

**EVALUATION OF REACTOR KINETIC PARAMETERS
WITHOUT THE NEED FOR PERTURBATION CODES***

M. M. Bretscher

Argonne National Laboratory
Argonne, Illinois 60439-4841 USA

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SEP 16 1997
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To be presented at the
1997 International Meeting on Reduced Enrichment
for Research and Test Reactors

October 5-10, 1997
Jackson Hole, Wyoming USA

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*Work supported by the U.S. Department of Energy
Office of Nonproliferation and National Security
under Contract No. W-31-109-ENG-38

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ABSTRACT

The analysis of research reactor transients depends on the effective delayed neutron fraction (β_{eff}), its family-dependent components ($\beta_{\text{eff},i}$), the prompt neutron lifetime (l_p), and the decay constants (λ_i) for each delayed neutron family. Beginning with ENDF/B-V data, methods are presented for accurately calculating these kinetic parameters within the framework of diffusion theory but without the need for a perturbation code. For heavy water systems these methods can be extended to include the delayed photoneutron component of β_{eff} . However, a separate calculation is needed to estimate the fractional loss of fission product gamma rays, energetic enough to dissociate the deuteron, from leakage, energy degradation and absorption in fuel and structural materials. These methods are illustrated for a light-water Oak Ridge Research Reactor (ORR) LEU core and for a heavy-water Georgia Tech Research Reactor (GTRR) HEU core where calculated and measured values of the prompt neutron decay constant (β_{eff}/l_p) are compared.

1. INTRODUCTION

Safety analyses of research reactors include a determination of control rod worths and kinetic parameters upon which the dynamic performance of the reactor depends. Methods for calculating control rod worths are discussed in a recently up-dated technical report.¹ This paper focuses on procedures for accurately calculating the prompt neutron lifetime (l_p) and family-dependent effective delayed fission neutron fractions ($\beta_{\text{eff},i}$) without the need for a perturbation theory code. This β_{eff} method may be extended to include delayed photoneutrons in heavy water reactors.

These techniques have been used to calculate the prompt neutron lifetime and family-dependent effective delayed fission neutron fractions for an H_2O -cooled and -reflected Oak Ridge Research Reactor (ORR) with an LEU core and for the D_2O Georgia Tech Research Reactor (GTRR) with HEU fuel. The β_{eff} results agree with VARI3D² perturbation calculations and calculated prompt neutron decay constants (β_{eff}/l_p) compare favorably with directly measured values based on reactor noise and pile oscillator techniques. These computational methods and applications are discussed below.

2. COMPUTATIONAL METHODS

2.1 Prompt Neutron Lifetime

The $1/v$ insertion method³ is a simple but accurate way to calculate the prompt neutron lifetime. If the entire reactor (including the reflector) is perturbed by a dilute and uniform distribution of a purely $1/v$ neutron absorber, the fractional change in the eigenvalue is

$$\delta k / k_p = k_o \int_V \left[\sum_j \phi_j \phi_j^* \delta \Sigma_a \right] dV / PD = N \sigma_{ao} v_o k_o \int_V \left[\sum_j \phi_j \phi_j^* / v_j \right] dV / PD = N \sigma_{ao} v_o l_p$$

where N is the concentration (atoms/b-cm) of the $1/v$ absorber whose absorption cross section is σ_{ao} for neutrons of speed v_o . The eigenvalues for the perturbed and unperturbed configurations are k_p and k_o , respectively. The prompt neutron lifetime is obtained from this equation in the limit as N approaches zero.

$$l_p = \lim_{N \rightarrow 0} l_p' = \lim_{N \rightarrow 0} \frac{\delta k}{k_p} / N \sigma_{ao} v_o$$

A good approximation for a purely $1/v$ absorber is ^{10}B for which $\sigma_{ao} = 3837$ barns at $v_o = 2200$ m/sec. To determine l_p from this equation a multigroup diffusion theory code, such a DIF3D⁴, is used to calculate k_o and k_p eigenvalues for very dilute concentrations of ^{10}B (in the range of 10^{-9} to 10^{-8} atoms/b cm) and tight eigenvalue convergence requirements. For this purpose infinitely dilute ^{10}B multigroup cross sections need to be generated for each reactor region (homogenized fuel, side plates, control rods and control rod channels, reflectors, etc.). The prompt neutron lifetime is obtained by linearly extrapolating the concentration-dependent l_p' values to a zero ^{10}B concentration.

