

The SMAC Modal Parameter Extraction Package

Randall L. Mayes and Scott E. Klenke
 Experimental Structural Dynamics Department
 Sandia National Laboratories
 Albuquerque, New Mexico 87185-0557

ABSTRACT

The objective of many modal testing programs is to identify modal parameters of complex systems for model validation purposes. Often this objective is difficult to achieve in an efficient manner due to the complexity of the system being tested. Traditional modal extraction techniques perform well under certain conditions; however, when moderately damped modes and/or high modal density is characteristic of the system response, accurate identification of the modal parameters is challenging. A modal extraction package, called "Synthesize Modes and Correlate" or SMAC, has been developed to address these concerns. This extraction technique calculates reciprocal modal vectors based on frequency response function (FRF) measurements. The FRFs are multiplied by a reciprocal modal vector to synthesize a single degree-of-freedom (SDOF) FRF. The correlation coefficient compares the synthesized SDOF FRF against an analytical SDOF FRF. Frequency and damping values are optimized to obtain the maximum correlation coefficient, which corresponds to the true system root. A quadrature fit algorithm is then used to generate shapes based on the SMAC modal parameters. Examples are provided utilizing the SMAC package on experimental data from a moderately damped system and one with high-modal density application.

NOMENCLATURE

FRF: frequency response function
 SDOF: single degree-of-freedom
 ω_r : natural frequency of r^{th} mode
 ω_j : j^{th} frequency line of FRF
 ζ_r : damping coefficient of r^{th} mode
 Φ : mode shape matrix
 ψ : reciprocal modal (weighting) vector
 $H_p(\omega)$: analytical generalized coordinate FRF

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U.S. Department of Energy under Contract DE-AC04-94AL85000.

A_r : modal residue coefficient
 j : $\sqrt{-1}$

INTRODUCTION AND MOTIVATION

At Sandia National Laboratories there are several model validation programs. There is increasing desire to be able to extract modes over higher frequency bands than in the past, and one program requires the extraction of modal parameters for moderately damped (up to 5 percent damping) systems.

Traditional approaches to modal parameter extraction sometimes break down when the damping gets high and/or the modal density increases. Also when an analyst is attempting to extract a difficult mode, the process is often nonconverging. That is, if the analyst varies the parameters he provides to the extraction algorithm -e.g., frequency band, number of roots, particular set of sensor data, etc.- the root extracted may or may not be a better extraction than the last attempt.

The "Synthesize Modes and Correlate" (SMAC) algorithm was first presented at the 1998 IMAC^[1]. SMAC is very different from other modal extraction algorithms as it is based on modal filtering rather than an assumed representation for the time or frequency domain data. Since that time "user in the loop" optimization code has been written which has proven effective in providing a converging solution to a particular root. The SMAC algorithm has proven to be effective for large channel count tests (over 100 channels) in extracting the roots for moderately damped, weakly excited, and modally dense data. In its current implementation for a single reference, it cannot separate modes with perfectly repeated roots, but it has shown the capability of extracting roots at almost the same frequency but with significant damping differences -e.g., 236.5 Hz-0.24 percent damping and 236 Hz-5 percent damping. Traditional fitters often can obtain the low damping root, but cannot extract the higher damping root with almost the same frequency.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

After the basic theory for SMAC is presented below, two applications of the algorithm with real hardware will be presented. The first moderate-damping application is a system with approximately 50 modes in the bandwidth with up to 5 percent damping. The second application has at least 70 modes in the bandwidth, but damping is always below 3 percent. In addition to the improved implementation of the SMAC root finder, coding has been written to extract the mode shapes based on quadrature fit, and this is described herein.

THEORY

The SMAC algorithm is based upon the modal filtering approach rather than an assumed matrix polynomial or FRF form. This means that there must be at least as many accelerometers as there are active modes in the structure. They should be placed so that the associated experimental mode shape matrix is well conditioned for inversion. Since the algorithm is not based on a matrix polynomial, there are no computational roots, eliminating one set of decisions the analyst must make in deciding on the true system roots.

