Microscopic Calculations of Λ Single Particle Energies

Q. N. Usmani[a*] and A. R. Bodmer[b*]

[a]Department of Physics, University Putra Malaysia, 3400 Selangor, Malaysia
and
Department of Physics, Jamia Millia Islamia, New Delhi 110 025, India

[b]Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

Λ binding energy data for total baryon number $A \leq 208$ and for Λ angular momenta $\ell_\Lambda \leq 3$ are analyzed in terms of phenomenological (but generally consistent with meson-exchange) ΛN and ΛNN potentials. The Fermi-Hypernetted-Chain technique is used to calculate the expectation values for the Λ binding to nuclear matter. Accurate effective ΛN and ΛNN potentials are obtained which are folded with the core nucleus nucleon densities to calculate the Λ single particle potential $U_\Lambda(r)$. We use a dispersive ΛNN potential but also include an explicit ρ dependence to allow for reduced repulsion in the surface, and the best fits have a large ρ dependence giving consistency with the variational Monte Carlo calculations for ${}^5_\Lambda He$. The exchange fraction of the ΛN space-exchange potential is found to be 0.2–0.3 corresponding to $m^*_\Lambda \simeq (0.74-0.82)m_\Lambda$. Charge symmetry breaking is found to be significant for heavy hypernuclei with a large neutron excess, with a strength consistent with that obtained from the $A = 4$ hypernuclei.

1. INTRODUCTION

We analyze the Λ single particle (sp) data in terms of semi-phenomenological ΛN and ΛNN potentials which are generally consistent with meson-exchange models, with the ΛN potential constrained by the low energy scattering data [1]. The experimental Λ sp data cover a wide range of hypernuclei [2] with baryon number $A \leq 208$ and orbital angular momentum $\ell_\Lambda \leq 3$. Among other approaches to the Λ data, ours is perhaps closest to that of Millener, Dover and Gal [3] who use a local density approximation based on phenomenological (zero-range) Skyrme forces which brings out important qualitative features. However, in the present work the effect of the fringing field (FF) due to the finite range of the potentials is included whereas in our earlier work [4] this was not treated adequately. We show that the FF plays an important role. We also include a ΛN charge symmetry breaking (CSB) component. For heavy hypernuclei (HN) with a substantial

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DISCLAIMER

neutron excess, the CSB contribution is significant and consistent with that obtained from the $A = 4$ HN [5].

Our approach is microscopic and based on calculating the Λ binding to nuclear matter $D(\rho, k_\Lambda)$ for a Λ with momentum k_Λ and for densities $\rho \leq \rho_0$ using the Fermi-Hypernetted-Chain (FHNC) approach [1,6]. The associated ΛN and ΛNN effective potentials are calculated in lowest order of the cluster expansion but with corrections from the higher-order clusters. These effective potentials are folded into the core-nucleus densities $\rho(r)$ to give the sp potential $U_\Lambda(r)$. The effective mass arises from the space exchange part of the ΛN potential which gives the k_Λ dependence. The B_Λ are then calculated using a sp Schrödinger equation. Core distortion effects are neglected as they were found to be small in earlier calculations [7]. Our procedure should provide an excellent approximation to an exact microscopic approach, thus providing a well-founded phenomenology for analysis of the sp data.

2. THE ΛN AND ΛNN POTENTIALS

We use the same central Urbana-type 2π ΛN potential as in Ref. [1], the attractive part being due to the strong tensor OPE component acting twice. The short-range part of the ΛN potential is parameterized with a repulsive Saxon-Wood potential. Thus, our charge symmetric ΛN potential is

$$V_{\Lambda N}(r) = V(r) + V^x_{\Lambda N}\,, \qquad V^x_{\Lambda N} = -\epsilon V(r)(1 - P_x)\,. \tag{1}$$

The direct potential is

$$V(r) = W_0\Big[1 + exp\Big\{(r - R)/a\Big\}\Big]^{-1} - V_{2\pi}\,, \qquad V_{2\pi} = V_0 T^2_\pi(r)\,, \tag{2}$$

where $W_0 = 2137$ MeV, $R = 0.5$ fm, $a = 0.2$ fm, and r is in fm. $T_\pi(r)$ is the OPE tensor potential shape modified with a cut off (Ref. [1] for details). $V_0 = 6.15 \pm 0.05$ MeV fits the low energy Λp scattering data. P_x is the space-exchange operator; ϵ determines the strength of $V^x_{\Lambda N}$ relative to the direct potential. The Λp forward to backward ratio scattering data determines ϵ quite poorly $\approx 0.1 - 0.4$ and we therefore treat ϵ as a free parameter.

