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THRESHOLD RESUMMATION OF SOFT GLUONS IN HADRONIC REACTIONS – AN INTRODUCTION*

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Abstract

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THRESHOLD RESUMMATION OF SOFT GLUONS IN HADRONIC REACTIONS – AN INTRODUCTION

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I discuss the motivation for resummation of the effects of initial-state soft gluon radiation, to all orders in the strong coupling strength, for processes in which the near-threshold region in the partonic subenergy is important. I summarise the method of "perturbative resummation" and its application to the calculation of the total cross section for top quark production at hadron colliders. Comments are included on the differences between the treatment of subleading logarithmic terms in this method and in other approaches.

1 Introduction and Motivation

In inclusive hadron interactions at collider energies, $t\bar{t}$ pair production proceeds through partonic hard-scattering processes involving initial-state light quarks q and gluons g. In lowest-order perturbative quantum chromodynamics (QCD), at $\mathcal{O}(\alpha_s^2)$, the two partonic subprocesses are $q+\bar{q}\to t+\bar{t}$ and $g+g\to t+\bar{t}$. Calculations of the cross section through next-to-leading order, $\mathcal{O}(\alpha_s^3)$, involve gluonic radiative corrections to these lowest-order subprocesses as well as contributions from the $q+g$ initial state [1]. In this paper, I describe calculations that go beyond fixed-order perturbation theory through resummation of the effects of gluon radiation [2,3,4] to all orders in the strong coupling strength α_s.

The physical cross section is obtained through the factorization theorem

$$\sigma_{ij}(S,m)=\frac{4m^2}{S}\int_0^{\frac{S}{4m^2}-1} d\eta\Phi_{ij}(\eta,\mu)\hat{\sigma}_{ij}(\eta,m,\mu). \tag{1}$$

The square of the total hadronic center-of-mass energy is S, the square of the partonic center-of-mass energy is s, m denotes the top mass, μ is the usual factorization and renormalization scale, and $\Phi_{ij}(\eta,\mu)$ is the parton flux. The variable $\eta=\frac{s}{4m^2}-1$ measures the distance from the partonic threshold. The indices $ij\in\{q\bar{q},gg\}$ denote the initial parton channel. The partonic cross section $\hat{\sigma}_{ij}(\eta,m,\mu)$ is obtained either from fixed-order QCD calculations [1], or, as described here, from calculations that include of resummation [2,3,4] to all orders in α_s. I use the notation $\alpha\equiv\alpha(\mu=m)\equiv\alpha_s(m)/\pi$. The total physical cross section is obtained after incoherent addition of the contributions from the $q\bar{q}$ and gg production channels.

Comparison of the partonic cross section at next-to-leading order with its lowest-order value reveals that the ratio becomes very large in the near-threshold region. Indeed, as $\eta \rightarrow 0$, the "K-factor" at the partonic level $\hat{K}(\eta)$ grows in proportion to $\alpha \ln^2(\eta)$. The very large mass of the top quark, and the correspondingly small value of α notwithstanding, the large ratio $\hat{K}(\eta)$ makes it evident that the next-to-leading order result does not necessarily provide a reliable quantitative prediction of the top quark production cross section at the energy of the Tevatron collider. Analogous examples include the production of hadronic jets that carry large values of transverse momentum, the production of pairs of supersymmetric particles with large mass, and the pair-production of a fourth-generation quark, such as the postulated b'.

2 Gluon Radiation and Resummation

The origin of the large threshold enhancement may be traced to initial-state gluonic radiative corrections to the lowest-order channels. I remark that I am describing the calculation of the inclusive total cross section for the production of a top quark-antiquark pair, i.e., the total cross section for $t + \bar{t} +$ anything. The partonic subenergy threshold in question is the threshold for $t + \bar{t} +$ any number of gluons. This coincides with the threshold in the invariant mass of the $t + \bar{t}$ system for the lowest order subprocesses only.

For $i + j \rightarrow t + \bar{t} + g$, the variable z is defined through the invariant $(1 - z) = \frac{2k \cdot p_t}{m^2}$, where k and p_t are the four-vector momenta of the gluon and top quark. In the limit that $z \rightarrow 1$, the radiated gluon carries zero momentum. After cancellation of soft singularities and factorization of collinear singularities in $\mathcal{O}(\alpha_s^3)$, there is a left-over integrable large logarithmic contribution to the partonic cross section associated with initial-state gluon radiation. This contribution is often expressed in terms of "plus" distributions. In $\mathcal{O}(\alpha_s^3)$, it is proportional to $\alpha^3 \ln^2(1 - z)$. When integrated over the near-threshold region $1 \geq z \geq 0$, it provides an excellent approximation to the full next-to-leading order physical cross section as a function of the top mass [3].

