Title: Knudsen Layer Reduction of Fusion Reactivity

Author(s): Molvig, Kim
Hoffman, Nelson M.

Intended for: Publish popular article in NNSA Stockpile Stewardship Quarterly

Knudsen Layer Reduction of Fusion Reactivity

Kim Molvig and Nelson M. Hoffman

(The work is a collaboration with Eric M. Nelson, Brian J. Albright, Evan Dodd, George Zimmerman, and Ed Williams. The idea of Knudsen losses of fuel ions was first suggested by Robert B. Webster)

Los Alamos National Laboratory, Los Alamos, NM 87506

Introduction

Over-prediction of fusion energy yield has been a persistent enigma in inertial fusion systems for over 50 years. This paper presents a new piece of physics to help to resolve this puzzle. Our best (clean) theory and code predictions have systematically calculated higher energy yield from fusion than observed in experiments by factors of two and more. This yield over-prediction has traditionally been attributed to the "mix" of impurities into the fuel. Many models of such mix have been developed, refined and applied over the years. Nonetheless, current mix models fall short of being fully satisfactory. This work is motivated by the belief that it is missing physics beyond "mix" that is needed to account for yield over-prediction (although we certainly expect mix to play a role and have combined it with the new physics in simulating experiment). The Knudsen layer effect we describe briefly here (and published in *Physical Review Letters* [1]) is such a piece of physics that fundamentally reduces the fusion reactivity by depleting the high energy tail population of fuel ions that are primarily responsible for the yield.

Basic Theory of Fusion Reactivity and the Knudsen Layer

Nuclear fusion reactions at energies below several hundred keV involve quantum tunneling through the Coulomb barrier. The energy dependence of the fusion cross section, $\sigma_{fus}(E)$, primarily determined by the Gamow tunneling probability and thus proportional to,

$$\exp\left(-\sqrt{E_G / E}\right) \; ; \; E_G \equiv 2\pi^2 Z_1{}^2 Z_2{}^2 \left(e^2 / \hbar c\right) \mu c^2$$

where, E, is the center of mass energy, and, E_G, is the Gamow energy. For DT fusion the Gamow energy is 1183 keV. This is a rapidly rising function of energy. In the reactivity integral, this rapid increase of cross section competes with the rapid decrease of the *thermal equilibrium* ion distribution function, $f_i \approx \exp(E / T_i)$. This competition results in the Gamow peak in fusion reactivity plotted in Figure 1. The Gamow peak energy is, $\frac{E_0}{T_i} = \left(\frac{E_G}{4T_i}\right)^{1/3}$, well above the energy of the thermal ions.

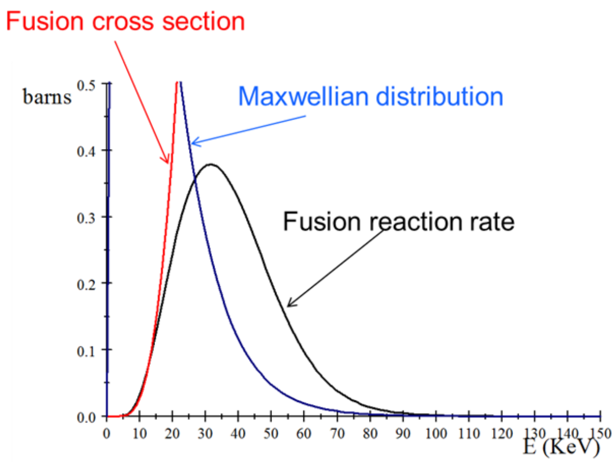

Figure 1: Competing energy dependences of Maxwellian ion distribution and fusion cross section determind Gamow peak in the fusion reactivity.

Inertially confined fusion systems typically have plasma fuel enveloped by a cold non-reacting region or "wall." Typically, thermal ions will have mean free paths for Coulomb scattering that are short compared to the distance to this wall. But owing to the Coulomb cross section energy variation going as, $\sigma_{Coul} \propto 1/E^2$, the mean free path for the fusing ions at the Gamow peak can be 10 to 50 times longer. These ions can reach the wall before undergoing a thermalizing collision and be lost, resulting in a depletion of the high energy tail ions that account for most of the fusion reactivity. In practical terms the way this works is shown in Figure 2. A reference display point at 80% radius is indicated. Half the fuel volume lies closer to the wall than this point. The circles surrounding the point indicate the mean free path for thermal and Gamow peak energy ions. The 2 keV case requires the zoomed in image to even see the radii, while at 10 keV temperatures, although the thermal ions are still not reaching the wall, a high fraction of the the Gamow peak ions do. This picture illustrates how Knudsen layer effect operates in practice. At ignition temperatures (as low as 2 keV in some systems) no effect will be seen. But as fusion proceeds to the higher temperatures of propagating burn (10 keV and above), one can expect significant reduction of fusion reactivity and therefore a reduction in fusion yield.