Meirovitch^[2] is credited with the original development of modal filters, but Shelley^[3] revived and advanced the uses in the early '90's. Reciprocal modal vectors, which are simply vectors of the modal filter matrix, have been extracted to calculate rigid body forces at Sandia National Labs by several researchers^[4,5,6,7]. It has also been shown that reciprocal modal vectors can be generated from a finite element model (FEM) to produce enhanced frequency response functions (FRFs) from experimental data^[8].

The SMAC theory^[1] is repeated here for clarity. A modal filter is a matrix of weighting vectors, sometimes called reciprocal modal vectors, that transforms system response parameters from physical coordinates into modal coordinates. Each reciprocal modal vector transforms the physical coordinates into one associated modal coordinate. The concept is extended to FRFs in the following way. Beginning with the modal substitution

$$[\Phi]\{\ddot{p}\} = \{\ddot{x}\}, \quad (1)$$

where p is the vector of modal coordinates, Φ is the mode shape matrix, and x is the vector of physical coordinates. To obtain the modal coordinates alone,

$$\{\ddot{p}\} = [\Phi]^+ \{\ddot{x}\} \quad (2)$$

and then transposing,

$$\{\ddot{p}\}^T = \{\ddot{x}\}^T [\Phi]^+{}^T. \quad (3)$$

If one defines $\{\Psi\}$ as the reciprocal modal vector that is one of the columns of $[\Phi]^+{}^T$ then transforms Equation (3) into the frequency domain and divides by a single input force, one obtains

$$\{H_p\} = [H_x]\{\Psi\}, \quad (4)$$

where $\{H_p\}$ is a SDOF FRF, $[H_x]$ is a set of experimental FRFs from a single input, and $\{\Psi\}$ is the reciprocal modal vector. The matrices in Equation (4) are expanded as

$$\{H_p\} = \left\{ \begin{array}{c} \frac{-A_r \omega_1^2}{\omega r^2 + j2\zeta_r \omega r \omega_1 - \omega_1^2} \\ \frac{-A_r \omega_2^2}{\omega r^2 + j2\zeta_r \omega r \omega_2 - \omega_2^2} \\ \vdots \\ \frac{-A_r \omega_{NS}^2}{\omega r^2 + j2\zeta_r \omega r \omega_{NS} - \omega_{NS}^2} \end{array} \right\}$$

$$[H_x] = \begin{bmatrix} H_1(\omega_1) & H_2(\omega_1) & \dots & H_{NO}(\omega_1) \\ H_1(\omega_2) & H_2(\omega_2) & \dots & H_{NO}(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ H_1(\omega_{NS}) & H_2(\omega_{NS}) & \dots & H_{NO}(\omega_{NS}) \end{bmatrix}$$

$$\{\Psi\} = \left\{ \begin{array}{c} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_{NO} \end{array} \right\}. \quad (5)$$

NS is the number of spectral lines, NO is the number of measurement locations in the experimental accelerance FRF matrix, ωr is the modal frequency, ω_j is the frequency at line j , ζ_r is the viscous damping ratio, A_r is the residue coefficient for the mode under consideration, and Ψ_i is a scalar.

In traditional modal filtering both the experimental response and weighting vector are known, and one solves for the uncoupled modal response. The proposed technique requires knowledge only of the experimental data and synthesis of an assumed SDOF analytical FRF, $\{H_p\}$ (here the assumption is based on real modes). By assuming a natural frequency and damping coefficient, $\{H_p\}$ is created in Equation (5). The residue coefficient is arbitrarily set to unity. This FRF is multiplied by the pseudo-inverse of the experimental data to create a least-squares weighting vector

-i.e., the calculated reciprocal modal vector, or mathematically

$$\{\Psi\} = [H_x]^+ \{H_p\} \quad (6)$$

If the assumed modal frequency and damping correspond to an active mode of the system, the weighting vector will be a true reciprocal vector. The vector will be orthogonal to all modes except for the one with frequency ω_r , and a SDOF FRF can be synthesized from the right-hand side of Equation (4) (see the first plot of Figure 1). However, if the assumed modal frequency is not a mode of the system, the weighted sum will not be able to create a reciprocal vector, and consequently will provide a poor synthesis of the analytical SDOF FRF (see the second plot in Figure 1).

