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Title:	Parametric Model of Debris Penetration Through Sump Strainers with Concurrent Filtration and Shedding
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Parametric Model of Debris Penetration Through Sump Strainers with Concurrent Filtration and Shedding

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Recirculation Theory

Two primary debris fates are of key interest to the risk-informed resolution analysis: (1) accumulation on the primary sump strainers with corresponding head loss, and (2) partial penetration of debris through the strainers with subsequent accumulation on fuel channel screens in the lower plenum. Accumulation on the core can impede flow, potentially causing local excessive heating and potentially limiting mixing to the point that boron precipitation can occur. Two opportunities exist for diverting debris away from the core: (1) a fraction of the ECCS recirculation flow feeds containment spray systems directly from the common header, (2) a fraction of debris-laden flow can bypass the core and proceed directly to the break without accumulating debris on the fuel assembly screen. Both of these pathways extend the time necessary to build debris levels of concern on either the strainers or the fuel assemblies. A schematic of flow diversion paths that affect the time-dependent debris inventory in the containment pool is shown in Figure 1.

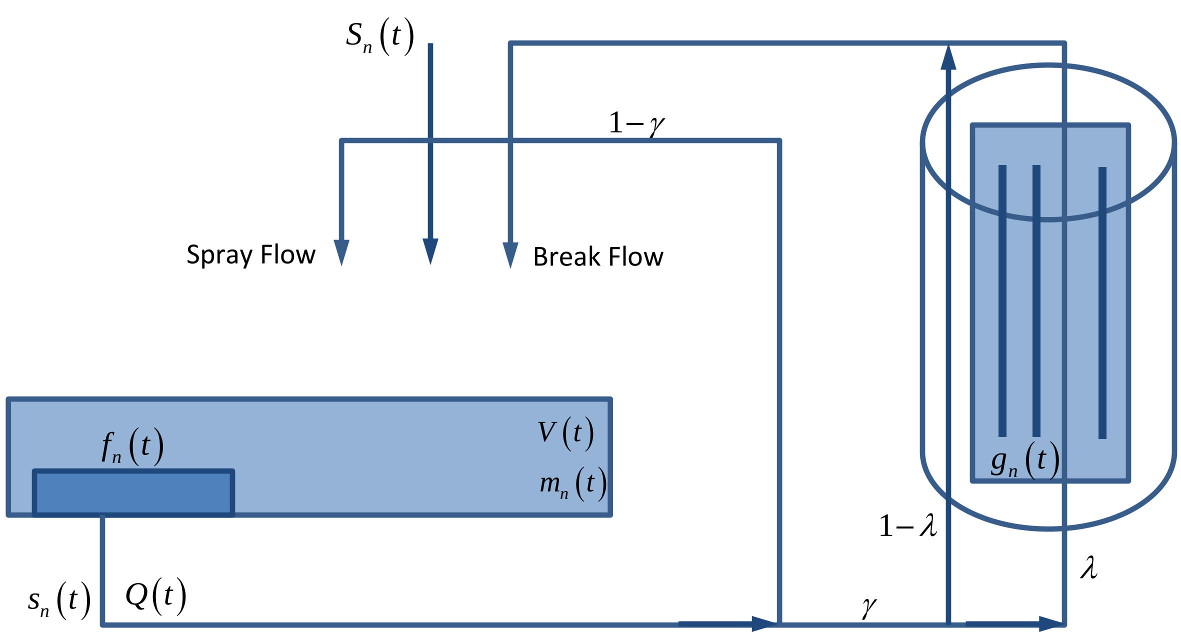

Figure 1. Schematic of recirculation fractions contributing to debris concentration in the containment pool.

For plants like STP that do not have direct impingement of LOCA generated debris on the sump strainers, it is sufficient to assume uniform mixing of suspendable fractions of all debris types in the containment pool. In plant geometries where significant transport time actually exists between the introduction of debris to the main body of water and arrival on the sump screen, the assumption of instantaneous uniform mixing effectively accelerates debris accumulation in a conservative manner.

Further assumptions implemented in this analysis include:
(1) ignore RCS volume so that instantaneous mixing applies to the containment pool volume alone and maintains conservatively high concentrations,
(2) ignore RCS transport times so that debris passes back to the pool instantly through sprays and through the break,
(3) no hold up or partial circulation of debris within the RCS except for possible capture on the fuel assemblies,
(4) no interaction between debris types when suspended in the pool,
(5) possible interaction between debris types in the accumulated beds,
(6) debris follows flow with no drift or lag relative to the fluid.

The differential rate of change for each debris type in the pool under the assumption of a single homogeneous mixing volume is

$$\frac{dm_n}{dt} = S_n - f_n \frac{Q}{V} m_n - g_n \gamma \lambda \left(1 - f\right) \frac{Q}{V} m_n + s_n - g_n \gamma \lambda s_n \, , \tag{1}$$

where all properties can be time dependent and have the following definitions:
$m_n(t) \equiv$ mass of debris type n suspended in the pool $\{\text{kg}\}$
$V(t) \equiv$ total liquid volume of the pool $\{\text{m}^3\}$
$Q(t) \equiv$ volumetric flow rate passing through strainer $\{\text{m}^3/\text{s}\}$
$S_n(t) \equiv$ source rate for initial introduction of debris type n $\{\text{kg/s}\}$
$s_n(t) \equiv$ shedding rate for debris type n from existing bed $\{\text{kg/s}\}$
$\gamma(t) \equiv$ fraction of Q going to safety injection, $(1-\gamma)$ goes to spray
$\lambda(t) \equiv$ fraction of injection volume passing through the core
$f_n(t) \equiv$ filtration efficiency for debris n at the strainer
$g_n(t) \equiv$ filtration efficiency for debris n at the core
$M_n^S(t) \equiv$ cumulative mass of debris n on the strainer (see below)
$M_n^C(t) \equiv$ cumulative mass of debris n on the core (see below)

In Eq. (1), the first term on the RHS denotes time-dependent source behavior like coatings degradation and chemical product formation that introduces material to the pool for the first time. Debris that recirculates back to the pool is not tracked as a source. Indeed, under the present assumption of zero RCS volume, debris that would actually recirculate through the ECCS system never leaves the containment pool volume.

The second term of Eq. (1) describes the initial filtration of all material arriving at the strainer as a function of time. The unknown filtration function $f_n(t)$ rises from some initial value f_0 inherent to the clean strainer geometry and approaches the value of 1.0 as the strainer module loads completely. In this context, “filtration” means that debris penetration is substantially impeded compared to the fluid velocity. Material that is released from the strainer bed via shedding is represented as a separate function $s_n(t)$. For conservative simplicity, no shedding will be permitted from a debris bed formed on the fuel assemblies.

The third term of Eq. (1) describes capture on the fuel assemblies of debris that penetrates directly through the strainer with the water flow. The unknown filtration function $g_n(t)$ rises from some initial value g_0 inherent to the clean fuel screen and approaches a value of 1.0 as the screens load completely. This function may be set to 1.0 for conservative capture of all debris passing through the core, or it may be assumed that filtration of debris at the fuel screen is similar to filtration of debris at the recirculation strainer.

The fourth and fifth terms of Eq. (1) describe, respectively, long-term shedding of debris from the preestablished bed and partial capture of this material on the fuel-assembly screen. Shedding from a bed depends on the mobility of smaller debris elements through the porous lattice and on the total volumetric flow exposure available to assist migration. Shedding requires the existence of a debris bed, so of course, the actual time-dependent shedding rate depends on the history of debris accumulation at the strainer. It is expected that shedding will continue for a significant time after complete strainer coverage is attained. Schematically, the two penetration contributions from direct passage (the complement of filtration) and shedding might look something like the curves presented in Figure 2.