The ground and excited states of ${}^{4}_{\Lambda}H$ and ${}^{4}_{\Lambda}He$ show charge symmetry breaking which is also expected on theoretical grounds. Earlier, we analyzed these data with a phenomenological CSB potential which we found to be effectively spin-independent [5]. Here we use a spin-independent CSB potential since in any case any spin dependence gives a negligible A^{-1} contribution for large A. Thus

$$V^{CSB}_{\Lambda N} = \tau_3\, V^{CSB}_0\, T^2_\pi(r)\,, \tag{3}$$

where $V^{CSB}_{\Lambda N} \approx -(0.5 \pm 0.005)$ MeV from $A = 4$. For heavy HN with an appreciable neutron excess CSB makes a relatively small but quite significant contribution.

A central ΛN potential fitted to low-energy scattering strongly overbinds heavy HN [1,8]. However the TPE ΛN potential could be strongly suppressed from propagator modifications by other nucleons. Such effects have been demonstrated in coupled-channel

reaction-matrix calculations [8–10] which can give large repulsive contributions. We represent such suppression effects by a phenomenological repulsive ΛNN potential

$$V_{\Lambda NN} \equiv V^D_{\Lambda NN} = W T^2_\pi(r_{\Lambda 1}) T^2_\pi(r_{\Lambda 2}) \,, \tag{4}$$

where $r_{\Lambda i}$ is the ΛN_i separation and where for $W > 0$ the contribution is repulsive.

Another (TPE) ΛNN potential $V^{2\pi}_{\Lambda NN}$ arises from the exchange of a pion between the Λ and each of two different nucleons and involves spin dependent and tensor contributions. Variational calculations using Monte Carlo (VMC) techniques for ${}^{5}_{\Lambda}He$ [1,11–13] and ${}^{16}_{\Lambda}O$ [14] show that the contribution of $V^{2\pi}_{\Lambda NN}$ is attractive, primarily because of the associated ΛNN correlations. The net contribution of $V^{2\pi}_{\Lambda NN} + V^D_{\Lambda NN}$ can then be mildly repulsive or even attractive for light HN. For heavier HN and for the Λ binding to nuclear matter at normal density $D(\rho_0)$ there are indications that this contribution will be repulsive. However for smaller ρ it may become less repulsive or even attractive. This feature was not present in our earlier analysis[4] due to inadequate treatment of the ΛNN correlations generated by $V^{2\pi}_{\Lambda NN}$. We therefore take a more phenomenological approach which allows the ΛNN potential to be more repulsive for at larger ρ or equivalently for larger A. We implement this by multiplying $V^D_{\Lambda NN}$ by a density dependent factor $F_\beta(\rho)$

$$\begin{aligned} F_\beta(\rho) &= \left[1 - exp\left(-\beta\rho^2/\rho_0^2\right)\right] \Big/ \left[1 - e^{-\beta}\right] \text{ for } \rho < \rho_0 \\ &= 1 \text{ for } \rho \geq \rho_0 \,. \end{aligned} \tag{5}$$

$F_\beta(\rho)$ makes the ΛNN contribution less repulsive for $\rho < \rho_0$ and thus gives effectively more repulsion for large A for which there is relatively less surface.

3. CALCULATION OF THE Λ SINGLE-PARTICLE ENERGIES

The B_Λ are calculated from a sp Schrödinger equation involving the sp potential $U_\Lambda(r)$ and the effective mass $m^*_\Lambda(r)$. The calculation of these in terms of the ΛN and ΛNN potentials is described below and is based on the local density approximation with $D(\rho, k_\Lambda)$ calculated variationally using the FHNC technique [1,6]. Thus for $A \to \infty$

$$-D(\rho, k_\Lambda) = \frac{\langle \Psi^{(A)} \, H^{(A)} | \Psi^{(A)} \rangle}{\langle \Psi^{(A)}, \Psi^{(A)} \rangle} - \frac{\langle \Psi^{(A-1)} | H_N^{(A-1)} | \Psi^{(A-1)} \rangle}{\langle \Psi^{(A-1)}, \Psi^{(A-1)} \rangle} \tag{6}$$

where $H^{(A)}, \Psi^{(A)}$ are the Hamiltonian and wave function of the HN and $H_N^{(A-1)}, \Psi^{(A-1)}$ those of the core nucleus. The variational wave functions are

$$\Psi^{(A)} = e^{i\vec{k}\cdot\vec{r}_\Lambda} F \Psi^{(A-1)} \tag{7}$$

where F contains the ΛN and ΛNN correlations

$$F = \left[\prod_{i=1}^{A-1} f_{\Lambda N}(r_{\Lambda i}) \prod_{i<j}^{A-1} f_{\Lambda NN}\left(\vec{r}_{\Lambda i}, \vec{r}_{\Lambda j}, \vec{r}_{ij}\right) \right] \tag{8}$$

