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Dynamical Systems in Circuit Designer's Eyes

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Motivation

- Measurements and simulations become more and more powerful
- Present some insights based on combination of theory, measurements, and modeling
- Present some open questions

Contents

- Examples of nonlinear circuit design
- Design: focus on theory and engineering methods (as opposed to numerical analysis)

  "Goal of computing is insight, not numbers"

- Modeling related to measurements
  - Volterra series, ANN, resistive f:R -> R
  - f: R -> R vs. F: X -> X

Useful examples

1. Linear Resonator $$\ddot{x}+\alpha\dot{x}+\omega^2 x=u(t)$$

2. Van der Pol $$\ddot{x}+\varepsilon f'(x)\dot{x}+\omega^2 x=u(t)$$

3. Duffing $$\ddot{x}+\alpha\dot{x}+g(x)=u(t)$$

4. Pendulum $$\ddot{x}+\alpha\dot{x}+\sin(x)=u(t)$$

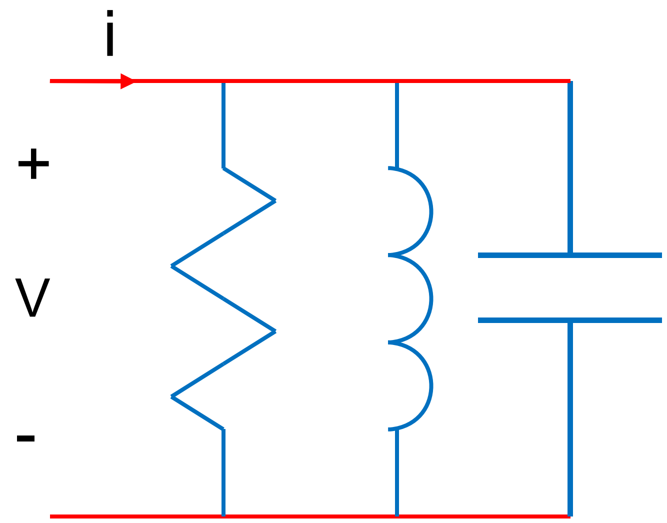
Resonator

$v(t)=Ve^{st}$; $i(t)=Ie^{st}$

$V_L = sL\ I_L$

$I_C = sC\ V_C$

$V_R = R\ I_R$

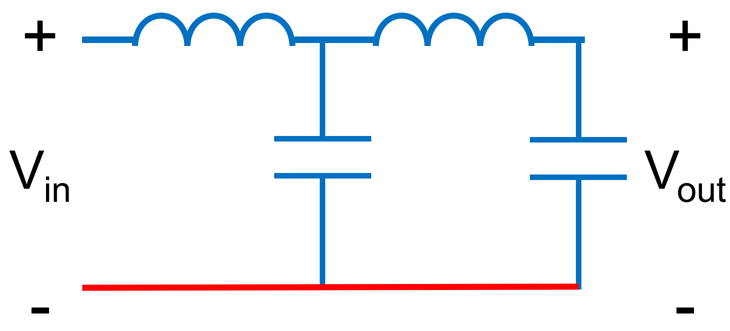


$$Z(s)=\frac{V}{I}=\frac{\alpha s}{s^2+\alpha s+\omega^2}$$

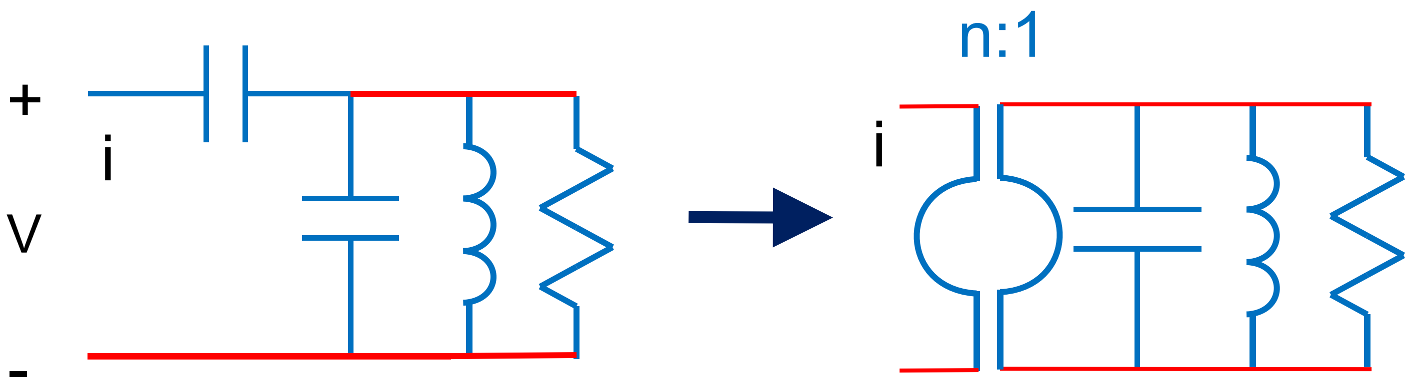
$$\ddot{x}+\alpha\dot{x}+\omega^2 x=u(t)$$

