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Title: P24 Plasma Physics Summer School 2012 Los Alamos National Laboratory Summer lecture series for students

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Intended for: P24 Plasma Physics Summer School
Web

P24 Plasma Physics Summer School
2012
Los Alamos National Laboratory
Summer lecture series for students

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1. Tom Intrator, P24 LANL: Kick off, Introduction - What is a plasma?
2. Bruno Bauer, Univ. Nevada–Reno: Derivation of plasma fluid equations
3. Juan Fernandez, P24 LANL Overview of research being done in p-24 –
4. Tom Intrator, P24 LANL: Intro to dynamo, reconnection, shocks
5. Bill Daughton X-CP6 LANL: Intro to computational particle in cell methods
6. Kirk Flippo, P24 LANL: High energy density plasmas
7. Thom Weber, P24 LANL: Energy crisis, fission, fusion, non carbon fuel cycles
8. Tom Awe, Sandia National Laboratory: Magneto Inertial Fusion
9. Yongho Kim, P24 LANL: Industrial technologies

What is a plasma?

The other 99% of the universe

T. Intrator

P-24 Plasma Physics

2012 Jun 13

P-24 Plasma Physics Summer School

Center for Non Linear Studies conference room

Wednesdays 1pm-2pm

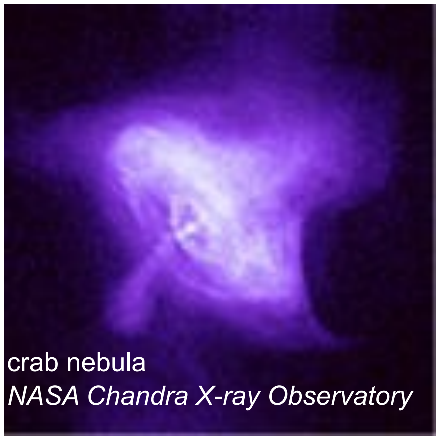
Abstract

This introduction will define the plasma fourth state of matter, where we find plasmas on earth and beyond, and why they are useful. There are applications to many consumer items, fusion energy, scientific devices, satellite communications, semiconductor processing, spacecraft propulsion, and more. Since 99% of our observable universe is ionized gas, plasma physics determines many important features of astrophysics, space physics, and magnetosphere physics in our solar system. We describe some plasma characteristics, examples in nature, some useful applications, how to create plasmas. A brief introduction to the theoretical framework includes the connection between kinetic and fluid descriptions, quasi neutrality, Debye shielding, ambipolar electric fields, some plasma waves. Hands-on demonstrations follow. More complete explanations will follow next week.

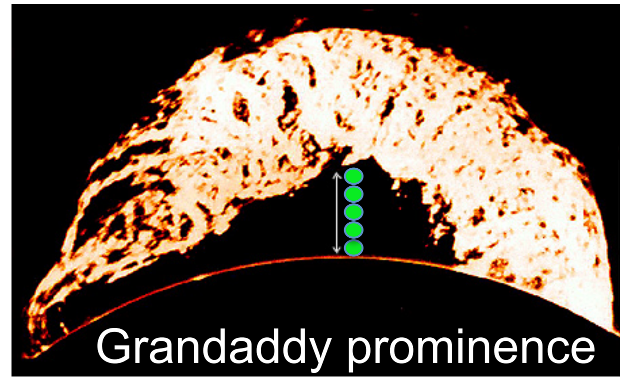
Outline

- What is a plasma?
- Different examples
- How to create plasmas?
- Theoretical framework
  - Particles & fluids
  - Maxwell's equations
  - Debye shielding, sheath
  - Magneto hydrodynamics
  - Waves
  - Bruno Bauer next week will amplify on these and more
- Some applications
- Summary

Large Scale Plasma Structure in the Universe

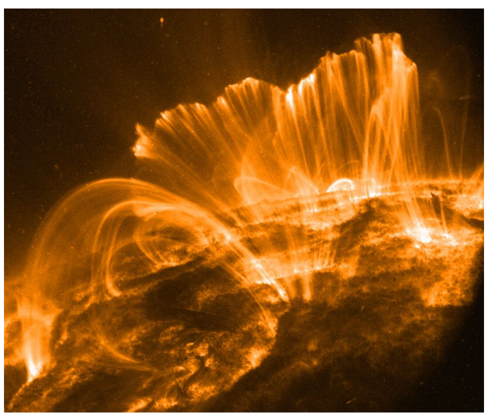


Crab Pulsar. This image combines optical data from Hubble (in red) and X-ray images from Chandra X-ray Observatory (in blue).



Arrow is 64,000km, 5 earths high, earth diameter = 12756km

June 4, 1946 High Altitude Observatory…
http://solar-center.stanford.edu/compare



TRACE satellite VUV images

Coronal arches, magnetic structures

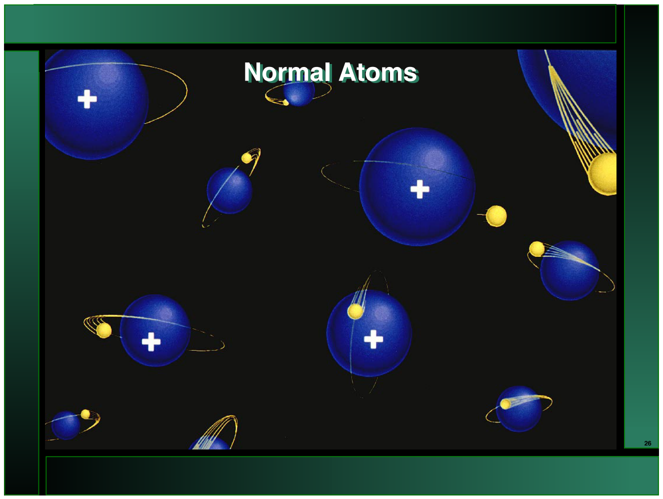
What is a plasma?

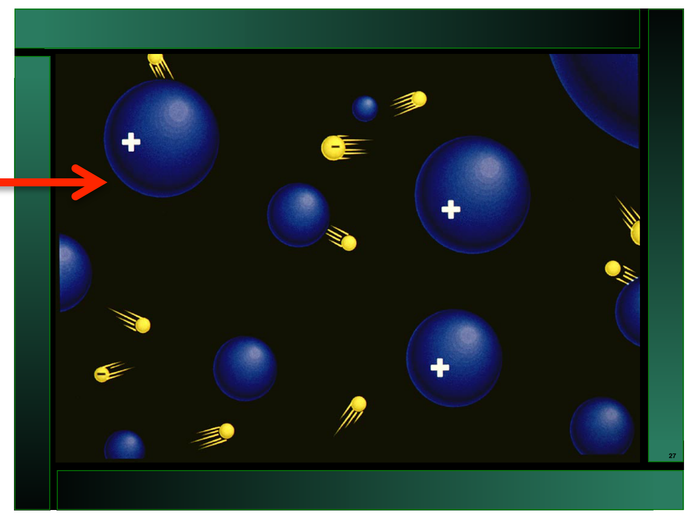
- Plasma state is the "fourth" state of matter
- As a solid is heated
  - Bonds between adjacent molecules loosen => solid (1/40 eV)
  - More heat => loosens up the lattice => Liquid state
  - More heat => neighboring bonds are broken => gas
  - More heat => molecular collisions => dissociate to atoms
  - More heat => collisions knock off electrons => plasma (>2eV)
    - neutral particles, ions & electrons
    - $k_B T \approx$ few eV, equivalent to chemical bonds
    - 99% of the universe
- solid => liquid => gas => plasma

Plasma science is interdisciplinary

- Plasma astrophysics: most of our universe
- Planetary magnetospheres
- Fusion energy
  - Magnetic: tokamak, stellarator, compact toroids: MFE
  - Inertial: laser, Z-pinch, pulsed: IFE
  - Magneto-Inertial: MIF is in between
- High energy density plasmas, extreme states of matter
- Spacecraft thrusters
- Industrial applications: semiconductors
- Radiation generators
- Weapons

