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Chapter VI

PROJECT SHERWOOD:

ORIENTATION LECTURES PRESENTED AT

OAK RIDGE NATIONAL LABORATORY

NOVEMBER - DECEMBER 1955

VI. Mirror Machines and High Energy Injection

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PROJECT SHERWOOD: ORIENTATION LECTURES PRESENTED AT OAK RIDGE NATIONAL LABORATORY, NOVEMBER-DECEMBER, 1955

VI. MIRROR MACHINES AND HIGH ENERGY INJECTION

Albert Simon

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VI. MIRROR MACHINES AND HIGH ENERGY INJECTION

As was pointed out in Chap. IV, the essential starting problem of the Sherwood program is the question of what to do with the ends of the magnetic field lines. The Princeton approach is to wrap the field into a figure-eight geometry. An alternative solution was proposed by R. F. Post late in 1951. His suggestion was to maintain the linear uniform field produced by a solenoidal winding, but to cap off the ends by use of the magnetic mirror principle. A major part of the Sherwood research at Livermore is devoted to an investigation of the feasibility of this method.

The features of the proposed Livermore devices will be discussed in this chapter. One of the most interesting suggestions was that the starting point of the machine be the injection of a hot plasma from an ion source, rather than starting with a cold gas and then heating this to thermonuclear temperatures. A substantial part of the Sherwood project at the Oak Ridge National Laboratory is devoted to research and development relating to high current, high energy ion sources. An interesting alternative is to use these ion sources with basic plasma experiments, rather than gadgets, in mind. There is always the possibility of growing a hot plasma (which has not yet been done), even though it be for a short time, and studying its behavior. These considerations will be discussed in the section entitled "High Energy Injection."

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Magnetic Mirrors

The magnetic mirror principle is an old and well known phenomenon. It refers to the fact that charged particles which are moving in a magnetic field tend to be reflected from regions of higher-than-average field. It was shown in Chapter V [Eq. (5.43)] that a particle moves in a magnetic field so as to keep its magnetic moment μ a constant. Thus

$$\mu = \frac{mv_{\perp}^2}{B} = \text{constant.} \qquad (6.1)$$

The magnetic moment may be expected to be a constant under adiabatic conditions. That is, when the magnetic field varies slowly in time compared to the Larmor frequency and varies slowly in space over a distance of the order of the Larmor radius. Hence the name of "adiabatic invariant" for the magnetic moment. It has recently been reported that M. D. Kruskal has proven that the magnetic moment is indeed a constant to <u>all</u> orders of a perturbation expansion of the particle equations of motion.

Equation (6.1) may be used to illustrate the means by which a mirror reflects a particle. Consider the situation shown in Fig. 6.1. In the

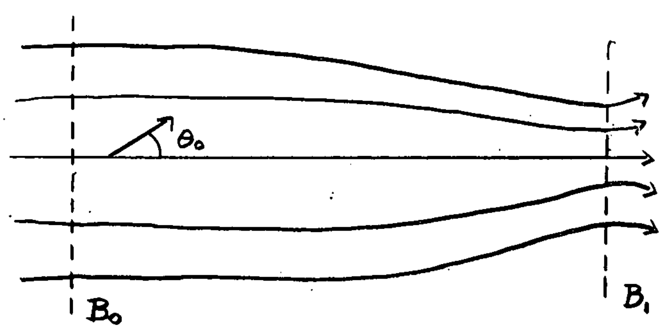

Fig. 6.1. Reflection by a Magnetic Mirror

region to the left, the magnetic field is uniform and has the magnitude B_o. The field increases on the right hand side to a maximum value denoted by B_1. Consider a charged particle in the left hand region whose velocity vector is at an angle θ_o to the field axis. Thus its velocity toward the mirror region is

$$v_{\parallel} = v \cos\theta_o,$$

and its perpendicular velocity is

$$v_{\perp} = v \sin\theta_o.$$

Since the force upon a moving charged particle in a magnetic field is at right angles to the particle motion, no work can be done. Hence the total kinetic energy of the particle must be conserved. Thus

$$\frac{1}{2} mv_{\parallel}^2 + \frac{1}{2} mv_{\perp}^2 = \frac{1}{2} mv^2 = \text{constant.} \qquad (6.2)$$

Furthermore, by Eq. (6.1), the magnetic moment will be invariant. Hence

$$\frac{mv_{\perp}^2}{B} = \frac{mv_{\perp}^{o2}}{B_o} = \frac{mv^2}{B_o} \sin^2\theta_o \qquad (6.3)$$

Divide Eq. (6.2) by the quantity B and substitute from Eq. (6.3). The following result is obtained:

$$\frac{v_{\parallel}^2}{B} = \frac{v^2}{B} - \frac{v^2 \sin^2\theta_o}{B_o}$$

Hence

$$v_{\parallel}^2 = v^2 \left(1 - \frac{B}{B_o} \sin^2\theta_o\right). \qquad (6.4)$$