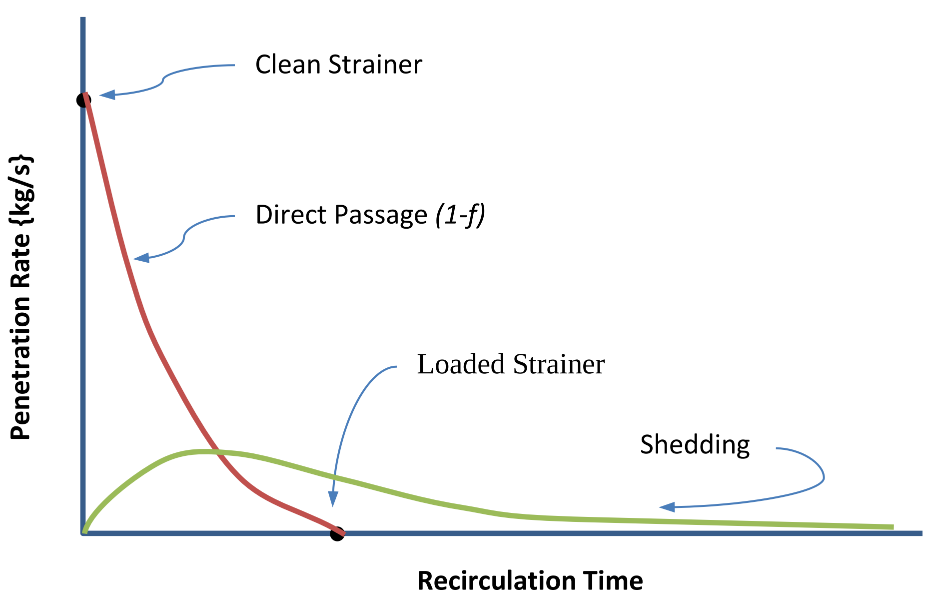

Figure 2. Schematic of penetration contributions from direct passage of debris through a noncontiguous debris bed and slower release attributed to shedding phenomena.

If only a fraction ν of all transportable debris is mobile enough to be released from the primary bed, and it is released with time constant $\hat{\eta}$ $\{1/\mathrm{s}\}$ at a rate proportional to the current inventory of "sheddable" debris m_s that is always assumed to be uniformly mixed in the bed, then a rate equation for sheddable inventory can be written as $\frac{dm_s}{dt} = \nu f \frac{Q}{V} m(t) - \hat{\eta} m_s(t)$ that has the solution $m_s(t) = \nu e^{-\hat{\eta}t} \int_0^t f(t') \frac{Q(t')}{V(t')} m(t') e^{\hat{\eta}t'} dt'$. The rate of shedding from this inventory is then

$$s_n(t) = \hat{\eta}_n m_s(t), \text{ or } \boxed{s_n(t) = \nu_n \hat{\eta}_n e^{-\hat{\eta}_n t} \int_0^t f_n(t') \frac{Q(t')}{V(t')} m_n(t') e^{\hat{\eta}_n t'} dt'}. \quad \textbf{(2)}$$

The integrand of Eq. (2) should be recognized as being related to the total debris accumulation up until time t of interest.

The release constant $\hat{\eta}$ in Eq. (2) can further be related to an effective debris migration velocity that is proportional to the fluid velocity. Knowing the migration velocity and the current thickness L determines the effective transit time through the bed so that $\hat{\eta} = \eta \frac{Q}{AL}$. The current thickness can be estimated knowing the total debris in the bed and an average bed density such that

$$\hat{\eta} = \eta \frac{QA\rho_{bed}}{AM_{total}} = \eta \frac{Q\rho_{bed}}{M_{total}}. \quad \textbf{(3)}$$

Thus, the rate of shedding expressed by Eq. (2) is intimately related to the total mass of debris present at any given time, and this dependency greatly complicates the solution of Eq. (1) for the mass of debris present in the containment pool. This complication can be handled in several ways including:

(1) Numerically solving Eq. (1) to address the embedded time integral needed for shedding,
(2) Dropping the shedding terms from Eq. (1) and solving Eqs. (1) and (2) independently in their separate regions of dominance,
(3) Adopting a more flexible definition of filtration that subsumes the fraction of penetration that occurs during shedding into an effective retention factor that tracks realistic aggregate debris penetration across the entire range of debris load.
(4) Assuming that the shedding process has an inherent time constant as described in Eq. (2) that is controlled more strongly by exit conditions from the bed than by migration time across the bed.

Option 1 will eventually be implemented together with the simplifying assumption of Option 4 to describe debris recirculation histories in the containment pool. Integration of Eq. (1) can only be performed if the filtration and shedding functions are known, so it is instructive to examine special cases of Eq. (1) that will be used to interpret penetration data collected for the purpose of defining $f_n(t)$.

Options 2 and 3 both require a solution of Eq. (1) in the form

$$\frac{dm_n}{dt} + hm_n = S_n \text{ where } h = \left[f_n + g_n \gamma \lambda \left(1 - f_n\right) \right] \frac{Q}{V} . \tag{4}$$

In this relation, h defines the operating history including flow rate, pool volume, spray operation , core bypass, and bed formation on both the strainer and the core inlet.

If the bed filtration and shedding histories are known, then time-dependent accumulation of debris n on the strainer can be expressed as

$$M_n^S(t) = \int_0^t \left[f_n(t') m_n(t') \frac{Q(t')}{V(t')} - s_n(t') \right] dt' . \tag{5}$$

Similarly, time-dependent accumulation of debris n on the core can be expressed as

$$M_n^C(t) = \int_0^t g_n(t') \gamma(t') \lambda(t') \left[\left(1 - f_n(t')\right) \frac{Q(t')}{V(t')} m_n(t') + s_n(t') \right] dt' \tag{6}$$

Note in Eqs. (5) and (6) that the prime notation denotes only a dummy integration variable; it is not a shorthand for differentiation. The current mass in either location depends on the entire previous history of all variables up to the present time t of interest, which requires a complete solution of Eq. (4).

Equation (4) is a first-order differential equation with nonconstant coefficients. It can be solved by introducing an integrating factor of $\exp\left(\int_0^t h(t')\,dt'\right)$ to form a perfect differential on the LHS such that

$$e^{\int_0^t h(t')dt'} \left(\frac{dm_n}{dt} \right) + e^{\int_0^t h(t')dt'} hm_n = S_n e^{\int_0^t h(t')dt'}$$

$$\frac{d}{dt}\left(m_n e^{\int_0^t h(t')dt'} \right) = S_n e^{\int_0^t h(t')dt'}$$

Note by the product differentiation rule that $\frac{d}{dx} fg = f \frac{dg}{dx} + g \frac{df}{dx}$ and that the differentiation of an integral by its upper limit gives $\frac{d}{dt}\int_0^t h(t')dt' = h(t)$. Integration of both sides over time requires an extra layer of bookkeeping for the time variation that will be provided by using double primes wherever needed to clarify the order of integration. Continuation from above yields the general solution

$$\int_0^t d\left(m_n e^{\int_0^{t'} h(t'')dt''}\right) = \int_0^t S_n e^{\int_0^{t'} h(t'')dt''} dt'$$

$$m_n(t) e^{\int_0^t h(t')dt'} - m_n(0) e^0 = \int_0^t S_n e^{\int_0^{t'} h(t'')dt''} dt'$$

$$\boxed{m_n(t) = \exp\left(-\int_0^t h(t')dt'\right)\left[m_n(0) + \int_0^t S_n(t') \exp\left(\int_0^{t'} h(t'')dt''\right) dt'\right]} \quad (7)$$

Note that t specifies any arbitrary time into the scenario and $t' \le t$ denotes all times from the beginning up to the present t of interest.

Equation (7) provides a very general solution to the first order rate equation that may be unfamiliar in its present form. If the operating history is constant such that f, g, Q and V never change then $\int_0^t h(t')dt' = h\int_0^t dt' = ht$ and Eq. (7) becomes

$$m_n(t) = \exp(-ht)\left[m_n(0) + \int_0^t S_n(t') \exp(ht') dt'\right]. \quad (8)$$

If the source rate is also constant, $\int_0^t S_n(t') \exp(ht') dt' = S_n \int_0^t \exp(ht') dt' = (S_n/h)\left[\exp(ht) - 1\right]$ and

$$m_n(t) = m_n(0)\exp(-ht) + \frac{S_n}{h}\left[1 - \exp(-ht)\right]. \quad (9)$$

Notice at $t = 0$, the contribution from the constant source is zero. This contribution rises with increasing time and approaches a steady-state asymptote of S_n/h.

Of equal interest is the case when the operating history is constant and a discrete batch of source debris is introduced very quickly at time t_m as a delta function such that $S_n(t) = S_{nm}\delta(t_m)$. In this case, $\int_0^t S_{nm}\delta(t_m)\exp(ht')dt' = S_{nm}\exp(ht_m)$ and Eq. (8) gives

$$m_n(t) = m_n(0)\exp(-ht) + S_{nm}\exp\left[-h(t - t_m)\right]. \quad (10)$$

Note that the second term only appears for times $t \ge t_m$, i.e., after the discrete debris batch is introduced. Because the original differential equation is first-order, or linear, in m_n, any number of discrete or continuous sources can be superimposed to give the total mass inventory as a function of time. For a superposition of discrete sources, Eq. (10) would be written as

$$m_n(t) = m_n(0)\exp(-ht) + \sum_{m=1} S_{nm} \exp\left[-h(t-t_m)\right], \tag{11a}$$

where the sources only contribute their respective exponential histories after $t \geq t_m$.