More engineering tricks

1. $$H(s) = \frac{1}{s^4 + \ldots + b_1 s + 1}$$

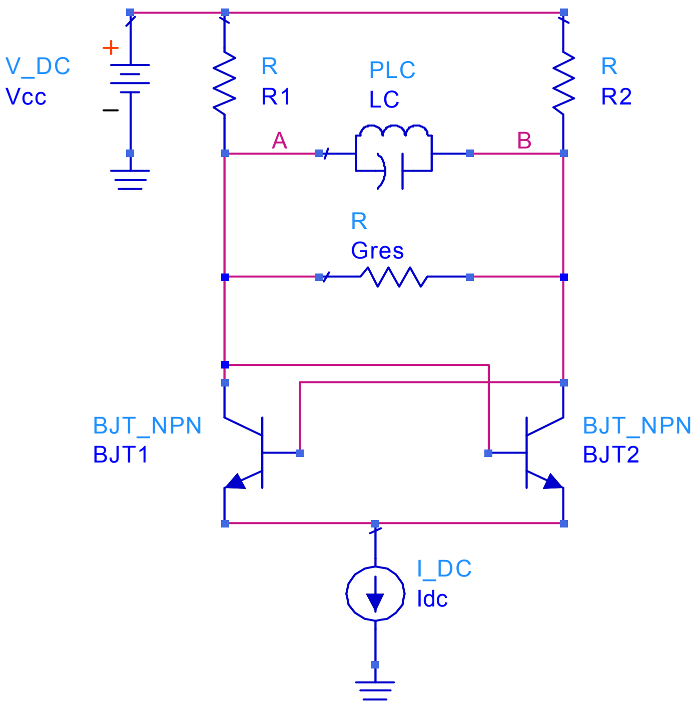


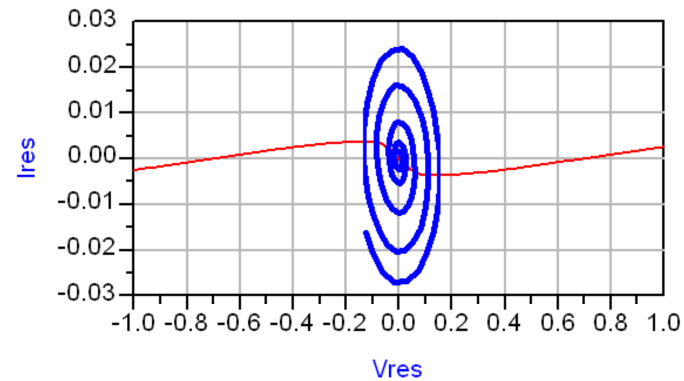
2. $$Z(\omega) \approx n^2 \frac{jRL\omega}{\left(1 - LC\omega^2 + j\omega L / R\right)}$$



Van der Pol

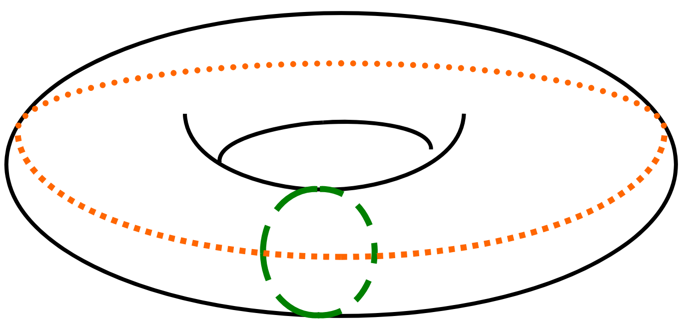
$$\ddot{x} + \varepsilon f'(x)\dot{x} + \omega^2 x = 0$$