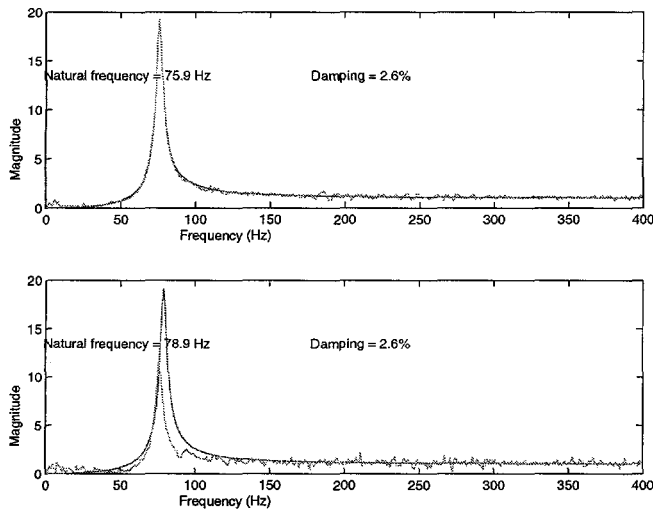


Figure 1 - Comparison of a Good and Poor Reconstruction with the Assumed Frequency and Damping (Smooth line is analytical SDOF FRF and jagged line is weighted sum of experimental FRFs)

Performance of the reconstruction attempts is monitored by measuring the correlation coefficient^[9] between the analytical FRF and the weighted sum of experimental FRFs. This correlation is focused on the region around the peak to enhance identification of the damping coefficient because the strongest effects of damping are seen near resonance. If the correlation coefficient between the analytical and experimental FRFs is a maximum near 1, then the assumed natural frequency and damping are taken to be a root of the system. This is an iterative approach where after one pair of frequency and damping values is evaluated, a new pair is assumed. In Figure 2, a typical correlation coefficient surface plot is shown, indicating that some optimization approach could be used to ascend to the local maxima and determine the associated frequency and damping. Notice that the indication of roots along the frequency axis is strong,

whereas the indication of roots along the damping axis is much less precise. This is consistent with the community experience that it is easier to identify the frequency accurately than the damping. In the next section it is shown that the appropriate number of frequency lines used in the correlation coefficient calculation can improve the roll-off of the surface in the damping direction, making the maxima more pronounced.

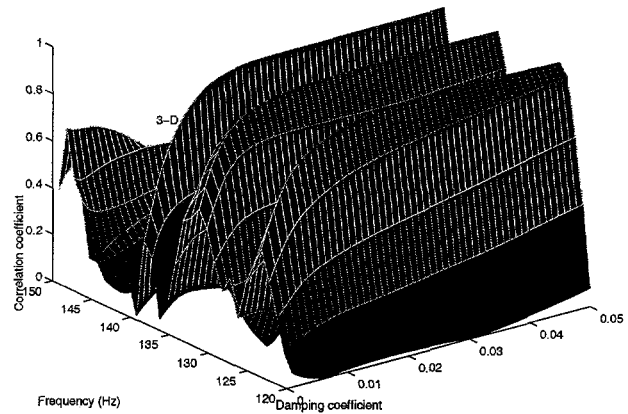


Figure 2 - 3-D Plot of Correlation Coefficient

IMPLEMENTATION

The algorithm was implemented with an assumption of real modes -i.e., the residue for H_p in Equation (5) is a real number. The solution of Equation (6) was constrained to obtain real valued weights, which decreases the solution time. The approach to obtain the roots has been improved over the presentation in the original paper^[1]. A classical mode indicator function (MIF), such as the normal MIF, power MIF or complex MIF, is used to determine a bandwidth of modal activity to investigate. Then a reasonable damping value is assumed, and the correlation coefficient between the vectors on each side of Equation (4) is calculated and plotted at each frequency line in the bandwidth under consideration. The peaks in the correlation coefficient are indications of the modes in the bandwidth as in Figure 3. In our experience only correlation coefficient peaks above 0.9 are worthy of investigation. The correlation coefficient plot can be made for any frequency resolution.

