Composite Fermion Approach to Strongly Interacting Quasi Two Dimensional Electron Gas Systems

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Final Technical Report

Closeout of Award No. DE-FG02-97ER45657

Grant ran from 08/01/97 – 11/30/2009

I. Introduction

A list of publications from 1997 through 2009 is attached to this report. Publications in standard journals surveyed by the Web of Science Citation Index can be pulled up electronically. Some conference proceeding from conference and workshops are published as a volume in a book series, and are more difficult to access, but a number have been included.

The major findings of our group during the period 1997 through 2009 rested on a few key papers published earlier. They are:

"Angular Momentum of Composite Fermion Excitations and the Band Structures of Quantum Hall Systems" Solid State Comm. 92, 865 (1994).

"Composite Fermion Excitations and the Electronic Spectra of Fractional Quantum Hall Systems," Phys. Rev. B 53, 9599 (1996).

"Fermi Liquid Shell Model approach to the Excitation Spectra in Fractional Quantum Hall Systems," Phys. Rev. Lett. 76, 3396 (1996).

The first two papers, coauthored by X. Chen, introduced the idea of an "effective monopole strength" Q* that acted as the effective angular momentum of the lowest shell of composite Fermions. This allowed us to predict the angular momentum of the lowest band of energy states for any value of the applied magnetic field simply by determining N_{QP} the number of quasielectrons or quasiholes in a partially filled CF shell and adding angular momenta of the N_{QP} Fermions excitations.

The third paper, coauthored by P. Sitko, S.N.Yi and K.S.Yi, treated the filled CF level as a "vacuum state" which could support QE and QH excitations. Numerical diagonalization of small systems allowed us to determine the angular momenta, the energy, and the pair interaction energies of these elementary excitations. The spectra of low energy states could then be evaluated in a Fermi liquid-like picture, treating the much smaller number of quasiparticles and their interactions instead of the larger system of N electrons with Coulomb interactions.

These ideas were applied over the next ten or twelve years to obtain a much more complete understanding of correlations and composite Fermions in quantum Hall systems.

II. Major Discoveries 1997-2009

1. We introduced the CF hierarchy picture and showed the relationship between Jain's very successful CF picture and Haldane's hierarchy. The latter had an implicit assumption that QPs would, like electrons in the lowest Landau level, be Laughlin correlated (i.e. avoid pair states with the largest pair repulsion).

2. We demonstrated that Haldane's assumption was not always valid, and that the QEs of the Laughlin state at filling factor $\nu=1/3$ were not Laughlin correlated.
3. We introduced the use of fractional parentage to this strongly interacting system, and used it in conjunction with a simple identity that I discovered, viz

$$\hat{L}^2 + N(N-2)\hat{l}^2 - \sum_{<i,j>}(\hat{l}_i + \hat{l}_j)^2 = 0$$

This identity says that the square of the total angular momentum operator for an N Fermion system, $\hat{L}$, plus N(N-2) times the square of the single particle angular momentum, $\hat{l}$, is equal to the sum over all pairs <i,j> of the square of $\hat{l}_i+\hat{l}_j$. This identity together with knowing that fractional percentage was a useful concept led to a surprising result. This result stated that the expectation value of $\sum_{<i,j>} (l_i + l_j)^2$ was exactly the same for every angular momentum multiplet |L α> that had the same value of the total angular momentum L.

4. An immediate consequence of this was the fact that a "harmonic" pseudopotential $V_H(L_2) = A + B\hat{L}_2^2$ which increased as the square of pair angular momentum, caused no correlations. Only the difference between the actual pseudopotential (i.e., the interaction energy of a pair as a function of pair angular momentum $\hat{L}_2$) caused correlations. Positive anharmonic behavior $\Delta V = V(L_2) - V_H(L_2) > 0$ caused Laughlin correlations, while $\Delta V < 0$ caused a tendency toward "pairing."
5. We also showed that according to Jain's mean field CF picture, the effective CF angular momentum would be $l^* = l - (N-1)$. I conjectured that the number of multiplets $G_{NI^*}(L)$ of total angular momentum L formed from N Fermions of angular $l^* = l - (N-1)$ would be a subset of $G_{NI}(L)$, the number formed from N Fermions of angular momentum l. This is the justification of Jain's CF picture. A subset $G_{NI^*}(L)$ is selected from $G_{NI}(L)$ which avoids pairs with angular momentum $L_2^{MAX} = 2l-1$, the highest energy pair state. The resulting subset $G_{NI^*}(L)$ has lower total angular momentum and lower energy. For $2l^* = N-1$, there is only a single multiplet with total angular momentum L=0. Jain's mean field picture is valid only when the interaction between pairs of Fermions has a positive anharmonic part at the pair angular momentum avoided in the Laughlin correlated state.
6. We showed that formation of pairs would occur when ΔV, the anharmonic part of $V(L_2)$, was negative (attractive). This occurs for QEs of the Laughlin $\nu = 1/3$ filled state, so no second generation of CFs occur at $\nu_{QE} = 1/3$. The observed $\nu = 4/11$ state cannot be a spin polarized Laughlin correlated state of $\nu_{QE} = 1/3$.
7. We did some very exciting work on negatively charged excitons, determining their energies, angular momenta, and pair-wise interactions for the ideal system in which electrons and holes resides on the same two dimensional plane in high magnitude field ($\hbar\omega_c$ >>Coulomb energy scale e^2/λ, where λ = magnetic length).
8. By relaxing the ideal conditions, we were able to address photoluminescenes (PL) in realistic systems. We showed that many experiments on PL were interpreted incorrectly. The strong PL peaks, interpreted as the neutral X^0, and the ground states of the singlet X_s^- and triplet X_t^- negatively charged exciton, were in fact the X^0, X_s^- and an excited state of the triplet. We called the triplet states X_{td}^- and X_{tb}^-(for bright and dark). The dark triplet didn't radiate because of angular momentum selection rules. A number of experimental papers on PL in the period 2001-2003 used our results to interpret (for the first time correctly) PL data.