Intermediate between a constant, persistent source rate and a discrete batch that is introduced at a point in time is a source that is added at a constant rate $S_{nm}/\Delta t_{m_{in}}$ $\{g/s\}$ for a fixed period of time $t_{m_1} \leq t \leq t_{m_2}$. To treat this condition during the source interval, Eq. (8) is written as

$$m_n(t) = e^{-ht}\left[m_n(0) + \int_{t_{m_1}}^{t} \frac{S_n}{\Delta t_{m_{in}}} e^{ht'} dt'\right] = m_n(0)e^{-ht} + \frac{S_n}{h\Delta t_{m_{in}}}\left(1 - e^{-h(t-t_{m_1})}\right) \tag{11b}$$

Again, the second term is rising towards a long-term asymptote that reaches a fixed value at the end of the source period; only decay occurs thereafter. For $t > t_{m_2}$

$$m_n(t) = m_n(0)e^{-ht} + \frac{S_n}{h\Delta t_{m_{in}}}\left(1 - e^{-h\Delta t_{m_{in}}}\right)e^{-h(t-t_{m_2})} \tag{11c}$$

A superposition of sources appearing at rates S_{nm} during nonoverlapping time intervals can be written as two terms $m = m_{decay} + m_{accum}$. The first term is analogous to discrete batch sources that only decay;

$$m_{n,decay}(t) = \exp(-ht)\left[m_n(0) + \sum_{m=1} \frac{S_{nm}}{h\Delta t_{m_{in}}}\left(1 - e^{-h\Delta t_{m_{in}}}\right)e^{ht_{m2}}\right] \text{ for times } t \geq t_{m_2}.$$

With some creative notation, this result can be written as

$$m_{n,decay}(t) = \exp(-ht)\sum_{m=0} \frac{S_{nm}}{h\Delta t_{m_{in}}}\left(1 - e^{-h\Delta t_{m_{in}}}\right)e^{ht_{m2}} \text{ for times } t \geq t_{m_2}, \tag{11d}$$

where $S_{n0} = hm_n(0)$, $\Delta t_0 = 0$, and the ratio $\left(1 - e^0\right)/\Delta t_0 \equiv 1$.

The second term accounts for accumulation of mass during the source addition periods

$$m_{n,accum}(t) = \frac{S_{nm}}{h\Delta t_{m_{in}}}\left(1 - e^{-h(t-t_{m_1})}\right) \text{ for times } t_{m_1} \leq t \leq t_{m_2}. \tag{11e}$$

Penetration Test Analysis

This section documents the evolution of theory related to the analysis of STP strainer module penetration tests conducted at Alden Research Laboratories. Initial simplistic assumptions involving instantaneous debris batch addition were used to derive formulas that emulate the collection of debris on filter media over recorded time intervals, which is essentially an integration of debris concentration over time. Analytic development is essential for understanding dependencies and building intuition about expected behavior. However, to accommodate complexities in the actual test procedure, numeric integration was found to be the most flexible and effective approach for extracting optimized model parameters from the data. Following subsections include a basic description of the test apparatus and the diagnostic objectives, a generic formulation of the desired time-dependent filtration function, analytic development, numeric integration, and results for an illustrative example problem.

Test Description

All previous equations that describe mass inventory in the containment pool can only be evaluated if all factors in the operating history $h(t)$ are known completely throughout the ECCS recirculation scenario, and this includes complete definitions of the filtration functions f and g. Considering the time delay introduced by shedding phenomena and the additional complication that filtration efficiency for one debris type n can depend on the presence of all other debris in the bed, it is not surprising that the most common assumption to date has been to assume 100% filtration of all material arriving on the sump strainer in order to obtain conservative estimates of head loss. Recent recognition that fibrous debris passing through the ECCS strainer can contribute to potential internal blockage conditions in the core has motivated a series of tests to measure the debris penetration history of an actual compact strainer module. Data from a carefully designed test can be used to determine the time-dependent details of $f(t)$ and $s(t)$ for use in the broader context of recirculation sump debris accumulation.

Basic attributes of the debris penetration tests include a large tank for introducing and mixing debris and a submerged strainer module of actual plant design. A recirculation pump draws water through the strainer at a specified velocity, and a manifold of bag filters downstream of the strainer allows time-integrated collection of debris that has penetrated the strainer during any desired period of time. The filter manifold is designed so that all of the flow passes through only one filter path at a time with 100% collection efficiency, which allows the flow to be diverted to a clean filter as desired. Additional isokinetic "grab" samples taken between the strainer and the bag filters permit slip stream sampling of debris concentration at more frequent intervals. Isokinetic sampling means that the extraction velocity at the inlet of the sample line matches the free stream velocity of the pipe. Mismatched sampling velocity introduces inertial bias that skews the sampled size distribution relative to the free stream size distribution.

The other key feature of the penetration test design is complete control over the source term history. Either discrete batches or a slow continuous stream of debris can be fed to the tank in order to vary the rate of arrival. Models for both source histories are provided in the previous section, and recall that the sources can be added in any combination as long as the concentration remains low enough that suspended debris from one source does not interfere with the transport of any other source. The other fundamental assumption in the model is uniform mixing throughout the volume. This is approximated by manual agitation of any debris that tends to settle, while taking care not to disturb the debris bed established on the test module. In the case of discrete batches, sampling intervals can be designed to relate changes in filtration response directly to the exponential accumulation from each batch. This is the test configuration that will be developed in this section.

Although the assumption of linearly additive debris concentrations in the pool may be reasonable for conservative transport estimation (agglomeration would enhance settling), there is some quantitative evidence that the concentration of debris arriving at the strainer does affect the degree of penetration. In the limit of very low debris concentration impinging on a clean strainer, one might imagine that individual fibers have a better chance of passing through than if they arrive in small tangled groups. Test variations are planned to explore the magnitude of this effect. The following analysis will be specific to the data from a single loading sequence, so an additional level of correlation may be needed to capture the quantitative significance of debris concentration on strainer filtration.

Similar observations hold for debris approach velocity. Because debris penetration through small holes in the strainer can be dominated by inertial forces, there may be some dependence of aggregate penetration on fluid velocity. Any dependence on velocity is likely to be most prevalent early in the strainer loading history where a preestablished fiber mat does not impede velocity driven pass through. An important velocity effect has been observed for a much higher range of flow conditions, but it is not certain that the trend will persist with equal importance into the sub 0.01 ft/s flow regime where directed momentum through the strainer becomes increasingly comparable to turbulent flow components in the pool. The test matrix is also designed to examine velocity as an additional correlate.

Under tank test conditions $g_n = \gamma = \lambda = 1$, and the operating conditions $h = Q/V$ are held constant. Also, the investigation will focus on fiberglass penetration only, so the subscript for debris type will be dropped. Equation (1) reduces to Eq. (4) that has a solution for multiple superimposed batch sources given by Eq. (11). Ideally, the test apparatus could be cleaned to a pristine condition between successive tests, but some residual material always remains circulating at the beginning of each new test sequence. This initial circulating debris inventory will be assigned to $m(0)$, and it will be treated as a discrete "batch" of fiber introduced unintentionally at time $t_0 = 0$. By collecting cumulative mass M_1 on a blank filter at time t_1 prior to any planned debris addition, the initial source term can be determined from

$M_1 = \int_0^{t_1} h S_0 e^{-ht} dt = S_0 \left(1 - e^{-ht_1}\right)$, so

$$S_0 = \frac{M_1}{1 - \exp\left(-ht_1\right)} \tag{12}$$

and the notation for current mass in the pool simplifies to

$$m(t)=\sum_{m=0} S_m \exp\left[-h\left(t-t_m\right)\right]=e^{-ht}\sum_{m=0} S_m e^{ht_m} \text{ for } t \ge t_m . \quad \textbf{(13)}$$

The result for initial latent source given by Eq. (12) assumes that no intermediate grab samples are taken during the blank filter collection period and that there is no retention of residual fiber on the strainer. Also, note in Eq. (13) that times t_m correspond to the introduction of each new debris batch.