VARI3D calculations for the prompt neutron lifetime are not as accurate as those obtained from this $1/v$ insertion method because this perturbation code uses a single set of group-dependent neutron velocities and not region-dependent values. Numerical values for the prompt neutron lifetime are given in Section 3 for a light-water ORR LEU core configuration and for a heavy-water GTRR HEU core. In this section VARI3D results are compared with those obtained by the $1/v$ insertion method.

2.2 Effective Delayed Fission Neutron Fractions, ${}^{DFN}\beta_{eff,i}$

Since the neutron multiplication factor for a reactor is proportional to the total fission yield (v), the multiplication factor for prompt neutrons only is proportional to $(v - {}^{DFN}E_{ave} v_d) = v(1 - {}^{DFN}\beta_{eff})$. Here v_d is the total delayed fission neutron yield and ${}^{DFN}E_{ave}$ is the average effectiveness of these delayed neutrons. The total effective delayed neutron fraction is therefore

$${}^{DFN}\beta_{eff} = (k - k_p)/k = \sum_i {}^{DNF}\beta_{eff,i}$$

where k is the eigenvalue for all neutrons and k_p is the eigenvalue for prompt neutrons only. Similarly, for the i_{th} family of delayed fission neutrons

$${}^{DFN}\beta_{eff,i} = (k - k_{p,i})/k \quad (1)$$

where $k_{p,i}$ is the multiplication factor without contributions from the i_{th} delayed neutron family.

To calculate $k_{p,i}$ the effects of the i_{th} family of delayed fission neutrons are removed from the multigroup cross section set. This is accomplished by adjusting fission yield and fission spectrum values⁵. For fissionable isotope m and energy group j , the adjusted fission yield is

$${}^m v_{p,j} = {}^m v_j - {}^m v_{d,i} \quad (2)$$

and the adjusted fission spectrum becomes

$${}^m \chi_{p,j} = ({}^m \chi_j - {}^m f_{d,i,j} {}^m v_{d,i} / {}^m v_j) {}^m N \quad (3)$$

where ${}^m N$ is a normalization factor chosen so that $\sum_j {}^m \chi_{p,j} = 1.0$ and where ${}^m f_{d,i,j}$ is the fraction of the i_{th} family of delayed fission neutrons emitted into energy group j .

Tables 1-4 provide ENDF/B-V delayed fission neutron data for ${}^{235}U$ and ${}^{238}U$ needed for the fission yield and fission spectrum adjustments in Eqs. 2 and 3. The delayed fission neutron fractions, ${}^m f_{d,i,j}$, are obtained by numerical integration of the probability functions $P(E')$ given in Tables 3 and 4. Note that $P(E') dE'$ is the probability that the delayed fission neutrons are emitted with energies (in MeV) between E' and $E' + dE'$.

For this study multigroup cross sections were generated using the WIMS-D4M code⁶. Fission yield and fission spectrum adjustments (Eqs. 2 and 3) were made by converting the initial multigroup cross section set from binary to ascii, making the required ${}^m v_{p,j}$ and ${}^m \chi_{p,j}$ changes, and converting the altered cross section set back to binary. Note that for WIMS-D4M the unaltered fission spectrum is that for ${}^{235}U$ only. Eigenvalues needed to determine $\beta_{eff,i}$ were calculated using these modified cross section sets and the DIF3D⁴ code.

