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Title: Recent Monte Carlo Results for Cold Atoms

Author(s): Joseph Carlson, Stefano Gandolfi, T-2
Alexandros Gezerlis, Michael Forbes, Univ. of Washington

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Title: Recent Monte Carlo Results for Cold Atoms

Abstract:

I discuss recent Diffusion Monte Carlo and Auxiliary-Field (lattice) Monte Carlo results for cold atoms. I focus on the equation-of-state and the contact parameter for equal masses, and then discuss the unequal mass case for both polarized and unpolarized systems.

Recent Monte Carlo Results for Cold Atoms

- Introduction
- Equal Mass at Unitarity
  - Improved DMC results
  - Initial lattice results
- Unequal Masses
  - Superfluid state
  - Normal (Polarized) state
  - Few-Particle states
- Future possibilities

Stefano Gandolfi (LANL), Alex Gezerlis (LANL/UW), Michael Forbes (UW)

Shiwei Zhang (W&M), Kevin Schmidt (ASU)

Cold Atoms near Unitarity

$$H = \sum_{i=1,n_l} \frac{-\hbar^2}{2m_l}\nabla_i^2 + \sum_{j=1,n_h} \frac{-\hbar^2}{2m_h}\nabla_j^2 + \sum_{i,j} V(r_{ij})$$

$$v(r) = -\frac{2}{m}\frac{\mu^2}{\cosh^2(\mu r)}$$

Take the limit $\mu \to 0$

Single scale in the problem: $k_F = (3\,\pi^2\,\rho)^{1/3}$

$E = \xi\, E_{FG} = \xi\,(3/5)\,k_F^2/\,(2m)$

$\Delta = \delta\, E_F = \delta\, k_F^2/\,(2m)$

$C = 8\,\pi^2\,\rho^2\,A^2 = \varsigma\,(2\,k_F^4\,/\,(5\,\pi))$

$g(r) \to A^2/\,r^2$

Control: phase diagram, `exotic' superfluids, high-energy RF response,...

Improved DMC calculations

Forbes, Gezerlis, Gandolfi (2010)
Gandolfi, Schmidt, Carlson (2010)

Upper Bound to the Energy
Applicable to polarized, unequal mass,...

$$
\begin{aligned}
\Psi_T &= \prod_{ij} f_{ij'} \Phi_{\mathrm{BCS}} \\
\Phi_{\mathrm{BCS}} &= \mathcal{A}[\phi(r_{11'})\phi(r_{22'})\ldots\phi(r_{nn'})]
\end{aligned}
$$

Canonical Ensemble
Dilute
Periodic Boundary Conditions

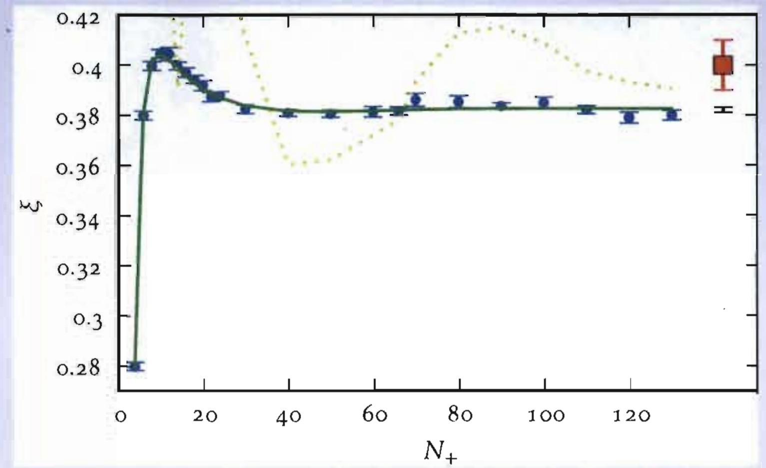
$$
\begin{aligned}
\phi(\mathbf{r}) &= \tilde{\beta}(r) + \sum_n a(k_n^2) \exp[i\vec{k}_n \cdot \vec{r}] \,, \\
\tilde{\beta}(r) &= \beta(r) + \beta(L-r) - 2\beta(L/2) \,, \\
\beta(r) &= [1 + cbr]\,[1 - \exp[-10br]] \frac{\exp[-br]}{br}
\end{aligned}
$$

Pair function
optimized variationally

$\frac{L^2}{4\pi^2}k^2$	$a(k^2)$	$\frac{L^2}{4\pi^2}k^2$	$a(k^2)$
0	0.00198	5	0.000190
1	0.00250	6	0.000200
2	0.00194	8	0.000167
3	0.00081	9	0.000163
4	0.00033	10	0.000120