Latent debris present in the test apparatus can be attributed to several sources including: (1) suspended matter described above, (2) random release from surfaces and internal crevices, (3) rinsing from tools and hands introduced during the test by stirring, and (4) airborne settling into the tank. Only component (1) can be measured with any accuracy, so all other spurious sources will be accepted as a penalty against the total fiber penetration estimate.

The key analytic goal of the penetration test campaign is to determine the functional behavior of $f(t)$ and $s(t)$ during the test, and then correlate them to the total mass of debris on the strainer as illustrated in Figure 2 so that the functional behavior can be applied generically to any fiber debris loading present on the prototypical strainer module. This goal will be accomplished by collecting cumulative downstream masses M_k on grab filters and bag filters over collection periods each beginning at t_{k_1} and ending at t_{k_2} . The index k denotes the sequential indices of each filter period that yields a cumulative mass data point. *It is assumed in the notation that no filter period ever spans the introduction of a new debris batch*. This convention determines the number of discrete source terms l that are included in the Eq. (13) summation. Index l corresponds to the debris batch added just prior to or simultaneous to the beginning of the filter period in question. Similarly, *it is assumed in the notation that no grab filter period ever spans two bag filter intervals*. Both of these assumptions could be relaxed at the expense of additional notation to describe partial time intervals that compose each filter period, but they impose relatively minor constraints on test series execution and yield significant modeling simplification. If it does become necessary to add multiple debris batches during the same bag filter interval, the equations can be evaluated over "virtual" bag filter times that comply with the convention and are then added to form an estimate of the actual filter span.

The concurrent use of grab filters and bag filters during a sampling period requires the application of flow split factors γ_k like that described in Figure 1 to assign flow to the core. The flow factor for the grab samples will be determined by the portion of total downstream flow that is diverted to the grab filter. The flow factor for bag filters will always be assigned a value of 1.0, and the predictive terms for any concurrent grab samples will then be subtracted. The data from each filter are independent and should never be added or subtracted. Only the predictive equations are modified to adjust for the portion of the flow that was received by each filter during each time integration period

Filtration Function

A generic functional form for the shedding rate has already been proposed in Eq. (2). This function has two additional unknown parameters η and ν that must be determined by comparison to data. Here the hat notation is dropped and the suggested relationship between migration velocity and bed thickness will not be pursued. In general, the shedding rate should behave like an exponential decay function that is modulated by the amount of debris in the bed, and the bed inventory is controlled by the unknown filtration function $f(t)$. Because filtration improves with the amount of debris on the screen, and because the amount of debris in the tank following a batch declines exponentially, one might expect the filtration function to increase in stages, behaving as an exponential complement following each batch addition. Of course, the total filtration efficiency per unit mass added can never exceed 1.0, and it should reach this value soon after the strainer module is covered uniformly.

One generic piece-wise continuous function that can reproduce the characteristics of a periodically increasing filtration efficiency is

$$f(t) = f_0 + \sum_{m=1}^{l-1} \alpha_m \left(1 - e^{-\beta_m \Delta t_m}\right) + \alpha_l \left[1 - e^{-\beta_l (t - t_l)}\right] = C_l + \alpha_l \left[1 - e^{-\beta_l (t - t_l)}\right]. \quad \textbf{(14)}$$

Here, f_0 is the initial filtration efficiency of the clean strainer module, the α_m are unknown scaling constants for each batch interval, and the β_m are unknown decay constants for each batch interval. The function is piecewise continuous, but it has independent behavior during each source interval. For a specific filter, only the most recent source batch $m = l$ is of interest, and all prior filtration intervals simply represent a constant C_l. Time dependence during the filtration interval is shown explicitly as the second term of Eq. (14) to support analytic integration. In addition to the unknown clean filtration efficiency f_0, this function introduces two unknown parameters for each discrete source interval.

A schematic of periodic behavior generated from this generic filtration function is shown in Figure 3. The shape of the periodic exponential complement suggests that grab samples would be most effective at adding resolution if they are initiated early in each source interval and spaced logarithmically rather than linearly if multiple samples can be obtained during a batch. Further examination of Eq. (14) also shows that the unknown parameters for early source periods become embedded in estimates for the later source periods through the constant C_l, so additional time resolution near the beginning of strainer loading will improve accuracy of the overall parametric fit.

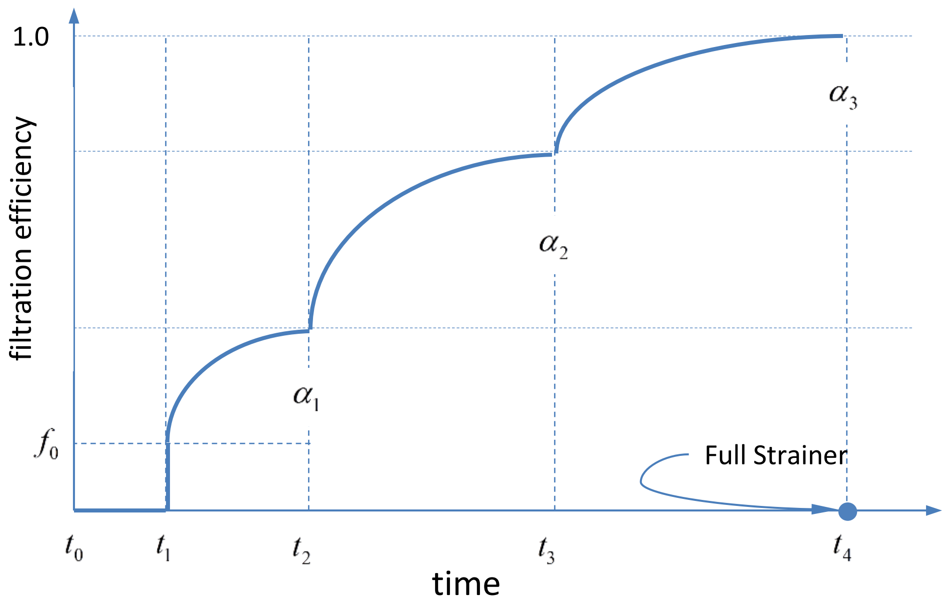

Figure 3. Schematic representation of a generic filtration function. Typical clean strainer filtration f_0 will be much greater than illustrated.

The time history of cumulative debris mass on the strainer given by Eq. (5) should resemble very closely the behavior of the filtration function until complete strainer loading is achieved. The final step of this analysis will be to correlate the time history of filtration to the time history of cumulative debris in order to obtain filtration as a function of strainer loading. Visual correlation can be accomplished by simply plotting f vs. M^S at a set of common evaluation times, but a final step of fitting or smoothing may be required to cast $f\left(M^S\right)$ in a form that is convenient for evaluation in the containment analysis.

Analytic Integration

Generic expressions for $s(t)$, $m(t)$, and $f(t)$ can now be substituted from Eqs. (2), (13), and (14), respectively, into Eq. (6) to form a predictive equation for the mass accumulated during a filtration period. Substitution will be accomplished in stages to improve traceability. First, recast Eq. (6) in a form that represents mass accumulation over an arbitrary interval;

$$M_k = \gamma_k h \left\{ \int_{t_{k_1}}^{t_{k_2}} m(t)\,dt - \int_{t_{k_1}}^{t_{k_2}} f(t)\,m(t)\,dt + \nu\eta \int_{t_{k_1}}^{t_{k_2}} e^{-\eta t} \left[\int_0^t f(t')\,m(t')\,e^{\eta t'}\,dt' \right] dt \right\} \quad (15)$$

The three terms of Eq. (15) represent mass arriving at the strainer, mass filtered by the strainer, and mass shed from the strainer, respectively. Filter mass can then be discussed as $M_k = I_1 + I_2 + I_3$.