2.3 Effective Delayed Photoneutron Fraction, ${}^{DPN}\beta_{eff}$

For heavy water reactors there are both delayed fission neutrons (DFN) and delayed photoneutrons (DPN) which contribute to the total effective delayed neutron fraction. In principle, the β_{eff} - method discussed in the previous section for finding ${}^{DFN}\beta_{eff}$ can be modified to determine ${}^{DPN}\beta_{eff}$. Since delayed photoneutrons are not present in the initial cross section set, their effects must be added in the adjusted cross section set. For this case

$${}^{DPN}\beta_{eff} = (k_a - k)/k_a \quad (4)$$

where k_a is the eigenvalue based on the cross section set adjusted for delayed photoneutron effects. The adjusted fission yield becomes

$${}^m v_{a,j} = {}^m v_j + F_\gamma v_{dpn} \quad (5)$$

Table 1

Delayed Neutron Data (ENDF/B-V) for the Thermal Fission of ^{235}U

| Family i | Decay Const. λ_i (sec ⁻¹) | Yield - $v_{d,i}$ neutrons/fiss | Relative Yield | Fraction $\beta_i = v_{d,i} / v^*$ |
|-------------|--|------------------------------------|-------------------|---------------------------------------|
| 1 | 1.272E-2 | 6.346E-4 | 0.038 | 2.604E-4 |
| 2 | 3.174E-2 | 3.557E-3 | 0.213 | 1.460E-3 |
| 3 | 1.160E-1 | 3.140E-3 | 0.188 | 1.288E-3 |
| 4 | 3.110E-1 | 6.797E-3 | 0.407 | 2.789E-3 |
| 5 | 1.400E+0 | 2.138E-3 | 0.128 | 8.773E-4 |
| 6 | 3.870E+0 | 4.342E-4 | 0.026 | 1.782E-4 |
| Total: | | 1.670E-2 | 1.000 | 6.853E-3 |

* For ^{235}U thermal fission, $v = 2.4367$ total neutrons/fission.

Note: For $E_n \geq 7.0$ MeV, v_d (total) = 9.000E-3 delayed neutrons/fission but with the above relative yields.

Table 2

Delayed Neutron Data (ENDF/B-V) for ^{238}U Fission
Induced by a Prompt-Neutron Spectrum

| Family i | Decay Const. λ_i (sec ⁻¹) | Yield - $v_{d,i}$ neutrons/fiss | Relative Yield | Fraction $\beta_i = v_{d,i} / v^*$ |
|-------------|--|------------------------------------|-------------------|---------------------------------------|
| 1 | 1.323E-2 | 5.720E-4 | 0.013 | 2.055E-4 |
| 2 | 3.212E-2 | 6.028E-3 | 0.137 | 2.166E-3 |
| 3 | 1.390E-1 | 7.128E-3 | 0.162 | 2.561E-3 |
| 4 | 3.590E-1 | 1.707E-2 | 0.388 | 6.133E-3 |
| 5 | 1.410E+0 | 9.900E-3 | 0.225 | 3.556E-3 |
| 6 | 4.030E+0 | 3.300E-3 | 0.075 | 1.186E-3 |
| Total: | | 4.400E-2 | 1.000 | 1.581E-2 |

* For $v = 2.7836$ total neutrons/fission.

Note: For $E_n \geq 9.0$ MeV, v_d (total) = 2.600E-2 delayed neutrons/fission but with the above relative yields.

