DoE Grant ER64440: Final Report
Collaborative Research: Robust climate projections and stochastic stability of dynamical systems

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1 Summary

In accordance with the original proposal, the project focused on conceptual exploration of El Niño/Southern Oscillation (ENSO) variability and sensitivity using a Delay Differential Equation model developed in the project. We have (i) established the existence and continuous dependence of solutions of the model (ii) explored multiple models solutions, and the distribution of solutions extrema, and (iii) established and explored the phase locking phenomenon and the existence of multiple solutions for the same values of model parameters. In addition, we have applied to our model the concept of pullback attractor, which greatly facilitated predictive understanding of the nonlinear model's behavior. The project has supported several graduate students in the Department of Mathematics and Statistics at the University of Nevada, Reno. The project has resulted in five peer-reviewed papers, and eight presentations at international meetings .

2 Background

The physical growth mechanism of El-Niño/Southern-Oscillation (ENSO) is quite well understood: is due to the positive atmospheric feedbacks on equatorial SST anomalies via the surface wind stress, cf. Bjerknes [1969]. Still, ENSOs unstable quasi-periodic behavior prevents its robust predictions, even at subannual lead times. Conceptual numerical modeling plays a prominent role in understanding ENSO variability and developing forecasts. To simulate, understand and predict such complex phenomena this project explores a full hierarchy of models, from "toy" (conceptual) via intermediate to fully coupled general circulation models (GCMs) [Neelin *et al.*(1998), Ghil and Robertson(2000), Dijkstra and Ghil(2005)]. Initiated in the 1980s, the study of conceptual ENSO models has significantly contributed to shedding new light on many aspects of ENSO, including its quasi-periodic behavior, onset of instabilities, phase locking, power spectrum, and interdecadal variability. This project focuses on theoretical and numerical exploration of a conceptual modeling approach that deals with a simplified picture of ENSO dynamics yet allows one to achieve a rather comprehensive understanding of ENSOs underlying mechanisms and their interplay. The findings of this part of the project will be consistently applied across the full modeling hierarchy.

3 Results

3.1 Model and its properties

Recall that during this project we introduced a nonlinear DDE with additive, periodic forcing:

$$h'(t) = -\tanh[\kappa\, h(t-\tau)] + b\cos(2\pi\, t),\ t \geq 0, \tag{1}$$

$$h(t) = \phi(t) \text{ for } t \in [-\tau, 0), \quad \phi(t) \in X. \tag{2}$$

where $h'(t) = \mathrm{d}h(t)/\mathrm{d}t$, $t \geq 0$, and the parameters τ, κ and b are all real and positive. The function $h(t)$ represents the thermocline depth deviations from the annual mean in the eastern Tropical Pacific; accordingly, it can also be interpreted roughly as the regional SST. The equations (1)-(2) is a simplified one-delay version of the two-delay model considered by Tziperman *et al.* ([Tziperman *et al.*(1994)]); it includes two mechanisms essential for ENSO variability: a delayed, negative feedback via the function $\tanh(\kappa\, z)$, and periodic external forcing. The past 30 years of research have shown that ENSO dynamics is governed, by and large, by the interplay of these nonlinear mechanisms and that their simplest version can be studied in periodically forced Boolean delay systems [Saunders and Ghil(2001), Ghil *et al.*(2008a)] and delay differential equations (DDE) [Suarez and Schopf(1998), Battisti and Hirst(1989), Tziperman *et al.*(1994)]. Before this project, DDE model studies of ENSO have been limited to linear stability analysis of steady-state solutions, which are not typical in forced systems; case studies of particular trajectories; or one-dimensional (1-D) scenarios of transition to chaos, where one varies a single parameter while the others are kept fixed. A major obstacle for the complete bifurcation and sensitivity analysis of DDE models lies in the complex nature of DDEs, whose analytical and numerical treatment is considerably harder than that of their ordinary differential equation (ODE) counterparts. This project made first steps toward comprehensive, theoretical and numerical, exploration of conceptual models related to ENSO dynamics. To do so, we developed appropriate software and described the model behavior in the three-dimensional (3-D) space of its physically relevant parameters. A key result was establishing two regimes of variability, stable and unstable, separated by a sharp neutral curve in parameter space [Ghil *et al.*(2008b), Zaliapin and Ghil(2010)]. We also obtained an existence and uniqueness theorem, as well as continuous, but possibly steep, dependence on model parameters; see the next subsection and [Ghil *et al.*(2008b)]. We explored the model behavior within the following parameter ranges: $0 \leq \tau \leq 2$ yr, $0 < \kappa < \infty$, $0 \leq b < \infty$.