The wave function of nuclear matter is

$$\Psi^{(A-1)} = \prod_{i<j}^{A-1} f_{NN}(r_{ij})\Phi^{(A-1)}(1,2\cdots A-1)\,, \tag{9}$$

where $\Phi^{(A-1)}$ is the uncorrelated Fermi gas wave function at density ρ. Details of the correlation factors $f_{\Lambda N}$, f_{NN}, $f_{\Lambda NN}$ and the calculational methods are given in Refs. 1 and 2. In the previous, as well as the present study, the effect of f_{NN} is negligible in lowering the energy its value being effectively unity.

$D(\rho, k_\Lambda)$ can be written as

$$D(\rho, k_\Lambda) = -\frac{\hbar^2 k_\Lambda^2}{2m_\Lambda} + D_0^{\Lambda N}(\rho) + D_x^{\Lambda N}(\rho, k_\Lambda) + D^{\Lambda NN}(\rho)\,, \tag{10}$$

where $D_0^{\Lambda N}$, $D_x^{\Lambda N}$ and $D^{\Lambda NN}$ are the direct ΛN, the exchange ΛN and ΛNN contributions.

$$D_0^{\Lambda N} = -\langle V\rangle + \langle T_{\Lambda N}\rangle = -\rho\int \tilde{V}_{\Lambda N}\, d\vec{r} \equiv -\rho t_0(\rho)\,. \tag{11}$$

The corresponding direct effective interaction is

$$\tilde{V}_{\Lambda N} = g_{\Lambda N}\left[V_{\Lambda N}(r) - \frac{\hbar^2}{4\mu_{\Lambda N}}\nabla_\Lambda^2\,\ell_n\, f_{\Lambda N}(r)\right], \tag{12}$$

where $g_{\Lambda N}$ is the ΛN distribution function obtained in the FHNC approximation and $\mu_{\Lambda N}$ is the ΛN reduced mass. For a Skyrme interaction (zero range potential) t_0 is independent of ρ, whereas for our ΛN potential $\langle T\rangle/\rho$ and $\langle V\rangle/\rho$ decrease slightly with ρ.

The ΛN space-exchange contribution is

$$D_x^{\Lambda N} = \rho\int V_{\Lambda N}^x\,(\epsilon, k_F, r\,)d\vec{r}, \tag{13}$$

where k_F is the Fermi momentum and where

$$V_{\Lambda N}^x\,(\epsilon, k_F, r\,) = \epsilon V_{\Lambda N}(r)\Big[g_{\Lambda N}(r)D_F(k_F r)j_0(k_\Lambda r) - g_{\Lambda N}\Big]\,. \tag{14}$$

Here $D_F = 3j_1(x)/x$ is the Slater function, and j_0 and j_1 are the zero and first order spherical Bessel functions. The distribution functions, $g_{\lambda d}$ and $g_{\Lambda N}$ *etc.* are defined in Refs. 1 and 6. For small k_Λ we retain terms only up to k_Λ^2, and then

$$V_\Lambda^x\,(\epsilon, k_F, r) = \tilde{V}_{\Lambda N}^x\,(k_F, r) - \epsilon\rho b k_\Lambda^2 \;\text{ with }\; b = \frac{1}{6}\,V_{\Lambda N}(r)\int g_{\Lambda N}(r)D_F(k_F r)r^2 d\vec{r}\,. \tag{15}$$

The corresponding effective ΛN exchange potential is

$$V_{\Lambda N}^x(k_F, r\,) = \epsilon V_{\Lambda N}(r)\Big[g_{\Lambda N}(r)D_F(k_F r) - g_{\Lambda N}\Big]\,. \tag{16}$$

The term proportional to k_Λ^2 in Eq. (15) is combined with the kinetic energy term of Eq. (10) to give

$$\frac{\hbar^2 k_\Lambda^2}{2m_\Lambda^*} = \frac{\hbar^2 k_\Lambda^2}{2m_\Lambda} - \epsilon\rho b\, k_\Lambda^2\,, \tag{17}$$

where m_Λ^* is the effective mass.