Table 3

**Delayed Neutron Energy Distributions, $P(E')$, for ^{235}U Fission
(ENDF/B-V)**

| E' - MeV | Family-Dependent $P(E')$ Values | | | |
|--------------------------|---------------------------------|----------|----------|--------------|
| | Family 1 | Family 2 | Family 3 | Families 4-6 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0797 | 2.7004 | 1.0986 | 2.1511 | 1.3288 |
| 0.0886 | 2.7752 | 1.1069 | 2.6107 | 1.3659 |
| 0.0984 | 3.0909 | 1.1482 | 2.3564 | 1.4401 |
| 0.1094 | 3.2529 | 1.2087 | 2.1296 | 1.4864 |
| 0.1215 | 3.3900 | 1.2528 | 1.9948 | 1.5080 |
| 0.1350 | 3.4066 | 1.2473 | 1.4831 | 1.5297 |
| 0.1500 | 3.3734 | 1.2721 | 1.3483 | 2.0396 |
| 0.1667 | 2.6921 | 1.2886 | 1.1797 | 2.0303 |
| 0.1852 | 2.9496 | 1.1261 | 0.8886 | 1.7707 |
| 0.2058 | 3.0784 | 1.0959 | 0.8304 | 1.7243 |
| 0.2287 | 2.4927 | 0.9940 | 1.1062 | 1.6162 |
| 0.2541 | 2.0440 | 1.4648 | 1.0235 | 1.5297 |
| 0.2823 | 1.2837 | 1.5419 | 1.0235 | 1.5977 |
| 0.3137 | 0.7935 | 1.2969 | 1.1859 | 1.4710 |
| 0.3485 | 0.7603 | 1.3271 | 1.5137 | 1.2021 |
| 0.3872 | 1.0428 | 1.3189 | 1.3728 | 1.1878 |
| 0.4303 | 0.5692 | 1.4923 | 1.2165 | 1.0321 |
| 0.4781 | 0.2742 | 1.3354 | 1.1797 | 1.0136 |
| 0.5312 | 0.2160 | 1.3024 | 0.9438 | 0.9858 |
| 0.5902 | 0.2368 | 0.8508 | 0.7998 | 0.7293 |
| 0.6558 | 0.2493 | 0.8866 | 0.6067 | 0.4790 |
| 0.7287 | 0.3282 | 0.7710 | 0.4750 | 0.7015 |
| 0.8096 | 0.2617 | 0.5204 | 0.3493 | 0.5408 |
| 0.8996 | 0.1412 | 0.2698 | 0.3554 | 0.3307 |
| 0.9995 | 0.0789 | 0.2203 | 0.2880 | 0.1452 |
| 1.1106 | 0.0748 | 0.0881 | 0.2298 | 0.1236 |
| 1.2340 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| E'_{ave} (MeV): | 0.275 | 0.426 | 0.418 | 0.402 |

Table 4

**Delayed Neutron Energy Distributions, $P(E')$, for ^{238}U Fission
(ENDF/B-V)**

| E' - MeV | Family-Dependent $P(E')$ Values | | | | |
|-------------------|---------------------------------|----------|----------|----------|--------------|
| | Family 1 | Family 2 | Family 3 | Family 4 | Families 5-6 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0797 | 2.7533 | 0.7684 | 2.3911 | 1.4081 | 1.8365 |
| 0.0886 | 3.1993 | 0.6656 | 2.3730 | 1.4308 | 2.1273 |
| 0.0984 | 3.3331 | 0.8388 | 2.2644 | 1.6283 | 2.2769 |
| 0.1094 | 4.3104 | 0.8523 | 2.0745 | 1.4696 | 2.3101 |
| 0.1215 | 4.2941 | 0.8712 | 2.1649 | 1.7804 | 2.3267 |
| 0.1350 | 2.9763 | 1.1959 | 1.4292 | 2.0653 | 2.1273 |
| 0.1500 | 2.3154 | 1.2121 | 0.9407 | 2.2692 | 1.9030 |
| 0.1667 | 1.3827 | 1.3528 | 1.3176 | 1.8937 | 1.8032 |
| 0.1852 | 2.0477 | 1.3014 | 1.6976 | 1.8128 | 1.6204 |
| 0.2058 | 2.1369 | 1.3420 | 1.6192 | 1.8160 | 1.6287 |
| 0.2287 | 2.3073 | 1.3014 | 1.5468 | 1.7383 | 1.5207 |
| 0.2541 | 1.7355 | 1.2581 | 1.1307 | 1.6671 | 1.3877 |
| 0.2823 | 1.4435 | 1.3528 | 0.7508 | 1.5765 | 1.3213 |
| 0.3137 | 0.7299 | 1.3609 | 0.5488 | 1.5797 | 1.3213 |
| 0.3485 | 0.9367 | 1.9264 | 0.6030 | 1.4243 | 1.3462 |
| 0.3872 | 1.3097 | 1.9129 | 0.7930 | 1.0747 | 1.3046 |
| 0.4303 | 0.8475 | 1.8453 | 0.8111 | 1.1848 | 1.1883 |
| 0.4781 | 0.6407 | 1.6180 | 0.7568 | 1.1492 | 1.0221 |
| 0.5312 | 0.6245 | 1.0417 | 0.7930 | 1.0618 | 0.8559 |
| 0.5902 | 0.2230 | 0.6710 | 0.8684 | 0.7801 | 0.7728 |
| 0.6558 | 0.1946 | 0.5438 | 0.9377 | 0.1748 | 0.6316 |
| 0.7287 | 0.3122 | 0.7332 | 0.7236 | 0.4402 | 0.4737 |
| 0.8096 | 0.1784 | 0.4302 | 0.5669 | 0.3075 | 0.2992 |
| 0.8996 | 0.1338 | 0.2841 | 0.4704 | 0.2687 | 0.1745 |
| 0.9995 | 0.0973 | 0.2597 | 0.3045 | 0.2428 | 0.1330 |
| 1.1106 | 0.0892 | 0.1136 | 0.2804 | 0.1327 | 0.0665 |
| 1.2340 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| E'_{ave} (MeV): | 0.284 | 0.438 | 0.434 | 0.379 | 0.359 |