A detailed numerical exploration of model solutions has found (i) Numerous scenarios relevant to the ENSO physics, including quasi-periodic El Niño/La Niña events, interdecadal variability, and patterns reminiscent of Madden-Julian oscillations or westerly wind bursts; (ii) The phase locking of solutions to the seasonal cycle: local temperature maxima and minima tend to occur at the same position within this cycle, which is a characteristic feature of the observed El Niño events; (iii) Parametric instabilities in the location of extrema; (iv) Co-existence of multiple solutions for the same parameter values in certain parameter ranges; (v) Scenario by which the model goes from simple (period-1) to more complicated (period-k) solutions.

3.2 Pullback attractor (PBA) and steep response to small parameter changes

During this year, we have applied to our model the PBA concept ([Ghil *et al.*(2008c)]) and first demonstrated that its dynamics — whether periodic or quasi-periodic — occurs on a

2-D torus, which is driven by the time-periodic forcing; see Fig. 2. This behavior reflects the competition between ENSO's two oscillatory mechanisms: the seasonal forcing and the self-sustained one due to the delayed feedbacks. Such an interpretation is much harder to obtain from the complex, parameter-sensitive dynamics of the model using more traditional approaches.

Furthermore, the novel tools of PBAs and of time-dependent invariant measures help understand the model's parameter sensitivity and its nonlinear dynamics. We have shown, in effect, that many of the model's statistics — such as the mean or maxima of model solutions — can vary abruptly with respect to small parameters variations [Ghil *et al.*(2008b), Zaliapin and Ghil(2010)]. It can be shown rigorously that these statistics depend on time-dependent invariant measures μ_t that are supported by the PBA $\mathcal{A}(t)$ (Chekroun, Zaliapin, & Ghil, 2010, in preparation).

For instance, let $S(t,s)$ denote the two-time flow associated with Eq. (1) in some appropriate phase space. Then the time average $(t-s)^{-1}\int_s^t \phi(S(u,s)h_0)\mathrm{d}u$, where ϕ is some observable (*i.e.* metric) of the system, converges as $s \to -\infty$ to the ensemble average with respect to the time-dependent invariant measure μ_t at the frozen time t. Sensitive dependence in model statistics is thus equivalent to sensitive dependence in the time-dependent invariant measure μ_t.

Figures 1,2 illustrate the forward attractor (blue points) and pullback attractor (red points) for periodic and ergodic behavior of the model.

Figure 3 illustrates that a change of about 1 percent in parameters can lead to very different invariant measures. The latter, in turn, are associated with significant changes in the mean of model's solutions [Ghil *et al.*(2008b)]. A quantitative analysis shows that a change of 1 percent in parameters can lead to a change of almost 100 percents in the invariant measure, leading to high-sensitivity of the model's statistics.

These findings will help us to analyze to which extent this type of sensitivity is generic, by considering more realistic ENSO models that will include additional delayed feedbacks, both positive and negative. We will further explore the quasi-biennial and quasi-quadrennial modes of variability associated with ENSO. The previous project has established the existence of "Devil's bleachers" sin the DDE-ENSO model's dependence of periods on parameters. In particular, we found multi-dimensional "tongues" of constant, low-frequency periods, of 2, 3, 5 and 7 years. We will further study the dependence of the dominant modes on model parameters and explore the dynamics when these parameters change slowly in time, as expected in the context of global warming.

4 Project publications

Papers

1. Ghil, M., I. Zaliapin, and S. Thompson: A delay differential model of ENSO variability: Parametric instability and the distribution of extremes. Nonlin. Prococces Geophys., 15, 417-433, 2008.

2. Zaliapin, I. and M. Ghil: A delay differential model of ENSO variability, Part 2: Phase locking, multiple solutions, and dynamics of extrema. Nonlin. Processes Geophys., 17, 123-135, 2010.

3. Zaliapin, I. and M. Ghil: Another Look at Climate Sensitivity. Nonlin. Processes Geophys., 17, 113-122, 2010.

4. Ghil, M. and I. Zaliapin: El Niño/Southern Oscillation: Impacts, Modeling and Forecasts, In Encyclopedia of Natural Hazards, P. Bobrowsky (Ed.), Springer, 2010, in review.