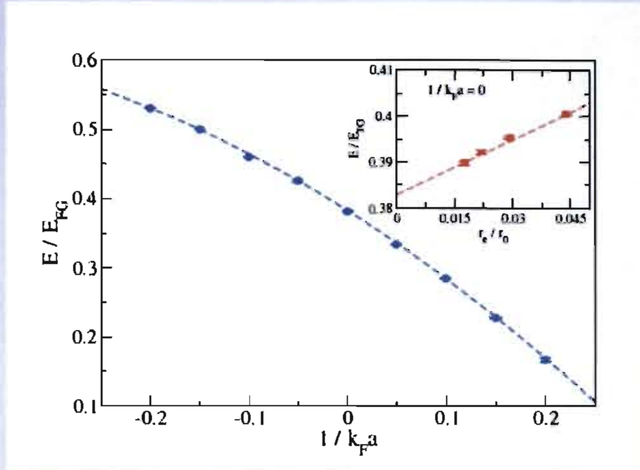
Finite Volume Effects

Forbes, Gezerlis, Gandolfi (2010)



EOS near unitarity

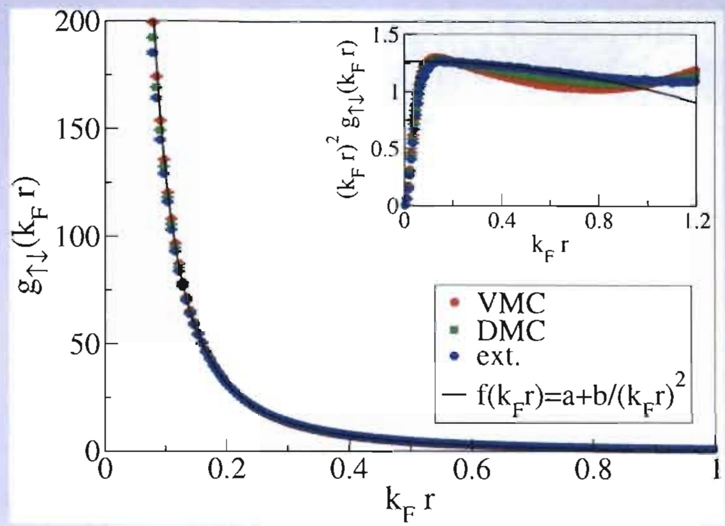
Gandolfi, Schmidt, Carlson (2010)



$$\frac{E}{E_{FG}} = \xi - \frac{\zeta}{k_F a} - \frac{5\nu}{3(k_F a)^2} + \dots ,$$

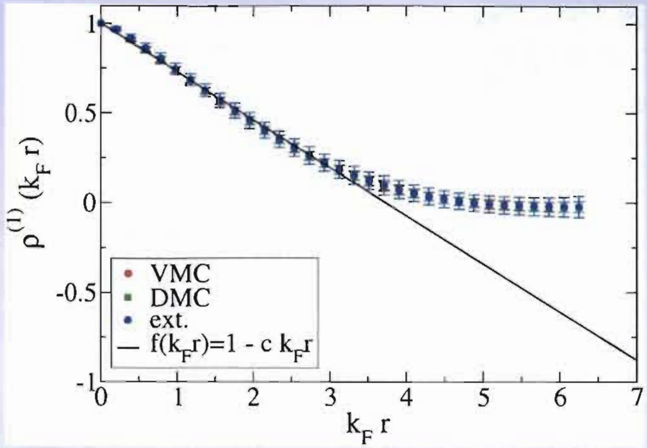
$\xi = 0.383(1)$ $\quad$ $\zeta = 0.901(2)$ $\quad$ $\frac{C}{k_F^4} = \frac{2\zeta}{5\pi} = 0.1147(3)$

Pair Distribution Function



$g_{\uparrow\downarrow}(r) \to \frac{9\pi}{20}\zeta(k_F r)^{-2}$ $\qquad \zeta = 0.897(2)$

Radial one-body density matrix



$$S_{\uparrow\downarrow}(k) - \frac{1}{2} = \frac{2\pi^2 n A^2}{k}\left[1 - \frac{1}{4\pi a k}\right] + \ldots . \qquad \zeta = 0.895(16)$$

