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Title: Recent Monte Carlo Results for Cold Atoms

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Intended for: Strong Interactions: From Methods to Structures
Bad Honnef, Germany
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Workshop Title: Strong Interactions: From Methods to Structures
Place: Bad Honnef, Germany, Feb 12-16, 2011

Title: Recent Monte Carlo Results for Cold Atoms

Abstract:

I discuss recent Diffusion Monte Carlo and Auxiliary-Field (lattice) Monte Carlo results for cold atoms. I focus on the equation-of-state and the contact parameter for equal masses, and then discuss the unequal mass case for both polarized and unpolarized systems.

Recent Monte Carlo Results for Cold Atoms

- Introduction

- Equal Mass at Unitarity

 - Improved DMC results

 - Initial lattice results

- Unequal Masses

 - Superfluid state

 - Normal (Polarized) state

 - Few-Particle states

- Future possibilities

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Shiwei Zhang (W&M), Kevin Schmidt (ASU)

Cold Atoms near Unitarity

$$H = \sum_{i=1, n_l} \frac{-\hbar^2}{2m_l} \nabla_i^2 + \sum_{j=1, n_h} \frac{-\hbar^2}{2m_h} \nabla_j^2 + \sum_{i,j} V(r_{ij})$$

$$v(r) = -\frac{2}{m} \frac{\mu^2}{\cosh^2(\mu r)}$$

Take the limit $\mu \rightarrow 0$

Single scale in the problem: $k_F = (3 \pi^2 \rho)^{1/3}$

$$E = \xi E_{FG} = \xi (3/5) k_F^2 / (2m)$$

$$\Delta = \delta E_F = \delta k_F^2 / (2m)$$

$$C = 8 \pi^2 \rho^2 A^2 = \varsigma (2 k_F^4 / (5 \pi))$$

$$g(r) \rightarrow A^2 / r^2$$

Control: phase diagram, 'exotic' superfluids, high-energy RF response,...

Improved DMC calculations

Upper Bound to the Energy

Applicable to polarized, unequal mass,...

Forbes, Gezerlis, Gandolfi (2010)

Gandolfi, Schmidt, Carlson (2010)

$$\Psi_T = \prod_{ij} f_{ij} \Phi_{\text{BCS}}$$

$$\Phi_{\text{BCS}} = \mathcal{A}[\phi(r_{11'})\phi(r_{22'})\dots\phi(r_{nn'})]$$

Canonical Ensemble

Dilute

Periodic Boundary Conditions

$$\phi(r) = \bar{\beta}(r) + \sum_n a(k_n^2) \exp[i\vec{k}_n \cdot \vec{r}] ,$$

$$\bar{\beta}(r) = \beta(r) + \beta(L-r) - 2\beta(L/2) ,$$

$$\beta(r) = [1 + cbr] [1 - \exp[-10br]] \frac{\exp[-br]}{br}$$

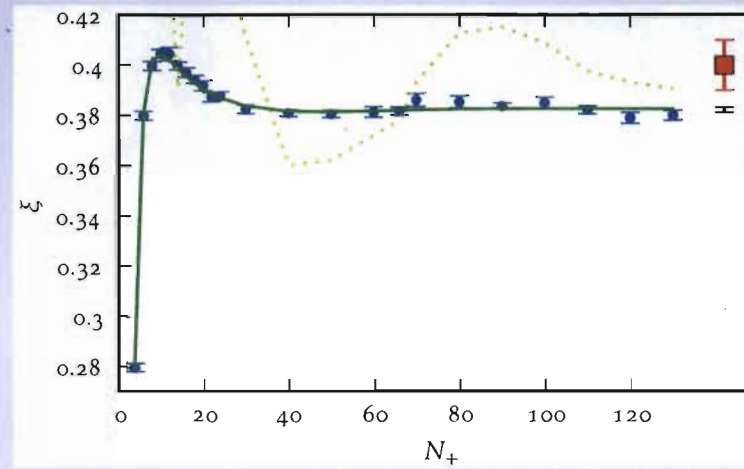
Pair function

optimized variationally

$\frac{L^2}{4\pi^2} k^2$	$a(k^2)$	$\frac{L^2}{4\pi^2} k^2$	$a(k^2)$
0	0.00198	5	0.000190
1	0.00250	6	0.000200
2	0.00194	8	0.000167
3	0.00081	9	0.000163
4	0.00033	10	0.000120