Normal atoms are charge neutral



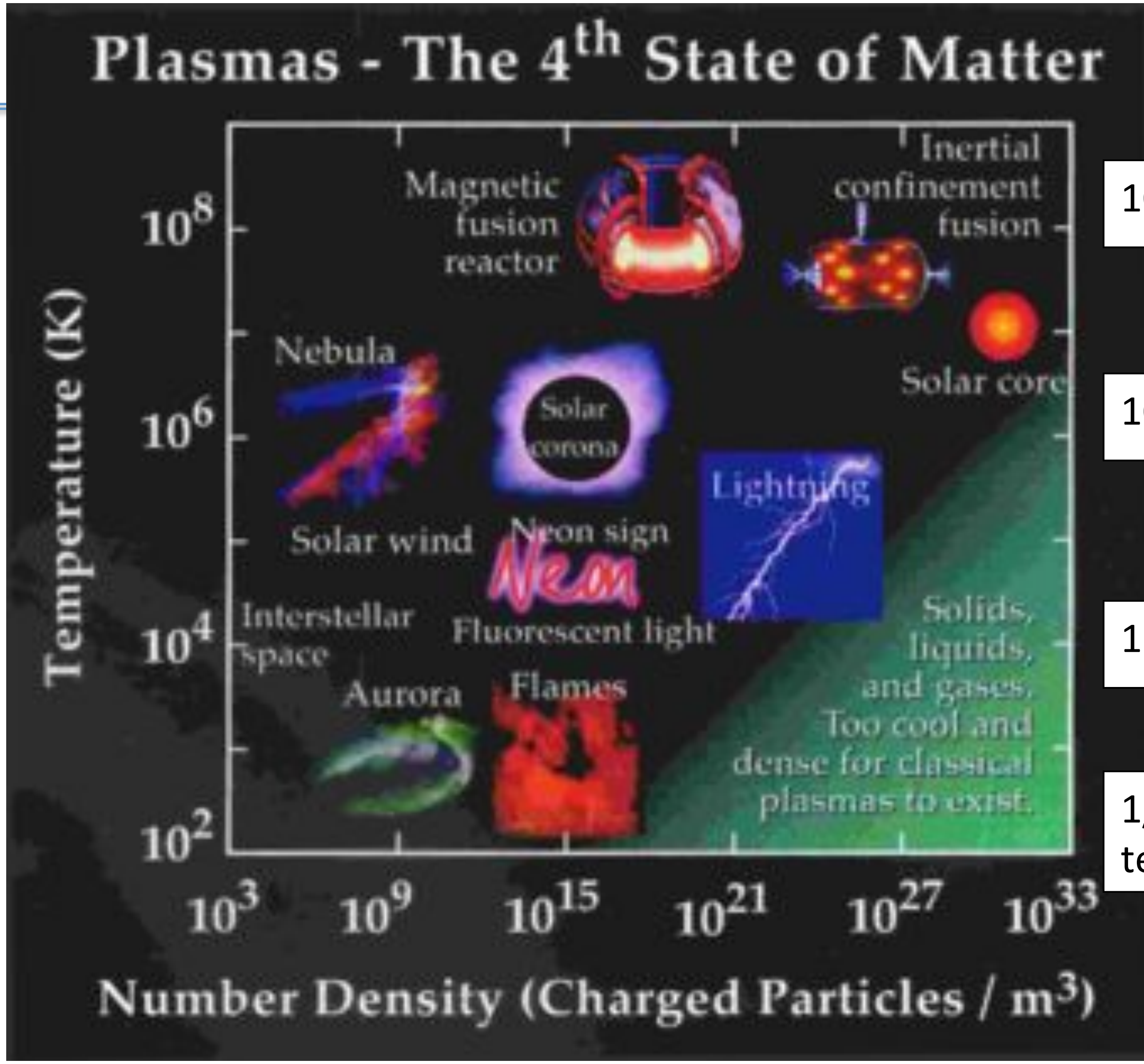


Add some thermal energy
electrons acquire enough energy to escape from atomic binding forces => ions & electrons

Fusion: Creating a Star on Earth
fusioned.gat.com/images/pdf/Slides01-67.pdf

plasmas differ from everyday materials

- With most materials, nearest neighbor interactions determine properties
- BUT: In a plasma,
  - charge separation generates electric fields
  - Charge flow generates currents and magnetic fields
  - These fields create "action at a distance" - Collective effects
  - A huge range of phenomena
    - Startling complexity
    - Many practical applications
    - Observable 99% of the universe is plasma



10 keV

100eV

1 eV

1/40 eV, room temperature

Plasma - πλασμα

- The term plasma was originally coined by Langmuir and Tonks in 1929
  - πλασμα
  - In Greek, this means *moldable substance* or jelly

- Today the term "plasma" is used quite generally to describe quasi neutral systems of charged particles
  - Plasma physics is the study of the its behavior
- Analogous to quantum mechanics - both wave and particle properties
  - Plasmas behave both as collections of particles as well as a fluid
  - Fluid and electromagnetic waves, flows

Sketch the history

- 1879 Crookes tube - "radiant matter"
- 1920's Langmuir- vacuum tubes
- 1930's Appleton - ionosphere
- 1950's thoughts about nuclear fusion
- 1960's solar wind, stellar interiors
- 1980's … industrial applications
    - Semiconductor industry
    - Plasma processing of materials
    - Plasma torch

Where do we find plasmas?

- On earth in nature
  - Lightning
  - Aurora
- in everyday life
  - Spark plug, arc welder, gas stove, fluorescent light bulb, …
- Beyond our earth's atmosphere
  - Magnetosphere pervades our solar system
  - Plasma system formed by the interaction of the earth's magnetic field and the solar wind
- Stars are made of plasma, including our sun
- Most of the universe is made up of plasma
  - Leave dark matter for the cognoscenti

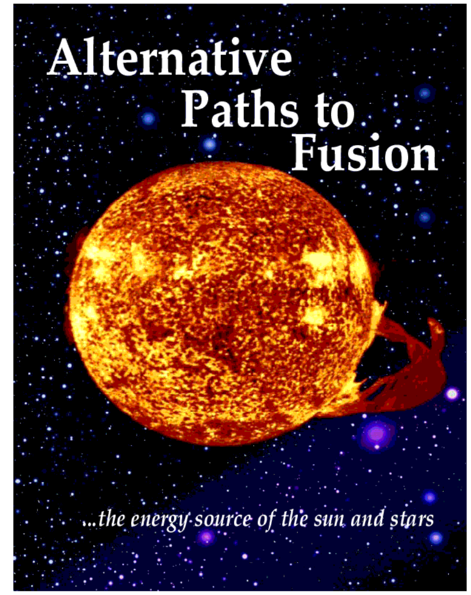
Northern lights - aurora borealis
electron acceleration from 1 R_E down polar field lines

How to make a plasma?

- Heating a container with gas inside is not practical
  - the container would need to be as hot as the plasma, i.e. ionized as well
- Typically in the laboratory a small amount of gas is heated and ionized. Power absorbtion can occur when you
  - Pass a current through it
  - Sprinkle a gas with fast electrons to collide with atoms
  - Shine radio, light waves through it
  - Shock it
- Container must be cooled, or insulated (e.g. with a magnetic field)

What are plasmas used for?

- Gas lasers, visible, X-ray wavelengths
- Plasma processing of materials
  - Plasma assisted chemistry
  - etching & deposition (semi-conductors, sunglasses)
- Lighting
- Next generation of particle accelerators



- Potential energy source from thermonuclear fusion
  - $D^+ + T^+ \Rightarrow \alpha(3.5\ \mathrm{MeV}) + n(14.1\ \mathrm{MeV}),\ T_i>10\mathrm{keV}$
  - Magnetic confinement, laser inertial confinement, Magneto Inertial Fusion
  - Non carbon fuel cycle
  - "greener" than fission breeder reactors

Plasma processing of materials

- Plasma chemistry
- Surface modification
- Semi conductors
  - Etching
  - Oxiding
- Plasma spray, torch
- Decontamination
- Gas lasers
- Solid state plasmas
- Fusion
- Space physics