It is clear from this result that the component of velocity along the field lines will decrease as the particle approaches the mirror region of higher field strength. In fact, the parallel velocity will go to zero, and hence the particle will be reflected, if the initial angle is large enough. Since the maximum field value in the mirror region is B_1, one can immediately write a critical equation for reflection;

$$\sin^2\theta_c = \frac{B_o}{B_1} . \qquad (6.5)$$

Any particle with an initial velocity vector which is at an angle to the field direction which is smaller than θ_c will escape through the mirror. Those with initial velocity angles which are greater than θ_c will be reflected from the mirror. Finally, if the morror ratio R is defined as the ratio of the field in the mirror to that in the uniform region, this result becomes

$$\sin\theta_c = \sqrt{\frac{1}{R}} \qquad (6.6)$$

A conceivable Mirror Machine will, of course, have mirrors at both ends as shown in Fig. 6.2.

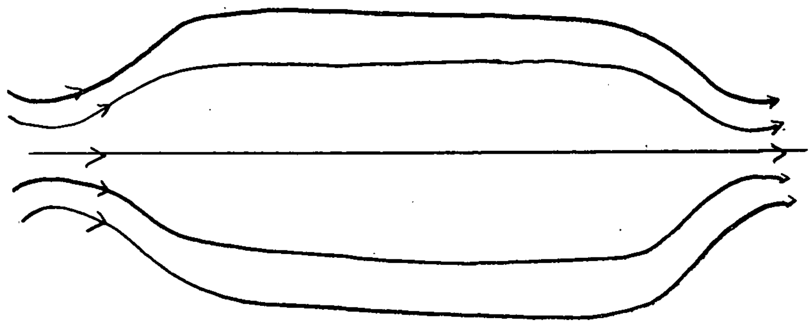

Fig. 6.2. The Mirror Machine

Diffusion Loss Through a Mirror

The most important quantity which is required for a discussion of the mirror machine is an estimate of the loss rate by diffusion through the mirrors. Upon the introduction of a hot plasma into a mirror machine, there will be the almost immediate loss of those particles whose velocity vectors lie in the two escape cones defined by the angle θ_c to the field axis. The resulting population in velocity space will be entirely depleted of velocities lying in this escape cone. The remaining particles will not remain trapped in the machine indefinitely. Owing to coulomb collisions, particles will sometimes acquire a new velocity, after a collision, which lies in the escape cone. As a result the particle will be lost. (It is assumed here that the mean free path is very long compared to the dimensions of the machine. This will almost certainly be so.) This diffusion in velocity space represents the most serious drain of particles and energy from the system and will be calculated in the next paragraph.

Assuming no spatial dependance of the particle density, the loss rate becomes:

$$\frac{dn}{dt} \cong - \frac{n^2}{2} \overline{\sigma v} P \tag{6.7}$$

where $\overline{\sigma v}$ is the coulomb collision rate for 90 deg scattering by multiple collisions and P is the probability of scattering into the escape cone. Just as in Chapt. V, the escape probability may be crudely estimated as the ratio of the surface area subtended by the escape cone on a unit sphere to the area of the entire sphere. The only difference between the two cases is that the escape area is a polar cap on each end of the sphere, while it was an annular region (see Fig. 5.4) in the case of the Stellarator. The probability P is now:

$$P = \frac{2\pi \int_0^{\theta_c} \sin\theta \, d\theta}{2\pi} = 1 - \cos\theta_c ,$$

or by Eq. (6.6),

$$P \cong 1 - \sqrt{1 - \frac{1}{R}} . \tag{6.8}$$

For a large mirror ratio, this becomes

$$P \cong \frac{1}{2R} . \tag{6.9}$$

Numerical estimates of the containment time are easily obtained. Eq. (6.7) may be written,

$$\frac{n}{t} \cong \frac{n^2}{2} (\overline{\sigma v}) P,$$

where t is the mean containment time. Hence

$$t \cong \frac{2}{n \overline{\sigma v} P} . \qquad (6.10)$$

Assume that $n \cong 10^{15}$ and $E = 10$ kev. The corresponding coulomb cross section for scattering through 90 deg by small angle collisions was shown to be 3000 barns, in Chapter II. Hence

$$t = \frac{2}{(10^{15})(3.10^{-21})(10^{8})P}$$

$$= \frac{2}{300 \text{ P}} \qquad (10 \text{ kev}) \qquad (6.11)$$

Now, by Eq. (6.8), the following values of P correspond to mirror ratios of 2 and 5, respectively.

$$P \cong 0.3 \qquad R = 2$$
$$\cong 0.1 \qquad R = 5 \qquad (6.12)$$

Hence, the corresponding containment times becomes:

$$t \cong .022 \text{ sec} \qquad R = 2$$
$$= .067 \text{ sec} \qquad R = 5 \qquad (6.13)$$