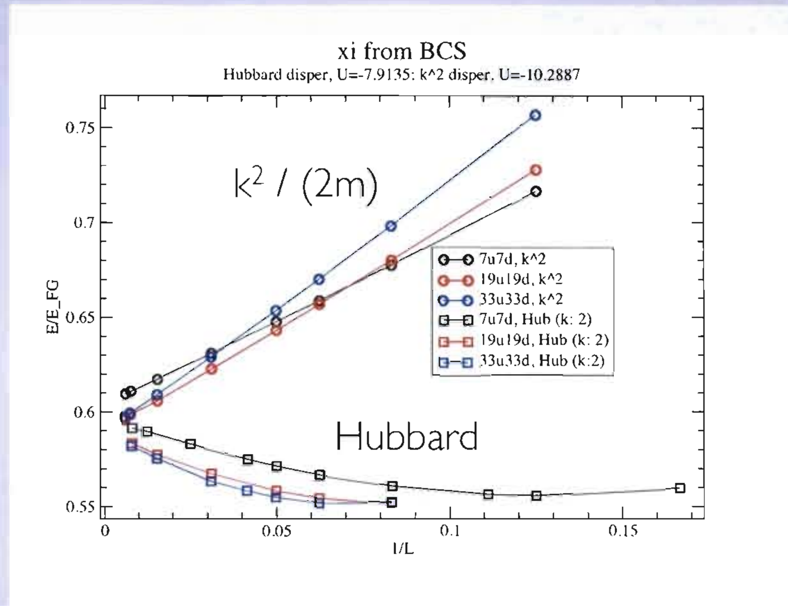
Lattice Approaches (in progress)

Equivalent to attractive Hubbard model in dilute limit
No sign problem, but dilute limit non-trivial
Canonical approach (more efficient for $T=0$)
Freedom in operators to more quickly approach continuum

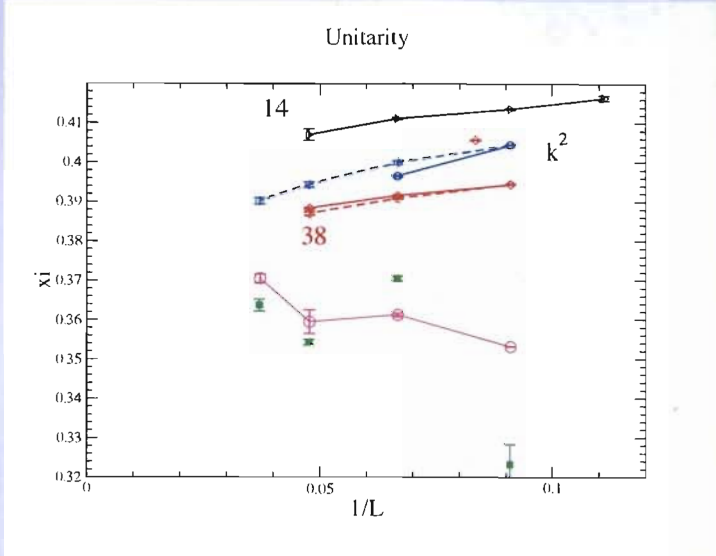
Example: Kinetic Energy

Hubbard model	(nearest neighbor hopping)
$k^2 / (2m)$	(easily evaluated via FFT)
$+ O(k^4)$	match 2-body spectra (effective range)

BCS on the lattice



Preliminary Results



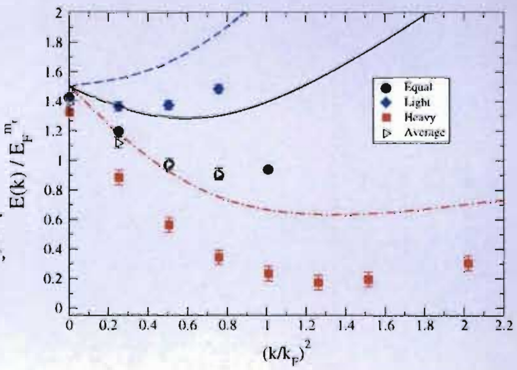
Unequal Masses

BCS solution unchanged for different reduced mass
Equal mass solution good starting point

$\xi\,(M/m = 6.5)$ - $\xi\,(M/m = 1) = -0.02$

In BCS theory

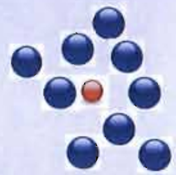
$$E_{h(l)}(k) = \frac{\xi_{h(l)}(k) - \xi_{l(h)}(k)}{2} + \sqrt{\left(\frac{\xi_h(k) + \xi_l(k)}{2}\right)^2 + \Delta^2(k)},$$