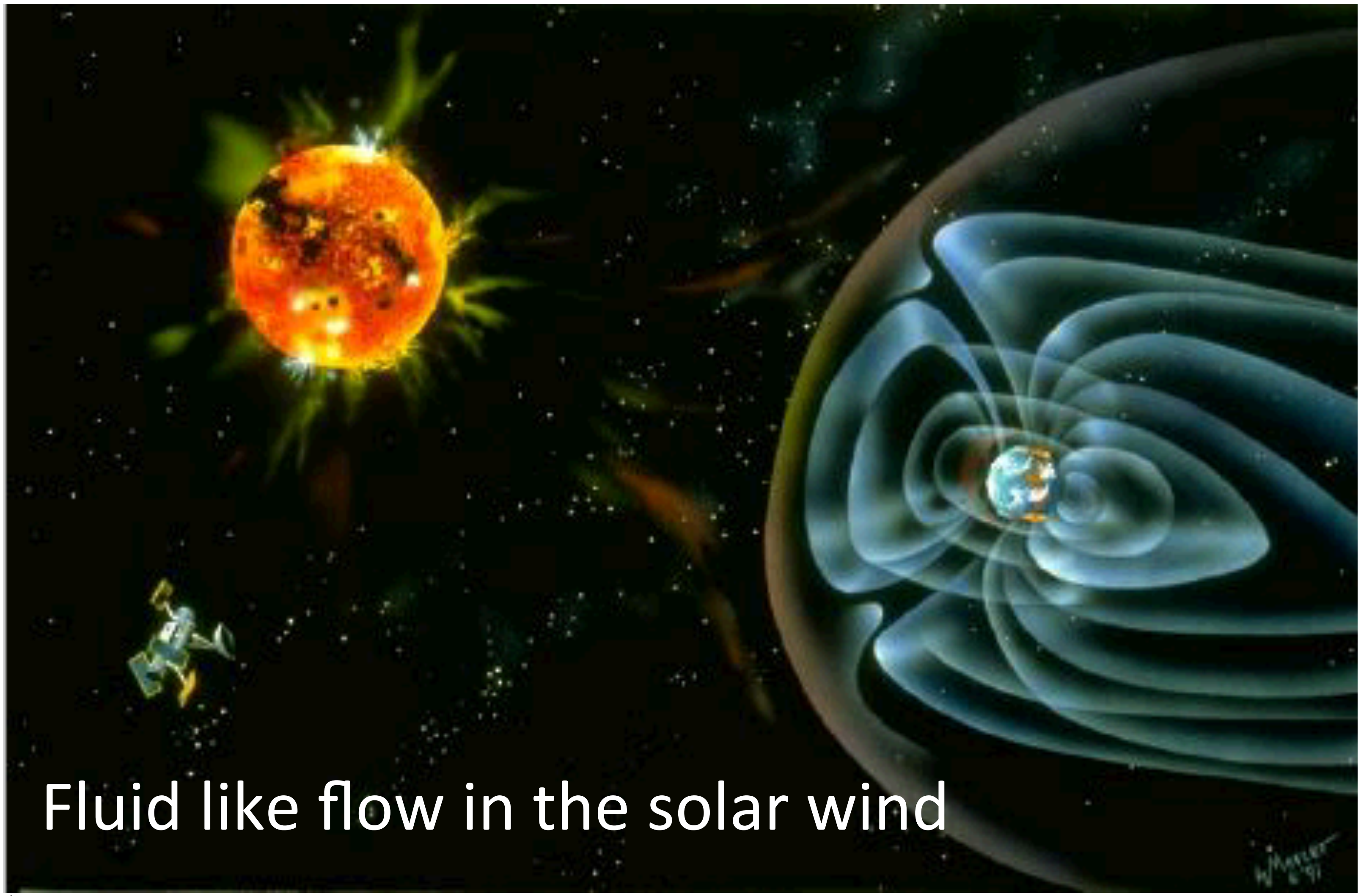
Why fusion plasma science?

- Dual nature of fusion research
  - *Grand Challenge* frontiers of science and technology
  - Plasma physics of most of the baryonic universe
  - Non-fossil energy source … could power large cities and industries
- LANL missions
  - Energy security
  - Threat reduction: neutrons for active interrogation, interpret satellite data
  - Pushes limits of multi scale computing
  - NWP: high energy density plasmas
- many other applications
  - Astro, space, solar physics
  - Advanced space propulsion

Theoretical framework is diverse

- Start with gas with *charged* particles
  - *Kinetic theory* of gases can be extended, include magnetic fields
  - Energy transfer between waves and particles
- *Many body problem* for inverse square fields
  - *Coulomb* for charged particles q/r^2
  - *Gravity* for stars mMG/r^2
- *Fluid dynamics* with high conductivity
  - *Magneto Hydro Dynamics* - (MHD)
  - *Flows* convect magnetic fields, relaxation, self organization
- Similarity to a solid
  - *Collective* wave modes are described by a *dielectric tensor*
  - Key properties- *polarization, dispersion* resemble solid state physics

3D view of solar wind, bow shock
probably typical of planetary magnetospheres



Fluid "feels" electromagnetic forces

- *Plasma fluid* includes
  - High electrical conductivity
  - free positive and negative charges < => electric fields
  - Charge movements <=> currents
  - Currents & magnetic fields must close on themselves
- Plasma *electric and magnetic* fields
  - This leads to a diverse "zoo" of properties
  - Self consistent structure is fascinating
  - Vorticity, particles <=> waves, turbulence, chaos …
  - Self organization

Maxwell's equations (SI units)

describe electromagnetic fields

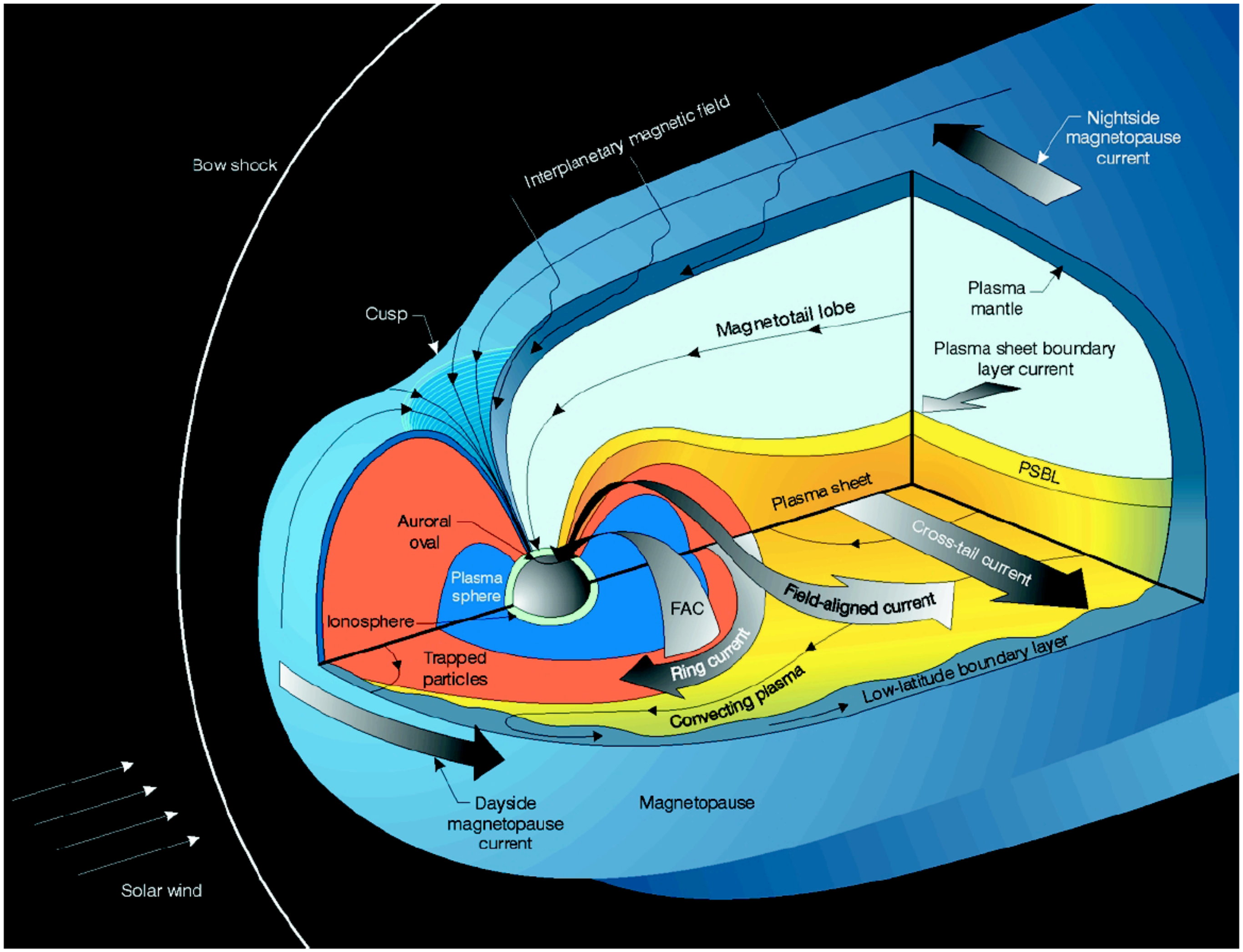
Faraday's Law	$\nabla \times E = -\dfrac{\partial B}{\partial t}$
Loop currents and magnetic field	$\nabla \bullet B = 0$
Poisson's eqn, charge density	$\nabla \bullet \varepsilon_0 E = e(n_i - n_e) = \rho$
Ampere's law	$\nabla \times B = \mu_0 J + \dfrac{1}{c^2}\dfrac{\partial E}{\partial t}$

