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Abstract

Implicit Monte Carlo Diffusion (IMD) [1] has been shown to be an efficient approach to evaluating radiative transfer in opaque media. This work demonstrates the implementation of IMD in a coupled system with the hydrodynamic equations. A radiating shock problem with a semi-analytic solution is used to demonstrate the ability of IMD to produce an accurate solution in a mixed frame.

Introduction

The original discretization developed by Fleck and Cummings [2] can accurately resolve the radiation fields without using a non-linear iteration. This set of semi-implicit equations creates a new type of interaction known as ”effective scattering” which represents the absorption and re-emission of a photon over a time step. In opaque media the effective scattering can begin to dominate particle interactions. In these thick scattering regions of a problem standard Monte Carlo approaches, such as IMC, become prohibitively expensive. As a result, many methods have been developed to accelerate the solution of these equations in opaque materials [1, 3].

Asymptotic analysis has shown that diffusion can accurately predict the radiative transport solution in a thick diffuse limit [4]. Implicit Monte Carlo Diffusion generates a statistical solution to a discretized set of diffusion equations using the Monte Carlo method.

Radiation Hydrodynamics

The frequency-independent mixed-frame radiation diffusion hydrodynamic equations can be written as [5]:

$$\frac{\delta \rho}{\delta t} + \nabla(\rho \nu) = 0 \tag{1}$$

$$\frac{\delta(\rho\nu)}{\delta t} + \nabla\left(\rho\nu^2 + p + \frac{1}{3}E\right) = 0 \tag{2}$$

$$\frac{\delta(\rho\epsilon_m)}{\delta t} + \nabla\left(\nu(\rho\epsilon_m + p)\right) = c\kappa_a f E - ca\kappa_a f T_m^4 - \frac{1}{3}\nu\nabla E \tag{3}$$

$$\frac{\delta E}{\delta t} + \frac{4}{3}\nabla\left(\nu E\right) - c\nabla\frac{1}{3\kappa_t}\nabla E = ca\kappa_a f T_m^4 - c\kappa_a f E + \frac{1}{3}\nu\nabla E \tag{4}$$

The variables in these equations are as follows: ρ is the material density, ν is the material velocity, p is the material pressure, E is the gray photon energy density, ϵ_m is the material energy density, c is the speed of light, a is the radiation constant, κ is the gray material opacity where the subscripts a and t denote the absorption and total opacity, f is the fleck factor, and T_m is the material temperature. This tightly coupled set of equations accounts for the conservation of mass (Eq. 1), conservation of momentum (Eq. 2), and the conservation of energy (Eq. 3 and Eq. 4).

In this work the radiative transfer equation is broken down into three separate equations via operator splitting; the radiation interactions:

$$\frac{\delta E}{\delta t} + -c\nabla\frac{1}{3\kappa_t}\nabla E = ca\kappa_a f T_m^4 - c\kappa_a f E, \tag{5}$$

the work done to the radiation field by the flow:

$$\frac{\delta E}{\delta t} + \frac{4}{3}\nabla(\nu)E = 0, \tag{6}$$

and the advection of the radiation field with the matter:

$$\frac{\delta E}{\delta t} - \nu\nabla E = 0. \tag{7}$$

The radiation interactions (Eq. 5) are evaluated via an IMD solver using a seconded order finite volume discretization of the diffusion operator. The equation for the work done to the radiation field (Eq. 6) can be transformed from a dependence on material velocity to dependence on material volume. Integrating this newly transformed equation over a time step results in the change in energy density associated with the work down to the radiation field by the fluid:

$$E^{n+1} = E^n\left(\frac{V^n}{V^{n+1}}\right)^{4/3}. \tag{8}$$

Finally the advection of the radiation field (Eq. 7) with the matter is handled by the Lagrangian hydrodynamics.

These split equations are solved by first evaluating Lagrangian hydrodynamics. The work added to the radiation field is evaluated using equation Eq. 8. The newly defined energy density is then used in the IMD simulation to evaluate the radiation diffusion equation. The resulting radiation field is then used to inform the hydrodynamics equations for then next time step.

Test Case

The semi-analytic radiation diffusion shock benchmark developed by Lowrie and Edwards [5] is used to verify the IMD method. This is a radiating shock problem with a Mach number of 45. The pre-shock and post shock conditions are listed in Table I. The material properties are listed in Table II.

Table I: Pre and post shock conditions.

Properties	Pre-Shock	Post-Shock	Units
ρ	1	6.426141724426906	[g/cc]
ν	0	$-4.817525855517521 \times 10^8$	[cm/s]
T	100	8.357785413124984	[keV]

Table II: Homogeneous material properties.

Properties	Value	Units
κ	$4.493983839817290 \times 10^8 \rho T^{-3.5}$	[cm$\hat{2}$/g]
κ_s	0.4006	[cm$\hat{2}$/g]
γ	5.0/3.0	Unitless
Cv	0.1446718127999906	[jrk/(g keV)]

Results

This work used the Kull [6] software package with the newly implemented IMD radiation package. The initial sonic point, material Mach number equal to 3, is set at $x = 2300$ [cm]. Where the problem domain is set such that $0 \le x \le 2500$ [cm] using 5000 initially equally spaced cells. The problem was ran to a final time of 4e-8 [sec] using 10e+6 IMD particle histories per time step. The time step size is confined such that $1 \times 10^{-13} \le \Delta t \le 1 \times 10^{-10}$.

The material temperature and radiation temperature profiles are shown in Fig. 1. The temperatures evaluated in our models are slightly depressed at the head of the shock wave.

Conclusions

This work implemented IMD into a mixed frame radiation hydrodynamics package. The method was tested using a semi-analytic radiative diffusion shock developed by Lowrie [5]. It was found that the IMD method produces an accurate result to this radiating shock test case.

Acknowledgements

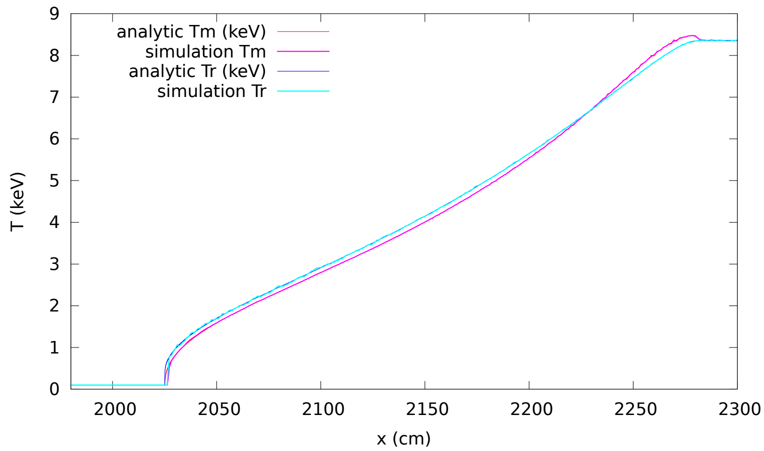

Figure 1: Material and radiation temperature profiles at 4e-8 [sec]

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