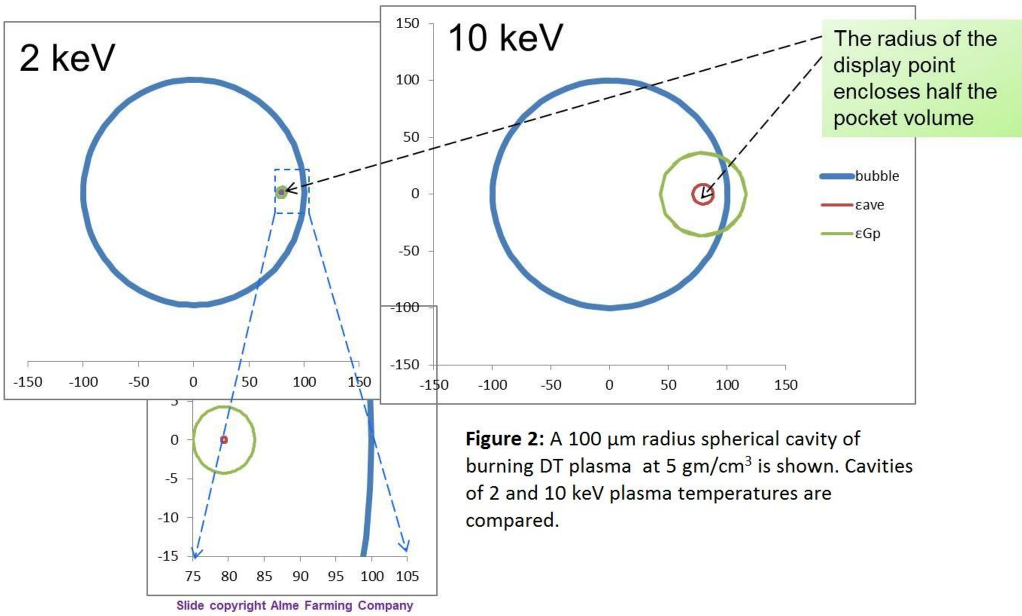

Figure 2: A 100 μm radius spherical cavity of burning DT plasma at 5 gm/cm³ is shown. Cavities of 2 and 10 keV plasma temperatures are compared.

We have developed a simple theory of the Knudsen tail effect by employing a *kinetic* spatial diffusion model, wherein the diffusion coefficient depended on particle energy and the diffusion operator was replaced by a local loss term of the same magnitude. We could then obtain a kinetic equation for the tail ion distribution function, and an approximate WKB solution.

$$C_D\left(f_i\right) \approx D\frac{\partial^2 f_i}{\partial x^2} = \frac{v^2}{\nu_{ii}\left(v\right)}\frac{\partial^2 f_i}{\partial x^2} \rightarrow -\frac{v^5}{\nu_{ii} v_{Ti}{}^3 L^2} f_i$$

$$0 = \frac{\partial}{\partial \varepsilon}\left[f + \frac{\partial f}{\partial \varepsilon}\right] - N_K^2 \varepsilon^3 f$$

Thus tail depletion depends on the Knudsen number, or ratio of thermal mean free path to distance from the wall.

$$N_K \equiv \frac{\lambda_i}{L} \approx 10^{-5}\frac{T_i^2}{\rho L}$$

$$f_K \approx \frac{2}{\sqrt{\pi + N_K \varepsilon^{3/2}}} \exp\left(-\varepsilon - 0.4 N_K \varepsilon^{5/2}\right)$$

From this simple expression one sees that while the Knudsen number may be small, the strong energy dependence can make it significant for energies around the Gamow peak.

From this distribution a fusion reactivity can be computed that is now a function of Knudsen

$$\left\langle \sigma_{fus} v \right\rangle \left(T_i, N_K\right) = \int_0^\infty \left(\sigma_{fus} v\right)\left(\varepsilon, T_i\right) f_K\left(\varepsilon, N_K\right) \varepsilon^{1/2} d\varepsilon$$

number. This implies a non-local reactivity that depends on distance from the wall.

Simulations of ICF capsule implosion experiments on OMEGA

We implemented this simple model in a radiation-hydrodynamics code and performed simulations of implosions of DT-filled ICF capsules conducted at the OMEGA laser [2] during 2005-2011. The implosions were selected to be highly diverse, spanning 2 1/2 orders of magnitude in observed yield and a factor of four in observed ion temperature. Fuel gas molar composition ranged from T:D = 0.55:1 to T:D = 585:1, more than three orders of magnitude. Several of the capsules were the deuterated-shell implosions and corresponding "reference" implosions analyzed by Wilson et al. [3]; these capsules give unambiguous evidence for ion-species mixing at the scale of an ion mean free path ("atomic mix"). Since it is clear that the deuterated shell yield requires mix we used both the Dimonte [4] mix model already in the code and the new combined Knudsen/mix model that contained both.