and the ^{235}U adjusted fission spectrum is

$$\chi_{a,j} = (\chi_j + f_{dpn,j} F_\gamma v_{dpn} / v_j) N. \quad (6)$$

In these equations F_γ is the fraction of fission product gamma rays emitted in the fuel with energies above the $D(\gamma, n)$ threshold (2.226 MeV) which are available for photoneutron production in the D_2O channels. Thus, $(1 - F_\gamma)$ is the fraction lost by absorption, energy degradation, and leakage effects. The total delayed photoneutron yield is v_{dpn} and the fraction of these emitted into energy group j is $f_{dpn,j}$.

Delayed photoneutron parameters for ^{235}U fission product gamma rays on D_2O are taken from Ref. 7 and are reproduced in Table 5. These data have been corrected for gamma ray absorption, energy degradation, and leakage and so represent maximum photoneutron yields for which $F_\gamma = 1$. To correct for incomplete saturation, the β -values in this table need to be multiplied by $[1 - \exp(-\lambda_j t_e)]$ where t_e is the effective irradiation time. For the heavy water GTRR a combination of ORIGEN⁸ and MCNP⁹ gamma source calculations were done to estimate F_γ , the spectrum of delayed photoneutrons in the D_2O channels, and the delayed photoneutron fraction $f_{dpn,j}$.

Table 5

Delayed Photoneutron Data* for ^{235}U Fission Product Gamma Rays on D_2O

| Family j | Half-Life | Decay Const. $\lambda_j (\text{sec}^{-1})$ | Photoneutron Yield - $v_{dpn,j}$ n/fiss (10^{-5}) | Relative Yield | Fraction $\beta_j (10^{-5})$ |
|----------|-----------|---|--|----------------|---------------------------------|
| 1 | 12.8 d | 6.26E-7 | 0.12 | 0.0005 | 0.05 |
| 2 | 53 h | 3.63E-6 | 0.25 | 0.0010 | 0.103 |
| 3 | 4.4 h | 4.37E-5 | 0.78 | 0.0032 | 0.323 |
| 4 | 1.65 h | 1.17E-4 | 5.65 | 0.0231 | 2.34 |
| 5 | 27 m | 4.28E-4 | 5.01 | 0.0205 | 2.07 |
| 6 | 7.7 m | 1.50E-3 | 8.14 | 0.0333 | 3.36 |
| 7 | 2.4 m | 4.81E-3 | 17.0 | 0.0695 | 7.00 |
| 8 | 41 s | 1.69E-2 | 49.5 | 0.2025 | 20.4 |
| 9 | 2.5 s | 2.77E-1 | 158.0 | 0.6463 | 65.1 |
| Total: | | | 244.45 | 1.0000 | 100.75 |

* Data taken from Ref. 7. These data have been corrected for gamma ray absorption, energy degradation, and leakage effects and so represent maximum photoneutron yields for which $F_\gamma = 1$.