Current conservation $\partial(en)/\partial t + \nabla \bullet J = 0$

Lorentz force equation $F = q(E + v \times B)$

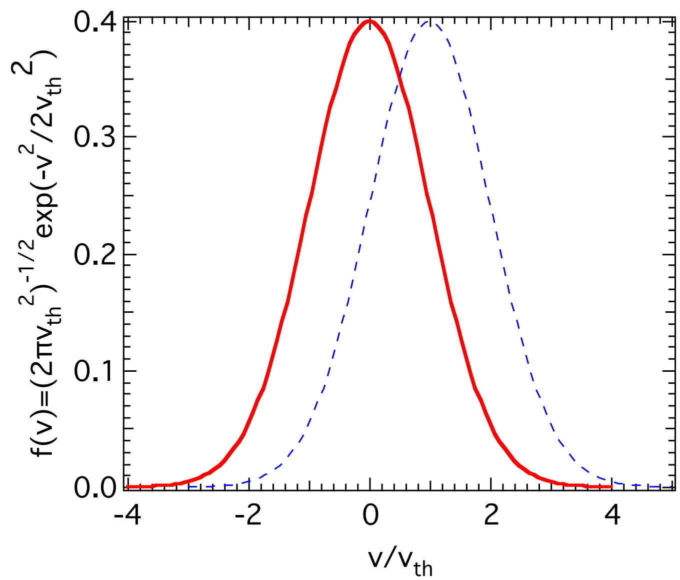
Earth + solar wind + dipole magnetic field

=> convecting field lines, bowshock, magnetotail, plasmoid



Link particle and fluid pictures

- Velocity distribution function
  - $f_v(\mathbf{x},t)$
  - Probability of a particle with a chosen velocity
  - Thermal jitter => Maxwellian distribution
  - $f(v) =$
    - $(2\pi v_{th}^2)^{-1/2} \exp(-(v-v_D)^2/2v_{th}^2)$
    - $T = mv_{th}^2$



Averaged fluid quantities

- Density $n/n_0 = \int f(v)\,dv$
- Thermal velocity $V_{th}^2 = \langle v^2 \rangle = \int v^2 f(v)\,dv$
- Drift velocity $v_D = \langle v \rangle = \int v\, f(v)\,dv$

Charge neutrality & Debye length

- Consider a plasma, that initially has
  - Thermo dynamic equilibrium: Maxwell-Boltzmann distribution
  - uniform charge density $n_{i0} \approx n_{e0}$
  - zero electric field
- How does the charge density behave when we attempt to change it?
- Suppose electron density drops from n_{e0} => $(1-\delta)n_{e0}$, $n_i(x) \approx$ constant
- If there were no change in electron density the equation for potential would be
  - $E = -\nabla\phi$
  - $\nabla \bullet \varepsilon E = e(n_i - n_e)$ divergence of $D = \varepsilon E$ = free charge density
  - $-\varepsilon\nabla^2\phi = e\,\delta n$

– $n_e(\phi, v) = n_{e0} \exp[-(e\phi/T + v^2/2v_{th}^2)]$

Charge neutrality & Debye length

- Maxwellian electron distribution f(energy) in 1 dimension
  - $-\varepsilon\, d^2\phi/dx^2 = e\,\{n_{i0} - n_{e0}\exp[-(e\phi/T + v^2/2v_{th}^2)]\}$ & integrate over velocity
  - $n(x)/n_{e0} = \exp[-(e\phi(x)/T_e)]$
  - expand the ODE in terms of small ϕ/T,
    - $-\varepsilon\, d^2\phi/dx^2 \approx e[n_{i0} - n_{e0}(1-\phi/T)]$
    - $d^2\phi(x)/dx^2 \approx -\, e\,\delta n_{e0}\,(e\phi(x)/T)\,/\,\varepsilon$
- Exponentially decaying solutions, *shielded Yukawa potential*
  - $\phi(x) \approx \exp[-x/\lambda_D]$
  - $e\phi(x)/T \approx \exp[-x/\lambda_D]$ normalize this equation to be dimensionless
  - $\lambda_{De} = [\varepsilon_0 T/(e^2 n_e)]^{1/2}$ *Debye length* $\lambda_{De}(\mathrm{cm}) = 740\,[T_e(\mathrm{eV})/n(\mathrm{cm}^{-3})]^{1/2}$
  -
- *Debye length* λ_D is the characteristic exponential decay *length*
  - Potential and electric fields from a point charge are shielded out by the sea of other charged particles on λ_{De} scales
  - definition of a plasma is that many particles exist in a *Debye sphere*
    - $n\,\lambda_D^3 \gg 1$
    - Validate statistical assumption of small departures from equilibrium

Quasi neutrality, ambipolar electric fields, sheath, boundary layer problem

– $e\phi(x)/T \approx \exp[-x/\lambda_D]$

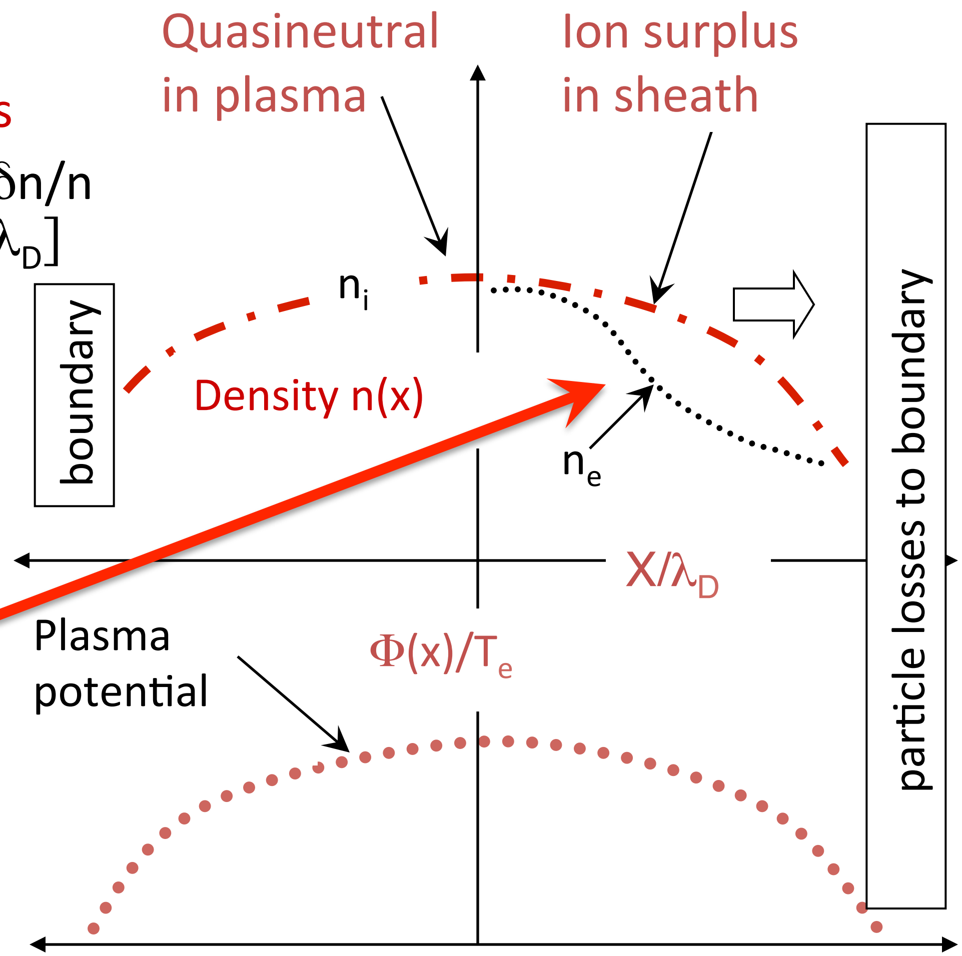
– Φ/T concave down for ion surplus

– $d^2[e\phi(x)/T]/dx^2 \approx -(x^2/\lambda_D{}^2)[e\phi/T]\, \delta n/n$
$\approx (x^2/\lambda_D{}^2) \exp[-x/\lambda_D]$