Although a fixed-order $\mathcal{O}(\alpha^4)$ calculation of $t\bar{t}$ pair production does not exist, universality of the form of initial-state soft gluon radiation may be invoked, and the leading logarithmic structure at $\mathcal{O}(\alpha^4)$ may be appropriated from the next-to-next-to-leading order calculations of massive lepton-pair production ($l\bar{l}$), the Drell-Yan process. In the near-threshold region, the hard kernel becomes

$$\mathcal{H}_{ij}^{(0+1+2)}(z,\alpha) \simeq 1 + 2\alpha C_{ij} \ln^2(1-z) + \alpha^2 \left[2C_{ij}^2 \ln^4(1-z) - \frac{4}{3} C_{ij} b_2 \ln^3(1-z) \right]. \tag{2}$$

The coefficient $b_2 = (11C_A - 2n_f)/12$; the number of flavors $n_f = 5$; $C_{q\bar{q}} = C_F = 4/3$; and $C_{gg} = C_A = 3$. The leading logarithmic contributions in each order of perturbation theory are all positive in overall sign so that the leading logarithm threshold enhancement keeps building in magnitude at each fixed order of perturbation theory.

The goal of gluon resummation is to sum the series in $\alpha^n \ln^{2n}(1-z)$ to all orders in α in order to obtain a more trustworthy prediction. This procedure has been studied extensively for the Drell-Yan process, and good agreement with data is achieved. In essentially all resummation procedures, the large logarithmic contributions are exponentiated into a function of the QCD running coupling strength, itself evaluated at a variable momentum scale that is a measure of the radiated gluon momentum. The set of purely leading monomials $\alpha^n \ln^{2n}(1-z)$ exponentiates directly, with α evaluated at a fixed large scale $\mu = m_t$ as may be appreciated from a glance at Eq. (2). This simple result does not mean that a theory of resummation is redundant, even if only leading logarithms are to be resummed. Indeed, straightforward use of the exponential of $\alpha 2C_{ij} \ln^2(1-z)$ would lead to an exponentially divergent integral (and therefore cross section) since the coefficient of the logarithm is positive. The naive approach fails, and more sophisticated resummation approaches must be employed.

Different methods of resummation differ in theoretically and phenomenologically important respects. Formally, if not explicitly in some approaches, an integral over the radiated gluon momentum z must be done over regions in which $z \to 1$. Therefore, one significant distinction among methods has to do with how the inevitable "non-perturbative" region is handled.

The method of resummation employed in my work with Harry Contopanagos [3] is based on a perturbative truncation of principal-value (PV) resummation [5]. This approach has an important technical advantage in that it does not depend on arbitrary infrared cutoffs. Because extra scales are absent, the method permits an evaluation of its regime of applicability, i.e., the region of the gluon radiation phase space where leading-logarithm resummation should be valid. We work in the $\overline{\mathrm{MS}}$ factorization scheme.

Factorization and evolution lead directly to exponentiation of the set of large threshold logarithms in moment (n) space in terms of an exponent E^{PV}. The function E^{PV} is finite, and $\lim_{n\to\infty} E^{PV}(n, m^2) = -\infty$. Therefore, the corresponding partonic cross section is finite as $z \to 1$ ($n \to +\infty$).

The function E^{PV} includes both perturbative and non-perturbative content. The non-perturbative content is not a prediction of perturbative QCD. Contopanagos and I choose to use the exponent only in the interval in moment space in which the perturbative content dominates. We derive a perturbative asymptotic representation of $E(x, \alpha(m))$ that is valid in the moment-space interval

$$1 < x \equiv \ln n < t \equiv \frac{1}{2\alpha b_2}. \tag{3}$$

The interval in Eq. (3) agrees with the intuitive definition of the perturbative region in which inverse-power contributions are unimportant: $\frac{\Lambda_{QCD}}{(1-z)m} \leq 1$.

The perturbative asymptotic representation is

$$E_{ij}(x,\alpha) \simeq E_{ij}(x,\alpha,N(t)) = 2C_{ij} \sum_{\rho=1}^{N(t)+1} \alpha^{\rho} \sum_{j=0}^{\rho+1} s_{j,\rho} x^j \ . \tag{4}$$

Here

$$s_{j,\rho} = -b_2^{\rho-1}(-1)^{\rho+j} 2^{\rho} c_{\rho+1-j}(\rho-1)!/j!\ ; \tag{5}$$

and $\Gamma(1+z) = \sum_{k=0}^{\infty} c_k z^k$, where Γ is the Euler gamma function. The number of perturbative terms $N(t)$ in Eq. (4) is obtained[3] by optimizing the asymptotic approximation $\left| E(x,\alpha) - E(x,\alpha,N(t)) \right|$ = minimum. Optimization works perfectly, with $N(t) = 6$ at $m = 175$ GeV. As long as n is in the interval of Eq. (3), all the members of the family in n are optimized at the same $N(t)$, showing that the optimum number of perturbative terms is a function of t, i.e., of m only.

