the paper, while the dark circles carry current in the opposite direction. The resultant magnetic field is shown, along with a dotted line which is an axis of revolution for the entire figure. In practice, of course, the entire figure would be wrapped around into a torus so as to seal off the ends. Note that the plasma fills the inside volume (around the dotted line) and would tend to stretch magnetic lines if it starts to leak through these lines.

The major flaw in this device is the very large particle leaks occuring at the juncture between opposite windings. These leaks look like mirrors but are even worse in that the leak zone is a line extending all the way around the tube rather than a single point as in the Mirror device itself. A simple calculation by H. Snyder has shown that the resultant leak rate is very much larger than that of the mirror. Hence, the device would seem to be economically unattractive.

A possible way out of this difficulty is to reduce the losses by moving the Picket Fence rapidly in the axial direction. This device is known as the "Moving Picket Fence." One way of achieving this effect is to interchange the directions of the currents in the windings at high frequency. The main drawback is the very large amounts of RF power which would be needed for this purpose. The same effect may possibly be achieved more economically by superimposing an RF field of relatively low power on the DC field. The result would be a rapid vibration of the magnetic field at the points of leakage.

<u>Cusp</u>. The cusp device, which is being investigated by the New York University group, is basically similar to the Picket Fence. A sketch is

shown in Fig. 7.6. Oppositely directed currents in the end coils produce a magnetic field which has the proper curvature for stability. Note that mirror leaks, as well as cusp leaks, exist in this device.

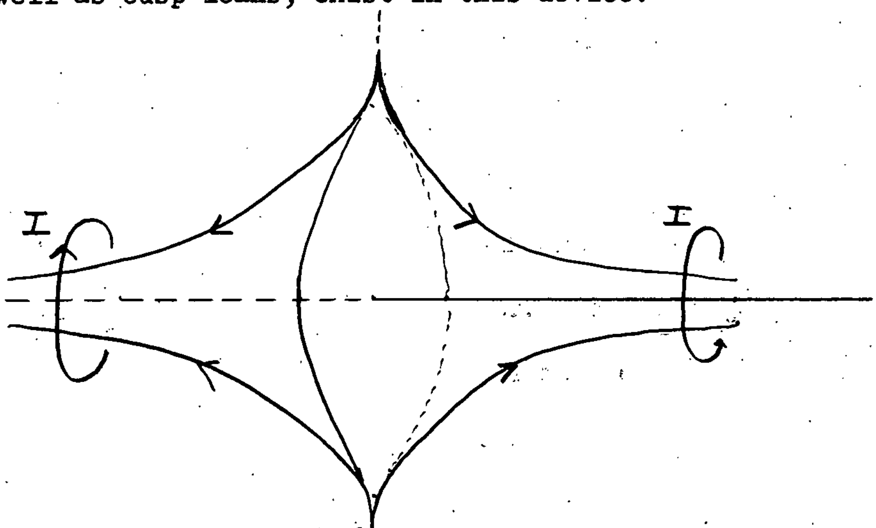

Fig. 7.6. A Cusp Device

The geometry shown in Fig. 7.6 results from a rotation of a two-dimensional hypocycloid about an axis. Other interesting cusp geometries result from other rotations of this basic two-dimensional figure.[27] It is probable that all of these devices will remain economically unattractive because of the large particle leak rates.

Ion and Electron Streams. A proposal by W. H. Bennett[28] envisions the use of sustained deuterium ion streams at currents exceeding the minimum value for magnetic self-focusing. The ions are accelerated in a Thomas-type cyclotron and are built up in a circulating orbit near the outer edge of the

27. H. Grad, Conference on Controlled Thermonuclear Reactions, TID-7520, p. 99 (Sept., 1956).
28. W. H. Bennett, Proposed Thermonuclear Investigation, NRL Report 4479, RD 466 (Dec. 1, 1954).

device. Since the loss rate is presumably small, only low density streams need be considered. Bennett states that these low density streams of high velocity particles are not subject to "kink" instability.

The Russians[29] have recently proposed the use of relativistic stabilized electron beams in the design of high energy accelerators. Although their proposals all refer to high-energy particle accelerators, it seems likely that these considerations were originally motivated by research in the field of controlled thermonuclear reactions.