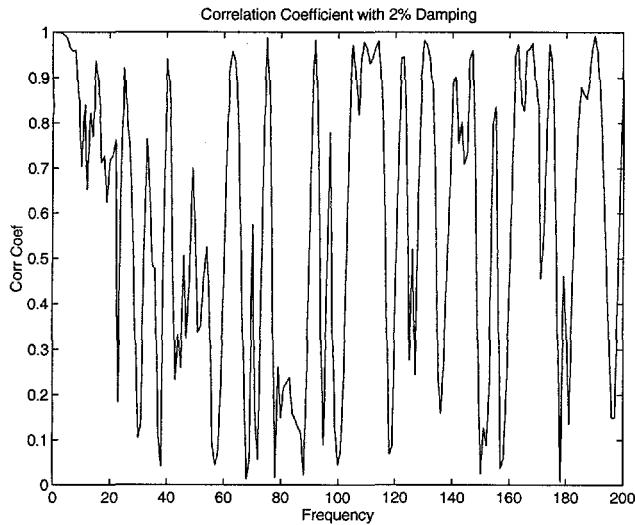


Figure 3 - SMAC Correlation Coefficient Plot

The analyst then focuses on a particular root to extract in the bandwidth. Two routines, which operate in the same manner, are used to converge on the root. In the first routine a narrow frequency band around the suspected root is selected. The correlation coefficient is calculated for an assumed damping at five equally spaced points. Then a parabola is fit to the 5 points, and the frequency of the maximum point of the parabola is calculated. This becomes the first estimate of the natural frequency as shown in Figure 4.

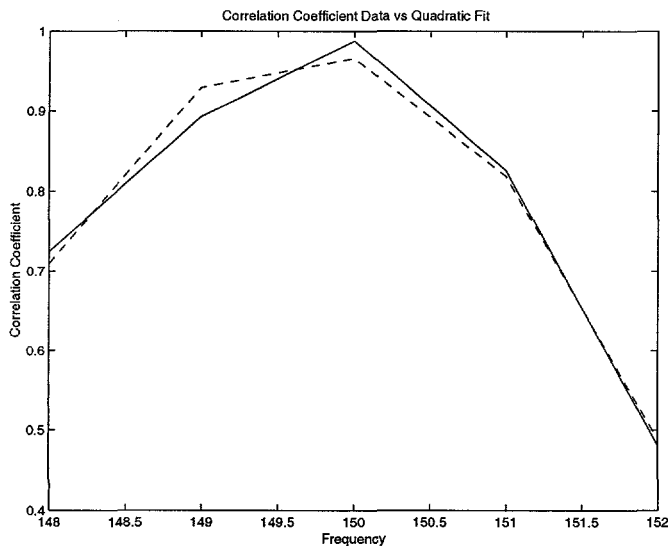


Figure 4 - Quadratic Fit (Dashed) to Correlation Coefficient (Solid) versus Frequency (Estimated 2 Percent Damping)

Then the second routine is executed, which is exactly like the first except the abscissa is now a range of damping ratios, and the frequency is assumed to be the estimate just calculated above. A best estimate of damping is then obtained as shown in Figure 5. The analyst oscillates back and forth between these two routines until he is satisfied with

the convergence upon the root. In general, converged roots will have a correlation coefficient above .99 for active modes of the test. Then the root is recorded. The other roots are obtained similarly.

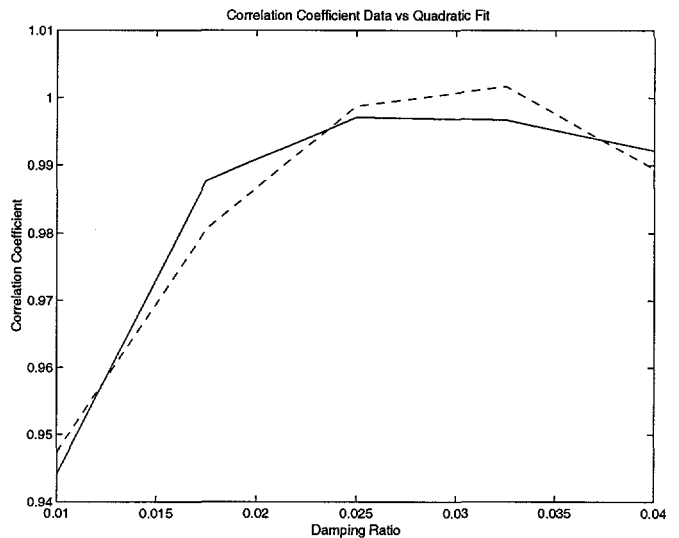


Figure 5 - Quadratic Fit (Dashed) to Correlation Coefficient (Solid) versus Damping (Estimated Frequency 149.7 Hz)