Resummation is completed in a finite number of steps. When the running of the coupling strength α is included up to two loops, all monomials of the form $\alpha^k \ln^{k+1} n$, $\alpha^k \ln^k n$ are produced in the exponent of Eq. (4). We discard monomials $\alpha^k \ln^k n$ in the exponent because of the restricted leading-logarithm universality between $t\bar{t}$ production and massive lepton-pair production, the Drell-Yan process.

The moment-space exponent that we use is the truncation

$$E_{ij}(x,\alpha,N) = 2C_{ij} \sum_{\rho=1}^{N(t)+1} \alpha^{\rho} s_{\rho} x^{\rho+1}, \tag{6}$$

with the coefficients $s_{\rho} \equiv s_{\rho+1,\rho} = b_2^{\rho-1} 2^{\rho}/\rho(\rho+1)$. This expression contains no factorially-growing (renormalon) terms. One can also derive the perturbative expressions, Eqs. (3), (4), and (5), without the principal-value prescription, although with less certitude [3].

After inversion of the Mellin transform from moment space to the physically relevant momentum space, the resummed hard kernel takes the form

$$\mathcal{H}_{ij}^{R}(z,\alpha) = \int_0^{\ln(\frac{1}{1-z})} dx e^{E_{ij}(x,\alpha)} \sum_{j=0}^{\infty} Q_j(x,\alpha) \, . \tag{7}$$

The leading large threshold corrections are contained in the exponent $E_{ij}(x,\alpha)$, a calculable polynomial in x. The functions $\{Q_j(x,\alpha)\}$ arise from the analytical inversion of the Mellin transform from moment space to momentum space. These functions are expressed in terms of successive derivatives of E. Each Q_j contains j more powers of α than of x so that Eq. (7) embodies a natural power-counting of threshold logarithms. However, only the *leading* threshold corrections are universal. Final-state gluon radiation as well as initial-state/final-state interference effects produce subleading logarithmic contributions that differ for processes with different final states. Accordingly, there is no physical basis for accepting the validity of the particular subleading terms that appear in Eq. (7). Among all $\{Q_j\}$ in Eq. (7), only the very leading one is universal, Q_0, and it is the only one we retain. Hence, Eq. (7) can be integrated explicitly, and the resummed partonic cross sections become

$$\hat{\sigma}_{ij}^{R;pert}(\eta,m) = \int_{z_{min}}^{z_{max}} dz e^{E_{ij}(\ln(\frac{1}{1-z}),\alpha)} \hat{\sigma}'_{ij}(\eta,m,z). \tag{8}$$

The derivative $\hat{\sigma}'_{ij}(\eta,m,z) = d(\hat{\sigma}_{ij}^{(0)}(\eta,m,z))/dz$, and $\hat{\sigma}_{ij}^{(0)}$ is the lowest-order $\mathcal{O}(\alpha_s^2)$ partonic cross section expressed in terms of inelastic kinematic variables. The lower limit of integration, z_{min}, is fixed by kinematics. The upper limit, $z_{max} < 1$, well specified within the context of our calculation, is established by the condition of consistency of leading-logarithm resummation. It is derived from the requirement that the value of all subleading contributions $Q_j, j \geq 1$ be negligible compared to the leading contribution Q_0. The presence of z_{max} guarantees that the integration over the soft-gluon momentum is carried out only over a range in which poorly specified non-universal subleading terms would not contribute significantly even if retained. We cannot justify continuing the results of leading-logarithm resummation into the region $1 > z > z_{max}$.

To obtain the physical cross section, we insert the resummed expression Eq. (8) into Eq. (1) and integrate over η. Perturbative resummation probes the threshold down to $\eta \geq \eta_0 = (1-z_{max})/2$. Below this value, perturbation theory

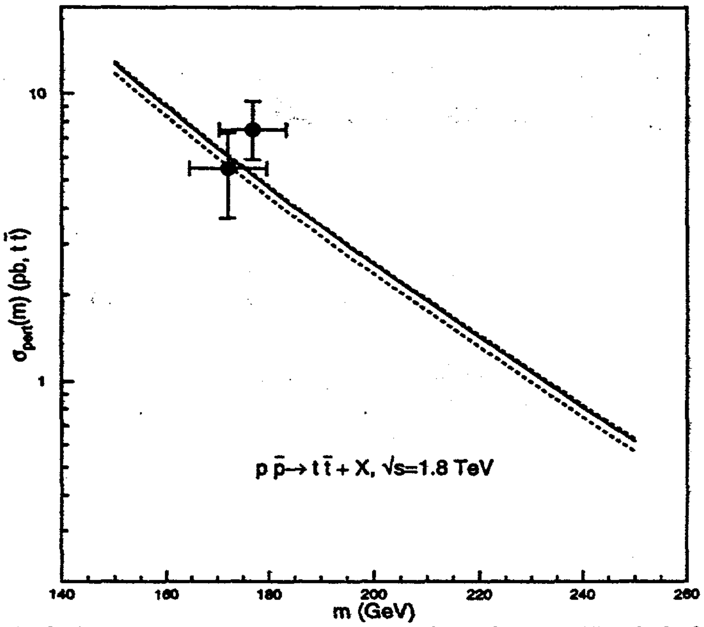

Figure 1: Inclusive total cross section for top quark production. The dashed curves show the upper and lower limits while the solid curve is our central prediction. CDF and D0 data are shown.

