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Chap. IV

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Chap. IV. The Problem of the Ends

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IV. THE PROBLEM OF THE ENDS

One of the most obvious ways of eliminating the ends of a magnetic field is to wrap the field lines around into a toroidal shape. However, even before this geometry is considered, the possibility of simply using a long solenoid should be considered. These two geometries are considered in the next two sections.

Solenoid Length

It is always possible to conceive of a solenoid which is so long that leakage of particles and heat transfer to the ends becomes negligible. An estimate of the required length is easily obtained. It has been demonstrated that the diffusion and heat transfer coefficients in the direction of the field lines are larger by a factor of $(\omega\tau)^2$ than the corresponding coefficients at right angles to the field direction. Since the diffusion time and heat transfer losses vary as ℓ^2, it is clear that a solenoid whose length is larger than its radius by a factor of $\omega\tau$ will have equal diffusion and heat conduction losses in the two directions. Thus, the necessary length L is

$$L \gtrsim (\omega\tau)\ell$$
$$\gtrsim 3.3 \times 10^5 \ell$$

Even if the tube radius is only 10 cm, which is only borderline as far as containment time is concerned, the required length becomes

$$L \gtrsim 33 \text{ km}$$

Such dimensions seem outside the realm of feasible devices, especially when it is realized that the volume must be highly evacuated and filled with a magnetic field of 20 kilogauss or larger.

The Torus: Particle Drift in a Inhomogeneous Field

The idea of eliminating the ends of a magnetic field by wrapping it into a torus is a rather obvious one and was proposed back in 1945 by Robert R. Wilson. At first glance, this trick appears to eliminate containment problems. The trouble is that the magnetic field in a toroidal geometry is necessarily nonuniform, as is illustrated in Fig. 4.1. By

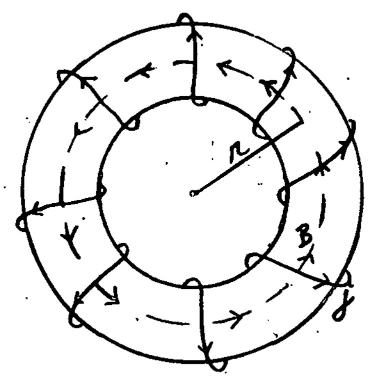

Fig. 4.1

Toroidal Field Produced by Solenoidal Windings

Maxwell's equations, the integral of the curl $\vec{H}$ over the area contained inside the dotted line can be written as

$$\int (\nabla \times \vec{H}) \cdot d\vec{A} = \oint \vec{H} \cdot d\vec{\ell} = \int 4\pi \vec{j} \cdot dA = \text{constant}$$

Hence

$$\oint \vec{H} \cdot d\vec{\ell} = 2\pi r H = \text{constant}$$

and

$$H = \text{constant}/r \qquad (4.1)$$

Thus the field in a torus is nonuniform and falls off as the reciprocal power of the radius r. Unfortunately, a charged particle in a nonuniform

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magnetic field experiences a drift in a direction which is at right angles to both the field gradient and the field direction itself. Fermi called attention to the existence of this phenomenon (which had been known to astrophysicists for some time) immediately after Wilson's suggestion, and showed that the resultant drift rates were enormously faster than could be tolerated. To demonstrate this fact, it is necessary to derive an expression for the drift rate in an inhomogeneous field.

It will be assumed in this derivation that the magnetic field is entirely in the positive z-direction and varies in magnitude in the x-direction only. The equations of motion of a charged particle in the x-y plane take the form:

$$m \frac{dv_x}{dt} = \frac{e}{c} v_y H(x) \tag{4.2a}$$

$$m \frac{dv_y}{dt} = -\frac{e}{c} v_x H(x) \tag{4.2b}$$

It is convenient to define a new coordinate s, defined by the relation

$$s = \int_0^t \frac{eH(x)}{mc} dt \tag{4.3}$$

Note that x is an implicit function of t. Note further that the integral in Eq. (4.3) cannot be evaluated, in general, since to do so would require knowledge of the particle's orbit, which is as yet unknown. Nevertheless, this change of variable makes possible a series solution for the motion.

By means of Eq. (4.3), we may rewrite Eq. (4.2) as follows:

$$\frac{dv_x}{ds} = v_y \tag{4.4a}$$

$$\frac{dv_y}{ds} = -v_x \tag{4.4b}$$

These equations may be solved immediately to yield:

$$v_x = A \sin s + B \cos s \qquad (4.5a)$$

$$v_y = A \cos s - B \sin s \qquad (4.5b)$$

The constants may be immediately identified as the initial values of the components of the velocity at t = 0. Thus, since when t = 0, s = 0, one has

$$B = v_{xo}$$

$$A = v_{yo}$$

Furthermore, by squaring and adding Eq. (4.5) it is clear that

$$v_x^2 + v_y^2 \equiv v_\perp^2 = A^2 + B^2 = v_{xo}^2 + v_{yo}^2 = v_{\perp o}^2 \qquad (4.6)$$

Hence, the scalar velocity, or what is the same thing, the energy, is a constant of the motion. This result is obvious since a magnetic field, for which the force is always at right angles to the particle velocity, can do no work on the particle.