2.4 Decay Constants for Delayed Fission Neutron Families

Delayed fission neutron decay constants (λ_i) are given in Tables 1 and 2 for fission in ^{235}U and ^{238}U . For a given reactor a weighted sum of the isotopic decay constants is needed. The appropriate weighting factor is ^mF which is defined as the fraction of all fissions which occur in isotope m.

3. NUMERICAL ILLUSTRATIONS

3.1 Light-Water ORR LEU Core 179-AX5

These methods have been used to calculate I_p , β_{eff} and its family-dependent components for an Oak Ridge Research Reactor (ORR) core that was operated in 1986 as part of the Whole-Core LEU $\text{U}_3\text{Si}_2\text{-Al}$ Fuel Demonstration¹⁰. This H_2O -cooled and -reflected core (179-AX5) is fully described in Ref. 10 (pp.3-7, 79-81). The $\text{U}_3\text{Si}_2\text{-Al}$ fresh LEU core consisted of 14 standard 19-plate fuel elements (340 g ^{235}U /element) and four 15-plate fuel follower control elements (200 g ^{235}U /element) arranged in a 5 x 4 lattice with two vacant corner positions. For the diffusion calculations the four cadmium box-type shim/safety rods were withdrawn to the experimentally-determined critical elevation. Calculated and measured kinetic parameters for the ORR 179-AX5 core are summarized in Table 6 below.

Table 6

Comparison of Calculated and Measured Kinetic Parameters in the Light Water ORR Core 179-AX5 with 18 Fresh LEU Fuel Elements

| Parameter | VAR13D Perturbation Code | Calc'd (C) by Present Methods | Experiment (E) | C/E |
|--|--------------------------------|-------------------------------------|-----------------|-------|
| ${}^{\text{DFN}}\beta_{\text{eff}, 1}$ | 3.044E-4 | 3.033E-4 | | |
| ${}^{\text{DFN}}\beta_{\text{eff}, 2}$ | 1.694E-3 | 1.707E-3 | | |
| ${}^{\text{DFN}}\beta_{\text{eff}, 3}$ | 1.493E-3 | 1.512E-3 | | |
| ${}^{\text{DFN}}\beta_{\text{eff}, 4}$ | 3.246E-3 | 3.279E-3 | | |
| ${}^{\text{DFN}}\beta_{\text{eff}, 5}$ | 1.030E-3 | 1.047E-3 | | |
| ${}^{\text{DFN}}\beta_{\text{eff}, 6}$ | 2.120E-4 | 2.166E-4 | | |
| Total = ${}^{\text{DFN}}\beta_{\text{eff}}$ | 7.980E-3 | 8.064E-3 | | |
| $I_p - \mu\text{sec}$ | 38.18 | 41.55 | | |
| $\alpha \equiv \beta_{\text{eff}}/I_p\text{-sec}^{-1}$ | 209.0 | 194.1 | 192.3 ± 1.2 | 1.009 |

Table 6 shows that the family-dependent $\beta_{\text{eff},i}$ values obtained by these cross section adjustment methods agree with the VARI3D perturbation calculation to within about 1%. The VARI3D result for the prompt neutron lifetime is 38.18 $\mu\text{-sec}$ which is smaller and not as accurate as that obtained by the 1/v insertion method because the perturbation code uses a single set of group-dependent neutron velocities and not region-dependent values. The prompt neutron decay constant α is the ratio of the effective delayed neutron fraction β_{eff} to the prompt neutron lifetime τ_p . Using reactor noise techniques, Ragan and Michalczo¹¹ measured α for the ORR 179-AX5 core. Based on the non-perturbation methods discussed above, the calculated value of α is in good agreement with the measured one.

3.2 Heavy-Water GTRR HEU Core

The 5-MW heavy-water Georgia Tech Research Reactor (GTRR) is described in Ref. 12. Each of the HEU MTR-type fuel assemblies contains 16 fueled and 2 unfueled plates and about 188 g ^{235}U . Fuel assemblies are spaced 6 inches apart in a triangular array. During the startup phase of the GTRR, the prompt neutron decay constant ($\beta_{\text{eff}}/\tau_p$) was measured by the pile oscillator technique while β_{eff} was determined using a method of reactor noise analysis and an absolute calibration of reactor power. The measurements were done for a fresh 14-element core and are described in Ref. 13. Since this report does not include the elevation of the control rods during the measurements, they were not modeled in the calculations described below.