Binding of One Heavy or One Light

$B(H) = 0.36\ E_F(L)$

effective mass ~1.0



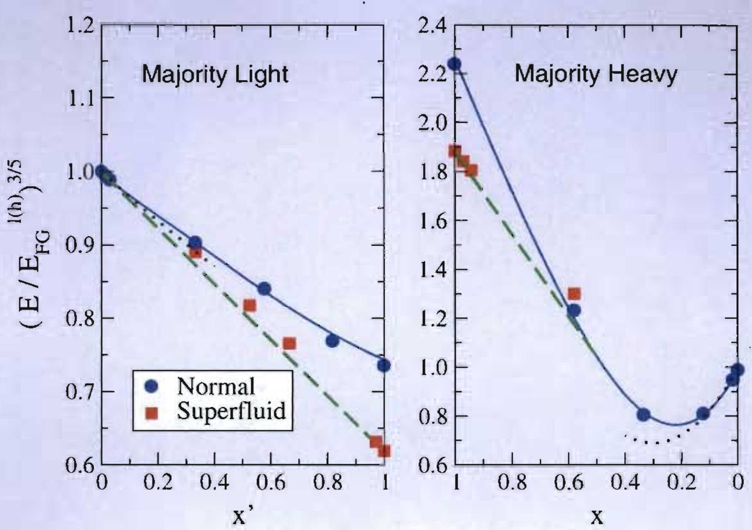
$B(L) = 2.3\ E_F(H)$

effective mass ~ 1.3

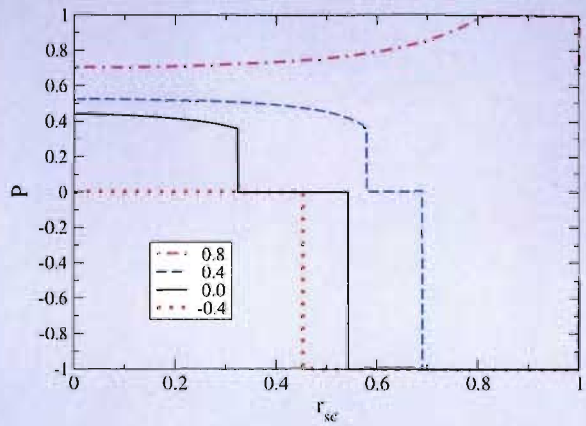
Agreement w/ previous calculations

R. Combescot et al., Phys. Rev. Lett. 98, 180402 (2007)

Polarized Systems

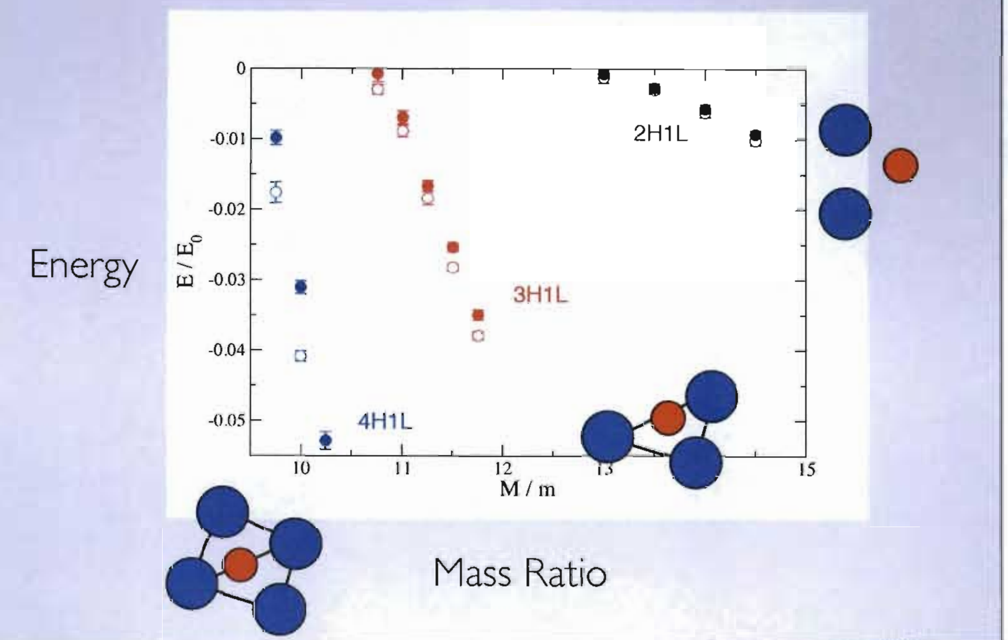


Local Density Approximation for Harmonic Trap



Polarization versus radius for different population imbalances

Non-Universal Behavior N-heavy 1-light



Future Possibilities

'Exotic' Superfluid States
Mixed (or other) Dimensions
Static Response & Finite Systems
Dynamic (RF) Response
Multiple Species
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