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Final Report:B595949 - Fast Solvers for Discrete Hodge Laplacians

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Fast Solvers for Discrete Hodge Laplacians

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The focus of this research proposal was on multilevel algorithms for the solution of discrete Hodge Laplacians and in particular on the construction of coarse spaces and fast solvers for graph Laplacians and related problems.

The subcontractor worked on polynomial smoothers for symmetric positive definite problems [KVZ12]. In this research a defect correction scheme for constructing the sequence of polynomials of best approximation in the uniform norm to 1/x on a finite interval with positive endpoints was derived. As an application, the two-level methods for scalar elliptic partial differential equation (PDE), where the relaxation on the fine grid uses the aforementioned polynomial of best approximation was considered. Based on a new smoothing property of this polynomial smoother that the Subcontractor proved and a proper choice of the coarse spaces, we obtain as a corollary, that the convergence rate of the resulting two-level method is uniform with respect to the parameters, coarsening ratio and variations in the matrix entries for special class of matrices.

In [VZ12] the subcontractor constructed a coarser version of the mixed graph Laplacian operator with the purpose to construct two-level, and by recursion, a multilevel hierarchy of graphs and associated operators..

The subcontractor has also worked on a class of multilevel preconditioners [KLM12] based on approximate block factorization for conforming finite element methods (FEM) employing quadratic trial and test functions. The Subcontractor considers diffusion problems governed by a scalar elliptic partial differential equation (PDE) with a strongly anisotropic coefficient tensor. The proposed method provides a high robustness with respect to non-grid-aligned anisotropy, which is achieved by the interaction of the following components: (i) an additive Schur complement approximation to construct the coarse-grid operator; (ii) a global block (Jacobi or Gauss-Seidel) smoother complementing the coarse-grid correction based on (i); and (iii) utilization of an augmented coarse grid, which enhances the efficiency of the interplay between (i) and (ii); The performed analysis indicates the high robustness of the resulting two-level method.

REFERENCES

[KVZ12] Johannes Kraus · Panayot Vassilevski · Ludmil Zikatanov Polynomial of Best Uniform Approximation to 1/x and Smoothing in Two-level Methods. *Computational Methods in Applied Mathematics*. Vol. 12 (2012), No. 4, pp. 448–468   2012

[VZ12] Panayot S. Vassilevski and Ludmil Zikatanov. Commuting projections on graphs. To appear in *Numerical Linear Algebra with Applications*, 2013.

[KLM13] Johannes Kraus, Maria Lymbery, Svetozar Margenov. Robust multilevel methods for quadratic finite element anisotropic elliptic problems. To appear in *Numerical Linear Algebra with Applications*, 2013.