Further progress can now be made by assuming that the magnetic field does not vary appreciably in magnitude in a distance of the order of the Larmor radius. In that case, it is permissible to expand the expression for the field strength in a power series in the field gradient,

$$H(x) = H(o) + \left.\frac{dH}{dx}\right)_o x + \ldots \qquad (4.7)$$

and keep only the lowest terms. By use of Eq. (4.7), Eq. (4.3) may be rewritten as

$$s \cong \frac{eH(o)}{mc} t + \frac{e}{mc} H'(o) \int_o^t x \, dt + \ldots \qquad (4.8)$$

The superscript prime on H denotes a spatial derivative of H.

Using Eq. (4.8), Eq. (4.5b) may be rewritten as

$$v_y = A \cos \left\{ \frac{eH(o)}{mc} t + \frac{eH'(o)}{mc} \int_o^t x \, dt \right\} - B \sin \left\{ \frac{eH(o)}{mc} t + \frac{eH'(o)}{mc} \int_o^t x \, dt \right\} \qquad (4.9)$$

Now

$$\begin{cases} \sin \left[\theta + x\right] = \sin\theta + x \cos\theta + \ldots . \\ \cos \left[\theta + x\right] = \cos\theta - x \sin\theta + \ldots . \end{cases} \qquad (4.10)$$

where the omitted terms are of higher order in x. Hence Eq. (4.9) may be rewritten as:

$$v_y \cong A \left\{ \cos\omega_o t - \sin\omega_o t \, (\omega_o)' \int_o^t x \, dt \right\} - \left\{ B \sin\omega_o t + \cos\omega_o t \, (\omega_o)' \int_o^t x \, dt \right\} \qquad (4.11)$$

where $\omega_o = eH(o)/mc$ and $(\omega_o)'$ represents the same expression with H(o) replaced by H'(o).

For the expression to be evaluated consistently, the value of x used in the integral should be of zero order in an expansion in powers of the field gradients. This result is easily obtained from the zero order expansion of Eq. (4.5a). Thus

$$v_x \cong A \sin\omega_o t + B \cos\omega_o t$$

$$\therefore x = \int v_x dt = -\frac{A}{\omega_o} \cos\omega_o t + \frac{B}{\omega_o} \sin\omega_o t + c \qquad (4.12)$$

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where c is the constant of integration. The origin of coordinates about which the field expansion has been made may always be chosen so that

$$x = -\frac{A}{\omega_o} \text{ at } t = 0$$

In this case c = 0 and

$$x = -\frac{A}{\omega_o}\cos\omega_o t + \frac{B}{\omega_o}\sin\omega_o t \tag{4.13}$$

Next,

$$\int_0^t x\,dt = -\frac{A}{\omega_o^2}\sin\omega_o t - \frac{B}{\omega_o^2}\cos\omega_o t + \frac{B}{\omega_o^2} \tag{4.14}$$

Substituting this expression in Eq. (4.11), one obtains

$$v_y = A\cos\omega_o t - B\sin\omega_o t + \frac{A^2}{\omega_o^2}(\omega_o)'\sin^2\omega_o t + \frac{B^2}{\omega_o^2}(\omega_o)'\cos^2\omega_o t$$

$$+ \frac{2AB}{\omega_o^2}(\omega_o)'\sin\omega_o t\cos\omega_o t - \frac{AB}{\omega_o^2}(\omega_o)'\sin\omega_o t - \frac{B^2}{\omega_o^2}(\omega_o)'\cos\omega_o t \tag{4.15}$$

After averaging over time, the result becomes

$$\overline{v_y} = \frac{A^2 + B^2}{2\omega_o^2}(\omega_o)' \equiv \frac{v_\perp^2 mc}{2eH^2}\frac{dH}{dx}. \tag{4.16}$$

Similarly, the first order result for v_x may be shown to be entirely periodic in time, and thus

$$\overline{v_x} = 0 \tag{4.17}$$

In vector terms, these two results may be written as

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$$v_D = -\frac{mv_\perp^2 c}{2eH^3} \nabla H \times \vec{H} \qquad (4.18)$$

This proof may be pushed even further, although the details will not be given here, to show that no further drifts occur even if second derivatives of the field magnitude in the x- and y-directions are considered.

One final drift remains. The coordinate system has been chosen so that the z-axis is in the direction of the magnetic field. This implies that $\left.\frac{dH}{dz}\right|_0 \equiv 0$. However, the second derivative may not vanish and to be consistent with the results above, the possible influence of such a term must be considered. The effect of this term is found rather easily. The second derivative in z corresponds to a curvature of the magnetic field lines. A particle moving along a curved path experiences a centrifugal force which acts in every way as an actual external force does. An external force produces a net drift as has already been shown in Eq. (3.6). Thus,

$$F_{centri} = \frac{mv_\parallel^2}{R} \vec{r} \qquad (4.19)$$

where $\vec{r}$ is a unit vector in the direction of curvature of the field. The radius of curvature is R and $v_\parallel$ is the particle velocity along the field lines. Substituting this expression in Eq. (3.6), one has

$$v_D = \frac{cmv_\parallel^2}{eH^2R} \vec{r} \times \vec{H} \qquad (4.20)$$

Equations (4.18) and (4.20) constitute the expression for the drift velocities of a particle in an inhomogeneous magnetic field.