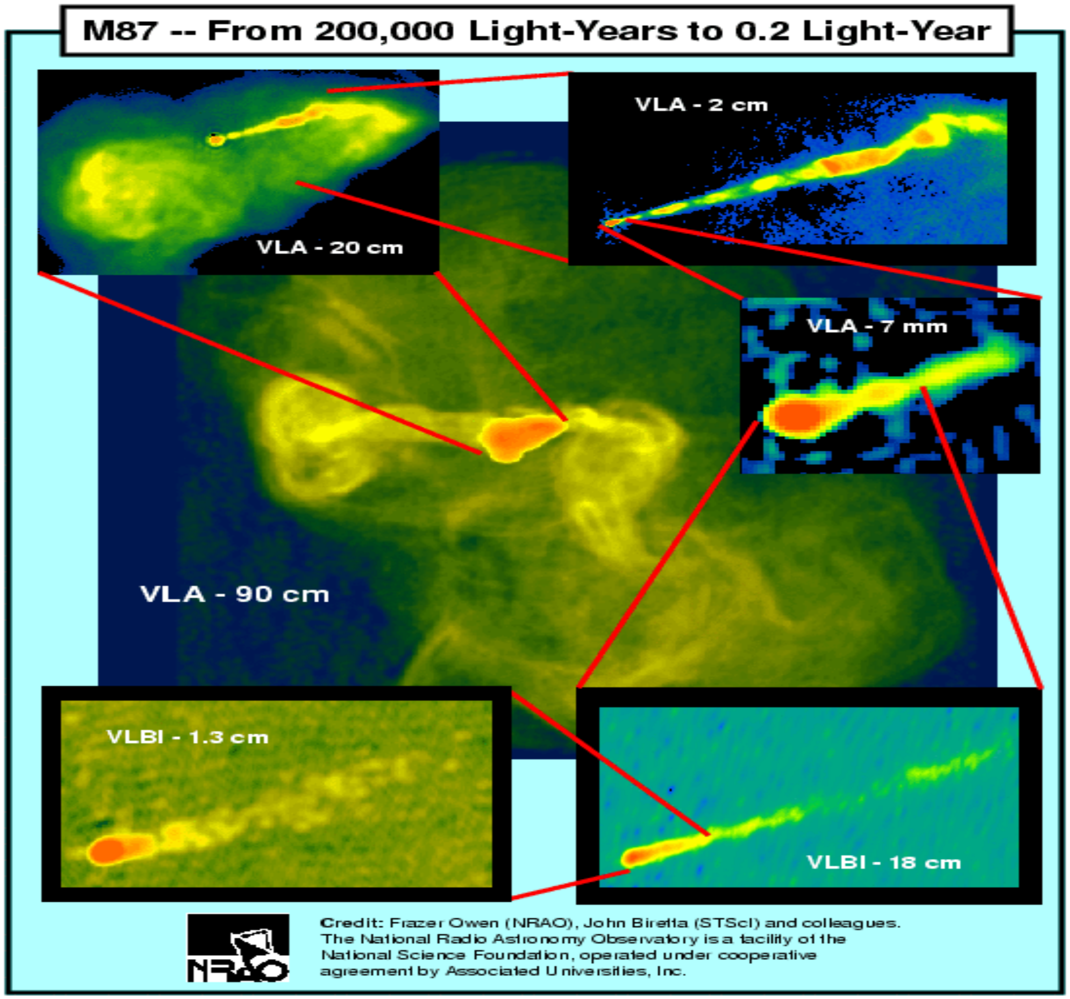
– $\approx \exp[-e\phi/T]$

- scales with density difference between mobile electrons n_e and heavy ions n_i

- electrons escape faster than ions but are held back by self consistent electric field
- Scale length is λ_D



Galactic jets - fluid flow and radiation powered by annihilation of magnetic fields (reconnection)?



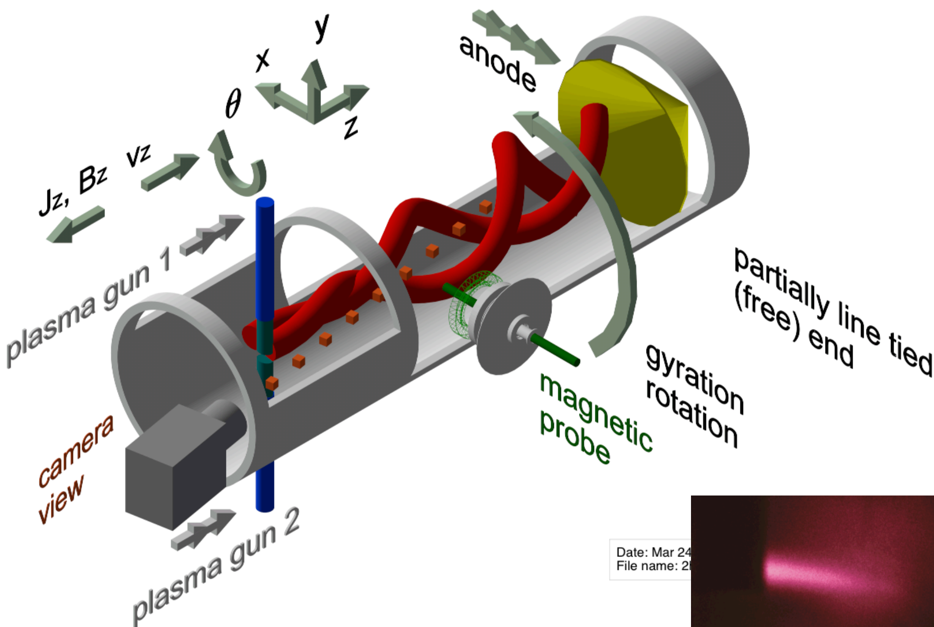
Magneto Hydro Dynamic approximation

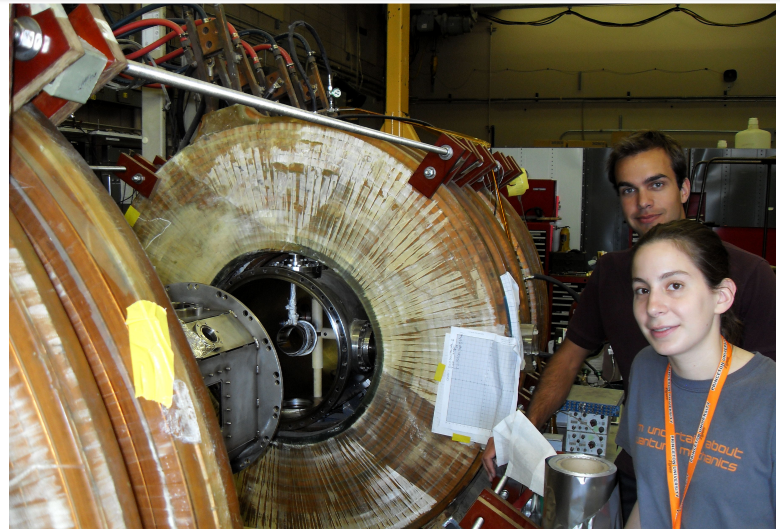
- If a magnetized plasma has disturbances that are slow enough and large enough spatial scale
  - Mobile electrons can prevent the buildup of any electric field that moves with the plasma
    - Fields are "stuck" in plasma reference frame - *frozen flux* theorem
  - Higher frequency modes of oscillation and wave propagation do not play a role
  - Behavior simplifies substantially
- Magneto Hydro Dynamic (MHD) approximation