One improvement to be noted from the original implementation⁽¹⁾ is that there are an optimum number of frequency lines over which the correlation coefficient should be calculated. Typically, we used 11 lines in the original paper, but it was found that 41 lines were better for the data that will be reported here. The best way to determine the number of lines is to get an estimate of a root, and plot the correlation coefficient as a function of damping for the estimated frequency. The optimum number of lines will result in a high correlation coefficient at the true root and a rapid monotonic roll-off of the correlation coefficient at damping values away from the true root, Figure 6.

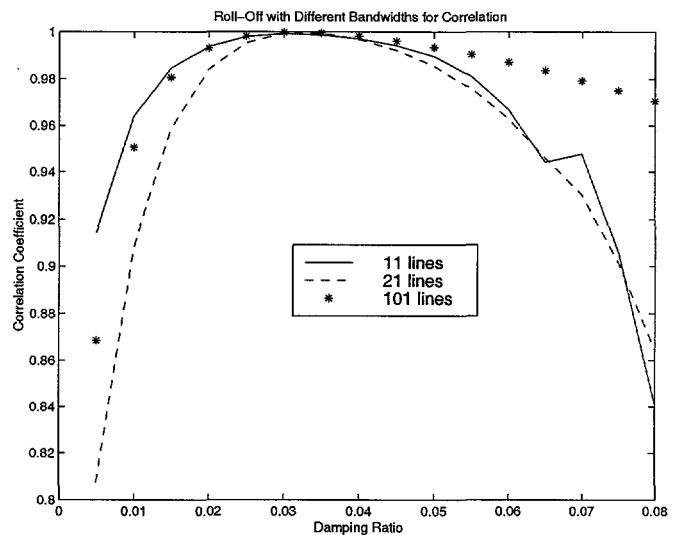


Figure 6 - Correlation Coefficient Rolloff versus Damping

Too few lines can result in discontinuities as seen for the 11-line plot. Once this optimum is found for a mode near the center of the frequency bandwidth, it is generally good for the entire data analysis. Adding a few extra lines is good insurance against discontinuities and does not significantly impact the roll-off.

The implementation of SMAC extracts one root at a time. For modes that are otherwise difficult to extract, the implementation provides a converging process. With commercial extraction algorithms one never knows whether the next attempt at the extraction of a difficult mode will be any better than the last attempt. SMAC convergence on strong well-separated modes takes very few iterations. Closely spaced modes take more iteration, but it is clear when the point of diminishing returns has been reached. The modal density can get so high that the optimization based on the correlation coefficient surface is not effective. A flat plot with very high correlation coefficient characterizes this and SMAC will fail in such a bandwidth.

THEORY OF THE MODE SHAPE FITTER

A new addition to the SMAC extraction package is the mode shape-fitting code. This algorithm is based on a quadrature fit of the data and utilizes the IMAT™ software to generate a shape universal file^[10] for each mode of interest. Previously, a technique^[11] was used that assumed that the FRF could be represented with real modes. The current mode shape algorithm extends this approach to a least squares fit of the data. To establish the matrix for the fit, four frequency lines are selected near each resonant frequency estimated by SMAC. By eliminating all the frequency lines except those closest to each estimated root, errors in the residue calculations from nearby modes that are not fit can be minimized. The residue amplitudes are the solution to the following problem with $4n$ equations and n unknowns:

$$\text{Imag} \begin{Bmatrix} \mathbf{H}_k(\omega_1) \\ \mathbf{H}_k(\omega_2) \\ \vdots \\ \mathbf{H}_k(\omega_{4n}) \end{Bmatrix} = \quad (7)$$