is not to be trusted. For $m = 175$ GeV, we determine that the perturbative regime is restricted to values of the subenergy greater than 1.22 GeV above the threshold ($2m$) in the $q\bar{q}$ channel and 8.64 GeV above threshold in the gg channel. The difference reflects the larger color factor in the gg case. The value 1.22 GeV is comparable to the decay width of the top quark, a natural definition of the perturbative boundary and by no means unphysically large.

3 Physical cross section

Other than the top mass, the only undetermined scales are the QCD factorization and renormalization scales. A common value μ is adopted for both. In Fig. 1, our total cross section for $t\bar{t}$-production is shown as a function of top mass in $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV. The central value is obtained with the choice $\mu/m = 1$, and the lower and upper limits are the maximum and minimum of the cross section in the range $\mu/m \in \{0.5, 2\}$. At $m = 175$ GeV, the full width of this uncertainty band is about 10% . As is to be expected, less variation with μ is evident in the resummed cross section than in the next-to-leading order cross section. In estimating uncertainties, Contopanagos and I do not consider explicit variations of the non-perturbative boundary, expressed through z_{max}. For a fixed m and μ, z_{max} is obtained by enforcing dominance of the universal leading logarithmic terms over the subleading ones. Therefore, z_{max} is *derived* and is not a source of uncertainty. At fixed m, the boundary necessarily varies as μ and thus α vary.

Contopanagos and I calculate $\sigma^{t\bar{t}}(m = 175 \text{ GeV}, \sqrt{S} = 1.8 \text{ TeV}) = 5.52^{+0.07}_{-0.42}$ pb, in agreement with data [6]. This cross section is larger than the next-to-leading order value by about 9%. The top quark cross section increases quickly with the energy of the $p\bar{p}$ collider. We determine $\sigma^{t\bar{t}}(m = 175 \text{ GeV}, \sqrt{S} = 2 \text{ TeV}) = 7.56^{+0.10}_{-0.55}$ pb. The central value rises to 22.4 pb at $\sqrt{S} = 3$ TeV and 46 pb at $\sqrt{S} = 4$ TeV.

Extending our calculation to larger values of m at $\sqrt{S} = 1.8$ TeV, we find that resummation in the principal $q\bar{q}$ channel produces enhancements over the next-to-leading order cross section of 21%, 26%, and 34%, respectively, for $m = 500$, 600, and 700 GeV. The reason for the increase of the enhancements with mass at fixed energy is that the threshold region becomes increasingly dominant. Since the $q\bar{q}$ channel also dominates in the production of hadronic jets at very large values of transverse momenta, we suggest that on the order of 20% of the excess cross section reported by the CDF collaboration [7] may be accounted for by resummation.

4 Other Methods of Resummation

Two other groups have published calculations of the total cross section at $m = 175$ GeV and $\sqrt{S} = 1.8$ TeV: $\sigma^{t\bar{t}}(\text{LSvN}^2) = 4.95^{+0.70}_{-0.40}$ pb; and $\sigma^{t\bar{t}}(\text{CMNT}^4) = 4.75^{+0.63}_{-0.68}$ pb. From a numerical point of view, ours and theirs all agree within their estimates of theoretical uncertainty. However, the resummation methods differ as do the methods for estimating uncertainties. Both the central value and the band of uncertainty of the LSvN predictions are sensitive to their arbitrary infrared cutoffs. To estimate theoretical uncertainty, Contopanagos and I use the standard μ-variation, whereas LSvN obtain theirs primarily from variations of their cutoffs. It is difficult to be certain of the central value and to evaluate theoretical uncertainties in a method that requires an undetermined infrared cutoff.