The containment time for both mirror ratios is inadequate. Furthermore, mirror ratios larger than 5 are probably unrealistic in an actual gadget. The trouble is that the mirror is quite leaky compared to a Stellarator, for example. This may be seen by comparing the escape probabilities of Eq. (6.12) with the corresponding value for a Model D Stellarator [see Eq. (5.17)]. Recent calculations by D. Judd et al, to be described in the next paragraph, have yielded even more pessimistic estimates for the containment time. Hence, thinking on the mirror project has been confined to plasma energies in the region of 100 kev rather than 10 kev. In this region, the coulomb cross section is reduced by a factor of 100 although the particle velocities are increased by $\sqrt{10}$. Furthermore, the particle density is reduced to 10^{14} for fixed value of the magnetic pressure [see Eq. (2.20) and following discussion]. Hence the new containment time estimate becomes:

$$t = \frac{2}{(10^{14})(3.10^{-23})(3.3.10^{8})P}$$

$$= \frac{2}{P} \qquad \text{(100 kev)}. \qquad (6.14)$$

Thus,

$$t = 6.7 \text{ sec} \qquad R = 2$$

$$= 20 \text{ sec} \qquad R = 5 \qquad (6.15)$$

It should be noted that the nuclear reaction time for the D-T reaction is only slightly changed from its value at 10 kev. Although the D-T reaction rate is increased by a factor of eight over its value at 10 kev (see Table 2.1), the particle density is reduced by a factor of 10. Hence, the nuclear time is only slightly changed and the mirror containment is now entirely adequate.

The containment times for a pure D-D reactor are the same as in the D-T case. It is clear that the containment at 10 kev is entirely inadequate for the D-D reaction [see Eq. (2.22)]. However, at 100 kev, the D-D reaction cross section has increased over its value at 10 kev by a factor of 35 (see Table 2.1). Hence the maximum desired containment time is now about 3 sec rather than 10 sec. Thus, from Eq. (6.15), it is conceivable that a mirror machine could operate on the D-D reaction at 100 kev.

Previous reference has already been made to the improved calculation of mirror losses by Judd, MacDonald, and Rosenbluth.[9] The starting point for this calculation is the spatially independent Boltzmann equation

$$\frac{\partial f_1}{\partial t} = \int (f_o' f_1' - f_o f_1)\, v \, \frac{d\sigma}{d\Omega}\, d\vec{c}_o \tag{6.16}$$

where f is the distribution function in velocity space and where $d\sigma/d\Omega$ is the Rutherford differential cross section. The velocity vector is denoted by $\vec{c}_o$ and v is the relative velocity of collision. Since the coulomb scattering is predominantly small angle scattering, the integrand may be expanded in a Taylor series in the vector increments of velocity $\delta\vec{c}_o = \vec{c}_o' - \vec{c}_o$ and $\delta\vec{c}_1 = \vec{c}_1' - \vec{c}_1$. Mirror losses are incorporated into the equation by assuming that particles whose velocity angles fall within the escape cone are immediately lost from the system. This leads to the boundary condition

$$f(c^2, \theta) = 0 \qquad \theta \leq \theta_c \tag{6.17}$$

where θ_c is the critical angle. It is also assumed in the derivation that the distribution function is factorable, as follows,

$$f(c^2,\theta,t) = h(c^2,t)\, g(\cos\theta),$$

and that $g(\cos\theta)$ is nearly isotropic outside of the escape cone. The resulting expression for the particle loss rate is:

$$\frac{dn}{dt} \cong - n^2 \frac{4\pi}{3} \frac{e^4}{m^2} \left(\overline{\frac{1}{v}}\right)\left(\overline{\frac{1}{v^2}}\right) \lambda_o \,\ell n \left(\frac{1}{\sin\theta_m}\right) \tag{6.18}$$

where θ_m is the minimum scattering angle in the laboratory system, m is the ion mass and

$$\lambda_o \cong \frac{1}{\log_{10} R} \tag{6.19}$$

Here R is the mirror ratio. The bars over the expressions in Eq. (6.18) denote the averages over the ion velocity distribution.

It is instructive to compare the result of Eq. (6.18) with the crude calculation illustrated by Eqs. (6.7) and (6.8). If Eq. (2.9) is substituted for the coulomb cross section, this result becomes:

$$\frac{dn}{dt} = - \frac{n^2}{2}\, 2\pi \frac{e^4}{m^2} \left(\overline{\frac{1}{v^3}}\right) \ell n \left(\frac{b_{max}}{b_{min}}\right) \cdot P \tag{6.20}$$

Comparing this with Eq. (6.18), one sees that the scattering probability P has been replaced by

$$P \longrightarrow \frac{4}{3} \lambda_o = \frac{4}{3\,\ell n_{10} R} \tag{6.21}$$

The arguments of the log terms have been assumed comparable and the product of the averages has been taken equal to the average of the product. By Eq. (6.12), it is seen that the loss rate is increased by the following factors

$$\frac{P_{JUDD}}{P} = \frac{4.5}{.3} = 15 \qquad R = 2$$

$$= \frac{1.9}{0.1} = 19 \qquad R = 5 \qquad (6.22)$$

These factors are very likely over-estimates of the actual effect. For one thing, the assumption of near isotropy of the angular distribution of velocity vectors can be expected to give an overly large loss rate, since the population would be depleted near the escape cone in the actual situation.