5. Ghil, M., Zaliapin, I. and Chekroun M.: Understanding ENSO variability and its extrema: A delay differential equation approach. AGU Monograph "Observations, Modeling and Economics of Extreme Events," 2010, in review.

Conference presentations

1. Zaliapin, I., M. Ghil, and S. Thompson (2007) A delay differential model of ENSO variability: parametric instability and the distribution of extremes, EOS Trans. AGU, 88(52), Fall Meet. Suppl. Abstract NG32A-02.

2. Zaliapin, I., M. Ghil, and S. Thompson (2007) A delay differential model of ENSO variability: Instabilities and the distribution of extremes. Proc. Climate Change Prediction Program Meeting, Indianapolis, September 17-19, 2007.

3. Ghil, M. and I. Zaliapin (2007) Extreme events: Some theoretical and practical considerations, Eos Trans. AGU, 88(23), Jt. Assem. Suppl., Abstract U32B-01 (INVITED)

4. Zaliapin, I. and M. Ghil (2007) A differential delay model of ENSO variability: quantitative predictability and structural instability, European Geosciences Union, General Assembly, Vienna, Austria, April 15-20, EGU2007-A-10437, NH8.01/NP4.04-1MO1O-001.

5. Zaliapin, I. and M. Ghil (2008) A delay differential model of ENSO variability: Extreme values and stability analysis. Proceedings of the International Symposium Topical Problems of Nonlinear Wave Physics 2008, Section Global and Synoptic Nonlinear Processes in the Atmosphere, Nizhny Novgorod, Russia, July 20-26, 2008, Abstract 3-52, pp.100-101.

6. Ghil, M., M. Chekroun, E. Simonnet, and I. Zaliapin (2008) Robust climate projections and stochastic structural stability of dynamical systems. Joint Mathematics Meeting of AMS, San Diego, CA, January 6-9, Abstract 1035-37-1713.

7. Zaliapin, I. and M. Ghil (2009) A delay differential model of ENSO variability: Extreme values and stability analysis, 2009 EGU General Assembly, April 19-24, Vienna, Austria, Session: CL55/NP8.4 Chaotic and Stochastic Climate Dynamics, Abstract EGU2009-6597.

8. Chekroun, M. D., I. Zaliapin, and M. Ghil (2009) A delay differential model for El-Nino/Southern Oscillation (ENSO): Pullback attractors, phase locking, and multiple solutions. EOS Trans. AGU, 90(52), Fall Meet. Suppl. Abstract NG13A-1088.

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[Zaliapin and Ghil(2010)] Zaliapin, I. and M. Ghil: A delay differential model of ENSO variability, Part 2: Phase locking, multiple solutions, and dynamics of extrema. Nonlin. Processes Geophys., 17, 123-135.