For the dispersive ΛNN potential of Eq. (4) we obtain

$$D^{\Lambda NN}(\rho) = \frac{1}{2} W\rho^2 \int T_\pi^2(r_{\Lambda 1}) T_\pi^2(r_{\Lambda 2}) g_3(r_{\Lambda 1}, r_{\Lambda 2}, r_{12}) d\vec{r}_1 d\vec{r}_2 d\vec{r}_\Lambda \tag{18}$$

where g_3 is the three-body distribution function defined in Refs. [1 and 6].

With the density modification of Eq. (5) the ΛNN contribution becomes

$$D_\beta^{\Lambda NN}(\rho) = F_\beta(\rho) D^{\Lambda NN}(\rho)\,. \tag{19}$$

The well depth D_Λ, its components , and m_Λ^* all at ρ_0 are shown in Table 3.

For the calculation of U_Λ the use of the various effective interactions is quite involved because of their density dependence. However for short-range correlations this density dependence is quite weak and the approximation

$$g_{\Lambda N}, g_{\Lambda d} \approx f_{\Lambda N}^2, g_{NN} \approx f_{NN}^2 \approx 1 \tag{20}$$

is very good. The effective interactions then become

$$\tilde{V}_{\Lambda N} = f_{\Lambda N}^2 \left[V_{\Lambda N}(r) - \frac{\hbar^2}{4\mu_{\Lambda N}} \nabla_\Lambda^2 \ell n f_{\Lambda N}(r) \right] \tag{21}$$

$$\tilde{V}_{\Lambda N}^{ex}(k_F, \vec{r}) = \epsilon f_{\Lambda N}^2 V_{\Lambda N}(r) \left[g_{\Lambda d} D_F(k_F r) - 1 \right] \tag{22}$$

$$\tilde{V}_{\Lambda NN} = W f_{\Lambda N}^2(r_{\Lambda 1}) f_{\Lambda N}^2(r_{\Lambda 2}) T_\pi^2(r_{\Lambda 1}) T_\pi^2(r_{\Lambda 2}) \tag{23}$$

and with the density modification of Eq. (15)

$$\tilde{V}_{\Lambda NN}^{(\beta)} = F_\beta \tilde{V}_{\Lambda NN}\,. \tag{24}$$

The difference between the approximate and "exact" (excluding the negligible elementary diagrams and with $F_\beta \equiv 1$) FHNC results increase with ρ but differ by only a few % even at ρ_0 (Ref. [15]). These effective interactions are then folded with the core density $\rho(r)$ to give

$$U_\Lambda(r) = U_\Lambda^d(r) + U_\Lambda^x(r) + U_\Lambda^{\Lambda NN}(r) \tag{25}$$

$$U_\Lambda^d(r) = (A-1) \int a_d(\rho) \tilde{V}_{\Lambda N}\left(|\vec{r} - \vec{r}\,'| \right) \rho(r') d\vec{r}\,' \tag{26}$$

$$U_\Lambda^x(r) = (A-1) \int a_x(\rho) \tilde{V}_{\Lambda N}^x\left(|\vec{r} - \vec{r}\,'| \right) \rho(r') d\vec{r}\,' \tag{27}$$

$$U_\Lambda^{\Lambda NN}(r) = \frac{1}{2}(A-1)(A-2) \int a_{\Lambda NN}(\rho) \tilde{V}_{\Lambda NN}^{(\beta)}\left(|\vec{r} - \vec{r}\,'|, |\vec{r} - \vec{r}\,''| \right) \rho(r') \rho(r'') d\vec{r}\,' d\vec{r}\,''\,. \tag{28}$$

The density dependent factors a_d *etc.* correct the approximate effective interactions based on Eq. 20 so as to reproduce the "exact" results for the components of $D(\rho)$. As expected these factors are close to unity.

4. RESULTS AND DISCUSSION

We use four parameters: V_0, ϵ, W, and β to fit the data with $V_0 = 6.15 \pm 0.05$ constrained by Λp scattering. (From here on all energies are in MeV.) The exchange fraction ϵ is allowed to vary freely. The parameter β in $F_\beta(\rho)$ is varied between 0.1 and ∞, where $\beta = \infty$ corresponds to $F_\beta \equiv 1$, *i.e.* to Eq. (4). The strength W of the ΛNN potential was allowed to vary freely between 0 and 0.04. The strength of the CSB potential was fixed at $V_0^{CSB} = -0.05$, the value obtained from the $A = 4$ HN. The charge densities were taken from Ref. [16] and the matter densities ρ were obtained by unfolding with the proton charge density.