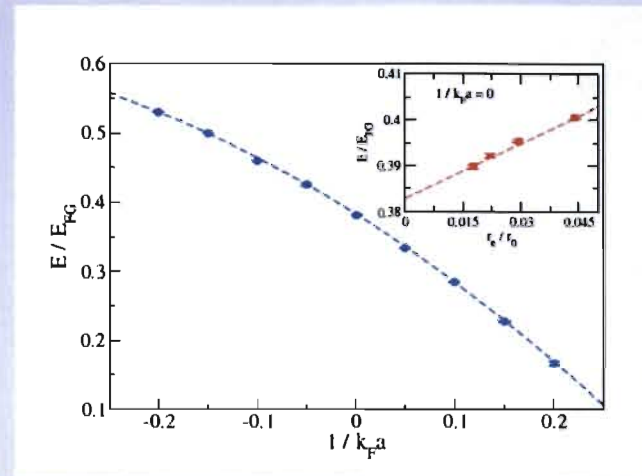
Finite Volume Effects

Forbes, Gezerlis, Gandolfi (2010)



EOS near unitarity

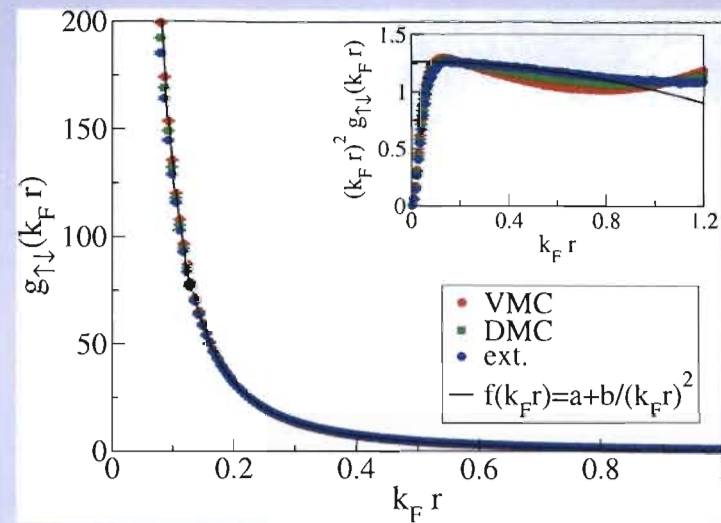
Gandolfi, Schmidt, Carlson (2010)



$$\frac{E}{E_{FG}} = \xi - \frac{\zeta}{k_F a} - \frac{5\nu}{3(k_F a)^2} + \dots$$

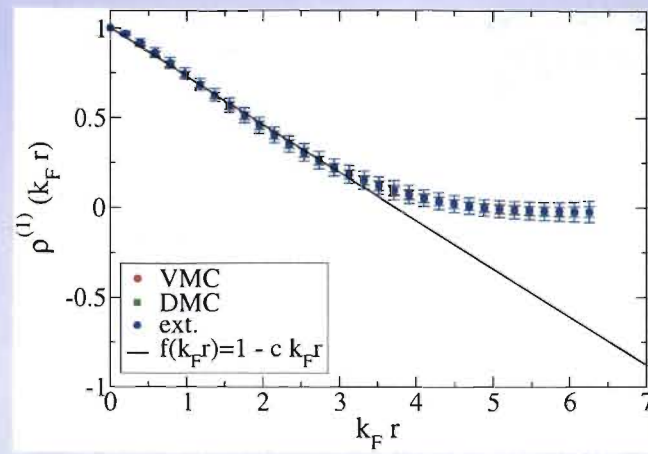
$$\xi = 0.383(1) \quad \zeta = 0.901(2) \quad \frac{C}{k_F^4} = \frac{2\zeta}{5\pi} = 0.1147(3)$$

Pair Distribution Function



$$g_{\uparrow\downarrow}(r) \rightarrow \frac{9\pi}{20} \zeta (k_F r)^{-2} \quad \zeta = 0.897(2)$$

Radial one-body density matrix



$$S_{\uparrow\downarrow}(k) - \frac{1}{2} = \frac{2\pi^2 n A^2}{k} \left[1 - \frac{1}{4\pi a k} \right] + \dots \quad \zeta = 0.895(16)$$

Lattice Approaches (in progress)