$$\text{Imag} \begin{Bmatrix} \mathbf{H}_{\text{ker}}^1(\omega_1) & \mathbf{H}_{\text{ker}}^2(\omega_1) & \cdots & \mathbf{H}_{\text{ker}}^n(\omega_1) \\ \mathbf{H}_{\text{ker}}^1(\omega_2) & \mathbf{H}_{\text{ker}}^2(\omega_2) & \cdots & \mathbf{H}_{\text{ker}}^n(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{\text{ker}}^1(\omega_{4n}) & \mathbf{H}_{\text{ker}}^2(\omega_{4n}) & \cdots & \mathbf{H}_{\text{ker}}^n(\omega_{4n}) \end{Bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_n \end{Bmatrix}$$

where the kernel function is given as

$$\{\mathbf{H}_{\text{ker}}^i\} = \left\{ \frac{-\omega_j^2}{\omega r^2 + j2\zeta_r \omega r \omega_j - \omega_j^2} \right\}, \quad (8)$$

and ω_j represents the chosen frequency lines of the FRF. Each FRF is then synthesized from the calculated residue coefficients and the kernel function over a prescribed frequency band

$$\{\mathbf{H}_{\text{syn}}\} = \begin{Bmatrix} \mathbf{H}_{\text{ker}}^1(\omega_1) & \mathbf{H}_{\text{ker}}^2(\omega_1) & \cdots & \mathbf{H}_{\text{ker}}^n(\omega_1) \\ \mathbf{H}_{\text{ker}}^1(\omega_2) & \mathbf{H}_{\text{ker}}^2(\omega_2) & \cdots & \mathbf{H}_{\text{ker}}^n(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{\text{ker}}^1(\omega_{NS}) & \mathbf{H}_{\text{ker}}^2(\omega_{NS}) & \cdots & \mathbf{H}_{\text{ker}}^n(\omega_{NS}) \end{Bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_n \end{Bmatrix}. \quad (9)$$

Comparisons of the synthesized and actual FRF data can be analyzed before the shape is produced. A search is completed on the data set to determine if a driving-point exists. If a true driving-point does not exist, a pseudo driving-point reference coordinate is selected from the available data set. A driving-point residue coefficient is then used to normalize the mode shape. The output of the code is a set of mode shape vectors with a modal mass equal to one. A typical comparison is shown in Figure 7. The imaginary part of an FRF measurement (solid line) is overlaid with the synthesized FRF (dashed line). Most of the bandwidth is fit very well in this particular example.

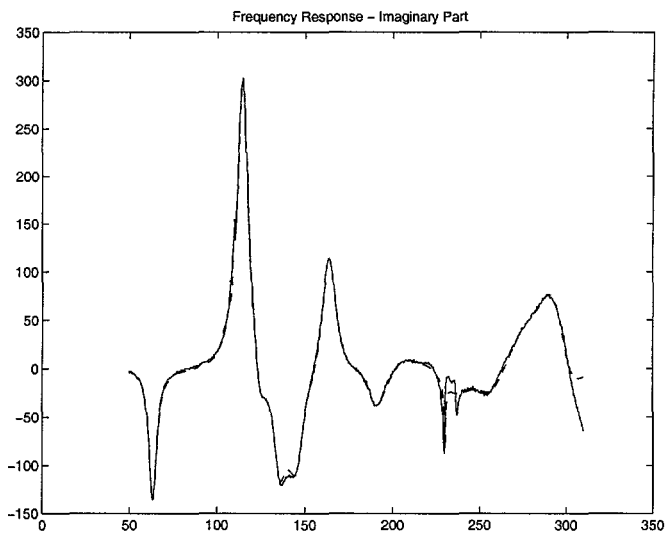


Figure 7 - Imaginary Part of Y FRF Compared with Synthesis

APPLICATION 1 - MODERATE DAMPING

In the first SMAC application, a FEM was used to design the excitation^[12] for this experiment. The goal was to excite and observe a certain class of modes of the structure. Specifically there were 11 modes of interest for this excitation but over 50 modes in the bandwidth. There were 124 accelerometers recorded. All of the modes of interest were extracted by the SMAC package, and the damping values ranged between 3 and 5 percent. However, some modes of the support structure had damping as low as 0.25 percent. Even though there were only 11 modes of interest

from this excitation set, SMAC identified parameters for 29 modes, several of which were local or weakly excited but had some effect on the FRFs. In order to assess the integrity of the modal fit using SMAC, a normal mode indicator function (NMIF) comparison was made between the measured FRF data and the synthesized FRFs. Figure 8 shows this comparison. This figure also includes a curve that results from removing the driving point FRF data from the NMIF calculation. These curves give an overall view of the modal fit and indicate very good agreement at the lower frequencies. At the higher frequencies the NMIF from the synthesized data compares more closely with the NMIF with the driving point data removed. This indicates that the driving-point FRF has a larger real part at the higher frequencies and can effect the NMIF calculation adversely.