Our fits are given in Table 1. To not give undue weight to the five very accurate B_Λ for the carbon HN we give each of these a nominal error of 0.5 MeV, comparable to those of intermediate and heavy HN. We calculate three different χ^2 per degree of freedom: (i) for all data points ($N = 24$); (ii) omitting the d state of ${}^{40}_{\Lambda}Ca$ ($N = 23$); and (iii) omitting the d state of ${}^{40}_{\Lambda}Ca$ and all five ${}_{\Lambda}C$ values ($N = 18$).

Table 1
Calculated B_Λ and χ^2_ℓ for selected fits. The top row gives V_0, W, β, ϵ for each fit. The B_Λ are shown for each ℓ value. The value in parentheses for ${}^{13}_{\Lambda}C$ is the χ^2 (per B_Λ) for all five ${}_{\Lambda}C$ states. The bottom row shows the χ^2_ℓ (per B_Λ), the value in parentheses is χ^2_d (per B_Λ) omitting ${}^{40}_{\Lambda}Ca(d)$. All energies in MeV.

	No FF	With FF	With FF	With FF
	6.2 .02 ∞ 0.35	6.2 .028 ∞ 0	6.2 .024 ∞ 0.11	6.2 .021 2 0.255
	s p d f	s p d f	s p d f	s p d f
${}^{11}_{\Lambda}C$	10.4	9.1	8.7	9.1
${}^{12}_{\Lambda}C$	10.9 0.4	9.7 1.0	9.4 0.5	9.5 0.3
${}^{13}_{\Lambda}C$	11.5 0.1 (0.6)	10.4 1.6 (4.6)	10.1 1.0 (5.6)	10.2 0.9 (4.1)
${}^{16}_{\Lambda}O$	13.3 1.9	12.1 3.4	11.9 2.6	12.3 3.5
${}^{28}_{\Lambda}Si$	16.9 6.9	15.8 9.4	16.0 8.7	16.1 7.4
${}^{32}_{\Lambda}S$	17.6 8.2	16.6 9.4	16.8 8.7	16.8 8.6
${}^{40}_{\Lambda}Ca$	19.0 10.3 1.4	18.2 11.3 4.0	18.6 10.7 3.0	18.9 10.8 2.8
${}^{51}_{\Lambda}B$	20.0 11.9 3.2	19.2 12.8 6.0	20.1 12.5 5.0	19.6 12.1 4.6
${}^{89}_{\Lambda}Y$	22.1 15.9 8.8 1.4	21.6 16.5 11.0 4.9	22.8 16.8 10.4 3.8	22.1 16.2 9.9 , 3.3
${}^{139}_{\Lambda}La$	23.8 18.9 13.0 6.5	23.8 19.5 14.6 9.5	25.2 20.1 14.6 8.7	24.8 19.7 14.1 18.2
${}^{208}_{\Lambda}Pb$	24.9 21.0 16.2 10.7	25.4 21.6 17.5 13.1	26.9 19.2 17.9 12.9	26.9 22.5 17.6 12.5
χ^2_e	2.2 0.9 0.6 (0.5) 1.13	2.5 1.7 14.0 (13) 4.9	3.2 0.6 5.8 (0.9) 1.7	2.2 0.4 4.4 (0.2) 0.6

With only a direct ΛN potential all the B_Λ are grossly overbound. Inclusion of ΛN space exchange still gives a very poor fit, and without the FF the fit is even worse. Also, even without the scattering constraints on V_0 no reasonable fits are possible. A ΛN potential alone is thus ruled out by the sp data.

To see the effect of the finite range of the ΛN and the ΛNN potentials, *i.e.* of the FF, we made calculations for $\epsilon = 0$ and $\beta = \infty$ which correspond to the ΛN potential without space-exchange and for the ΛNN potential without density modification. χ^2 for the best fits without FF ($V_0 = 6.2$, $W = 0.032$) and with FF ($V_0 = 6.2$, $W = 0.028$) are given in

Table 2. The dramatic improvement in χ^2 with a FF is quite striking and seems strong evidence for its reality.

Table 2
χ^2 (per B_Λ) for interactions with $V_0 = 6.2$ MeV. For each W (MeV) and β the optimum ϵ is shown. The three values of χ^2 are in decreasing order: χ^2 for all 24 B_Λ, omitting ${}^{40}_{\Lambda}Ca(d)$, and omitting ${}^{40}_{\Lambda}Ca(d)$ and all ${}_{\Lambda}C$. Values in parentheses are with a CSB potential $V_0^{CSB} = -0.05$ MeV.