The first integral of Eq. (15) is evaluated as follows:

$$\begin{aligned}\int_{t_{k_1}}^{t_{k_2}} m(t)\,dt &= \int_{t_{k_1}}^{t_{k_2}} \sum_{m=0}^{l} S_m e^{-h(t-t_m)} dt \\ &= \sum_{m=0}^{l} S_m e^{ht_m} \int_{t_{k_1}}^{t_{k_2}} e^{-ht} dt \\ &= \frac{1}{h}\left(e^{-ht_{k_1}} - e^{-ht_{k_2}}\right) \sum_{m=0}^{l} S_m e^{ht_m}\end{aligned} \tag{16}$$

The second integral of Eq. (15) is evaluated by recognizing that only the most recent segment of the filtration function varies in time during the integration interval:

$$\begin{aligned}\int_{t_{k_1}}^{t_{k_2}} f(t)\,m(t)\,dt &= \int_{t_{k_1}}^{t_{k_2}} \left\{C_l + \alpha_l\left[1 - e^{-\beta_l(t-t_l)}\right]\right\}\left(\sum_{m=0}^{l} S_m e^{ht_m} e^{-ht}\right) dt \\ &= \left\{(C_l + \alpha_l)\int_{t_{k_1}}^{t_{k_2}} e^{-ht} dt - \alpha_l e^{\beta_l t_l} \int_{t_{k_1}}^{t_{k_2}} e^{-(h+\beta_l)t} dt\right\} \sum_{m=0}^{l} S_m e^{ht_m} \\ &= \left\{\frac{C_l + \alpha_l}{h}\left(e^{-ht_{k_1}} - e^{-ht_{k_2}}\right) - \frac{\alpha_l}{h+\beta_l} e^{\beta_l t_l}\left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}}\right)\right\} \sum_{m=0}^{l} S_m e^{ht_m}\end{aligned} \tag{17}$$

The inner integral in the third term of Eq. (15) requires additional care to properly account for the time dependence of the integrand over all batch intervals. Consider the time decomposition

$$I_3 = \gamma_k h\nu\eta \int_{t_{k_1}}^{t_{k_2}} e^{-\eta t}\left[\int_0^{t_l} fme^{\eta t'} dt' + \int_{t_l}^{t_{k_1}} fme^{\eta t'} dt' + \int_{t_{k_1}}^{t} fme^{\eta t'} dt'\right] dt\,. \tag{18}$$

The second and third terms of Eq. (18) apply to the same single source interval l. Evaluate the second term to establish notation:

$$\int_{t_l}^{t_{k_1}} f(t')\,m(t')\,e^{\eta t'} dt' = \int_{t_l}^{t_{k_1}} f(t') \sum_{m=0}^{l} S_m e^{ht_m} e^{-(h-\eta)t'} dt'.$$

By comparison with Eq. (17) the result is

$$\int_{t_l}^{t_{k_1}} f(t')\,m(t')\,e^{\eta t'} dt' = \left\{\frac{C_l + \alpha_l}{h-\eta}\left[e^{-(h-\eta)t_l} - e^{-(h-\eta)t_{k_1}}\right] - \frac{\alpha_l \exp(\beta_l t_l)}{h-\eta+\beta_l}\left[e^{-(h-\eta+\beta_l)t_l} - e^{-(h-\eta+\beta_l)t_{k_1}}\right]\right\} \sum_{m=0}^{l} S_m e^{ht_m}$$

which is a constant with respect to time that will be defined as $D_{l2}\sum_{m=0}^{l} S_m e^{ht_m}$. This constant can now be integrated over the outer integral of Eq. (18) to obtain

$$\begin{aligned}\gamma_k h\nu\eta \int_{t_{k_1}}^{t_{k_2}} e^{-\eta t}\left[\int_{t_l}^{t_{k_1}} fme^{\eta t'}dt'\right]dt &= \gamma_k h\nu\eta D_{l2}\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\int_{t_{k_1}}^{t_{k_2}} e^{-\eta t}dt \\ &= \gamma_k h\nu D_{l2}\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left(e^{-\eta t_{k_1}} - e^{-\eta t_{k_2}}\right)\end{aligned} \quad \textbf{(19)}$$

The third part of Eq. (18) is not constant with respect to time, but it can be written similarly

$$\begin{aligned}\int_{t_{k_1}}^{t} fme^{\eta t'}dt' &= \int_{t_{k_1}}^{t} f(t')\sum_{m=0}^{l} S_m e^{ht_m} e^{-(h-\eta)t'}dt' \\ &= \left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left\{\frac{C_l+\alpha_l}{h-\eta}\left[e^{-(h-\eta)t_{k_1}} - e^{-(h-\eta)t}\right] - \frac{\alpha_l \exp(\beta_l t_l)}{h-\eta+\beta_l}\left[e^{-(h-\eta+\beta_l)t_{k_1}} - e^{-(h-\eta+\beta_l)t}\right]\right\} \\ &= \left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left[\begin{array}{l}\frac{C_l+\alpha_l}{h-\eta}e^{-(h-\eta)t_{k_1}} - \frac{\alpha_l \exp(\beta_l t_l)}{h-\eta+\beta_l}e^{-(h-\eta+\beta_l)t_{k_1}} \\ -\frac{C_l+\alpha_l}{h-\eta}e^{-(h-\eta)t} + \frac{\alpha_l \exp(\beta_l t_l)}{h-\eta+\beta_l}e^{-(h-\eta+\beta_l)t}\end{array}\right] \\ &= \left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left[D_{l3} - \frac{C_l+\alpha_l}{h-\eta}e^{-(h-\eta)t} + \frac{\alpha_l \exp(\beta_l t_l)}{h-\eta+\beta_l}e^{-(h-\eta+\beta_l)t}\right]\end{aligned}$$

where $D_{l3} = \frac{C_l+\alpha_l}{h-\eta}e^{-(h-\eta)t_{k_1}} - \frac{\alpha_l \exp(\beta_l t_l)}{h-\eta+\beta_l}e^{-(h-\eta+\beta_l)t_{k_1}}$. Integrating this result over the outer integral of Eq. (18) gives the result

$$\gamma_k h\nu\eta \int_{t_{k_1}}^{t_{k_2}} e^{-\eta t}\left[\int_{t_{k_1}}^{t} fme^{\eta t'}dt'\right]dt = \gamma_k h\nu\eta\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left\{\begin{array}{l}\frac{D_{l3}}{\eta}\left(e^{-\eta t_{k_1}} - e^{-\eta t_{k_2}}\right) - \frac{C_l+\alpha_l}{h(h-\eta)}\left(e^{-ht_{k_1}} - e^{-ht_{k_2}}\right) \\ +\frac{\alpha_l \exp(\beta_l t_l)}{(h+\beta_l)(h-\eta+\beta_l)}\left[e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}}\right]\end{array}\right\} \quad \textbf{(20)}$$

The first part of Eq. (18) must be evaluated separately for each source interval, but it should also yield a constant with respect to time. Separating the integral over each source and recalling that $f = 0$ prior to t_1 gives

$$\begin{aligned}\int_0^{t_l} fme^{\eta t'}dt' &= \sum_{j=1}^{l-1}\int_{t_j}^{t_{j+1}} f(t')m(t')e^{\eta t'}dt' \\ &= \sum_{j=1}^{l-1}\int_{t_j}^{t_{j+1}} f(t')\sum_{m=0}^{j} S_m e^{ht_m}e^{-(h-\eta)t'}dt' \\ &= \sum_{j=1}^{l-1}\left(\sum_{m=0}^{j} S_m e^{ht_m}\right)\left\{\begin{array}{l}\dfrac{C_j+\alpha_j}{h-\eta}\left[e^{-(h-\eta)t_j}-e^{-(h-\eta)t_{j+1}}\right] \\ -\dfrac{\alpha_j\exp\left(\beta_j t_j\right)}{h-\eta+\beta_j}\left[e^{-\left(h-\eta+\beta_j\right)t_j}-e^{-\left(h-\eta+\beta_j\right)t_{j+1}}\right]\end{array}\right\} \\ &= \left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\sum_{j=1}^{l-1}\frac{\sum_{m=0}^{j} S_m e^{ht_m}}{\sum_{m=0}^{l} S_m e^{ht_m}}\{\ \}_j \\ &= D_{l1}\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\end{aligned}$$

The outer integral of Eq. (18) can now be applied to obtain the result

$$\begin{aligned}\gamma_k h\nu\eta\int_{t_{k_1}}^{t_{k_2}} e^{-\eta t}\left[\int_0^{t_l} fme^{\eta t'}dt'\right]dt &= \gamma_k h\nu\eta D_{l1}\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\int_{t_{k_1}}^{t_{k_2}} e^{-\eta t}dt \\ &= \gamma_k h\nu D_{l1}\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left(e^{-\eta t_{k_1}}-e^{-\eta t_{k_2}}\right)\end{aligned} \tag{21}$$

The combined form for Eq. (18) is now obtained by collecting Eqs. (19), (20) and (21). The result is

$$I_3 = \gamma_k h\nu\left(\sum_{m=0}^{l} S_m e^{ht_m}\right)\left[\begin{array}{l}\left(D_{l1}+D_{l2}+D_{l3}\right)\left(e^{-\eta t_{k_1}}-e^{-\eta t_{k_2}}\right)-\dfrac{\eta\left(C_l+\alpha_l\right)}{h\left(h-\eta\right)}\left(e^{-ht_{k_1}}-e^{-ht_{k_2}}\right) \\ +\dfrac{\alpha_l\eta\exp\left(\beta_l t_l\right)}{\left(h+\beta_l\right)\left(h-\eta+\beta_l\right)}\left(e^{-\left(h+\beta_l\right)t_{k_1}}-e^{-\left(h+\beta_l\right)t_{k_2}}\right)\end{array}\right] \tag{22}$$