The group of Catani, Mangano, Nason, and Trentadue (CMNT)[4] calculate a central value of the resummed cross section (also with $\mu/m = 1$) that is less than 1% above the exact next-to-leading order value. Both they and we use the same universal leading-logarithm expression in moment space, but differences occur after the transformation to momentum space. The differences can be stated more explicitly if one examines the perturbative expansion of the resummed hard kernel $\mathcal{H}^R_{ij}(z, \alpha)$. If, instead of restricting the resummation to the universal leading logarithms, one uses the full content of $\mathcal{H}_{ij}^{R}(z,\alpha)$, she or he would find an analytic expression that is equivalent to CMNT's numerical inversion,

$$\mathcal{H}_{ij}^{R} \simeq 1 + 2\alpha C_{ij}\left[\ln^2(1-z) + 2\gamma_B \ln(1-z)\right] + \mathcal{O}(\alpha^2). \qquad (9)$$

In terms of this expansion, in our work we retain only the leading term $\ln^2(1-z)$ at order α, but both this term and the non-universal subleading term $2\gamma_B \ln(1-z)$ are retained by CMNT. If this subleading term is discarded in Eq. (9), the residuals $\delta_{ij}/\sigma_{ij}^{NLO}$ defined by CMNT increase from 0.18% to 1.3% in the $q\bar{q}$ production channel and from 5.4% to 20.2% in the gg channel. After addition of the two channels, the total residual δ/σ^{NLO} grows from the negligible value of about 0.8% to the value 3.5%. While still smaller than the increase of 9% that we obtain, the increase of 3.5% vs. 0.8% shows the substantial influence of the subleading logarithmic terms in the CMNT results.

Contopanagos and I judge that it is preferable to integrate over only the region of phase space in which the subleading term is suppressed numerically. Our reasons include the fact that the subleading term is not universal, is not the same as the subleading term in the exact $\mathcal{O}(\alpha^3)$ calculation, and can be changed if one elects to keep non-leading terms in moment space. The subleading term is negative and numerically very significant when it is integrated throughout phase space (i.e., into the region of z above our z_{max}). In our view, the results of a *leading-logarithm* resummation should not rely on subleading structures in any significant manner. The essence of our determination of the perturbative boundary z_{max} is precisely that below z_{max} subleading structures are also numerically subleading, whether or not these poorly substantiated subleading logarithms are included.

In the remainder of this section I offer a more systematic analysis [8] of the role played in the CMNT approach by non-universal subleading logarithms and show in some detail how their method and results differ from ours. I treat expansions of the resummed momentum-space kernel up to two loops. The corresponding cross sections are integrable down to threshold, $z_{max} = 1$ and $\eta = 0$. However, the effects of the various classes of logarithms are pronounced if one continues the region of integration beyond our perturbative regime.

In moment space, the exponent to two-loops is obtained from Eq. (4):

$$E_{ij}^{[2]}(x,\alpha) = g\alpha(s_{2,1}x^2 + s_{1,1}x + s_{0,1}) + g\alpha^2(s_{3,2}x^3 + s_{2,2}x^2 + s_{1,2}x + s_{0,2}), \quad (10)$$

with $g = 2C_{ij}$ and $x = \ln n$. One can perform the analytical Mellin inversion directly, beginning with Eq. (10). After a trivial integration, the results for the one- and two-loop hard kernels are

$$\mathcal{H}^{(1)} = x_z^2\alpha\{gs_{2,1}\} + x_z\alpha\{g(s_{1,1} + 2c_1 s_{2,1})\} \ , \tag{11}$$

and

$$\begin{aligned}\mathcal{H}^{(2)} &= x_z^4\alpha^2\{g^2 s_{2,1}^2/2\} + x_z^3\alpha^2\{gs_{3,2} + g^2(s_{2,1}s_{1,1} + 2c_1 s_{2,1}^2)\}\\ &+x_z^2\alpha^2\{g(s_{2,2} + 3c_1 s_{3,2}) + g^2(s_{1,1}^2/2 + 3c_1 s_{1,1}s_{2,1} + s_{2,1}s_{0,1} + s_{2,1}^2[6c_2 - \pi^2]\}\\ &+x_z\alpha^2\{g(s_{1,2} + 2c_1 s_{2,2} + s_{3,2}[6c_2 - \pi^2])\\ &+g^2(s_{0,1}s_{1,1} + 2c_1 s_{0,1}s_{2,1} + c_1 s_{1,1}^2 + s_{2,1}s_{1,1}[6c_2 - \pi^2]\\ &+s_{2,1}^2[12c_3 - 2\pi^2 c_1])\} \ .\end{aligned} \tag{12}$$

All the constants are defined in Eqs. (4) and (5). Equation (11) includes a leading logarithmic term, $x_z^2\alpha$, as well as a next-to-leading term, $x_z\alpha$.

The question to be addressed is whether it is justified and meaningful to retain all of the terms in Eqs. (11) and (12) in the computation of the resummed cross section. The issue has to do with what one intends by resummation of leading logarithms. Contopanagos and I use the term *leading logarithm* resummation to denote the case in which the moment space exponent, Eq. (10), contains only the constants $E_{LL} = \{s_{\rho+1,\rho}, 0\}$. This is also what is done in the CMNT method, and the exponent *in moment space* in their work is identical to that used for our predictions, Eq. (6). However, in contrast to our expression in momentum space, Eq. (8), the corresponding CMNT expression in momentum space includes the numerical equivalent of all terms in Eqs. (11) and (12) that are proportional to $s_{\rho+1,\rho}$.