		With FF	With FF	With FF	No FF	No FF
W		0.028	0.024	0.021	0.032	0.020
β						
0	$\epsilon =$	0.00	0.11	0.195	0.00	0.35
		3.82	2.52(217)	3.25	10.26	1.5(2.10)
		2.43	1.96(1.59)	3.14	9.22	1.54(2.16)
		1.84	0.94(0.64)	1.87	10.86	1.79(2.55)
4	$\epsilon =$	0.05	0.15	0.23		
		5.71	2.46	2.10		
		3.86	1.53	1.74		
		3.82	0.90	0.72		
2	$\epsilon =$	0.09	0.18	0.255		
		6.32	2.52	1.77(1.56)		
		4.36	1.45	1.30(1.07)		
		4.65	1.07	0.50(0.36)		
0.1	$\epsilon =$	0.16	0.24	0.305		
		6.31	2.67	1.66		
		4.54	1.61	1.22		
		5.13	1.42	0.58		

Our most comprehensive search was made for $V_0 = 6.2$ with FF. For $V_0 = 6.15$ and 6.10 less detailed searches were made. In Tables 1 and 2 only the results for $V_0 = 6.20$, the value preferred by the sp data, are given. The results for the other values of V_0 are similar; these results and those (for other values of the CSB strengths for $V_0^{CSB} = -0.1$ and $+.05$) will be presented elsewhere.

Tables 1 and 2 show, for each W and β, the optimum values of ϵ together with χ^2 for (i) all B_Λ, (ii) omitting ${}^{40}_{\Lambda}Ca(d)$, and (iii) omitting ΛC and ${}^{40}_{\Lambda}Ca(d)$. In Table 2 results including the CSB contribution are shown in parentheses for our best fits. The overall best fits with and without FF are quite good and are comparable in quality. However, omitting ${}_{\Lambda}C$ the fits with FF are better, and omitting the d state of ${}^{40}_{\Lambda}Ca$ makes them truly excellent. The implication of this will be discussed elsewhere. For the d state of ${}^{40}_{\Lambda}Ca$ all our fits with FF give $B_\Lambda \approx 3$ MeV as compared with the experimental $B_\Lambda \approx 1$ MeV, suggesting that the experimental B_Λ might be in error. Without the FF the B_Λ of the d state of ${}^{40}_{\Lambda}Ca$ is 1.4 MeV which is close to the experimental value. We emphasize

that the FF is demanded by the physics and that a fit without FF, however good, cannot be justified. Also, the best-fit values without FF are $W \approx 0.02$, $\epsilon \approx 0.35$ which are inconsistent with the results of VMC calculations [13] of ${}^{5}_{\Lambda}He$ which are discussed below. The fits without FF also worsen with inclusion of a CSB consistent with the $A = 4$ HN.

Fits can be somewhat comparable in quality for different V_0, ϵ, W, and β as considerable compensation may occur because decreasing V_0 or increasing ϵ, W, and β all correspond to more repulsion. Nevertheless our analysis very considerably restricts the parameters. Even without restriction to the preferred value $V_0 = 6.2$ but allowing the whole range 6.15 ± 0.05, we find $\epsilon \approx 0.25 \pm 0.05$, $W \approx 0.021$ and $\beta \approx 0.1$–2.0.

A new feature of our work is that $F_\beta(\rho) < 1$ is required with $\beta \approx 0.1$–2. This corresponds to a reduced repulsive ΛNN contribution from the surface. Since this is relatively more important for small A this implies an effectively less repulsive ΛNN potential for smaller A. This then is consistent with VMC calculations [13] of ${}^{5}_{\Lambda}He$ using a realistic Argonne v18 potential [17] which gives $B_\Lambda = 3.06 \pm .05$ for $V_0 = 6.2$, $\epsilon = 0.24$, $W = 0.01$ (experimental $B_\Lambda = 3.12 \pm .02$).