Equation (15), which predicts the mass that will be collected during a filter period, can now be assembled by substituting results from Eqs. (16), (17), and (22):

$$M_k = \gamma_k \left(\sum_{m=0}^{l} S_m e^{ht_m} \right) \left\{ \begin{aligned} & \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) - \left(C_l + \alpha_l \right) \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) + \frac{h\alpha_l}{h+\beta_l} e^{\beta_l t} \left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}} \right) \\ & + h\nu \left[\begin{aligned} & \left(D_{l1} + D_{l2} + D_{l3} \right) \left(e^{-\eta t_{k_1}} - e^{-\eta t_{k_2}} \right) - \frac{\eta \left(C_l + \alpha_l \right)}{h\left(h - \eta \right)} \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) \\ & + \frac{\alpha_l \eta \exp\left(\beta_l t_l \right)}{\left(h + \beta_l \right)\left(h - \eta + \beta_l \right)} \left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}} \right) \end{aligned} \right] \end{aligned} \right\}$$

Collecting terms gives a more compact form

$$M_k = \gamma_k \left(\sum_{m=0}^{l} S_m e^{ht_m} \right) \left\{ \begin{aligned} & \left[1 - \left(h - \eta\left(1 - \nu \right) \right) \frac{C_l + \alpha_l}{h - \eta} \right] \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) + h\nu \left(D_{l1} + D_{l2} + D_{l3} \right) \left(e^{-\eta t_{k_1}} - e^{-\eta t_{k_2}} \right) \\ & + \frac{h\alpha_l \exp\left(\beta_l t_l \right)}{\left(h + \beta_l \right)\left(h + \beta_l - \eta \right)} \left(h + \beta_l - \eta\left(1 - \nu \right) \right) \left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}} \right) \end{aligned} \right\}$$

The leading summation is related to the mass present in the pool at the start of the source batch during which the filter was taken. The actual data $M_k \{g\}$ can be divided by this sum to nondimensionalize the relationship in terms of scaled data $F_k = M_k \Big/ \sum_{m=0}^{l} S_m e^{ht_m}$. Recall that the flow fraction γ varies between grab and bag filters, so it will remain on the RHS.

After regrouping the terms $D_{l1} + D_{l2} + D_{l3}$, the final result is

$$\boxed{F_k = \gamma_k \left\{ \begin{aligned} & \left[1 - \left(h - \eta\left(1 - \nu \right) \right) \frac{C_l + \alpha_l}{h - \eta} \right] \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) + h\nu D_l \left(e^{-\eta t_{k_1}} - e^{-\eta t_{k_2}} \right) \\ & + \frac{h\alpha_l \exp\left(\beta_l t_l \right)}{\left(h + \beta_l \right)\left(h + \beta_l - \eta \right)} \left(h + \beta_l - \eta\left(1 - \nu \right) \right) \left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}} \right) \end{aligned} \right\},} \qquad \textbf{(23)}$$

for $k = 2, 3, \ldots$ where

$C_l = f_0 + \sum_{j=1}^{l-1} \alpha_j \left(1 - e^{-\beta_j \Delta t_j} \right)$, and

$$D_l = \left(\frac{C_l + \alpha_l}{h-\eta} - \frac{\alpha_l}{h-\eta+\beta_l} \right) e^{-(h-\eta)t_l} + \sum_{j=1}^{l-1} \frac{\sum_{m=0}^{j} S_m e^{ht_m}}{\sum_{m=0}^{l} S_m e^{ht_m}} \left\{ \begin{array}{l} \frac{C_j + \alpha_j}{h-\eta} \left[e^{-(h-\eta)t_j} - e^{-(h-\eta)t_{j+1}} \right] \\ -\frac{\alpha_j \exp(\beta_j t_j)}{h-\eta+\beta_j} \left[e^{-(h-\eta+\beta_j)t_j} - e^{-(h-\eta+\beta_j)t_{j+1}} \right] \end{array} \right\}.$$

Notice that Eq. (23) can be used to track the cumulative debris penetration by imagining a very large number of short successive filter periods with $\gamma_k = 1$. The cumulative penetration would then be the running sum of the individual increments.

Similarly, the history of mass accumulation on the strainer described by Eq. (5) can be viewed as the summation of a very large number of integration intervals that have solutions similar to the filter collection period treated above. Thus,

$$M^S(t_K) = \sum_{k=0}^{K} \int_{t_{k_1}}^{t_{k_2}} [fmh - s]\, dt \equiv \sum_{k=0}^{K} P_k \qquad \textbf{(24)}$$

where each of the increments has a familiar form

$$P_k = h \int_{t_{k_1}}^{t_{k_2}} fm dt - h\nu\eta \int_{t_{k_1}}^{t_{k_2}} e^{-\eta t} \left[\int_0^t fm e^{\eta t'} dt' \right].$$

In Eq. (24), the index k no longer corresponds to the filter data, but the discrete integration intervals are treated exactly the same, and each interval has a corresponding prior source batch l. The required integrals are given principally by Eqs. (17) and (22), and the result is

$$\boxed{P_k = \left[\sum_{m=0}^{l} S_m e^{ht_m} \right] \left\{ \begin{array}{l} \left[h - \eta(1-\nu) \right] \frac{C_l + \alpha_l}{h-\eta} \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) - \nu h D_l \left(e^{-\eta t_{k_1}} - e^{-\eta t_{k_2}} \right) \\ -\frac{h\alpha_l \exp(\beta_l t_l)}{(h+\beta_l)(h-\eta+\beta_l)} \left[h + \beta_l - \eta(1-\nu) \right] \left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}} \right) \end{array} \right\},} \qquad \textbf{(25)}$$

where C_l and D_l are defined as before.

The amount of debris passing through the strainer at the velocity of the water during a short observation period is

$$M_k^P = \int_{t_{k_1}}^{t_{k_2}} (1-f) hm dt = h \left\{ \int_{t_{k_1}}^{t_{k_2}} m dt - \int_{t_{k_1}}^{t_{k_2}} fm dt \right\}$$

These integrals are given by Eqs. (16) and (17), and the result is

$$M_k^P = \left(\sum_{m=0}^{l} S_m e^{ht_m} \right) \left\{ \left[1 - \left(C_l + \alpha_l \right) \right] \left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right) - \frac{\alpha_l}{h + \beta_l} e^{\beta_l t_l} \left(e^{-(h+\beta_l)t_{k_1}} - e^{-(h+\beta_l)t_{k_2}} \right) \right\}. \quad (26)$$

If Eq. (23) is use to track the time-dependent total penetration, then the time-dependent shedding portion can be estimated by subtracting Eq. (26) for each time increment.

A few observations regarding Eq. (21) may inspire strategies for optimizing this nonlinear predictive model for the unknown parameters $f_0, \alpha_m, \beta_m, \nu$, and η. First, recall that the "blank" filter $k = 1$ is used to determine the initial latent source term S_0, so it does not enter the data vector F_k. Second, if the fraction of sheddable debris $\nu = 0$, several terms disappear, and the equation reduces to the pure filtration solution. Third, pure filtration is a cumulative effect that could possibly be solved in a forward, time-progressive solution that uses near-by sequential data to determine α and β for the current source interval. The analogy for a linear system would be the progressive solution of a lower triangular coefficient matrix. Fourth, the factor $\left(e^{-ht_{k_1}} - e^{-ht_{k_2}} \right)$ on the first term is a constant specific to each datum and can be preevaluated.

Fifth, Eq. (21) describes collection of debris during a generic filtration interval where γ is constant during the collection period. To model a bag filter period with one or more intervening grab filters, first evaluate the formula using $\gamma = 1$, and then subtract the entire formula for *each* grab filter interval that is embedded within the bag filter period. If all filter masses are ordered in succession by retrieval time, the grab filter terms will have been preevaluated before they are needed for subtraction.