If expressed analytically, CMNT's corresponding "LL" hard kernels are

$$\mathcal{H}_{LL}^{(1)} = x_z^2\alpha g - x_z\alpha 2g\gamma_E, \tag{13}$$

and

$$\begin{aligned}\mathcal{H}_{LL}^{(2)} &= x_z^4\alpha^2 g^2/2 + x_z^3\alpha^2\{2gb_2/3 - 2\gamma_E g^2\}\\ &+x_z^2\alpha^2\{-2gb_2\gamma_E + g^2[3\gamma_E^2 - \pi^2/2]\}\\ &+x_z\alpha^2\{2gb_2[3\gamma_E^2 - \pi^2/2]/3 + g^2[\gamma_E\pi^2 - 2\gamma^3 - 4\zeta(3)]\} \ ,\end{aligned} \tag{14}$$

where $\zeta(s)$ is the Riemann zeta function; $\zeta(3) = 1.2020569$. Evaluating the expressions numerically for the $q\bar{q}$ channel, one obtains [8]

$$\mathcal{H}_{LL}^{(1)} = x_z^2\alpha \times 2.66666 - x_z\alpha \times 3.07848, \tag{15}$$

and

$$\mathcal{H}^{(2)}_{LL} = x_s^4\alpha^2 \times 3.55555 - x_s^3\alpha^2 \times 4.80189$$
$$-x_s^2\alpha^2 \times 33.88456 - x_s\alpha^2 \times 9.82479 \,. \tag{16}$$

Apart from the leading monomials that are the same as those in our approach, Eqs. (15) and (16) include a series of subleading terms, each of which has a significant negative coefficient. In practice, these subleading terms suppress the effects of resummation essentially completely. One of the effects of this suppression is that the resummed partonic cross section is *smaller* than its next-to-leading order counterpart in the neighborhood of $\eta = 0.1$, a region in which the next-to-leading order partonic cross section takes on its largest values. This point is illustrated in Fig. 3 of CMNT's second paper [4].

Although the specific set of subleading terms in Eqs. (15) and (16) is generated in the inversion of the Mellin transform, a case can be made that the terms are accidental. First, terms involving γ_E do not appear in the exact next-to-leading order calculation of the hard part, since they are removed in the specification of the $\overline{\mathrm{MS}}$ factorization scheme. Therefore, the term proportional to γ_E in Eq. (13) is suspect. Second, if the specific value of the subleading logarithm is extracted from the full $\mathcal{O}(\alpha^3)$ next-to-leading order calculation, one finds [8] $x_s\alpha(2g - 41/6)$ instead of the term $-x_s\alpha 2g\gamma_E$. Instead of the numerical coefficient 3.07848 in Eq. (15), one finds the smaller value 1.5 if the subleading logarithm of the exact $\mathcal{O}(\alpha^3)$ calculation is used. Thus, not only is the $\mathcal{O}(\alpha)$ subleading term retained in the CMNT approach different from that of the exact calculation, it is numerically about twice as large. Third, the results of a LL resummation should not rely on the subleading structures in any significant way. However, in the CMNT approach, Eq. (13), which is the one-loop projection of their resummed prediction, reproduces only 1/3 of the exact $\mathcal{O}(\alpha^3)$ enhancement, the other 2/3 being cancelled by the second (non-universal) term of Eq. (13). Although the goal is to resum the threshold corrections responsible for the large enhancement of the cross section at next-to-leading order, the CMNT method does not reproduce most of this enhancement.

Addressing questions associated with the γ_E terms, CMNT examine a type of NLL resummation in their second paper [4]. In this NNL resummation, the $\{s_{\rho+1,\rho}, s_{\rho,\rho}\}$ terms are retained in the exponent, Eq. (10). The corresponding hard kernels become

$$\mathcal{H}^{(1)}_{NLL} = x_s^2\alpha g, \tag{17}$$

and

$$\mathcal{H}^{(2)}_{NLL} = x_z^4\alpha^2 g^2/2 + x_z^3\alpha^2 2gb_2/3 - x_z^2\alpha^2 g^2[\gamma_E^2+\pi^2/2] - x_z\alpha^2\{gb_2[2\gamma_E^2+\pi^2/3]+g^2 4\zeta(3)\}\,. \tag{18}$$