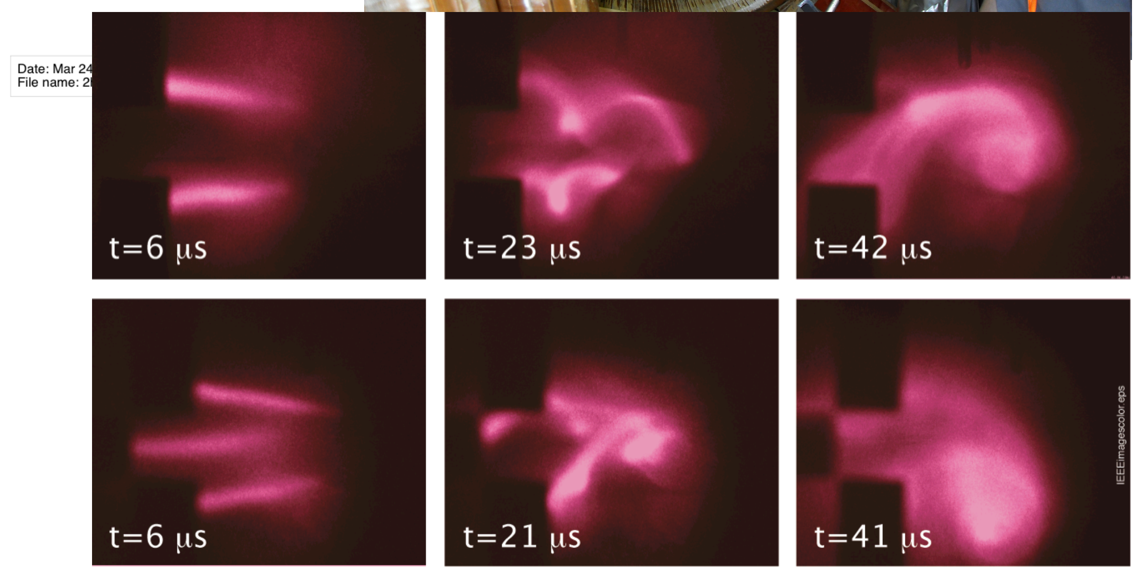
MHD force balance

- MHD model presumes a single fluid with mass density
  - $\rho = n_i m_i + n_e m_e \approx n(m_i + m_e) \approx n\, m_i$
- Charge density
  - $\sigma = (n_i - n_e)e$
- Mass velocity
  - $\mathbf{U} = (n_i m_i \mathbf{u}_i + n_e m_e \mathbf{u}_e)/\rho \approx \mathbf{u}_i + (m_e/m_i)\, \mathbf{u}_e$
- Current density
  - $J = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e) \approx ne(\mathbf{u}_i - \mathbf{u}_e)$
- Single fluid equation of motion
  - $\rho\, du/dt = \{\partial u/\partial t + u \cdot \nabla u\} = \sigma E + jxB - \nabla p$
- Resistive MHD includes finite resistivity
  - $\mathbf{E + uxB = \eta J + \{j \times B - \nabla p\} /en}$

Current ropes modeled in the laboratory





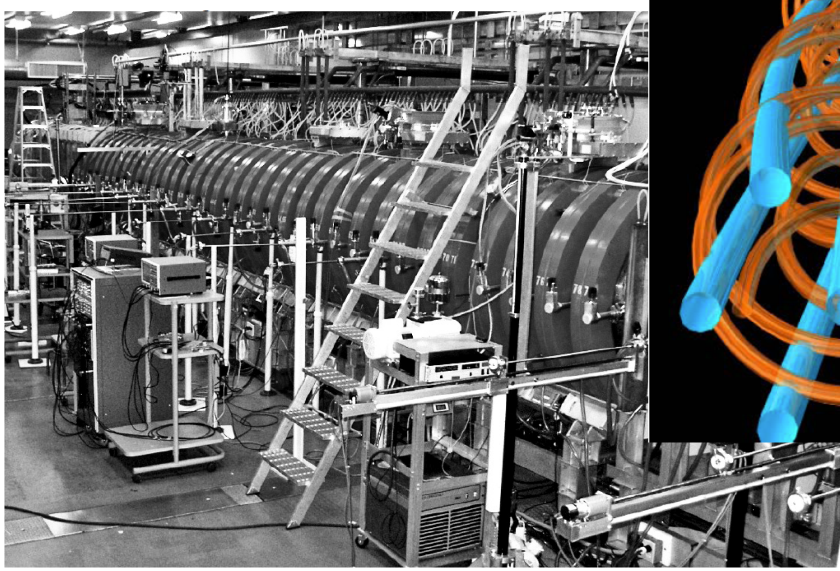


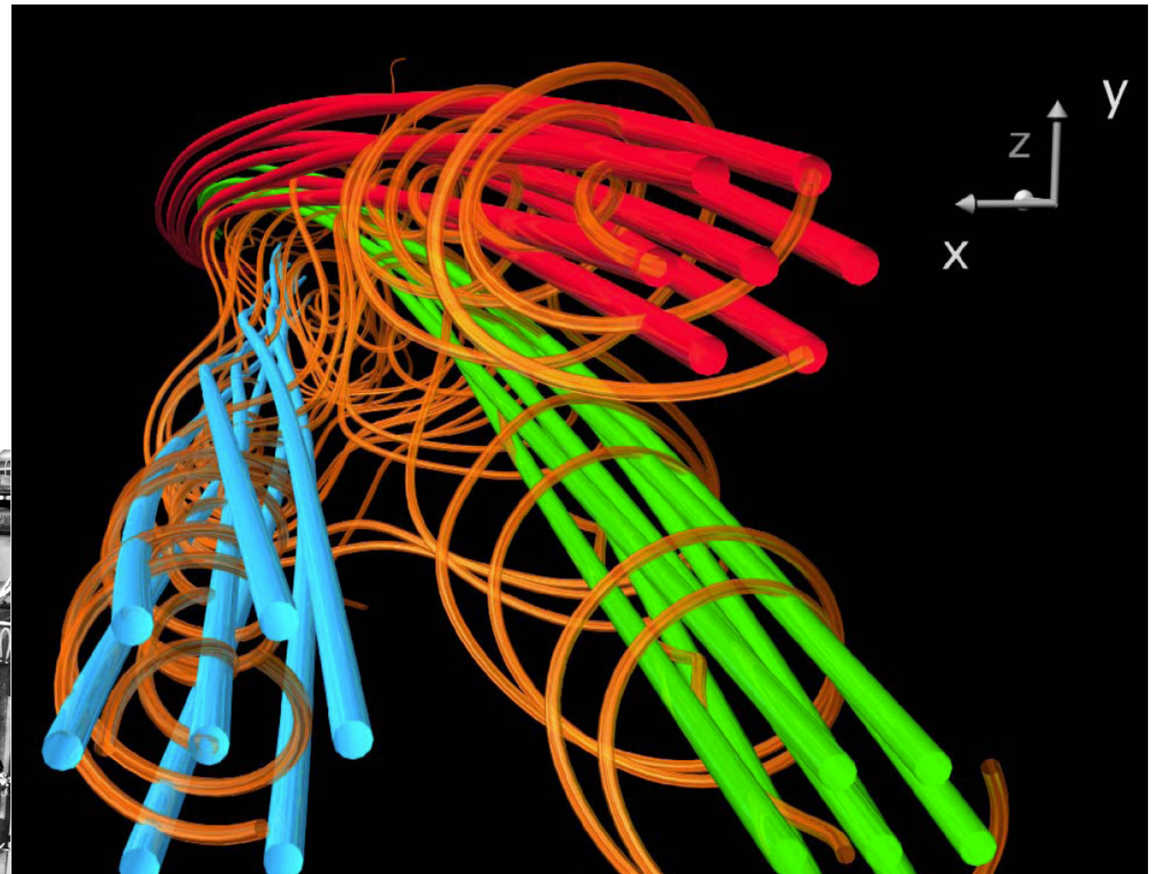
Alfven waves & MHD

- A notable property of MHD plasmas was discovered by Hannes Alfven in 1942
  - Michael Faraday had shown in 1800's that stresses in the magnetic field are equivalent to a pressure transverse to the B field and a tension along the field lines.
  - Alfven showed that certain waves can propagate along the field
  - Analogous to waves on a plucked string "Alfven waves"
  - Sound and Alfven speeds