Sixth, there is one initial condition, f_0, two free parameters for each source batch period, α_m and β_m, and two free parameters for shedding, ν and η. The inventory of unknown parameters suggests that there should be a least two data per filtration period, and preferably three. These data can consist of any combination of bag filters and grab filters taken during the batch interval. As noted earlier, additional data resolution should be applied preferentially to the early stages of strainer loading. If the system is severely underconstrained (fewer data than parameters), prior information regarding each parameter may be needed to obtain a unique optimum solution. Regarding data quality, there will be more sampling variability inherent to grab samples than there is associated with bag filters. Both factors may carry the same degree of measurement error, but sampling a partial flow stream at very low concentration has added spatial variability present in the signal being sampled.

Seventh, the formulas are evaluated for any arbitrary filter period that has only one source batch associated with the filter span. If it becomes necessary to add multiple batches of debris during a single bag filter, for example, the formulas can be applied over virtual filter intervals and then added to form an estimate of the cumulative mass during the actual filter period. In fact, the formulas apply equally well to 1-s time steps that can be added as needed to form composite estimates of any filter combination.

Finally, during algebraic manipulation, exponential factors were separated as constants that could be extracted from integrals and summations. This approach decoupled the usual appearance of exponential time differences into separate products of exponential absolute times. Numeric overflow/underflow problems may arise if appropriate time units are not chosen. For example, it would be unwise to choose time units of μs for a test that ran for many days. However, if numeric scaling is identified as an issue, it is perfectly acceptable to cast time constants in units of years, even if the test lasts for only a few hours. Ultimately, time is irrelevant to the desired correlation between filtration, shedding, and total debris mass on the strainer.

Several constraints are available for iterative solution of the nonlinear equation set given by Eq. (21). Because the raw data have been scaled, all predictions $F_k < 1$, and probably $F_k \ll 0.1$ in all but the earliest filters. Similarly, all $\alpha_m < 1$, $f_0 < 1$, and $\nu < 1$. Reasonable estimates of f_0 and ν are 0.8 and 0.2, respectively. Exponential decay constants β_m will have magnitudes on the same order as h because tank transport drives accumulation of debris at the strainer, but η will be much smaller, perhaps on the order of inverse days. By reference to Figure 3 it should be obvious that $C_l \leq 1$ for all source batches, and in fact, there may be point where it is practical to assume that $C_l = 1$ and $\alpha_l = 0$ for all following batches. The filtration function should approach 1.0 in a periodic asymptotic fashion, so it is reasonable to impose the constraint that successive α_m are monotonically decreasing. It might also be reasonable to assume as a prior condition that all $\alpha_m = 0$ and let data support positive values as needed.

Numeric Integration

Often, complications experienced during real experiments do not conform to idealized analytic assumptions. In this case, fluctuations in water volume and the addition of debris at approximately constant rate over a significant time interval make it extremely complicated to develop analytic formulas. However, numeric integration can handle many variations in detail. Write Eq.(6) for strainer mass accumulation over an arbitrary interval $t_{k_1} \leq t \leq t_{k_2}$. The interval is usually associated with collection on a filter designated by the index k but the notation applies equally well to a cumulative integral from 0 to t.

$$M_k = \gamma_k \left\{ \int_{t_{k_1}}^{t_{k_2}} m(t)h(t)dt - \int_{t_{k_1}}^{t_{k_2}} f(t)m(t)h(t)dt + \nu\eta \int_{t_{k_1}}^{t_{k_2}} e^{-\eta t} \left[\int_0^t f(t')m(t')h(t')e^{\eta t'}dt' \right] dt \right\}. \quad (27)$$

For the debris penetration test $h(t) = Q(t)/V(t)$. Flow rates are held approximately steady by a variable speed pump that is adjusted occasionally, but liquid volume can change by $\pm 5\%$. A history of water volume is needed for each test that tracks grab sample extraction and source volume addition. A piece-wise linear record of $h(t)$ is adequate. Debris mass history depends on the sequence of introduction, and it most likely to occur during nonoverlapping intervals where the feed rate is approximately constant. Recall that this behavior can be modeled as a sum of continuously decaying functions and accumulation functions defined by Eqs (11d) and (11e). The generic filtration function is given by Eq. (14).

If the entire time history is subdivided into very small increments and times for all actions during the test are known, then piecewise linear interpolation (trapezoidal) integration is sufficient to evaluate the individual terms of Eq. (27). Once the integrands have been evaluated at discrete times and individual panel areas are computed ($1/2\,\Delta t \times$height), the desired integral is simply the sum of time steps that lie between the limits. Integrals progressing in time are easily calculated as cumulative sums of the preevaluated discrete areas.

Accumulation on the strainer is given by an expansion of Eq. (5)

$$M^S(t)=\int_0^t f(t)m(t)h(t)\,dt-\nu\eta\int_0^t e^{-\eta t'}\left[\int_0^{t'} f(t'')m(t'')h(t'')e^{\eta t''}dt''\right]. \quad \textbf{(28)}$$

Simulation of Debris Load Correlation

A sample calculation was performed to build numeric subroutines for evaluating the basic functions derived above and for exercising the correlation of filtration and shedding to strainer loading. Table 1 lists the parameter values assumed for this exercise.

Figure 4 illustrates the pool mass history resulting from the addition of four discrete batches of debris. At any future time, the total inventory is the sum of all exponential tails resulting from prior source introduction. Although not clearly evident, this history properly accounts for the exponential accumulation of mass during constant-rate introduction periods.

Figure 5 illustrates the periodic filtration function resulting from the α and β parameters assumed in the example problem. The shape of the filtration function history will depend greatly on the number of debris batches added during the test, on the quantity of debris in each batch, and on the geometric response of the strainer module in the flow regime. Indeed, the prime objective of this analysis is to determine from data the shape of the filtration function.

Accumulation and shedding of mass from the strainer as defined by Eq.(26) are shown in Figure 6. In this example problem shedding is very minimal, and it is expected to be greater in actual test configurations. Another principal objective of this analysis is to determine both the fraction of sheddable debris and the release rate.

Eventual applications of the penetration test analysis will need a correlation of filtration as a function of debris quantity on the strainer that is independent of the time history in the tests from which it was derived. An example of this correlation is obtained by plotting $f(t)$ from Figure 5 against $M^S(t)$ from Figure 6 at common time points. Depending on fluctuations observed in data, final smoothing or fitting may be performed to aid later implementation of $f(M^S)$.

Table 1. Parameters assumed for graphical illustration.

```
% model parameters
nu = 0.1;                         % shedding coeff
eta = 4.6e-4;                     % shedding decay constant (min^-1)
f0 = 0.6;                         % initial strainer filtration efficiency
alpha = [ .2   .15  .075 0];      % alpha filtration params for each source
beta =  [0.08 0.07 0.06  0];      % beta filtration params for each source

% test parameters
Tm1 = [30  90 135 260];           % batch intro start time (min)
Tm2 = [40 100 145 270];           % batch intro end time (min)
Sm =  [20  40  30  10];           % batch size (g)
h = 0.01;                         % Q/V (min^-1), reciprocal turn-over period
delT = 1;                         % time step for evaluation (min)
M1 = 0.3;                         % blank filter mass (g)
gamgrb = 0.05;                    % flow fraction to a grab filter

% filter schedule
Tbag1 = [ 0 30  90 135 200 260];   % must start one bag period for each Tm1
Tbag2 = [30 90 135 200 260 330];   % last entry denotes test termination
Tgrb1 = [40 50 75 105 110 145 160 190 275 290 300];
Tgrb2 = [41 51 76 106 111 146 161 191 276 291 301];
```

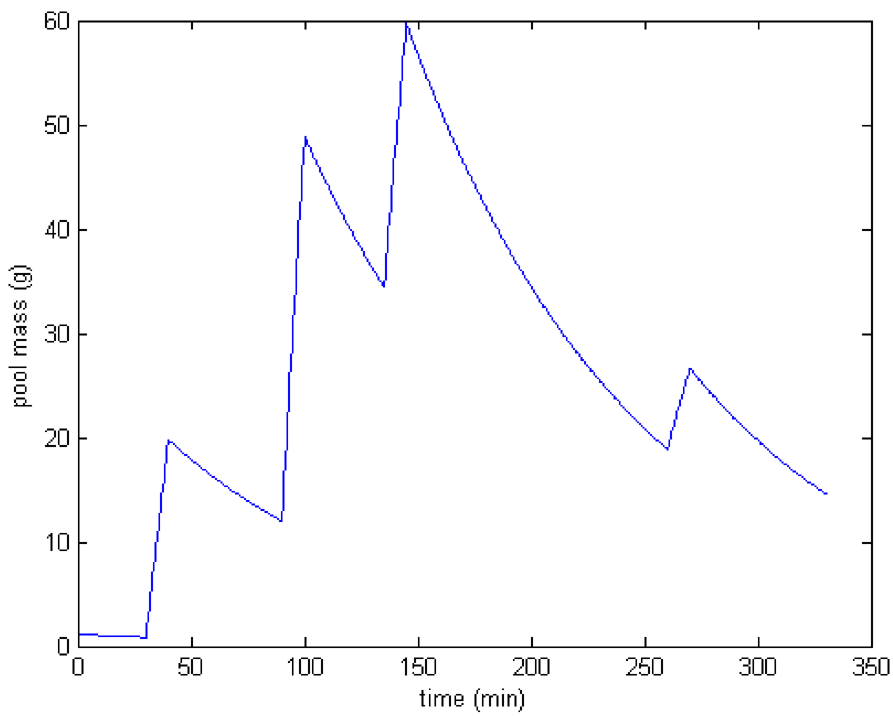

Figure 4. Time history of pool debris inventory for sample problem. Four unequal debris batches have been introduced over 10-min time periods.