Equation (17) is identical to the one-loop projection of our hard kernel. On the other hand, our two-loop projection contains only the first two terms of Eq. (18). The term proportional to $x_z^3\alpha^2$ is present in our case, along with the leading term proportional to $x_z^4\alpha^2$, because it comes from the leading logarithms in the exponent $E(n)$, through two-loop running of the coupling strength. In contrast to Eq. (14), Eq. (18) relegates the influence of the ambiguous constant coefficients to lower powers of x_z (but with larger negative coefficients). In the amended scheme, the unphysical γ_E terms are still present in the two-loop result, Eq. (18), along with π^2 and $\zeta(3)$ terms that may be expected but whose coefficients have no well defined physical origin. Recast in numerical form, Eqs. (17) and (18) become[8]

$$\mathcal{H}^{(1)}_{NLL} = x_z^2\alpha \times 2.66666, \tag{19}$$

and

$$\mathcal{H}^{(2)}_{NLL} = x_z^4\alpha^2 \times 3.55555 + x_z^3\alpha^2 \times 3.40739 - x_z^2\alpha^2 \times 37.46119 - x_z\alpha^2 \times 54.41253\,. \tag{20}$$

There is a significant difference between the coefficients of all but the very leading power of x_z in Eqs. (15) and (16) with respect to those in Eqs. (19) and (20), and the numerical coefficients grow in magnitude as the power of x_z decreases.

Using their NLL amendment, CMNT find that the central value of their resummed cross section exceeds the next-to-leading order result by 3.5% (both $q\bar{q}$ and gg channels added). This increase is about 4 times larger than the central value of the increase obtained in their first method, closer to our increase of about 9%. The reason for the significant change of the increase resides with the subleading structures, viz., in the differences between the LL version Eqs. (15) and (16) and the NLL version Eqs. (19) and (20). The subleading terms at two-loops cause a total suppression of the two-loop contribution (in fact, that contribution is negative), if one integrates all the way into what we call the non-perturbative regime. This suppression explains why an enhancement of only 3.5% is obtained in the amended method, rather than our 9%.

CMNT argue that retention of their subleading terms in momentum space is important for "energy conservation". By this statement, they mean that one begins the formulation of resummation with an expression in momentum space containing a delta function representing conservation of the fractional partonic momenta. In moment space, this delta function subsequently unconvolves the resummation. Therefore, when one inverts the Mellin transform to return to momentum space, the full set of logarithms generated by this inversion are required by the original energy conservation. This line of reasoning would be compelling *if the complete exponent $E(n)$ in moment space were known exactly*, i.e., if the resummation in moment space were exact in representing the cross section to all orders. However, the exponent is truncated in all approaches, and knowledge of the logarithms it resums reliably is limited both in moment and in momentum space. Hence, the set of logarithms produced by the Mellin inversion in momentum space should also be restricted. In our approach energy conservation is obeyed in momentum space consistently with the class of logarithms resummed. On the other hand, in the CMNT method, knowledge is presumed of all logarithms generated from the Mellin inversion, despite the fact that the truncation in moment space makes energy conservation a constraint restricted to the class of logarithms that is resummable, i.e., a constraint restricted by the truncation of the exponent $E(n)$. The two approaches would be equivalent provided a constraint be in place on the effects of subleading logarithms. This constraint is precisely our restriction $z_{max} < 1$, but no such constraint is furnished by CMNT. For this reason their results change significantly if one set of the logarithms generated in momentum space is adopted as "the set corresponding to energy conservation", and then compared with another set, produced by a different truncation of $E(n)$.

The essence of our determination of the perturbative regime, $z_{max} < 1$, is precisely that, in this regime, subleading structures are also *numerically subleading*, whether or not the classes of subleading logarithms coming from different truncation of the master formula for the resummed hard kernel are included. The results presented in Fig. 11 of our second paper [3], show that if we alter our resummed hard kernel to account for subleading structures but still stay within our perturbative regime, the resulting cross section is reduced by about 4%, within our band of perturbative uncertainty.

A criticism [4] is that of putative "spurious factorial growth" of our resummed cross section, above and beyond the infrared renormalons that are eliminated from our approach. The issue, as demonstrated in Eq. (29) of our second paper [3], can be addressed most easily if one substitutes any monomial appearing in Eq. (12), symbolically $\alpha^m c(l,m) \ln^l x_z$, into Eq. (8) and integrates over z:

$$\alpha^m c(l,m) \int_{z_{min}}^{1} dz \ln^l x_z = \alpha^m c(l,m)(1-z_{min}) l! \sum_{j=0}^{l} \ln^j(1/(1-z_{min})) \ . \quad (21)$$

For the purposes of this demonstration we set $\hat{\sigma}'_{ij} = 1$. The coefficients $c(l,m)$

can be read directly from Eq. (12). For the leading logarithmic terms,

$$c(2m, m) \propto 1/m!, \qquad (22)$$

where this factorial comes directly from exponentiation. After the integration over the entire z-range, the power of the logarithm in x_z becomes a factorial multiplicative factor, $l!$. The presence of $l!$ follows directly from the existence of the powers of $\ln x_z$ that are present explicitly in the finite-order result in pQCD and is therefore inevitable. If this exercise is repeated, but with the range of integration in Eq. (21) constrained to our perturbative regime, one obtains the difference between the right-hand-side of Eq. (21) and a similar expression containing z_{max}. The result is numerically smaller, but both of the pieces are multiplied by $l!$.