There is an additional loss mechanism which may be of importance. This is the possibility of ambipolar effects since the electrons, owing to their higher velocity, diffuse more rapidly through the mirrors. The resulting space charge would result in an electric field which could conceivably enhance the loss rate of ions from the system. This effect, if important, can be minimized by decreasing the electron temperature. It will be seen in the next section that a lag in electron temperature may be expected in normal operation. Calculations of ambipolar effects are now in progress at Livermore.

<u>Description of the Proposed Device</u>

The following sequence of operations are proposed for a possible mirror machine:

1. High energy injection and trapping.
2. Radial compression and heating.
3. Axial compression and heating.
4. Reaction and Randomizing.
5. Decompression.

These features will be discussed individually.

The original plans for the mirror machine called for a beam of high energy deuterons (or tritons) to be injected through the mirrors as the first stage of its cyclic operation. It is clear that a directed beam of particles whose velocity vector is at an angle to the field direction which is less than the critical angle will pass right through the mirror. These particles will continue right on out of the other mirror unless something is done in the interim which results in their being trapped in the device. Several schemes for this trapping exist.

One possibility is a uniform increase of the entire magnetic field during the injection process. As a consequence of the adiabatic invariance of the magnetic moment [see Eq. (6.1)], an increase in field strength increases the energy in the perpendicular motion and effectively increases the angle between the velocity vector and the field axis. If the field rises rapidly enough, trapping will result. An alternative scheme is one in which the mirror field grows in time, but the main field remains constant. Yet another possibility is to apply an RF field in resonance with the injected particles so as to increase the energy in the perpendicular motion. This last scheme would be severely limited by the problem of penetration of RF into a plasma.

The main difficulty with these original schemes is the inability of presently achievable ion sources to inject a sufficient quantity of plasma into the device during the time available. As a result, thinking has turned to the use of radial injection by either high energy neutral beams or molecular beams into the device. These features will be discussed in the section on high energy injection.

The second stage in the operation would be an increase in the magnetic field of the system throughout the length of the device. As was shown in Eq. (5.44), the square of the radius of the plasma varies inversely as the magnetic field. Hence, the plasma is radially squeezed and heated. The third step is a similar squeezing and heating but in the axial direction. This is accomplished by moving the magnetic mirrors toward each other. This mirror motion may be achieved either by mechanical or electrical means.

During and after the injection and compression, the plasma will become randomized through the mechanism of the coulomb collisions. At the same time nuclear reactions will occur. As the final stage of operation, the plasma is allowed to expand back out against the fields and as a result work is done on the field coils. This scheme constitutes a form of direct conversion of thermal energy into electrical power.

Let us consider some of the advantages and disadvantages of a mirror machine. One of the first advantages is the absence of drift effects, such as are found in the Stellarator, which tend to lead particles out of the device. Hence, it is unnecessary to devise such unproven features as scallops and figure-eights. This has an immediate consequence that it is not necessary to build a device which must produce enormous blocks of power in order to be economically successful. This could be a very important advantage. A third feature is that hot ion injection eliminates the problems associated with heating of an initially cold plasma. A fourth advantage is the natural way in which the sequence of operations lends itself to direct conversion of thermal energy to electrical energy.

Among the disadvantages, perhaps the most minor is the cyclic operation of the device compared to the steady state operation at Princeton. This usually results in poorer efficiency of operation. More serious is the problem of injecting sufficient plasma into the device. Present sources will not work for injection through the mirrors and, as will be shown in a later section, they even look marginal for radial injection. In addition, the economics are somewhat poorer. The fact that a particle energy of 100 kev is being used means that the particle density must be reduced to 10^{14}. As a result the specific energy yield in the plasma is reduced. The economic factors will be further discussed in the next section.

Before turning to this subject, it would be quite useful to point out the main reason for having an axial and radial compression of the plasma. The ions are injected with over 100 kev energy and therefore would end up near this temperature after thermalizing were it not for the presence of cold electrons. These electrons will come with the beam, somehow, in order that enormous space charges do not develop. The cross section for energy loss to these cold electrons is enormous, as was pointed out in Chapt. II. If the initial electron density is 10^{14} and the temperature is taken to be

100 volts, R. F. Post has calculated[14] that the deuterons would begin to lose energy exponentially with a half life $t = 5 \times 10^{-4}$ sec. This would represent a disastrous rate of loss from the ions were it not for the fact that the electron sink is a finite one. As energy is drained from the deuterons it goes into the electrons with a subsequent rise in electron temperature. The purpose of the axial and radial compression is to feed energy into the deuterons so as to compensate for the electron drain.

As the electron temperature rises, the energy transfer rate drops off as $T^{3/2}$. Thus at $T_e = 1$ kev, the *e*-folding time is now $t = 1.5 \times 10^{-2}$ sec, while the ion energy is now 99.9 kev. The e-folding time for energy input from the compression is of the order of the rise time of the magnetic field. Since this will be of the order of 10^{-2} sec or less, the compression will control the deuteron energy almost immediately.