Before applying these equations to the calculation of the drift velocities in a torus, it may be profitable to show that the drift velocities just derived can be understood on the basis of a qualitative picture similar to that presented in Chapter 3 for the drifts due to an external force. Consider a particle moving in an inhomogeneous field as sketched in Fig. 4.2.

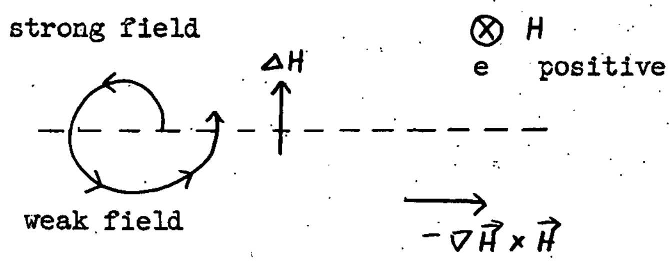

Fig. 4.2

Note that the radius of curvature is smaller in the strong field region and larger in the weak field region. The resultant drift is obvious.

Immediate use may be made of these results to evaluate the drift velocity in a torus. It has been demonstrated above that $H = \alpha/R$ where α is a constant and R is measured from the center of curvature of the torus. Hence

$$\nabla H = -\frac{\alpha}{R^2}\vec{r}$$

$$= -\frac{H}{R}\vec{r}$$

Thus, by Eqs. (4.18) and (4.20),

$$v_D = \frac{(mv_\perp^2 + 2mv_\parallel^2)c}{2eH^2R}\vec{r} \times \vec{H} \qquad (4.21)$$

This result may be put in more convenient form by recalling that for an isotropic gas $kT = mv_x^2 = mv_y^2 = mv_z^2$. Hence, $mv_\perp^2 = 2kT$ and $mv_\parallel^2 = kT$. Thus

$$|v_D| = \frac{2ckT}{eHR} \tag{4.22}$$

Note that the direction of drift is up out of the plane of the torus for one sign of the charge and in the opposite direction for the other. The magnitude of the drift is readily estimated using the standard conditions.

$$\begin{aligned} v_D &= \frac{2(3 \times 10^{10})(1.6 \times 10^{-12})(10^4)}{(4.8 \times 10^{-10})(2 \times 10^4)R} \\ &= 10^8/R \text{ cm/sec.} \end{aligned}$$

The drift time across the torus tube of radius r is then

$$t_D = \frac{rR}{10^8} \text{ sec}$$

Assuming a tube radius of 100 cm, the drift time is

$$t_D = R \times 10^{-6} \text{ sec}$$

Hence, it would require a torus having a radius of curvature of at least 1 km to obtain an average containment time of 0.1 sec. Such a device seems impractical.

As a final blow, it should be noted that if one attempts to correct for the ∇H drift by the application of an electric field in the direction perpendicular to the plane of the torus, there is then a drift at right angles to $\vec{E}$ and $\vec{H}$ which removes particles to the outside walls.

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As a historical note, it should be recorded that almost all of the items presented in the lectures up to this point were considered by a group at Los Alamos in 1945 and 1946. Soon after the torus was shown to be impractical, however, further work on the subject ceased, apparently as a result of the return of most of the members to the universities. Project Sherwood was born in 1951 as a result of two different suggestions for circumventing the containment problem which were contributed by L. Spitzer and J. L. Tuck. The details of these two proposals, as well as those of a third proposal made somewhat later by R. F. Post, will be presented in the next three lectures. Before turning to these, it may be interesting to consider some alternative proposals for achieving thermonuclear reactions which have arisen through the years and which have been uniformly unsuccessful.

Alternate Schemes

a. Sparks. A frequent proposal is that a gas be heated to thermonuclear temperatures by means of a high current transient discharge through it. The chief difficulties in this scheme are first, the inadequate containment time and second, the fact that this type of heating raises the electron temperature quickly but not the deuterons. The energy transfer rate from the electrons to the ions is rather slow and the system disperses long before the deuteron temperature has risen appreciably.

An interesting point in this regard is the fact that the famous (or infamous) project of R. Richter in Argentina was an attempt to make use of high current discharges in lithium deuteride gas. The avowed scheme was to make use of the tail of the Maxwell distribution to obtain reactions.

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The previous lectures have already shown how rapidly the reaction rate falls with temperature and how hopeless this approach is at the temperatures which could be achieved in this fashion.

b. Electrically Exploded Wires. This proposal has all the difficulties of the preceding one. In addition, the addition of high z components to the gas results in a rapid cooling owing to the increased bremsstrahlung.

c. Mechanical Shock Heating. Imploding charges and other such schemes will impart high velocities to the deuterons. However, when it is recognized that the thermal velocities corresponding to 10^8 °K are in the neighborhood of 10^8 cm/sec, it seems unlikely that such devices will be successful.