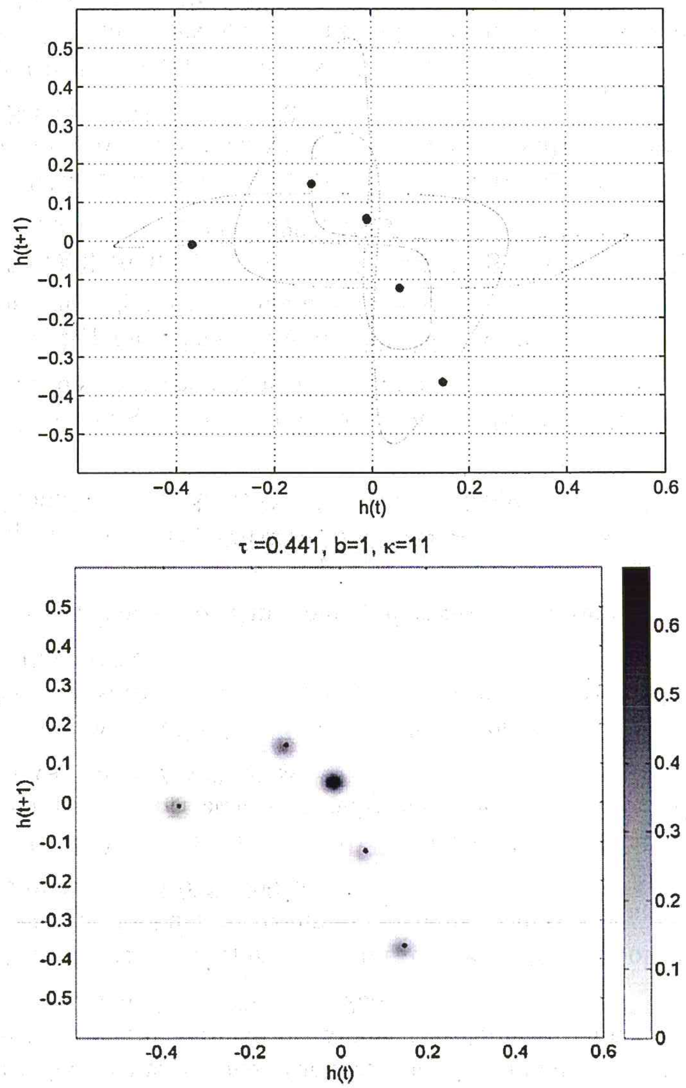

Figure 1: Top: Forward attractor (blue dots) and pullback attraactor (red dots) for the DDE system (1)-(2) for $b = 1, \kappa = 11, \tau = 0.441$. The model has period 5 at this point, and so the PBA consists of 5 points. Bottom: the measure on the PBA, estimated by a convolution of the empirical density of points on the PBA (black dots) and a gaussian kernel. Note: the 2-D representation in the bottom panel is used for visual convenience; the measure is concentrated on the set of 5 points.

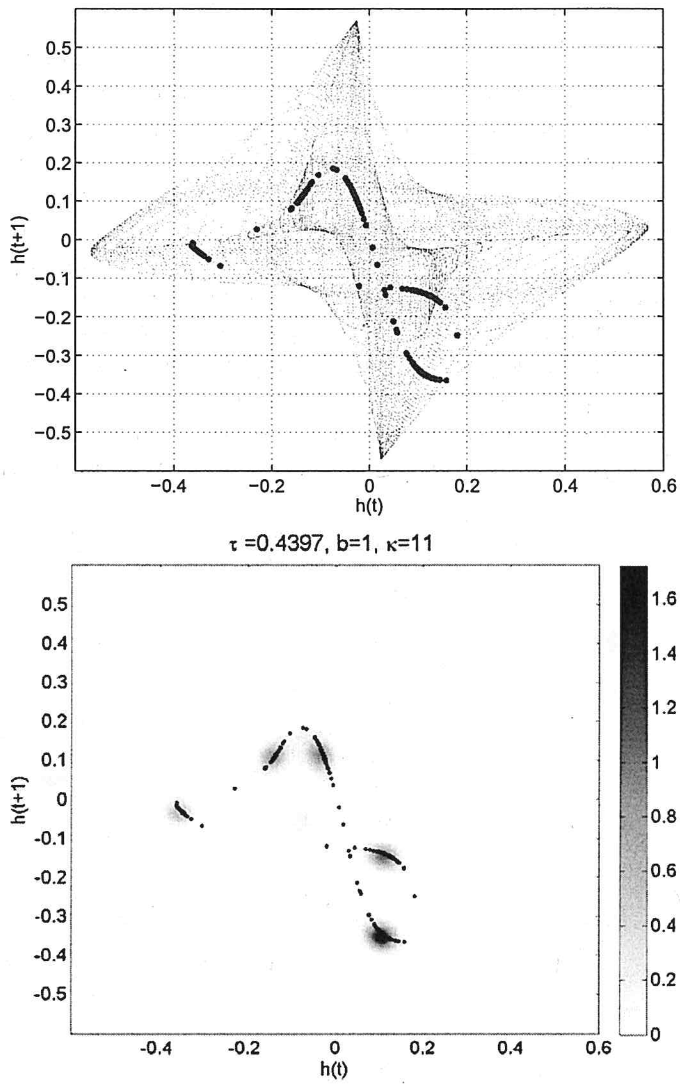

Figure 2: Top: Forward attractor (blue dots) and pullback attraactor (red dots) for the DDE system (1)-(2) for $b = 1, \kappa = 11, \tau = 0.4397$. The model has an ergodic behavior at this point, and so the PBA is located along a circle. Bottom: the measure on the PBA, estimated by a convolution of the empirical density of points on the PBA (black dots) and a gaussian kernel. We notice that the measure is still concentrated in 5 points. Note: the 2-D representation in the bottom panel is used for visual convenience; the measure is concentrated on a 1-D surface.