The final electron temperature will not be equal to 100 kev. Owing to the greater bremsstrahlung of the electrons, $\left[P \sim m^{-3/2}\right.$, see Eq. (2.3)$\left.\right]$ the final electron temperature will sit considerably below that of the ions, and in the neighborhood of 20-50 kev.

Some Economic Considerations

Many of the economic considerations are entirely similar to those already discussed in connection with the Stellarator. An expression for the magnet power is given by Eq. (5.59). Assuming B = 30 kg, s = 0.5 and the outer radius of the copper coil as twice the inner radius, $r_2/r_1 = 2$ and

14. R. F. Post, Sixteen Lectures on Controlled Thermonuclear Reactions, UCRL-4231 (Feb., 1954).

$$P_M \cong 11 \text{ kw/cm.} \tag{6.23}$$

Similarly, the nuclear yield is given by Eq. (5.62). At kT = 100 kev,

$$(\overline{\sigma v})_{DT} = 8 \times 10^{-16},$$

and

$$P_N \cong 0.04\ \beta^2 R^2 \text{ kw/cm,} \tag{6.24}$$

where R is the radius of the reaction tube and an electron temperature of 50 kev has been assumed. Assume that 30% of the nuclear power is recoverable and that 50% of this amount will be used to operate the magnet. Thus it is necessary that

$$(0.3)(0.5)P_N = P_M$$

and hence

$$\beta^2 R^2 = 1800 \tag{6.25}$$

If β is equal to its maximum possible value of unity, the minimum working radius is

$$R \geq 43 \text{ cm.} \tag{6.26}$$

The thermal power generated per unit length is 72 kw/cm and the salable power 15% of this which is 11 kw/cm or about 1 megawatt/meter. This is a factor of 3 less than the salable power per unit length of the Stellarator.

There may be a strong incentive to work with as small a value of β as possible. A maximum reasonable value of R may be determined by considering capital costs. The total weight of Cu per cm is

$$W = \pi({r_2}^2 - {r_1}^2)\ s\ d$$

where d is the density of copper. Assuming as before that the space factor $s = 0.5$, that $r_2/r_1 = 2$ and that $d = 8.9$, this becomes

$$W = 42\ R^2\ \text{gm/cm.}$$

If it is assumed that the cost of the copper is \$1 per pound installed, the capital investment in copper becomes:

$$C \cong 0.1\ R^2\ \text{dollars/cm.} \tag{6.27}$$

A reasonable capital investment cost is \$200 per kilowatt of salable electric power. Hence, it is desired that

$$\frac{0.1\ R^2}{11} = 200$$

or

$$R \leq 150\ \text{cm.} \tag{6.28}$$

Upon substituting this value in Eq. (6.25), it is seen that the minimum possible value of β is about 0.3. Hence, it will not be possible to operate with values of β appreciably smaller than unity.

Actually the economic situation is somewhat worse than sketched above. It has been assumed that the only losses are in the field windings. As has already been mentioned, the thermal investment in 100 kev particles is not negligible and should be included in the accounting. Assuming that particles must be supplied at a rate equal to their loss through the mirrors, the input power may be written as

$$P_{FUEL} \cong \frac{n^2}{2} \overline{\sigma v} \pi R^2 E P, \qquad (6.29)$$

where n is the number of ions (including both tritons and deuterons), $\overline{\sigma v}$ is the coulomb scattering rate, E the input thermal energy, and P the probability of loss through a mirror after a 90 deg deflection. Now, taking the electron temperature as 50 kw,

$$n = \frac{2}{3} \frac{\beta}{kT} \frac{B^2}{8\pi},$$

and hence Eq. (6.29) may be written as

$$P_{FUEL} \cong \frac{(\overline{\sigma v}) E B^4 P}{900 (kT)^2} \beta^2 R^2 \qquad (6.30)$$

As before, choose B = 30 kg and kT = 100 kg. Now, E = 3/2 kT and at 100 kev, σ = 30 barns and v = 3.3 x 10^8 cm/sec. Hence

$$P_{FUEL} \cong .0085 P \beta^2 R^2 \text{ kw/cm}.$$

Now, using the most pessimistic values of the escape probability, which are given in Eq. (6.22) one finds that

$$P_{FUEL} \cong .04\ \beta^2 R^2 \text{ kw/cm} \qquad R = 2$$

$$\cong .016\ \beta^2 R^2 \qquad R = 5$$

Hence, the thermal fuel investment is as large as the nuclear energy yield for a mirror ratio of 2 and is disturbingly close even for R = 5. These results were first noticed by Bing, Judd, McDonald, and Rosenbluth[15] who performed a more careful calculation. It should be remembered that the end loss calculations of Judd et al. may be overly pessimistic. However, it seems clear that the economic balance is tighter for the mirror machine than it appears to be for the Stellarator. Since the ratio of nuclear yield to power input in fuel varies as $\sqrt{kT}\ (\overline{\sigma v})_{DT}$, improvement may be obtained by going to high temperatures. In this case, larger radii will be necessary to keep the magnet power ratio favorable.