9. The PL discussed above was based on having a dilute electron system (with $\nu << {}^{1}/_{3}$). For ν close to ${}^{1}/_{3}$ (or any other FQH value), the X^- exciton can still form, but it is a negatively charged Fermion. Its correlations with the electrons in FQH state make it act as if it has an "effective charge" $\nu^* = -\frac{1}{3}e$. When QHs are present in the Laughlin state in which the correlated X^- resides, the X^- can bind one or two of the fractionally charged Laughlin quasiholes. This gives three quasiexciton states, QX^-, QX^0, and QX^+, made up of fractionally charged constituents. QX^- has charge ${}^{-e}/_{3}$; QX^0 has a Laughlin QH of charge ${}^{+e}/_{3}$ bound to the QX^-, and QX^+ has two Laughlin QHs bound. These three quasiexcitons will dominate the PL intensity as the dc magnetic field sweep from $\nu < \frac{1}{3}$ to $\nu > \frac{1}{3}$ and the number of Laughlin QHs changes.

III. To summarize our most important results:

1. We showed why the Jain mean field picture of composite Fermions gave a satisfactory description of the low energy spectrum even though the mean field energy scale was totally irrelevant at high magnetic field. We gave examples of when the CF mean field picture didn't work. We explained the physics behind the numerical results.
2. We studied excitons and exitonic complexes in quantum Hall systems, introduced the bright X_{bt}^- and dark X_{dt}^- triplet states, and demonstrated that photoluminescence (PL) spectre were being incorrectly interpreted (prior to our work) because the X_{dt}^- was not actually observed experimentally.
3. We introduced the idea of quasiexcitons QX^-, QX^0, QX^+ in quantum Hall systems. The objects composed of fractionally charged constituents should dominate PL near FQH states.

IV. Current Works

An outgrowth of our work up to 2009 are two important developments:

1) We generated a generalized CF picture which attaches Chern-Simons flux quanta to unbound electrons of charge $-e$ and to pairs of charge $-2e$. The magnitude of the effective CS flux multiplied by the effective charge (which senses the CS flux on other particles) must be the same for the individual electrons and for the electron pairs. This picture explains not only the value $2l = 2N - 3$ of the single particle angular momentum for an $L = 0$ incompressible state, but also explains the two lowest bands of excitations as CF pair quasiparticle and quasihole states or broken CF pairs resulting in two unbound electrons.
2) We have developed a heuristic argument for determining the value of the "finite size shift" in relation $2l = \nu^{-1}N - c_\nu$ that describes incompressible Hall states (which agrees with numerical studies).
3) We have used the values of c_ν to arrive at trial wavefunctions for electrons in a partially filled Fermion Landau level. For $\nu' = \nu - 2 = {}^{1}/_{2}$ (describing the $\nu = {}^{5}/_{2}$ state) we partition N electrons into groups g_A and g_B each containing $\frac{N}{2}$ electrons. The wavefunction

$$\Psi_{5/2} = \Pi_{i<j} z_{ij} \cdot S[\Pi_{i<j \in g_A} \Pi_{k<l \in g_B} (z_{ij} z_{kl})^2]$$

gives $2l = 2N - 3$ and should describe the $\nu = {}^{5}/_{2}$ state. For the Jain $\nu = {}^{2}/_{5}$ state we chose g_A to have $\frac{N}{2}$ - 1 particles and g_B to have $\frac{N}{2}$ + 1. Then a correlation function can be

constructed with different correlations between particles in g_A and g_B, which when symmetrized, corresponds to $2l = (\frac{5}{2})N - 4$, and describes the $\nu = 2/5$ state. In the equation, S is a symmetrizing operator which symmetrizes the product in brackets over all N particles. These trial wavefunctions remain to be tested by comparison to numerical solutions obtained by exact diagonalization.

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