Summary of Experimental Program

Experiments concerned with the pinch effect fall into two general catagories. One is the class of toroidal pinches. The pinch currents are produced by inductive action, and the experimental emphasis is on the study of confinement and stabilization. The second type is the linear pinch. Inductive techniques are not necessary here and the pinch current may be obtained by direct discharge through the tube. Linear pinches may only be used to study the short term (10^{-6} sec) behavior of pinches since the electrodes will seriously contaminate and perturb the plasma after a longer interval. Emphasis in the linear pinch experiments is on the study of the predictions of the Rosenbluth M-theory and the associated heating by rapid compression.

The Perhapsatron. This is the general name for a series of toroidal pinch devices which have been investigated at Los Alamos. The present device consists of a laminated transformer core linked by a toroidal tube. The pyrex tube is 7 cm in diameter and has a major diameter of 70 cm. The overall length is about 2 meters. The power supply was originally provided

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29. G. J. Budker, Relativistic Stabilized Electron Beam, CERN Symposium, Geneva, 1956.

by energy stored in RG-19/U cables. However, the present source is a bank of 38 capacitors each having a 1 microfarad capacity and charged to about 15 Kev. The capacitors are individually linked by a spark gap to a single turn laid along the tube and which serves as the primary of the transformer. The torus is the secondary.

Pre-ionization of the gas in the toroidal tube is accomplished by means of an RF oscillator operating at 500 watts. Initial gas pressures range from 4 to 500 microns. Upon discharge of the condensers, the current rises in about 10 μs. to a maximum value of about 40,000 amp. During this period bright pinches are seen in xenon and other heavy gases. Fainter pinches occur in hydrogen and deuterium. Smear camera observations show that several compressions occur; however, instabilities break up the discharge after a few microseconds. The observed times are in general agreement with the Kruskal-Schwarzchild theory.

Observations indicate that a maximum compression of about 30 occurs in the pinch. Spectroscopic observations yield a resulting temperature of about 50 ev. High energy gamma rays up to 200 kev have also been observed. Since the induced voltage is about 15 kev/turn, these must arise from runaway electrons which perform more than 10 complete circuits of the tube and then collide with the walls.

<u>Columbus I</u>. Columbus is the general name for the linear pinch devices at Los Alamos. Columbus I is itself a machine which has gone through a series of changes, with the particular model being distinguished by the addition of one or more superscript primes to the Roman numeral. The present device

consists of a quartz cylinder with a diameter of 6.5 cm and 33 cm in length. The energy source is a capacitor bank having a 36 μf capacity and charged to 17 Kev. The inductance of the system before pinching is 0.12 μh. The capacitors are discharged through a spark gap to an electrode at the top of the tube.

An important distinguishing feature of the Columbus devices is the high electric fields obtained. The average field is about 1 Kv/cm as compared with an average value of 100 volts/cm in the Perhapsatron. The Rosenbluth M-theory has shown that the velocity of contraction of the pinch, which is also 1/2 the velocity increment of the ions per collision with the wall, is proportional to the square root of the applied electric field. Hence, large electric fields are desirable. If a velocity corresponding to that of a deuteron at 10 kev is required ($\cong 10^8$ cm/sec) the necessary field is easily obtained from Eq. (7.28). It is

$$E_o = \frac{v^2\sqrt{4\pi\rho_o}}{c} .$$

Assume final density of about 10^{14}. Then if $v = 10^8$,

$$E_o = \frac{(10^{16})\left[4\pi \cdot 10^{14} \cdot 3.4 \times 10^{-24}\right]^{1/2}}{3 \times 10^{10}} \text{ esu}$$

$$\cong 6.5 \text{ Kv/cm.}$$

Hence, electric fields of the order of several kilovolts per centimeter are clearly desirable.

Upon discharge of the capacitor bank through Columbus I, peak currents of the order of 150,000 amp have been obtained. Good pinches of about 1 cm diameter are observed in deuterium and neutrons are emitted. The average number of neutrons per pulse is about 10^7. The neutrons are emitted about 1.3 μs after breakdown and persist for 0.3 μs. The onset of neutron emission seems to be closely correlated with the onset of instabilities in the pinch.

A great deal of careful research has gone into determining the origin of these neutrons. It is firmly established the sources of neutrons are concentrated along the axis of the tube and that the neutrons do not originate from bombardment of deuterons in the wall by accelerated deuterons in the gas. Recently, nuclear plate observations have indicated that the neutrons are emitted preferentially in the axial direction and that the center of mass of the colliding deuterons is not at rest but is moving toward the cathode. The corresponding deuteron energy, assuming the target at rest, is 34 Kev. Deuteron energies up to 200 Kev have been observed.