High Energy Injection

Consider the problem of injection through the mirrors. Suppose that a battery of ion sources are lined up, shoulder to shoulder, filling the cross sectional area of the machine just outside of the mirror and pointed into the device. Now there have been ion sources developed at Oak Ridge which yield currents of the order of 2 amps per square inch. Assume then, that as a result of the stacking, an average overall input current of about 1/2 amp per square inch can be achieved. Assume further, for the moment, that every ion which is injected is trapped and that the injection time T is of the

15. G. Bing, et al. Some Calculations of End Losses in Mirror Machines, published in "Conference on Controlled Thermonuclear Reactions," Princeton University, TID-7503 (Feb., 1956).

order of 1 millisecond. Then, if L is the distance between mirrors, the final ion density n becomes,

$$n = \frac{I\,T}{L} , \qquad (6.31)$$

where I is the source current. Now

$$I = \frac{1}{2} \text{ amps/in.}^2 = .08 \text{ amps/cm}^2$$

$$\cong 5 \times 10^{17} \text{ ions/cm}^2 \text{ sec}$$

Assuming L = 5 meters, one finds

$$n \cong \frac{5 \times 10^{17}\, 10^{-3}}{5 \times 10^{2}} = 10^{12} \text{ cm}^{-3}.$$

The final density is still a factor of 100 smaller than required for the operating state. However, magnetic compression will raise this value and improvement could also be achieved by pushing the injection time up somewhat. Although the final density is uncomfortably small, this is not the real difficulty with injection through the mirrors. The more essential difficulty is the total field rise which must be achieved if trapping is to occur. This will be calculated in the next paragraph.

Consider trapping by means of a uniform rise of field strength over the entire length of the mirror machine. Suppose that the ions are injected at an angle θ_o which differs by only a small amount δ from the critical angle θ_c. This is illustrated in Fig. 6.3. Here the dotted lines represent the

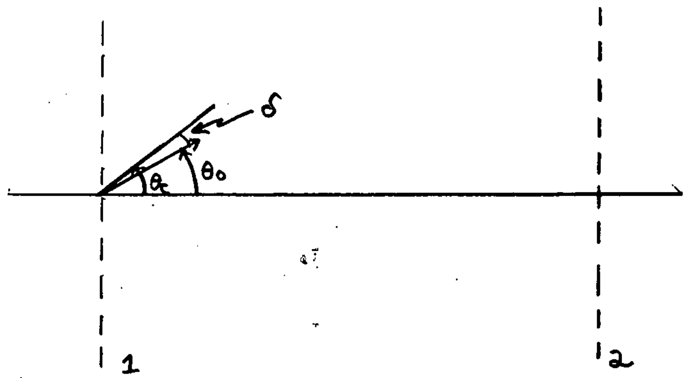

Fig. 6.3. Injection and Trapping

regions of maximum field strength in the two mirrors. Suppose that the field strength at 1 and 2 at the time of injection is denoted by B_o and that this quantity has risen to the value B by the time that the ion reaches the region of 2. Since

$$\sin\theta = \frac{v_\perp}{v} , \qquad (6.32)$$

it is clear from Eq. (6.3) that the new angle is

$$\sin\theta = \sqrt{\frac{B}{B_o}} \sin\theta_o . \qquad (6.33)$$

In order that trapping occur, it is necessary that this final angle be equal to or larger than the critical angle. Hence

$$\sqrt{\frac{B}{B_o}} = \frac{\sin\theta_c}{\sin\theta_o} = \frac{\sin\theta_c}{\sin(\theta_c - \delta)}$$

or

$$\sqrt{\frac{B}{B_o}} \simeq 1 + \delta \cot\theta_c + \ldots\ldots \qquad (6.34)$$

Now the time interval for traversal of the device from region 1 to region 2 is

$$T = \frac{L}{v \cos\theta_o} \cong \frac{L}{v \cos\theta_c} . \qquad (6.35)$$

Hence the final field B is,

$$B - B_o = \dot{B} \frac{L}{v \cos\theta_c} \qquad (6.36)$$

where $\dot{B}$ is the rate of change of magnetic field. Eq. (6.34) may be rewritten as

$$\frac{B}{B_o} \cong 1 + 2\delta \cot\theta_c , \qquad (6.34)$$

and Eq. (6.36) as

$$\frac{B}{B_o} = 1 + \frac{\dot{B}}{B_o} \frac{L}{v \cos\theta_c} . \qquad (6.38)$$

Combining Eqs. (6.37) and (6.38) yields the condition

$$\frac{\dot{B}}{B_o} \cong \frac{2v \cos^2\theta_c}{L \sin\theta_c} \delta . \qquad (6.39)$$

Finally, by use of Eq. (6.6) this condition may be written as

$$\frac{\dot{B}}{B} \cong \frac{2v}{L} \frac{R - 1}{\sqrt{R}} \delta \qquad (6.40)$$