3 flux ropes in LAPD

Large Plasma Device
UCLA
W. Gekelman





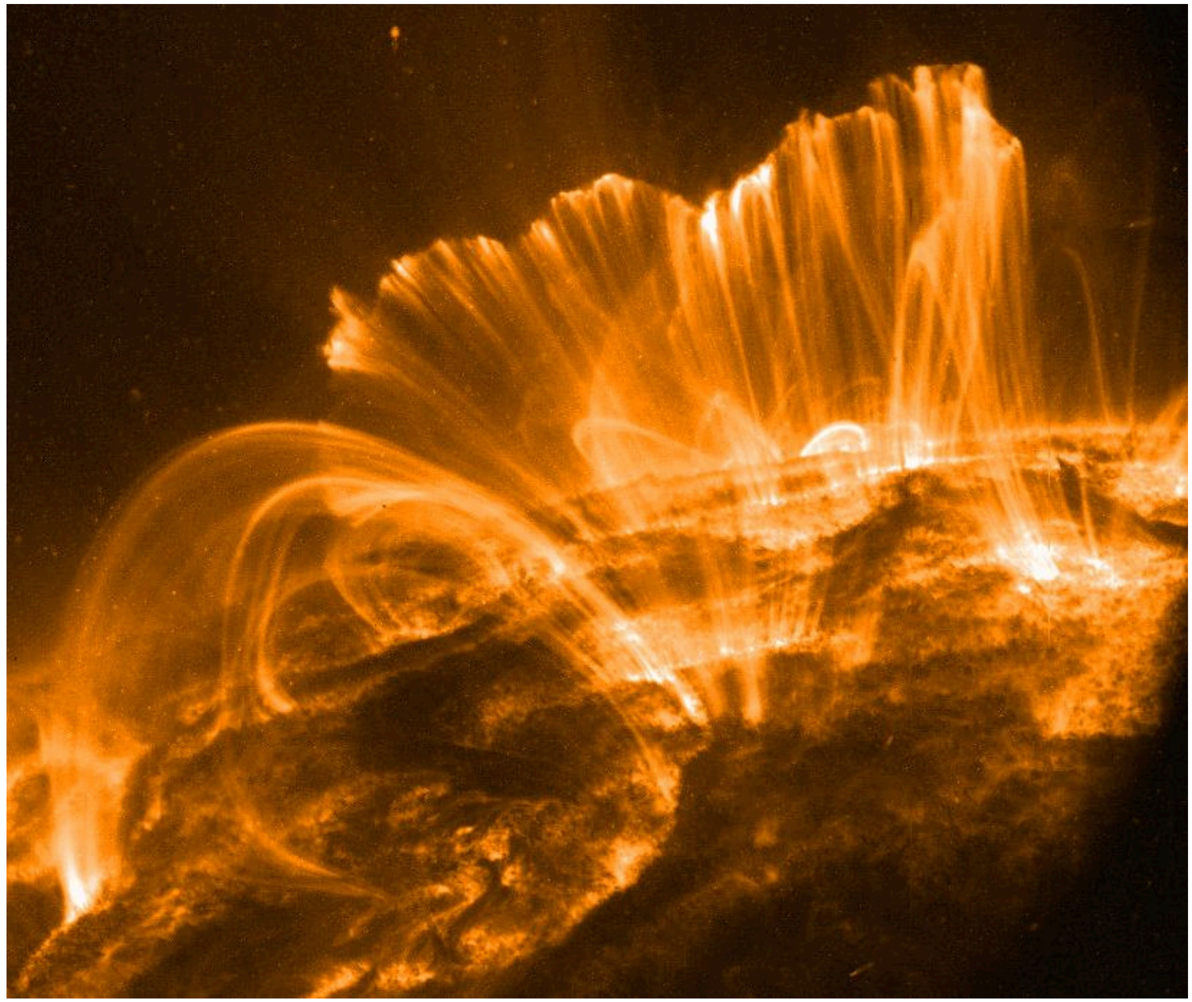
- B lines (RGB)
- J lines (orange)
- Gekelman et al, ApJ2012

Complex systems

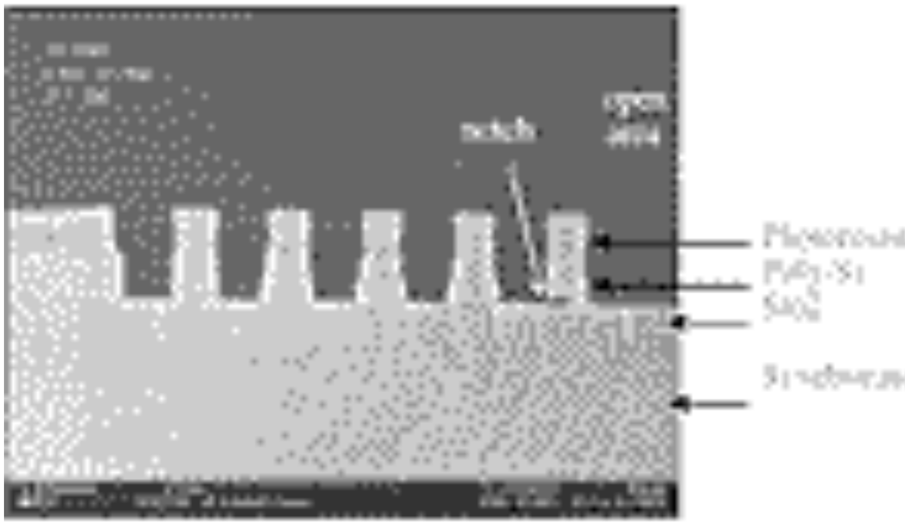
- Plasma systems can exhibit *self organized* behavior of high complexity
- Self organization occurs in many arenas
  - Space & astrophysics, biosystems, self assembly of micro and nano components, protein folding
  - *Selective decay* processes, thermodynamics
    - Dissipation of some invariant on small scales (eg energy in eddies, turbulence)
    - Persistence of other quantities on larger spatial scales (e.g. helicity)
- *Chaos* … a cottage industry

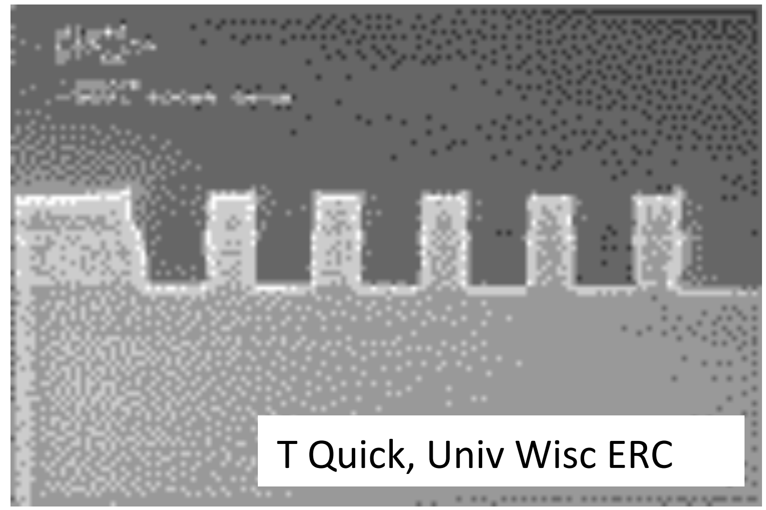
Solar arcades: fluid & magnetic structure

TRACE spacecraft image, NASA



Plasma etching of silicon





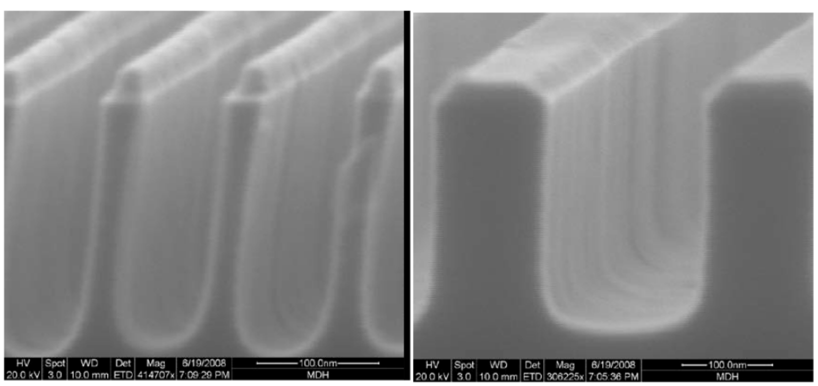


Fig II.2 Cross sectional SEM of a pseudo Bosch etch of 30 nm (left) and 100 nm (right) silicon ridges masked using approximately 55 nm of PMMA. This etch was terminated before the onset of mask erosion at 256 nm.

Henry_ICP ETCHING OF SILICON PhD thesis
Caltech 2010
30nm & 100nm features

Summary

- Plasmas are pervasive - 99% of observable universe
- Many examples in everyday life, applications, technology, nature
- Fluids with electrical and magnetic forces
- Experimentally and computationally accessible
- Basic physics is still unfolding - an adventure

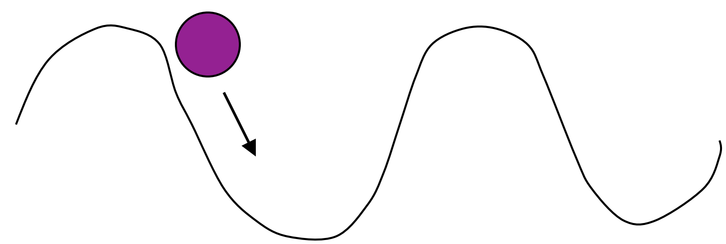
Extra slides

Plasma waves *vs* free space waves

- charged particles are tightly coupled to electric and magnetic fields in the plasma
- Wave properties depend on the the dielectric and magnetized medium
    - Charged particles can move freely along a B field
    - Movement across the B field is inhibited
    - Anisotropic wave propagation and properties