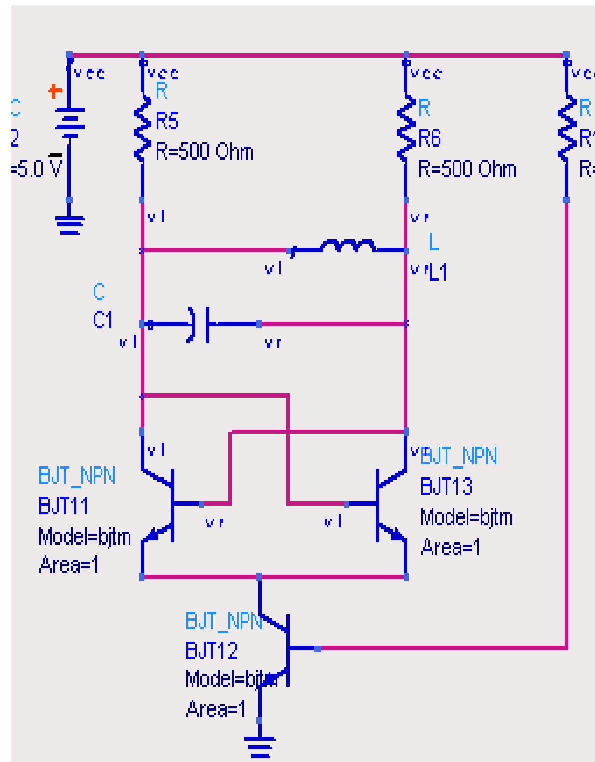


More dimensions

$$x(t) = A(\theta)\cos(\theta) + h(\theta, t, \varepsilon)$$



$$\dot{\theta} = \omega + \Theta(\theta, h(\theta), t, \varepsilon)$$



IC oscillator

Nonlinear design

- Harmonic balance – Galerkin method
- Measurements $y_n(A)$ and Models:
  - Harmonic balance
  - Volterra series
  - Neural networks
- Insight from harmonic balance
- Averaging