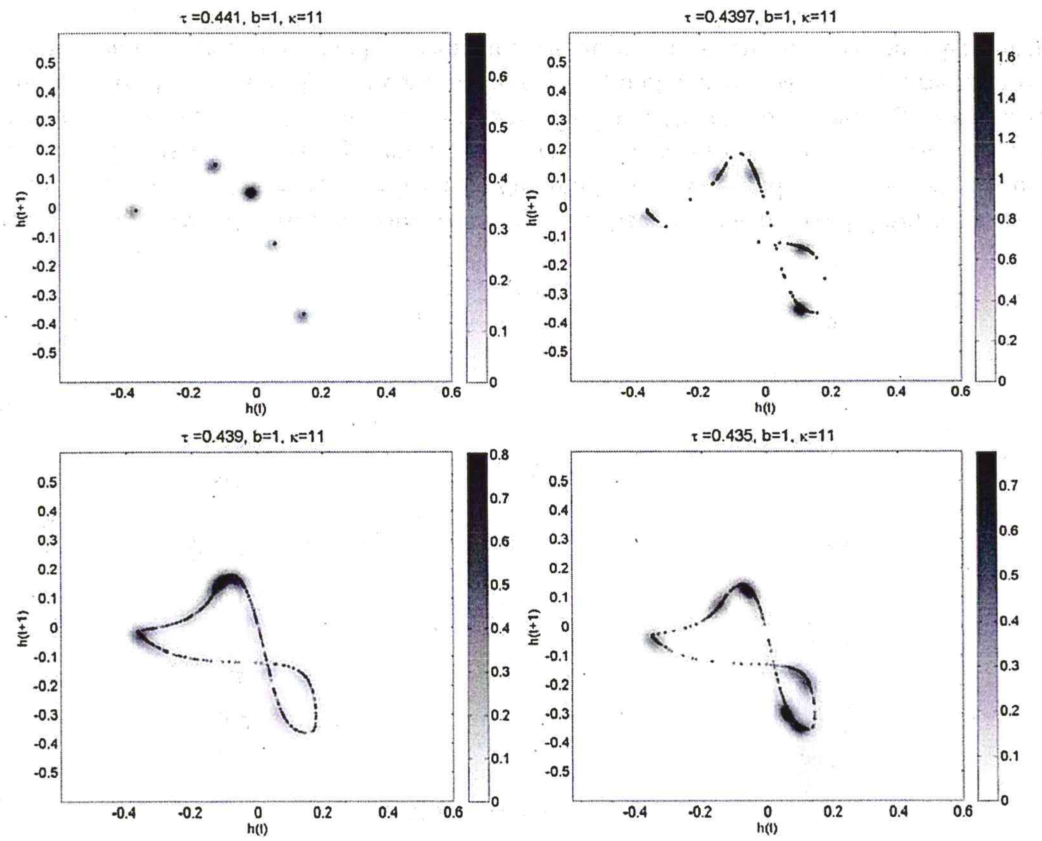

Figure 3: The pullback attractor (small black dots) for the ENSO-DDE model (1) for fixed $b = 1$ and $\kappa = 11$. The oceanic wave dalay τ is changing as: (a) $\tau = 0.441$, (b) $\tau = 0.4397$, (c) $\tau = 0.439$, and (d) $\tau = 0.437$; the invariant measure supported by the PBA is represented by a grey scale. The model is quasi-periodic in panels (b), (c) and (d), while it has period 5 in panel (a), where the PBA consists of 5 points. In each case the invariant measure supported by the PBA is estimated by a convolution of the empirical density of points on the PBA and a gaussian kernel, and is represented by a grey scale. The panels illustrate a change of less than 1 % in the parameter value from (a) to (b), (b) to (c), etc., whereas the corresponding changes in the invariant measures are 50 % and 97 %, in the L_1 norm (*i.e.*, in mean absolute value of difference). These results are in agreement with the high-sensitivity observed in the the model's statistics [Ghil *et al.*(2008b)], and illustrate how the concepts of pullback attractors and time-dependent invariant measures they support are relevant for encoding such sensitivity. from [Chekroun, Zaliapin, & Ghil, 2010, in preparation].

Ella Chavez

From: Ella Chavez
Sent: Friday, March 11, 2011 3:01 PM
To: Ilia Zaliapin
Cc: Eric B Herzik
Subject: Final Technical Report

Importance: High

Non Compliance with Sponsored Project Requirement: Submission of the Final Technical Report

Agency: DOE
Agency #: DE-FG02-07ER64440
UNR #: 1320 114 2779, Robust Climate Projections and Stochastic Stability of Dynamical Systems
End Date: 07/14/10 Due: 10/14/10
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