Equivalent to attractive Hubbard model in dilute limit

No sign problem, but dilute limit non-trivial

Canonical approach (more efficient for $T=0$)

Freedom in operators to more quickly approach continuum

Example: Kinetic Energy

Hubbard model (nearest neighbor hopping)

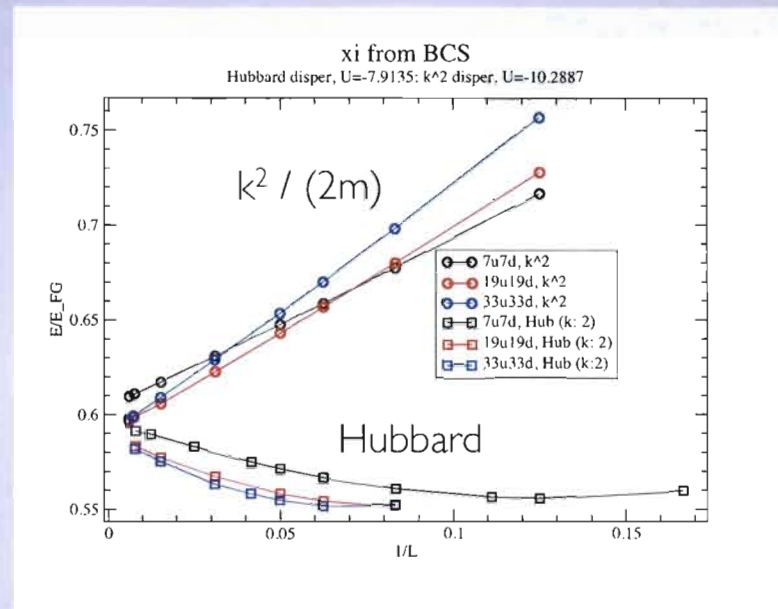
$$k^2 / (2m)$$

(easily evaluated via FFT)

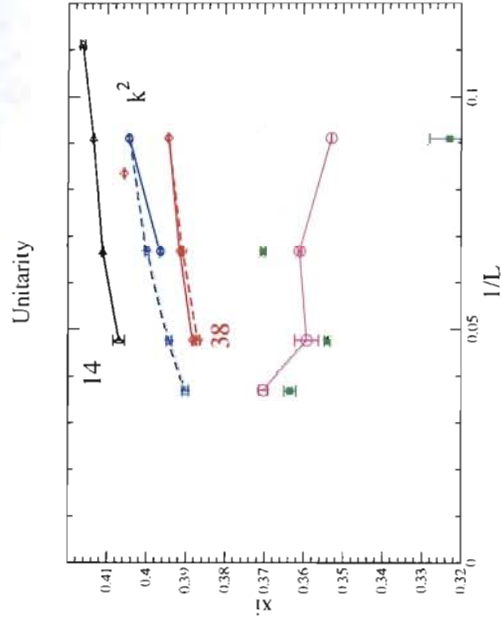
$$+ O(k^4)$$

match 2-body spectra (effective range)

BCS on the lattice



Preliminary Results



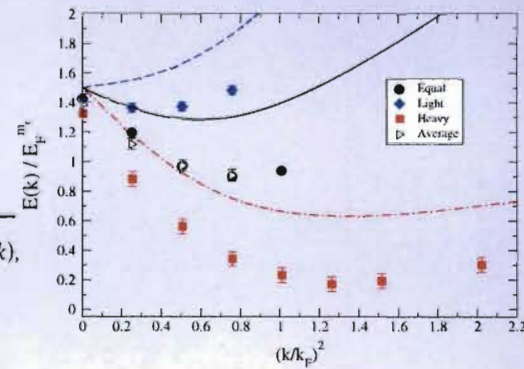
Unequal Masses

BCS solution unchanged for different reduced mass
 Equal mass solution good starting point

$$\xi (M/m = 6.5) - \xi (M/m = 1) = -0.02$$

In BCS theory

$$E_{h(l)}(k) = \frac{\xi_{h(l)}(k) - \xi_{l(h)}(k)}{2} + \sqrt{\left(\frac{\xi_h(k) + \xi_l(k)}{2}\right)^2 + \Delta^2(k)},$$