Using region-dependent 7-group WIMS-D4M ENDF/B-V cross sections, the prompt neutron lifetime was calculated for the 14-element core using the 1/v insertion method. From Eqs. 1-3 and data in Tables 1-4, ${}^{\text{DFN}}\beta_{\text{eff},i}$ was calculated for each delayed fission neutron family.

An estimate of the effective delayed photoneutron fraction was obtained in the following manner. A combination of ORIGEN⁸ and MCNP⁹ calculations was used to estimate the spectrum of the delayed photoneutrons in the D_2O coolant channels and to determine F_γ which, as mentioned earlier, is needed to correct the maximum photoneutron yields (Table 5) for γ -ray attenuation from leakage, absorption, and energy degradation effects. The ORIGEN calculation determined the energy distribution of fission product gamma rays after shutdown with energies above the deuterium (γ, n) threshold (2.226 Mev). For this calculation the irradiation time was long enough so that all the delayed photoneutron precursor activities were saturated prior to shutdown. This gamma ray energy distribution was used as a source term in the meat of each fuel plate in subsequent MCNP Monte Carlo photon calculations. For these γ -calculations the entire 14 fuel element assembly with thick axial and radial D_2O reflectors was modeled. Energy-dependent gamma ray fluxes resulting from the ORIGEN source terms were calculated for the D_2O coolant channels and for the heavy water surrounding each fuel element. For convenience, these fluxes were combined to determine the average gamma ray flux in the heavy water regions in and around the core. Two Monte Carlo calculations were needed to determine F_γ . The first was done using normal atom densities for all the materials and with leakage boundary conditions applied at the external surfaces. To determine the amount of gamma ray attenuation from leakage, energy degradation, and absorption in the core, a second Monte Carlo calculation was needed where fuel meat, clad, and side plate atom densities were reduced to effectively zero and where reflective boundary conditions were used to eliminate γ -leakage effects. F_γ is the ratio of the fission product gamma ray fluxes in the D_2O

regions from these two MCNP calculations for energies above 2.226 MeV. In this way it was found that $F_\gamma = 0.557$. The energy spectrum of the delayed photoneutrons was calculated from the energy-dependent γ -fluxes in the D_2O channels and from the $D(\gamma, n)$ cross sections for the photodisintegration of the deuteron (see Ref. 7, p.143). From this spectrum the delayed photoneutron fraction $f_{dpn,j}$ was calculated as was the average energy of the delayed photoneutrons which was found to be 0.51 Mev. This average energe is larger but not very different from the average energy of the delayed fission neutrons (see Tables 3 and 4). With these values for F_γ and $f_{dpn,j}$ and data in Table 5 Eqs. 4-6 were used to estimate ${}^{DPN}\beta_{eff}$.

Table 7 summarizes the calculations for the prompt neutron lifetime and the effective delayed neutron fractions (fission and photo). The effective delayed fission neutron fractions are in very good agreement with the VARI3D perturbation results. However, VARI3D does not calculate effective delayed photoneutron fractions. For reasons stated earlier, the VARI3D calculation for the prompt neutron lifetime is too small.