The CSB calculations are implemented through the change $V_0 \rightarrow V_0 + V_0^{CSB}$ in the charge symmetric ΛN potential, Eq. (2). To a good approximation the change in B_Λ due to CSB is then

$$\Delta B_\Lambda = V_0^{CSB} B'_\Lambda \left(\frac{N-Z}{A-1} \right); \quad B'_\Lambda = \frac{dB_\Lambda}{dV_0}. \tag{29}$$

Since ΔB_Λ is proportional to the neutron excess, we include CSB only for the four heaviest HN ("heavies"): ${}^{51}_{\Lambda}V$, ${}^{91}_{\Lambda}Y$, ${}^{139}_{\Lambda}La$, ${}^{208}_{\Lambda}Pb$. The derivatives B'_Λ, shown in Table 4, were obtained numerically and are almost the same for all our best-fit potentials. Also shown are the ΔB_Λ for $V_0^{CSB} = -0.05$, essentially the value obtained in Ref. [5] for $A = 4$. The parameters of the best fits without CSB are effectively unchanged if we omit the heavies. This implies that with CSB the fits including the heavies are not spuriously distorted. For each of the best fit interactions with FF, there is a definite preference for a negative V_0^{CSB} consistent with –0.05. The relative improvements in the χ^2 (with FF) are quite appreciable and significantly improve the overall fits. The resulting χ^2 are shown in parentheses in Table 2. The best fit without FF is worsened with a negative V_0^{CSB} since in this case the B_Λ for the heavies are already too small without CSB.

Table 3
Well depth, effective mass and related quantities. All energies in MeV, ρ in fm^{-3}.

Values for $\rho_0 = 0.165$ fm^{-3}										
V_0	W	β	ϵ	D_Λ	$D_0^{\Lambda N}$	$-D_x^{\Lambda N}$	$-D^{\Lambda NN}$	m^*_Λ/m_Λ	D_{max}	ρ_{max}
6.2	.02	2.0	.255	30.4	72.5	9.4	32.7	0.78	33.9	.124
6.2	.02	0.1	.305	28.4	72.5	11.2	32.9	0.74	36.5	.126
6.15	.02	0.1	.260	27.5	69.5	9.4	32.6	0.78	33.5	.126
6.1	.02	0.1	.200	28.7	66.5	7.1	30.8	0.82	35.4	.128

Table 4
CSB quantities. ΔB_Λ in MeV.

			B'_Λ				$\Delta B_\Lambda(V_0^{CSB}$ = -0.05)			
	$N-Z$	$\frac{N-Z}{A-1}$	s	p	d	f	s	p	d	f
${}^{51}_{\Lambda}V$	4	0.08	39	30	20		.16	.12	.08	
${}^{89}_{\Lambda}Y$	10	0.11	42	35	28	20	.24	.20	.16	.11
${}^{139}_{\Lambda}La$	24	0.17	46	40	35	28	.40	.35	.30	.24
${}^{208}_{\Lambda}Pb$	43	0.21	48	44	39	34	.50	.46	.41	.35

5. CONCLUSIONS

Our main results are as follows:

1. The FF due to the finite range of the ΛN and ΛNN potentials has a major effect.

2. The best fits are for $F_\beta < 1$ ($\beta \approx 0.01$–2) corresponding to a large ΛNN ρ dependence which translates into an A-dependent strength W which is nicely consistent with ≈0.01 MeV for ${}^{5}_{\Lambda}He$ and becomes ≈0.02 MeV for nuclear matter. This indicates an effective ΛNN dispersive potential which becomes increasingly repulsive for larger A and whose probable interpretation is in terms of a mixture of dispersive and two-pion exchange ΛNN potentials, the latter giving an attractive contribution for small A (as shown by VMC calculations) but a more repulsive contribution for large A because of the associated correlations.

3. A somewhat large ΛN strength $V_0 \simeq 6.2$ MeV is preferred but is consistent with Λp scattering.

4. $D(\rho)$ shows the characteristic "saturation" feature, which is a consequence of the repulsive ΛNN constribution, first noted by Millener *et al.* [3]. The maximum of $D(\rho)$ occurs at ≃0.125 fm^{-3} (Table 3).

5. The well depth for our best fits is $D_\Lambda \equiv D(\rho_0) \simeq 29 \pm 1$ MeV (Table 3).

6. The sp data restricts the space-exchange fraction to $\epsilon \simeq 0.25 \pm 0.05$, compared to the very poorly determined values 0.1–0.4 obtained from Λp scattering. Our corresponding effective mass is $m^*_\Lambda(\rho_0) \simeq (0.78 \pm 0.04) m_\Lambda$.