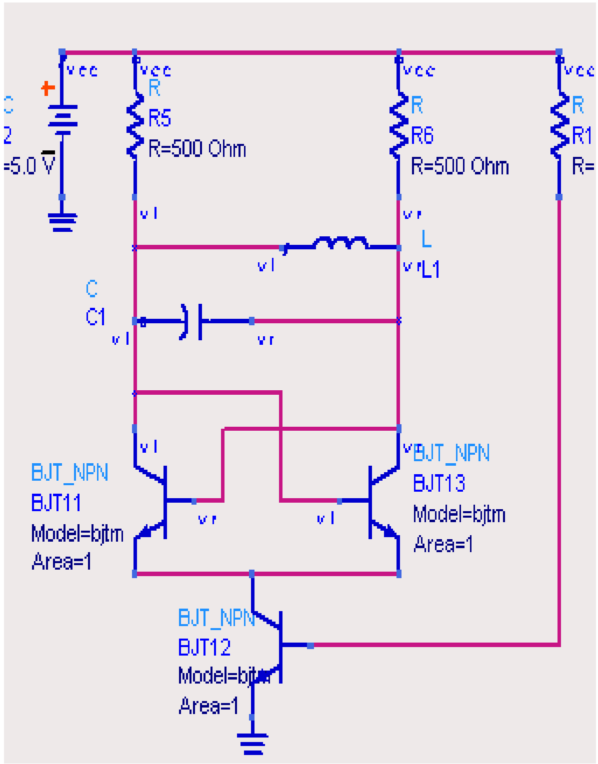
Harmonic balance

$$x(t) = \int h(t-\tau) f(x(\tau)) d\tau,$$

$$x(t) = \sum_n x_n e^{j\omega t},$$

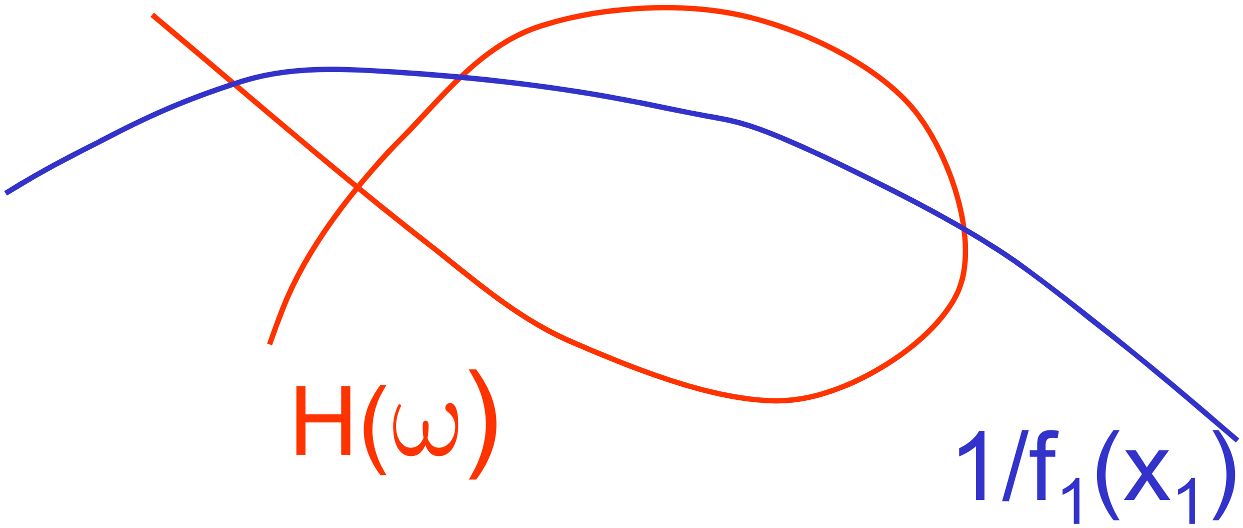
$$f(x(t)) = \sum_n f_n e^{jn\omega t}$$

$$x_n = \sum_n H(n\omega) f_n(x_0, x_1, \ldots)$$



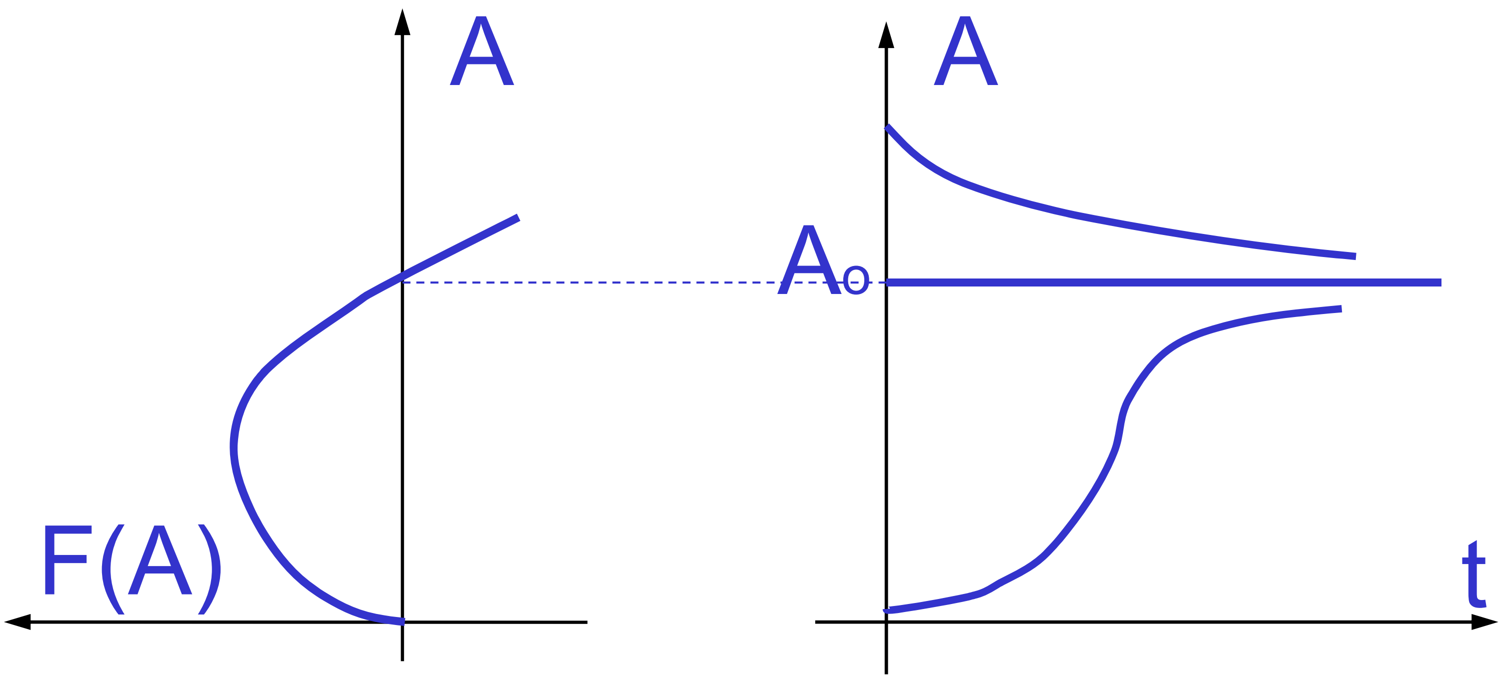
Describing function

$$x_1 = H(\omega) f_1(0, x_1, 0, \ldots)$$



$$\sum_{n \neq 1} |x_n|^2 \quad \text{is small}$$

Solution to $A' = F(A)/Q$



$$x(t) = A(t)\cos(\omega t + \varphi(t))$$

Summary

Phase plane is still very useful with proper models

Harmonic balance/describing function offers powerful insight (via the combination of simulation with circuit and ODE theory)

Measurement and simulation capabilities increased, especially harmonics measurements (since sinusoids are easy to generate)

What next: Volterra series, ANN, more needed
Error estimates, h*f(x) -> F: X -> X