| Parameter ^a | VARI3D Perturbation Code | Calc'd (C) by Present Methods | Experiment ^b (E) | C/E | |
|--|--------------------------------|-------------------------------------|----------------------------------|-------|--|
| | | | | | |
| ${}^{DFN}\beta_{eff, 1}$ | 2.713E-4 | 2.721E-4 | | | |
| ${}^{DFN}\beta_{eff, 2}$ | 1.516E-3 | 1.521E-3 | | | |
| ${}^{DFN}\beta_{eff, 3}$ | 1.337E-3 | 1.339E-3 | | | |
| ${}^{DFN}\beta_{eff, 4}$ | 2.896E-3 | 2.901E-3 | | | |
| ${}^{DFN}\beta_{eff, 5}$ | 9.110E-4 | 9.140E-4 | | | |
| ${}^{DFN}\beta_{eff, 6}$ | 1.851E-4 | 1.859E-4 | | | |
| $\Sigma_i {}^{DFN}\beta_{eff, i}$ | 7.116E-3 | 7.133E-3 | | | |
| ${}^{DPN}\beta_{eff}$ | | 5.734E-4 | | | |
| $\beta_{eff} \equiv {}^{DF}\beta_{eff} + {}^{DP}\beta_{eff}$ | | 7.706E-3 | 7.55E-3 $1\sigma > 3\%$ | 1.021 | |
| $l_p - \mu\text{sec}$ | 728.2 | 770.3 | 770.4 | 1.000 | |
| $\alpha \equiv \beta_{eff}/l_p - \text{sec}^{-1}$ | | 10.0 | 9.8 $1\sigma \approx (1-2\%)$ | 1.021 | |

^a Because this is an HEU system, the ${}^{DFN}\lambda_i$ decay constants are essentially those for ${}^{235}\text{U}$ fission given in Table 1. The ${}^{DPN}\lambda_i$ values for delayed photoneutrons are given in Table 5.

^b No errors are quoted in Ref. 13 for the β_{eff} and β_{eff}/l_p measurements. However, the measured value of β_{eff} depends on a combination of reactor noise analyses and an absolute calibration of the reactor power (or fission rate). Ref. 13 quotes an error of 5.6% for this calibration. Since β_{eff} depends inversely on the square root of the fission rate, the minimum error in the measurement of the effective delayed neutron fraction is 3%. Typical errors in pile oscillator measurements of β_{eff}/l_p are in the 1-2% range. The experimental value for the prompt neutron lifetime is inferred from the β_{eff} and β_{eff}/l_p measurements.

Although the calculated value for the total effective delayed neutron fraction is within the uncertainties of the measurement, the calculated prompt neutron decay constant falls somewhat outside the expected error bounds of the pile oscillator β_{eff}/l_p measurement. This suggests that there is more attenuation of the photoneutron fission product gamma ray precursors than are accounted for in the ORIGEN/MCNP calculations. Some of these gamma rays are lost in the GTRR cadmium shim/safety rods which were not included in the above calculations. This cadmium absorption would lower F_γ but probably not enough to reduce the β_{eff}/l_p C/E ratio to unity. However, these calculated kinetic parameters are thought to be accurate enough to analyze the transient behavior of the GTRR.

4. CONCLUSIONS

Non-perturbation methods are presented for calculating the effective delayed neutron fraction, its family-dependent components, and the prompt neutron lifetime. The β_{eff} method depends on a diffusion theory eigenvalue calculation in which the spectrum and the yield of fission neutrons are adjusted to remove delayed fission neutron contributions. Accurate results for the prompt neutron lifetime are obtained from the $1/v$ insertion method. These methods are illustrated by calculating β_{eff} and l_p values for the H_2O -moderated and H_2O -reflected ORR 179-AX5 core with LEU fuel and for the D_2O -moderated and D_2O -reflected GTRR core with HEU fuel. Family-dependent effective delayed fission neutron fractions obtained by these methods agree very well with VARI3D perturbation calculations for both the ORR and GTRR cores. For the ORR core the calculated prompt neutron decay constant (β_{eff}/l_p) was found to be in good agreement with the directly-measured value. For the GTRR, however, the calculated β_{eff}/l_p value, which includes delayed photoneutrons, is about 2.1% larger than the measured value.

The effectiveness of delayed photoneutrons in heavy water reactors depends on the attenuation of fission product gamma rays with energies greater than the binding energy of the deuteron and on the energy spectrum of photoneutrons in the D_2O . Using the ORIGEN code to determine a fission product γ -ray source distribution in the fuel meat, MCNP Monte Carlo photon calculations were performed to calculate the fission product gamma ray attenuation factor and to estimate, from $D(\gamma, n)$ cross sections, the photoneutron energy spectrum in the D_2O coolant channels. This procedure was used to calculate the effective delayed photoneutron fraction in the heavy-water GTRR with HEU fuel.

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