The beam from an ion source has an inherent angular spread which one finds very difficult to reduce below a few degrees. Hence the quantity δ

can probably be made no smaller than about 0.1 radians. Assuming a mirror ratio of 4 and a length of 5 meters, one obtains

$$\frac{\dot{B}}{B} \cong \frac{3(3.3 \times 10^8)}{5 \times 10^2} \; (0.1) \; = \; 2 \times 10^5$$

Integrating, this yields

$$B(t) \; = \; B(o) \; e^{(2 \times 10^5)t} \qquad (6.41)$$

Equation (6.41) indicates that after a millisecond, the field must have increased over its initial value by the enormous factor exp(200). Since the initial field value can hardly be less than about 2 kg (the Larmor radius of a 100 kev deuteron in a field of 2 kg is 30 cm) it is clear that this rise is impossible. In fact, since final fields of the order of 40 kg are about a reasonable limit it is clear that the total increase must be a factor of 20 ($=e^3$) or less. This would limit $\dot{B}/B$ to a value of 3×10^3 or less. Since this limit is a factor of 70 less than required for complete trapping, one would expect only about 1/70 of the ions to be trapped by the maximum field rise which can be maintained. Hence, injection through the mirrors has been discarded.

One possible scheme for injection is to use the method of molecular ion breakup suggested independently by John Luce at ORNL, and H. York at Berkeley. This technique is illustrated in Fig. 6.4. A beam of high energy D_2^+ ions are projected across a magnetic field. The molecule is then dissociated into an atomic ion and a neutral atom near the extremity of the orbit. The

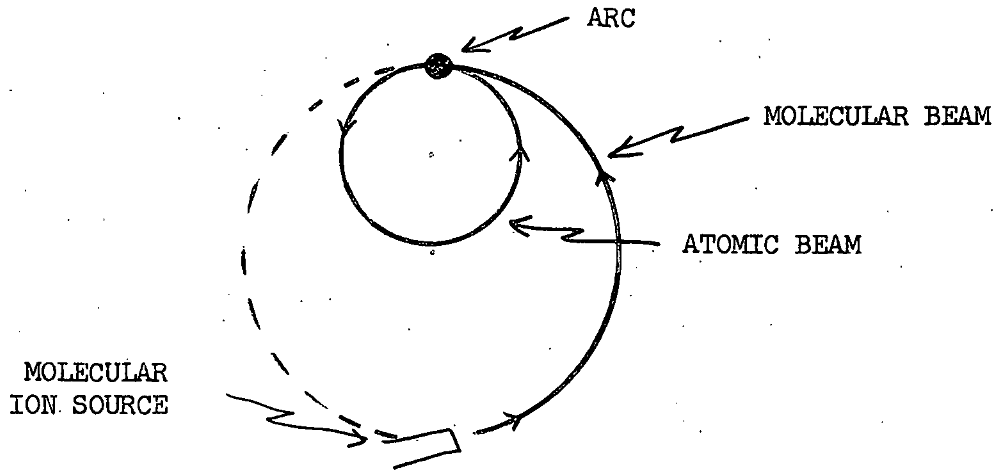

Fig. 6.4. Molecular Ion Breakup

resulting ion has about half the momentum of the molecule, half the Larmor radius and hence is trapped inside the field. The actual breakup in a Sherwood device would be caused by the plasma. The present experimental investigations of this method achieve breakup by means of a carbon arc[16] which is in the direction of the magnetic field and which intersects the molecular beam at a localized point.

The advantages of molecular injection are first that injection may be accomplished radially around the mirror machine rather than through the mirror. This allows more area for the ion sources. More importantly, little or no field rise is required for trapping. Perhaps the main objection to this method is the fact that the ions are deposited only 1 Larmor radius from the walls. This could lead to serious diffusion losses and sputtering, and hence some magnetic compression will be necessary. Plans are now under way at Oak Ridge to combine the features of molecular injection, arc

16. J. S. Luce, Ionization and Dissociation of Energetic Ions by a Carbon Discharge, ORNL-2219 (Nov., 1956).

breakup and magnetic mirrors into a small device which might enable one to grow a low density but high temperature plasma. The proposed device is called the DCX and would be used to investigate the physics of hot plasmas.

Two alternative radial injection schemes involve energetic neutral injection[17] and trapping of energetic particles by time-rising fields.[18] The first scheme is one in which D^+ ions would be accelerated to about 100 kev in a conventional accelerator, then are sent through a gas target from which about half the ions emerge neutral with very little scattering or energy loss. The neutral beam would then cross into the magnetic field and would be ionized and trapped by colliding with the plasma ions. In the second scheme, field rise times are still a problem, although not as bad as in the case of mirror injection since particles can be injected with a very small component of velocity in the field direction. Both methods are being investigated at Livermore.

Survey of Experimental Program

A listing of the experimental devices at Livermore is given below. A very brief description of the apparatus and some of the reported results are included. This table is based on Sherwood Conference reports, which are particularly sketchy on these points.