It is believed that these neutrons are not thermonuclear in origin but originate as a result of accelerations in the gas due to the large electric fields associated with instabilities. One explanation, due to S. Colgate,[30] makes use of the electric fields produced in the neck of the pinch when the "sausage" instabilities set in. This is illustrated in Fig. 7.7. Deuterons are accelerated across the neck as shown. Note that the electric field must be in the direction shown in order that the crossed electric and magnetic

30. S. A. Colgate, Neutron Production in the Pinch Due to Instability Breakup, UCRL-4702 (May 12, 1956).

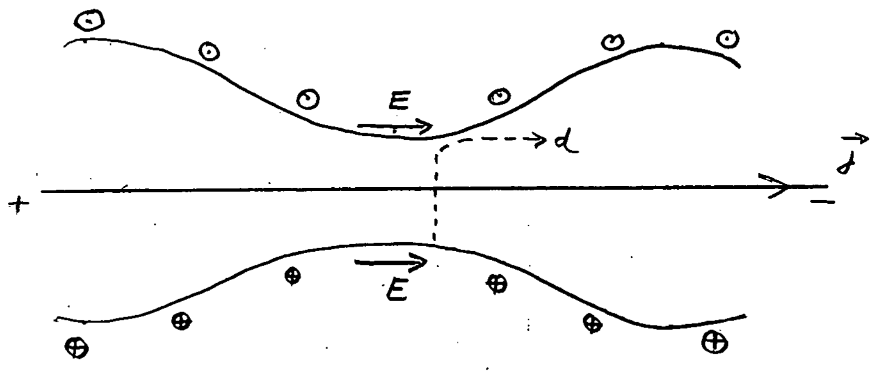

Fig. 7.7. Origin of Pinch Neutrons

field give the proper direction for the instability force. Hence, the center of mass motion of the deuterons is towards the cathode. An alternative explanation is due to J. Tuck.[31] In this explanation, the electric field in the neck becomes large enough to impart the observed energy to the deuterons by the radial motion given in the M-theory. When the neck becomes as wide as the sheath is thick, the deuterons that hit the opposite side of the neck are then diverted in the direction of the cathode by the combined action of the radial sheath electric field and the magnetic field. This trajectory is illustrated in Fig. 7.7 by the dotted lines. The analysis of Rosenbluth, which will not be given here, indicates this direction uniquely. Finally, the duration of the observed neutron bursts is much longer than would be calculated from a single "sausage" necking-off process. This difficulty is circumvented by proposing that the instabilities occur randomly throughout the length of the pinch during the emission period.

31. J. L. Tuck, Conference on Controlled Thermonuclear Reactions, TID-7520, p. 23 (Sept., 1956).

Columbus II. This is an enlarged and improved linear pinch machine which is under construction at Los Alamos. The device is designed to operate at 100 Kev with an energy storage of 10^5 joules. The intended implosion times are short, 1-2 μs, and hence very fast rise times are required. This in turn implies a low inductance system. A special low inductance capacitor has been designed for this purpose. This capacitor has 0.8 μf capacity at a maximum voltage of 100 Kv and an internal inductance less than 0.12 μh. The total number of these capacitors in Columbus II will be 25. The final device will have a short-circuit current of 2×10^6 amp with an initial rate of rise of current of 10^{12} amp/sec.

Berkeley Linear Pinch. The work on linear pinches at Berkeley has closely paralleled that at Los Alamos. Neutrons have also been observed, and investigations are continuing on the stabilizing effects of a longitudinal field. Particular interest at Berkeley is centered on possible uses of collapse heating.

Magnetic Induction Machine. A toroidal device is being designed at Los Alamos which will produce potential gradients similar to those in Columbus, i.e., 1 to 2 Kv per cm. An energy source of about 10^5 joules will be used, and the voltage will be induced in the same fashion as the Perhapsatron. A discharge current of 10^6 amp is anticipated.

Russian Pinch Work. Recent revelations[32] of Russian work on controlled thermonuclear reactions have been confined to pinch investigations. It is

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32. L. V. Kurchatov, report of Harwell talk presented in Nucleonics 14, 36 (1956).

apparent that the Russian results are strikingly similar to ours. They observe neutrons and assign the origin to instabilities just as we do. They have also developed a theory identical to the Rosenbluth M-theory, and observe the several radial oscillations before instability breakup. Further, it seems that the Russians[33] are aware of the importance of a longitudinal magnetic field for the stabilization of a plasma.

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33. L. A. Artsimovich, Lecture at Symposium on Electromagnetic Phenomena in Cosmical Physics, Stockholm (Aug. 1956) to be published in the Soviet Journal of Atomic Energy.