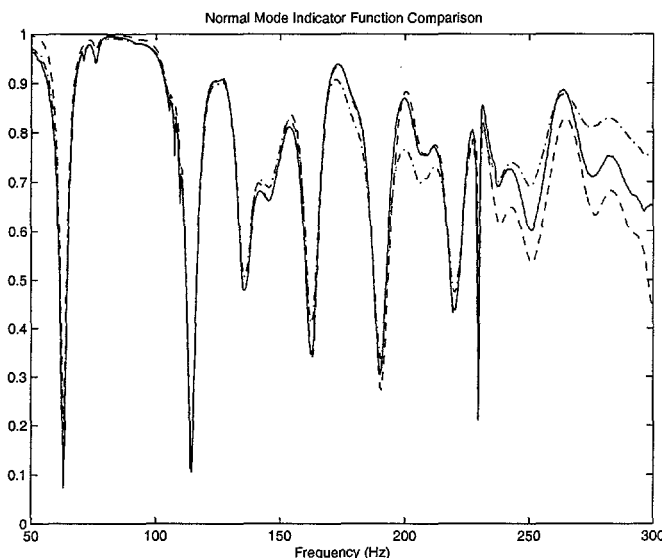


Figure 8 – Normal Mode Indicator Function Comparison
All Measured FRFs (Dot-Dash), Measured FRFs Without Driving Point (Solid) and Synthesized FRFs (Dashed)

APPLICATION 2 – HIGH MODAL DENSITY

In the second SMAC application, there were more 70 modes evident in the bandwidth based on the Multivariate Mode Indicator Function calculated for the test data set. In this example no FEM was available to aid in the test design, and 105 accelerometers were placed on the structure based on engineering judgment. The damping values calculated from this experiment were between 0.5 and 2.5 percent for almost all modes. Comparing NMIFs from the measured FRFs and synthesized FRFs assessed the integrity of the fit for this high modal density data. This comparison gives a very good overall assessment of the modal fit to a frequency of 1000 Hz. Figure 9 shows the NMIFs with the driving-point data removed.

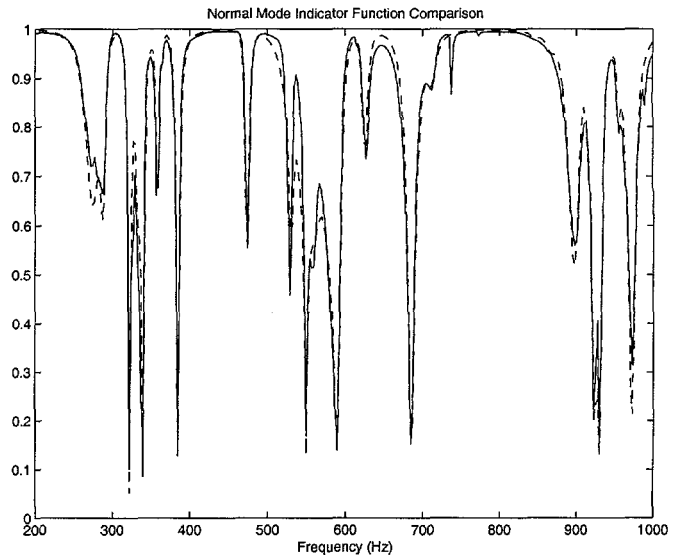


Figure 9 – Normal Mode Indicator Function Comparison
Measured FRFs Without Driving Point (Solid) and Synthesized FRFs (Dashed)

UNCERTAINTY

All accelerometer sensitivities were calibrated at the Sandia National Laboratories primary standards lab and are traceable to NIST with a 99 percent confidence interval of plus or minus 2.5 percent for a 100 Hz sine wave. The signal conditioning and data acquisition system were recently calibrated, and all channels were within 2 percent of the RMS value of a calibrated sine wave source at 100 Hz.