7. CSB effects are significant for heavy HN with a large neutron excess and are well consistent with the $A = 4$ HN.

Finally, we comment on the existing OBE potentials [18] and the associated (lowest-order) G-matrix calculations of the well depth [10]. All the potentials except ND (Nijmegen D) give either a very small attractive or small repulsive odd-state (*i.e.* p-wave) contribution $D_p \simeq$ –1.0 to 3.5 MeV. Only ND gives a significantly attractive odd-state contribution $D_p \simeq 8$ MeV comparable to that (≃ 7–12 MeV) obtained for our best-fit interactions. The "even-state" G-matrix contributions are to be compared with our values of $D_s + D^{\Lambda NN}$ (D_s is the s-state contribution to D) which are ≃ 16–21 MeV for our best-fit

interactions, whereas the G-matrix results are nearly all $\simeq$ 31–35 MeV. This suggests that these are missing about 10 MeV repulsion due to higher-order (3-body) contributions, in particular from $V^{2\pi}_{\Lambda NN}$. This would bring the G-matrix results for ND ($D_\Lambda = 40.5$ MeV) into good agreement with our best-fit potentials with FF. All the other OBE potentials, because of insufficiently attractive D_p, would give too small $D_\Lambda \simeq 20$ MeV. Our results thus show a definite preference for ND.

REFERENCES

1. A. R. Bodmer and Q. N. Usmani, Nucl. Phys. **A477** (1986) 621.
2. R. Chrien, Nucl. Phys. **A473** (1988) 705c; P. H. Pile *et al.*, Phys. Rev. Lett. **66** (1991) 2585; B. Povh, Prog. Part. Nucl. Phys. **5** (1980) 245; C. B. Dover and A. Gai, *ibid* **12** (1984) 171; T. Hasegawa *et al.*, Phys. Rev. **C53** (1996) 1210.
3. D. J. Millener, C. B. Dover, and A. Gal, Phys. Rev. **C38** (1988) 2700.
4. Q. N. Usmani and A. R. Bodmer, *Condensed Matter Theories*, eds. J. W. Clark, K. A. Shoaib and A. Sadig (Nova Science Publishers 1994) p. 395.
5. A. R. Bodmer and Q. N. Usmani, Phys. Rev. **C31** (1985) 1400.
6. Q. N. Usmani, Nucl. Phys. **A340** (1980) 397.
7. Q. N. Usmani and A. R. Bodmer, *Condensed Matter Theories*, eds. J. W. Clark, K. A. Shoaib and A. Sadig (Nova Science Publishers 1994) p. 381.
8. A. Gal, Advances in Nucl. Phys. **8** (1995) 1.
9. A. R. Bodmer and D. M. Rote, Nucl. Phys. **A169** (1971) 1; J. Dabrowski and W. Piechocki, Ann. Phys. **126** (1980) 317; W. Piechocki and J. Dabrowski, Ann. Phys. Polon. **B12** (1981) 475.
10. T. Motoba and Y. Yamamoto, Nucl. Phys. **A585** (1995) 29c; Y. Yamamoto and H. Bando, Prog. Theor. Phys. **73** (1985) 905; Y. Yamamoto, A. Reuber, H. Himeno, S. Nagata, and T. Motoba, Czech. J. Phys. **42** (1992) 1249; Jifa Hao, T. T. S. Kuo, A. Reuber, K. Holinde, J. Speth, and D. J. Millener, Phys. Rev. Lett. **71** (1993) 1498; Y. Yamamoto and H. Bando, Prog. Theor. **83** (1990) 254.
11. A. A. Usmani, Phys. Rev. **C52** (1995) 1773.
12. S. Murali, Ph. D. Thesis, Jamia Millia Islamia, New Delhi, India (1995).
13. Rita Sinha and Q. N. Usmani, private communication.
14. A. A. Usmani, S. C. Pieper, and Q. N. Usmani, Phys. Rev. **C51** (1995) 2347.
15. Q. N. Usmani and A. R. Bodmer, to be published.
16. H. D. Vries, C. W. deJager, and C. deVries, At. Data Nucl. Data Tables **37** (1987) 495.
17. R. B. Wiringa, Phys. Rev. **C43** (1991) 1585; R. B. Wiringa, V.J.V. Stoks, and R. Schiavilla, Phys. Rev. **C51** (1995) 38.
18. M. M. Nagels, T. A. Tijken, and J. J. deSwart, Phys. Rev. **D15** (1977) 2545; **20** (1979) 1633; P. M. Maessen, Th. A. Rijken, and J. J. deSwart, Phys. Rev. **C40** (1989) 2226; R. Buttgen, K. Holinde, B. Holzenkamp, and J. Speth, Nucl. Phys. **A450** (1986) 403c; A. Reuber *et al.*, Czech. J. Phys. **42** (1992) 1115; J. Speth, *The Bonn-Julich Meson-Exchange Interactions*, Proc. Hypernuclear Workshop, Jefferson Lab (1995), p. 5.