Binding of One Heavy or One Light



$$B(H) = 0.36 E_F(L)$$

effective mass ~ 1.0



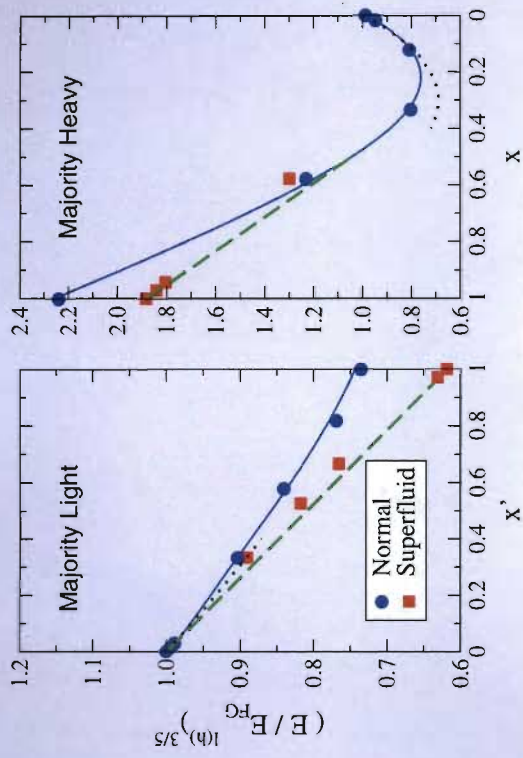
$$B(L) = 2.3 E_F(H)$$

effective mass ~ 1.3

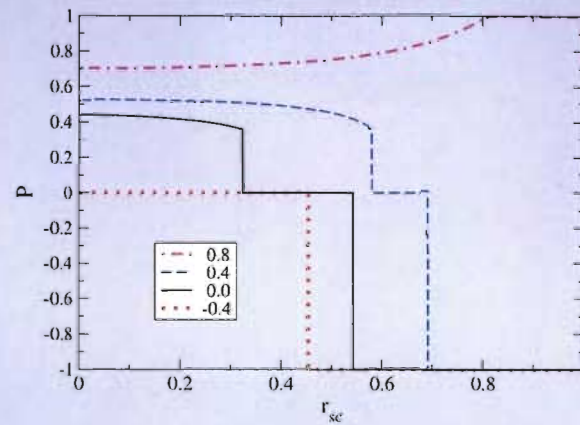
Agreement w/ previous calculations

R. Combescot et al., Phys. Rev. Lett. 98, 180402 (2007)

Polarized Systems

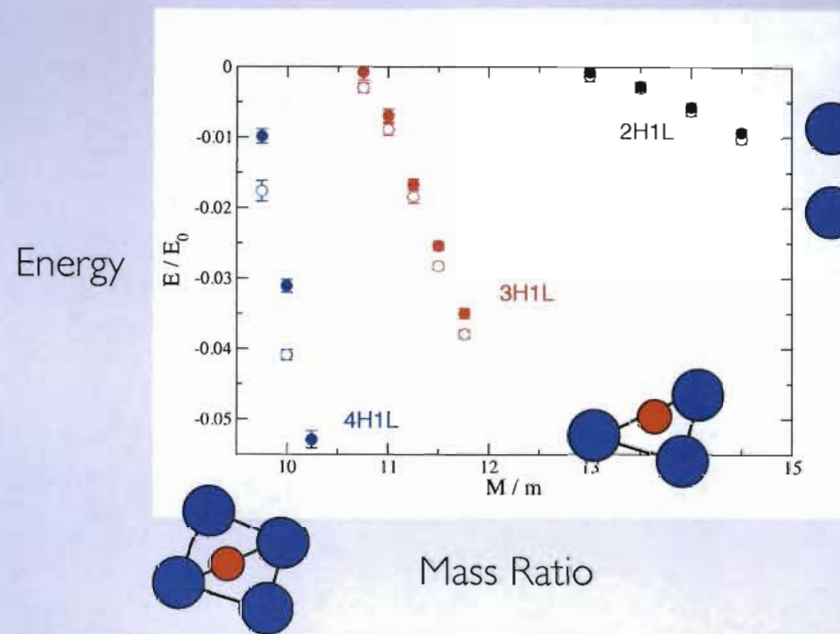


Local Density Approximation for Harmonic Trap



Polarization versus radius for different population imbalances

Non-Universal Behavior N-heavy I-light



Future Possibilities

'Exotic' Superfluid States

Mixed (or other) Dimensions

Static Response & Finite Systems

Dynamic (RF) Response

Multiple Species

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