The model contained two parameters, the mix initial scale length and the fraction of absorbed laser energy, that were varied to best fit the 11 shots simulated. The Knudsen model was used without adjustable parameters. We used a normalized error metric to measure the deviation of simulation and experiment,

$$D_3^2 \equiv \frac{1}{N}\sum_{\text{shots } k}^{N}\left(\frac{\log_{10} Y_{obs,k}^2 - \log_{10} Y_{sim,k}^2}{\sigma_{\log Y}^2} + \frac{T_{ion,obs,k}^2 - T_{ion,sim,k}^2}{\sigma_{T,ion}^2} + \frac{t_{bang,obs,k}^2 - t_{bang,sim,k}^2}{\sigma_{bang}^2}\right)$$

in addition to the temperature and yield measurements, we used the additional observed quantity, the bang time (time of peak neutron production rate), to characterize the simulation. For each model, several hundred 1D simulations were run for each of the eleven capsules in Figure 3, spanning a grid varying the two input parameters: mix scale length and fractional laser source. We then identified the values of the input parameters giving the smallest value of RMS D_3, averaged over all of the capsules in Figure 3. The result was that for the Knudsen/mix model, RMS D_3 had a minimum value of 2.2 at a mix scale length of 0.15 µm, while for the mix model alone, RMS D_3 had a minimum value of 4.1 at a mix scale length of 0.4 µm.

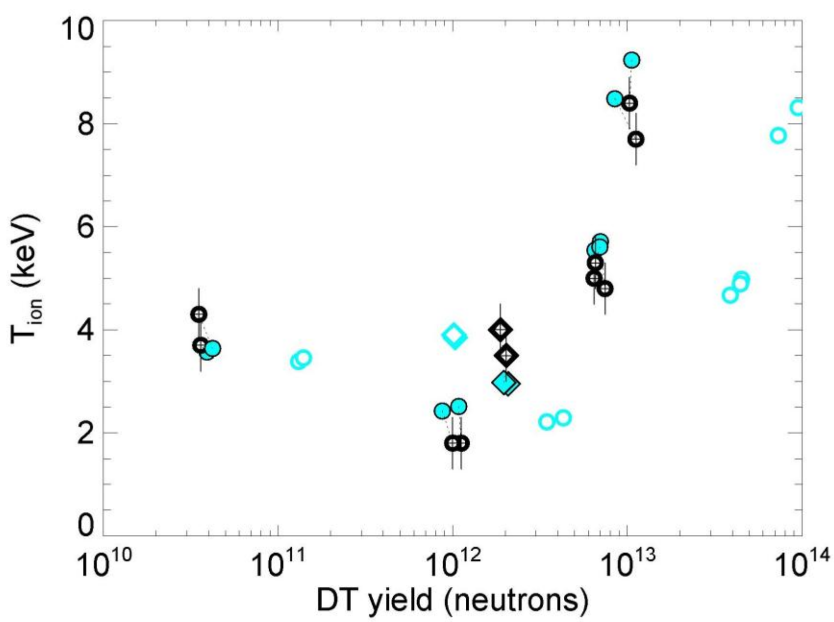

Figure 3. Comparison of observed (black symbols) and simulated (blue symbols) DT neutron yields and ion temperatures for the OMEGA capsules discussed in the text. Open blue symbols show "nominal" simulations. Filled blue symbols show simulations using Knudsen/mix model. Circles indicate ordinary plastic-shell capsules. Diamonds show deuterated-shell capsules. Knudsen/mix model gives markedly improved agreement with observations, compared to nominal.

For the hotter implosions, most of the observed yield reduction is accounted for by the Knudsen model. For cooler implosions, most of the yield reduction comes from mix, and the yield *increase* seen for the deuterated shells also comes from mix. Our Knudsen/mix model handles all of this variation automatically and naturally. The improved agreement between observations and simulations given by the Knudsen/mix model, compared to the mix model alone, is significant, indicating the greater explanatory and predictive power of the Knudsen/mix model.

In summary, we believe the Knudsen layer effect on reactivity to be an important new piece of physics in fusion burn that helps resolve the long standing enigma of over-prediction of yield. Much work needs be done to compute the effect accurately in complex geometry and with a much more detailed treatment of the "wall" interaction with the high energy fuel ions.

References

1. Molvig, K., Hoffman, N. M., Albright, B. J., Nelson, E. M., and Webster, R. B., "Knudsen Layer Reduction of Fusion Reactivity", Phys. Rev. Lett., **109**, 095001 (2012).
2. T. R. Boehly et al., Opt. Commun. **133**, 495 (1997).
3. D. C. Wilson, et al., Phys. Plasmas, **18**, 112707 (2011).
4. G. Dimonte, Phys. Plasmas, **7**, 2255 (2000).