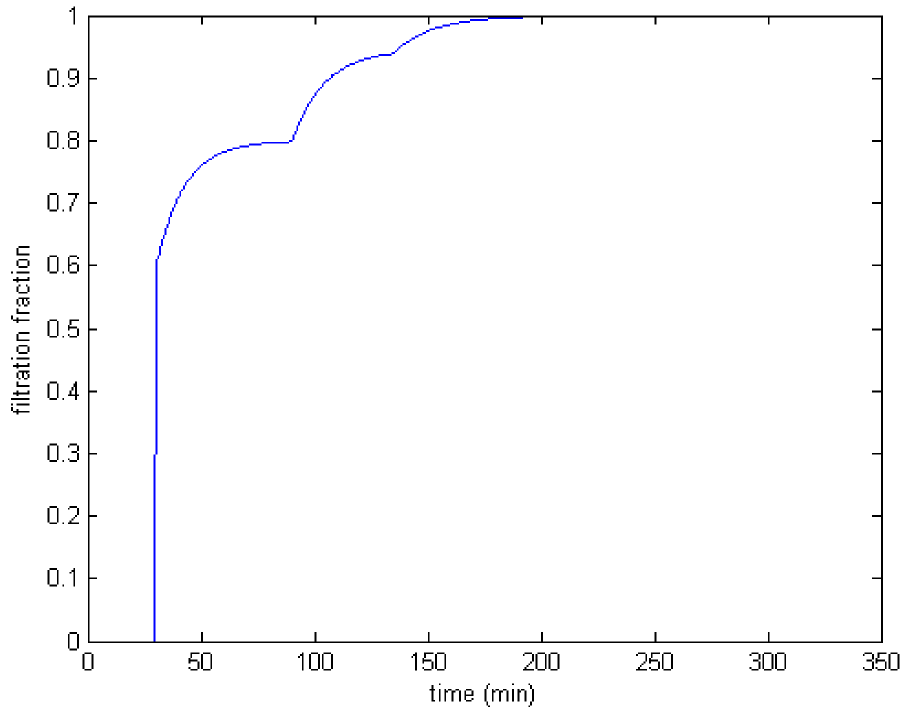

Figure 5. Example of time-dependent filtration function for assumed parameters.

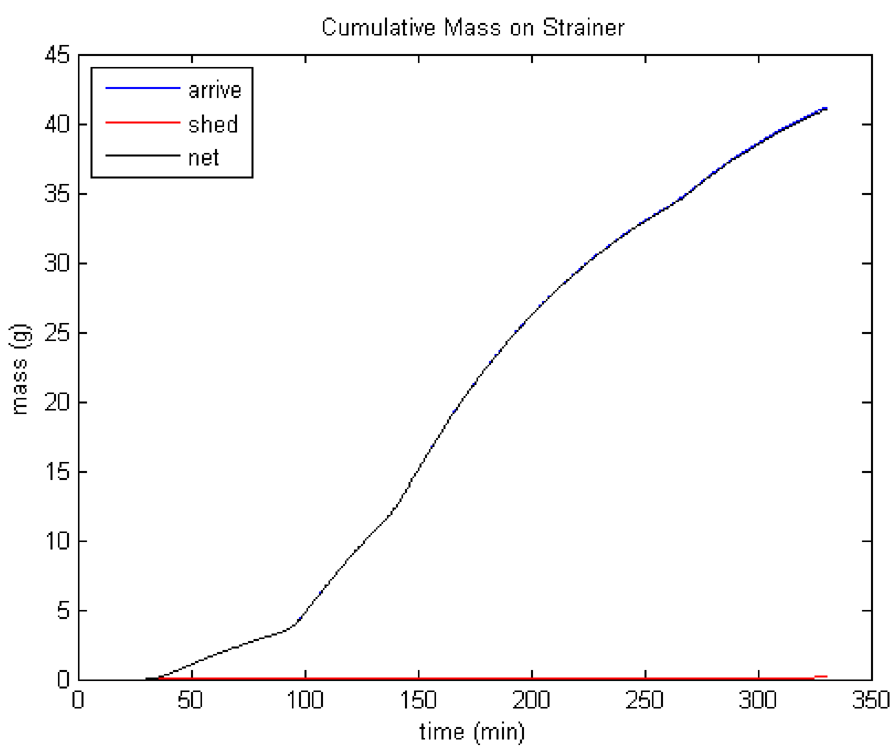

Figure 6. Example histories of mass arriving and shedding on the strainer.

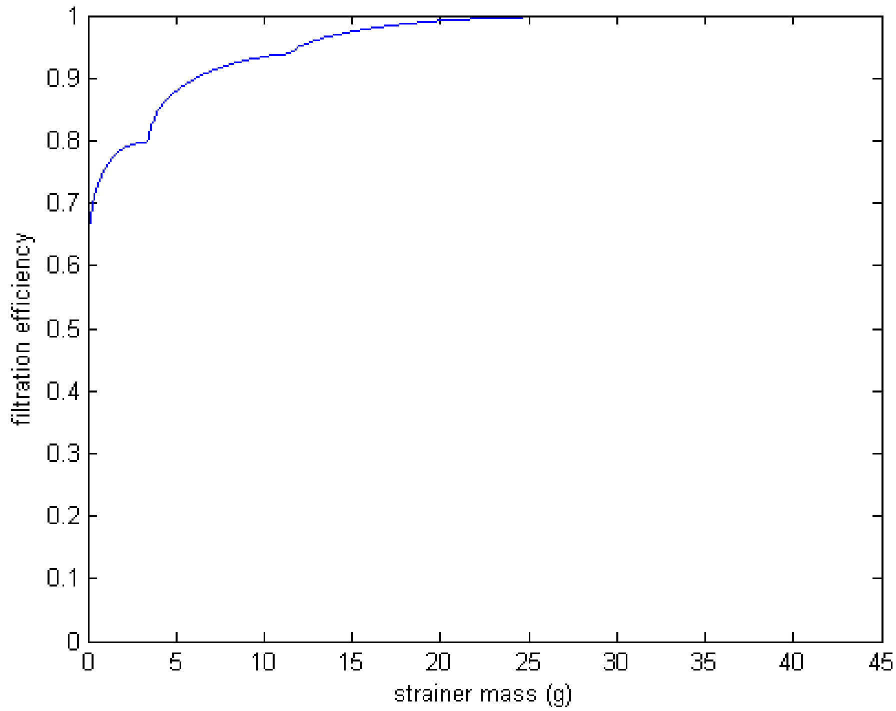

Figure 7. Example of correlation between assumed filtration function time history and accumulation of mass over several unequal batch intervals.

The final step in this example problem is to compute synthetic filtration data as defined by Eq. (27) according to the schedule defined in Table 1. Recall that bag filter accumulation estimated assuming $\gamma = 1$ must be corrected by subtracting any concurrent grab samples that have occurred. The sampling schedule in this example includes grab samples during each bag filter interval and one case where two bag filters span a single source period. The example does not include a case of multiple sources within a given bag filter period, but there is no inherent restriction in evaluating data of this type. Complications involved with analytic separation of the integrals into concurrent source and filter periods are now handled transparently by numeric evaluation of the integrands.

Table 2. Synthetic data for example problem.

	Bag (g)	Grab (g) (x 1000)
1	0.364	0.1191
2	0.4386	0.1132
3	0.543	0.5905
4	0.5664	0.5155
5	0.2869	0.4895
6		0.3735
7		0.3416
8		0.6141
9		0.2765
10		0.2607
11		0.1947