Table Top I. This device is a mirror machine utilizing pulsed magnetic fields. The peak mirror field is near 30 kg with a mirror ratio R adjustable

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17. E. J. Lauer, Energetic Neutral Injection Into Thermonuclear Machines, UCRL-4554 (Aug., 1955).

18. W. I. Linlor, High Energy Peripheral Injection Into Mirror Machines, UCRL-4569 (Sept., 1955).

from 2:1 to 4:1. The field rises in 600 μs and decays in 10 milliseconds. The device has a 6 in. ID and a length of 44 in.

Injection is by means of a deuterium loaded titanium spark source. This "hydride" source is of about 10 μs duration and delivers a plasma with energies in the range of 5 to 10 ev. These energies have been determined by time of flight and probe techniques.

The purpose of this experiment is to observe trapping and compression by time rising fields. Containment times of 300-400 μs have been reported. These times are comparable to the theoretical mirror containment times determined by Judd et al.[9]

<u>Table Top II</u>. This device has a somewhat larger peak mirror field than Table Top I and in addition, has a DC field for initial trapping of the plasma. The peak mirror field is 30 kg with a mirror ratio of 2:1. The DC field has a mirror value of 600 gauss and also has a mirror ratio of 2:1. The field rises in 650 μs and decays in 30 milliseconds. The device has a 6 in. ID and a length of 50 in.

Injection is from a "hydride" source. A base pressure of 10^{-6} mm Hg has been used. Probe measurements indicate that a plasma having an electron density of 10^{12} has been contained for a time of 200-300 μs. There is evidence that the plasma is compressed by the rising magnetic field. Soft x-rays having energies up to 20 kev appear for the duration of the containment. In addition, hard x-rays having energies up to 100 kev persist for much longer times. The hard x-rays are believed due to high energy electrons striking the walls after the containment is over.

Toy Top. Top Top has a peak mirror field of 250 kg with an R = 2:1. There is also trapping by a small DC field which may vary from 50-500 gauss with an R = 3:1. The field rises in 200 μs and decayse in 3 milliseconds. The device is quite small, 2 in. ID and a 12 in. length. The base pressure is about 10^{-7} mm Hg.

Observations have been made for field compressions ranging from a factor of 500 to 1300. In theory, this could lead to final energies of 2.5 to 6.5 kev for the plasma, which is provided by a hydride source. There is no good evidence for this temperature. Containment times of 3 milliseconds have been observed. Fast electrons are seen, as well as x-rays in the range from 10 to 200 kev.

Q-Cumber I. This is a DC machine having a central field of from 50-200 gauss. The mirrors have a maximum field of 3 kg and are individually variable. The diameter of the glass envelope is 6 in. The glass is coated with silver paint. The plasma source is of the usual hydride type.

Since there is no compression of an initially cold source, this device is intended only to study the behavior of a cold plasma. The very low initial fields make possible the attainment of a high β with relatively low ion energy and density. In addition, the variable mirror ratios allow a study of the efficiency of mirror trapping. Another interest is in the diffusion rate of charged particles across the magnetic field.

Results so far indicate quite clearly that mirrors are effective in the containment of a plasma. In addition, the diffusion rate is much lower than predicted by the Bohm formula (see Chapt. IX).

Q-Cumber II. In order to take full advantage of lower initial fields, it is advisable to use as large a tube radius as possible. Q-Cumber II has an 18 in. ID which narrows to 4 in. at the ends. The central field is 25 gauss. Containment times of 1.5 to 2 milliseconds have been observed. A $\beta = 0.1$ has been obtained. However, the neutral gas background was so large as to obscure interpretation of the containment. (The motivation in seeking a high value of β is to look for the instabilities which are expected to be present for β close to unity.)

Squash I. So named after its size which, in length at least, is as big as a squash court. The device is to have a 12 in. ID and a length of 18 ft and is to stand with its axis vertical. In a sense this device plays the same role for Livermore that Model C does for the Princeton group. That is, it is intermediate in size between the table top models and a power producer. As such, work has been temporarily shelved on this device until questions of stability, as well as adequate injection sources, are resolved.

The peak mirror field is to be about 80 kg with an R of 2:1. The rise time is to be from 5 to 10 milliseconds with a 200 millisecond decay rate. The total energy in the condenser bank is to be 10^7 joules. There is to be both axial and radial compression.

Saturn. This device has an equatorial ring source located on the median plane between two solenoid coils whose length is small compared to the coil radius. The source is located such that ϕ, the flux enclosed, is given by $\phi \leq 2\pi r^2 H$. The quantity r, in this betatron condition, is the source radius and H is the field at the source. If this condition is satisfied, the emitted particles will be accelerated toward the center.

The device has two solenoid coils of 12 in. ID located 12 in. apart. The center field is 6000 gauss with an R = 1.5. The source is a usual hydride one, and the field rises in 70 μs and decays in 900 μs. An electron density of 10^{13} is contained for about 700 μs. Compression of the plasma is observed and there is some indication of a final temperature of 50 ev.

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