CONCLUSIONS

The SMAC algorithm has been shown to be extremely robust in extracting not only well excited but also weakly excited modes with moderate or low damping from test data. Since the SMAC calculations can be made for frequencies between adjacent frequency lines of the FRFs, the code is not limited to the resolution of the FRF measurement. The SMAC root finder; in conjunction with the mode shape fitter, provides a good comparison of synthesized FRFs with the actual test data. There are no computational roots generated, so all efforts can be directed toward the true system roots.

A disadvantage is that it is not easy to distinguish well-excited modes from weakly excited modes with the correlation coefficient. For this reason it is of value to use SMAC in conjunction with accepted mode indicator functions to investigate the frequency ranges of interest.

The technique converges on roots with the currently implemented "user in the loop" optimization, as opposed to commercial fitters that provide an answer which may or may not be satisfactory during multiple parameter estimation attempts. SMAC cannot separate perfectly repeated roots but has been shown to separate very closely spaced roots in

frequency with large differences in damping. Since the algorithm is based on modal filtering, a reasonable number and distribution of sensors is required. Future enhancements to the SMAC extraction package will include calculation of complex modes and the extension to multiple input, multiple output compatibility.

REFERENCES

- [1] Mayes, Randall L. and Johansen, David D., "A Modal Parameter Extraction Algorithm Using Best-Fit Reciprocal Vectors", *Proceedings of the 16th International Modal Analysis Conference*, pp517-521, February 1998.
- [2] Meirovitch, L. and Baruh, H., "The Implementation of Modal Filters for Control of Structures", *Journal of Guidance, Control and Dynamics*, Vol. 8, No. 6, pp707-716, November-December 1985.
- [3] Shelley, S.J. and Allemang, R.J., "Calculation of Discrete Modal Filters Using the Modified Reciprocal Modal Vector Method", *Proceedings of the 10th International Modal Analysis Conference*, pp37-45, 1992.
- [4] Bateman, Vesta I., Mayes, Randall L. and Carne Thomas G., "Comparison of Force Reconstruction Methods for a Lumped Mass Beam", *Journal of Shock and Vibration*, Vol. 4, No. 4, pp231-239, 1997.
- [5] Mayes, R.L., "Measurement of Lateral Launch Loads on Re-Entry Vehicles Using SWAT", *Proceedings of the 12th International Modal Analysis Conference*, pp1063-1068, 1994.
- [6] Carne, Thomas.G., Mayes, Randy. L. and Bateman, Vesta I., "Force Reconstruction Using the Sum of Weighted Accelerations Technique - Max-Flat Procedure", *Proceedings of the 12th International Modal Analysis Conference*, pp1054-1062, 1994.
- [7] Gregory, D.L., Priddy, T.G. and Smallwood, D.O., "Experimental Determination of the Dynamic Forces Action on Non-Rigid Bodies," SAE Technical Paper Series, Paper No. 861791, Aerospace Technology Conference and Exposition, Long Beach, CA, October 1986.
- [8] Mayes, R.L. and Carne, T.G., "Extraction of Modal Parameters with the Aid of Predicted Analytical Mode Shapes", *Proceedings of the 14th International Modal Analysis Conference*, pp267-272, 1996.
- [9] Thomas, L.F. and Young, J.I., *An Introduction to Educational Statistics: The Essential Elements, Third Edition*, pp77, 1993.
- [10] *User Manual for Modal Analysis 9.0*, General Electric CAE International, 1985, pp. E3-9.
- [11] Mayes, Randall L., "A Multi-Degree-of-Freedom Mode Shape Estimation Algorithm Using Quadrature Response", *Proceedings of the 11th International Modal Analysis Conference*, pp1026-1034, 1993.
- [12] Carne, T.G. and Dohrmann, C.R., "A Modal Test Design Strategy for Model Correlation," *Proceedings of the 13th International Modal Analysis Conference*, pp927-933, 1995.