The factorial coefficient $l!$ is not the most important source of enhancement. For the leading logarithms at two-loop order, $l = 2m = 4$, and the overall combinatorial coefficient from Eqs. (21) and (22) is $(2m)!/m! = 12$. For comparison, at representative values of η near threshold, $\eta = 0.1$ and 0.01, the sum of logarithmic terms in Eq. (21) provides factors of 16.1 and 314.3, respectively. Similarly, the (multiplicative) color factors at this order of perturbation theory are $(2C_{ij})^2 = 7.1$ and 36 for the $q\bar{q}$ and gg channels, respectively. All of these features are connected to the way threshold logarithmic contributions appear in finite-order pQCD and how they signal the presence of the non-perturbative regime. Thus, preoccupation with the $l!$ factor seems misplaced [8].

"Absence of factorial growth" is based on the use by CMNT of Eq. (16) for their main predictions, an expression that contains non-universal subleading logarithms, all with significant negative coefficients. Mathematically, factorial growth is present for each of the powers of the logarithm in Eq. (21), since these monomials are linearly independent. Absence of factorial growth based on a numerical cancellation between various classes of logarithms, most of them with physically unsubstantiated coefficients, appears to us to be an incorrect use of terminology. In the CMNT approach the effects of resummation are suppressed by a series of subleading logarithms with large negative coefficients. If there is no physical basis for preference of Eqs. (13) and (14) over Eqs. (17) and (18), as CMNT appear to suggest, then the difference in the resulting cross sections can be interpreted as a measure of theoretical uncertainty. This interpretation would not justify firm conclusions of a minimal 0.8% increment in the physical cross section based on the choice of Eqs. (13) and (14).

The CMNT value for the inclusive top quark cross section at $m = 175$ GeV and $\sqrt{S} = 1.8$ TeV, including theoretical uncertainty, lies within our uncertainty band. Therefore, the numerical differences between our results for top quark production at the Tevatron have little practical significance. However, there are important differences of principle in our treatment of subleading contributions that will have more significant consequences for predictions in other processes or at other values of top mass and/or at other energies, particularly in reactions dominated by gg subprocesses.

5 Discussion and Conclusions

The advantages of the perturbative resummation method [3] are that there are no arbitrary infrared cutoffs and there is a well-defined region of applicability where subleading logarithmic terms are suppressed. When evaluated for top quark production at $\sqrt{S} = 1.8$ TeV, our resummed cross sections are about 9% above the next-to-leading order cross sections computed with the same parton distributions. The renormalization/factorization scale dependence of our cross section is fairly flat, resulting in a $9 - 10\%$ theoretical uncertainty. Our perturbative boundary of 1.22 GeV above the threshold in the dominant $q\bar{q}$ channel is comparable to the hadronic width of the top quark, a natural definition of the perturbative boundary.

Our estimated theoretical uncertainty of $9 - 10\%$ is associated with μ variation. An entirely different procedure to estimate the overall theoretical uncertainty is to compare our enhancement of the cross section above the next-to-leading order value to that of CMNT [4], again yielding about 10%. An interesting question is whether theory can aspire to an accuracy of better than 10% for the calculation of the top quark cross section. To this end, a mastery of subleading logarithms would be desirable, perhaps requiring a formidable complete calculation at next-to-next-to-leading order of heavy quark production, to establish the possible pattern of subleading logarithms, and resummation of both leading and subleading logarithms. An analysis in moment space of the issues involved in resummation of next-to-leading logarithms for heavy quark production is presented by Kidonakis and Sterman [9]. Inversion of the resummed moments to the physically relevant momentum space requires considerable work. Full implementation of the resummation of next-to-leading logarithms would reduce the difference somewhat between our results and those of CMNT and move the debate to the level of next-to-next-to-leading logarithms.

Our prediction falls within the relatively large experimental uncertainties. If a cross section significantly different from ours is measured in future experiments at the Tevatron with greater statistical precision, we would look for explanations in effects beyond QCD perturbation theory. These explanations might include unexpectedly substantial non-perturbative effects or new production mechanisms. An examination of the distribution in η might be revealing.

The all-orders summation of large logarithmic terms, that are important in the near-threshold region of small values of the scaled partonic subenergy, $\eta \rightarrow 0$, was described here for the specific case of top quark production at the Fermilab Tevatron collider. Other processes for which threshold resummation will also be pertinent include the production of hadronic jets that carry large values of transverse momentum and the production of pairs of supersymmetric particles with large mass.

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