Landau Damping

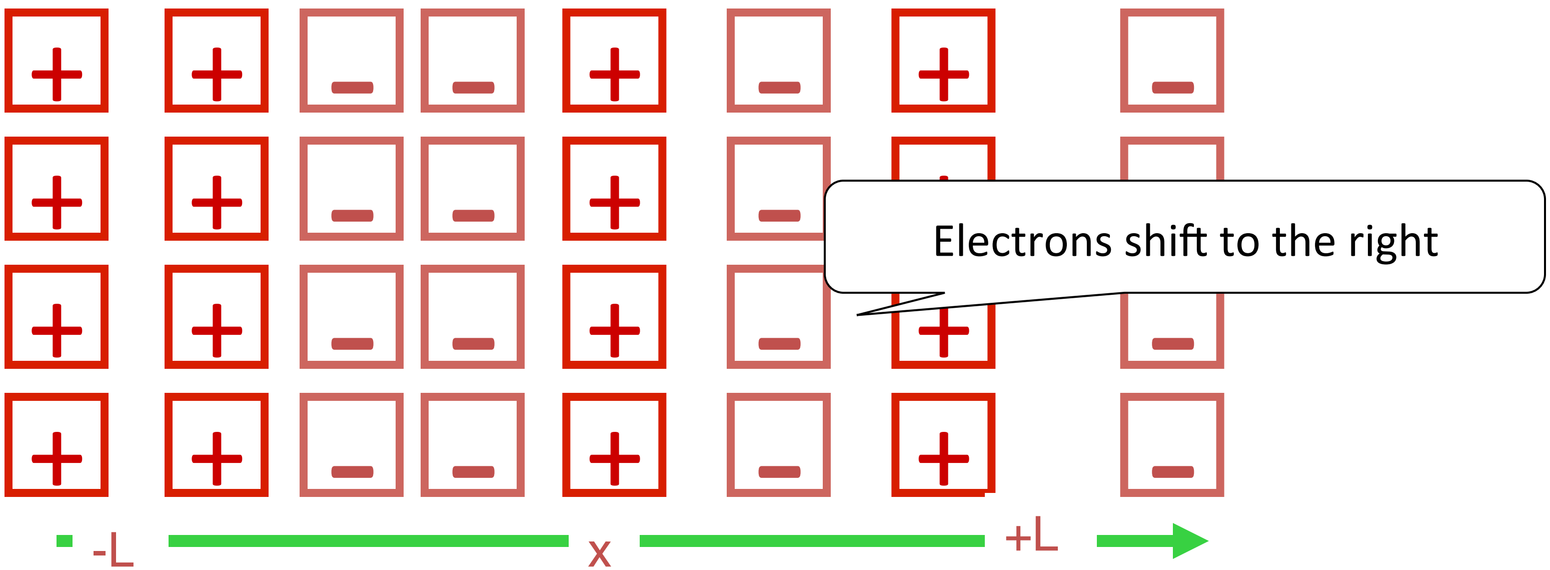
Wave propagation



- Particles "surf" on wave if $v_{\phi} > v_{particle}$
- Wave gains energy from particle if $v_{\phi} < v_{particle}$

Electron plasma oscillations

- Consider one dimensional motion
- Sheet of electrons centered at x
- Now move e- distribution to right, x' =x + ξ
  - Excess positive charge neξ per unit area
  - Field points to right $\mathbf{E} = ne\varepsilon_0\xi$



- E field points to right, because of charge separation

Electron plasma oscillations

- Equation of motion is $f=ma= m\, d^2x/dt^2$
- $-eE = m\, d^2\xi/dt^2$
  - Recall $E = ne\varepsilon_0\xi$ so that
  - $m\, d^2\xi/dt^2 = -e\,(ne\varepsilon_0\xi)$
  - $d^2\xi/dt^2 + e^2 n\, \varepsilon_0\xi/m = 0$

- The ODE for displacement ξ is then
  - $(d^2/dt^2 + \omega_{pe}^2)\,\xi = 0$
  - Where $\xi(t) = \xi_0\,[\exp(+i\omega_{pe}t) + \exp(-i\omega_{pe}t)]$
  - $\omega_{pe}^2 = n\, e^2\varepsilon_0/m$ is the electron plasma frequency

Stability

Unstable plasma systems without collisions

- Stability is one of the most important and recurring problems in plasma physics
- Ideal plasma has zero collisions
  - Certain situations may be theoretically conceivable
  - But they wont survive if they are unstable
  - Fusion devices, eg tokamaks, RFP,
  - Buoyancy instability for flux tubes trapped in suns photosphere
  - Two stream instabilities

Stability

Unstable plasma systems due to collisions

- Finite *resistivity* (collisions) enables the resistive decay of currents in the system
- This is normally very slow in astrophysical settings
- However Dungey realized (1953) that resistive diffusion can lead to strong concentrations of current that then speed up the diffusion process
  - Resistive instabilities
  - Tearing modes, reconnection, solar flares

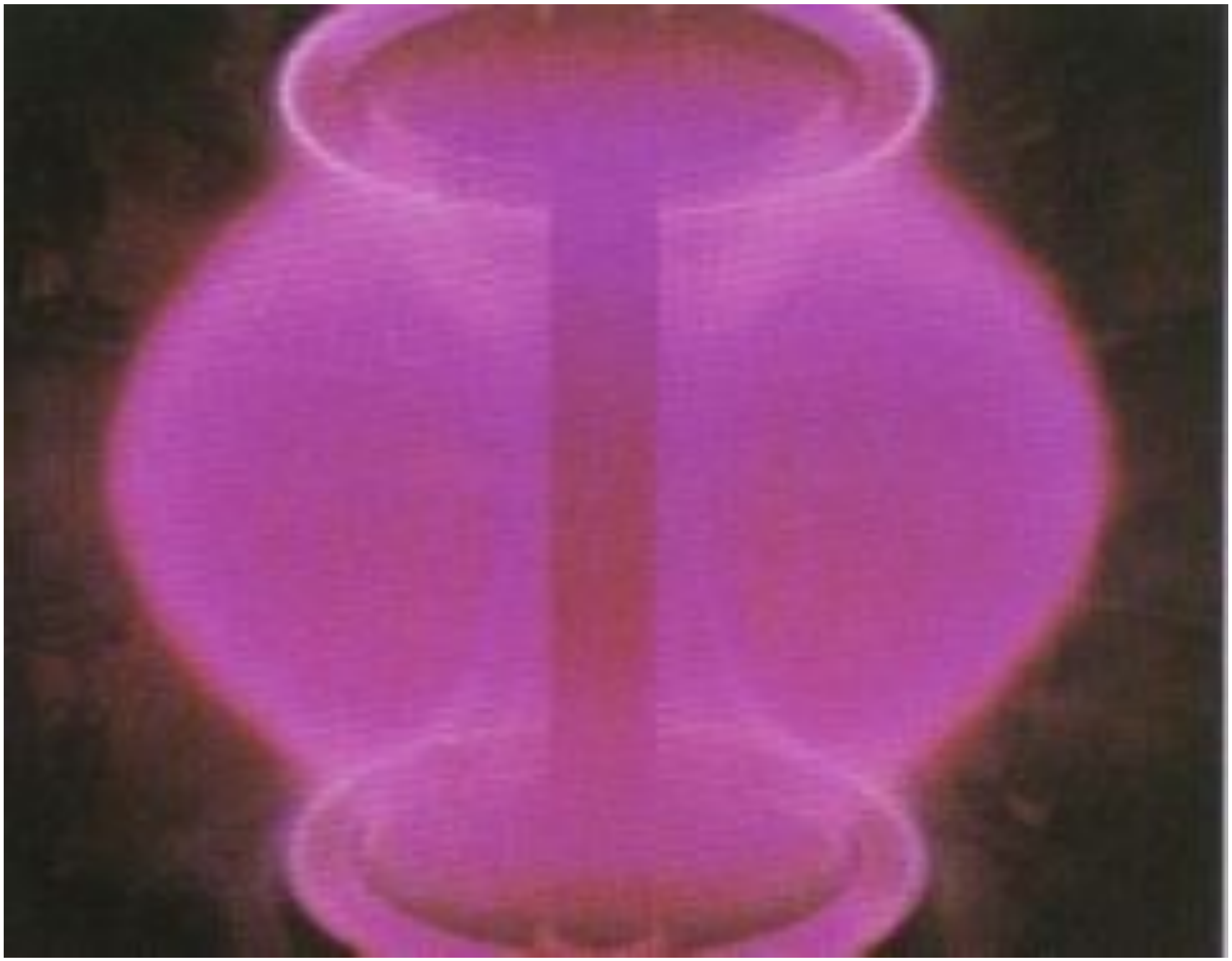
Complex systems

- Plasma systems can exhibit self organized behvior of high complexity
- Self organization occurs in many arenas
  - Space & astrophysics, biosystems, self assembly of micro and nano components, protein folding
  - *Selective decay* processes, thermodynamics
    - Dissipation of some invariant on small scales (eg energy in eddies, turbulence)
    - Persistence of other quantities on larger spatial scales (e.g. helicity)
- *Chaos* … a cottage industry

Confinement is not always perfect (or even good)

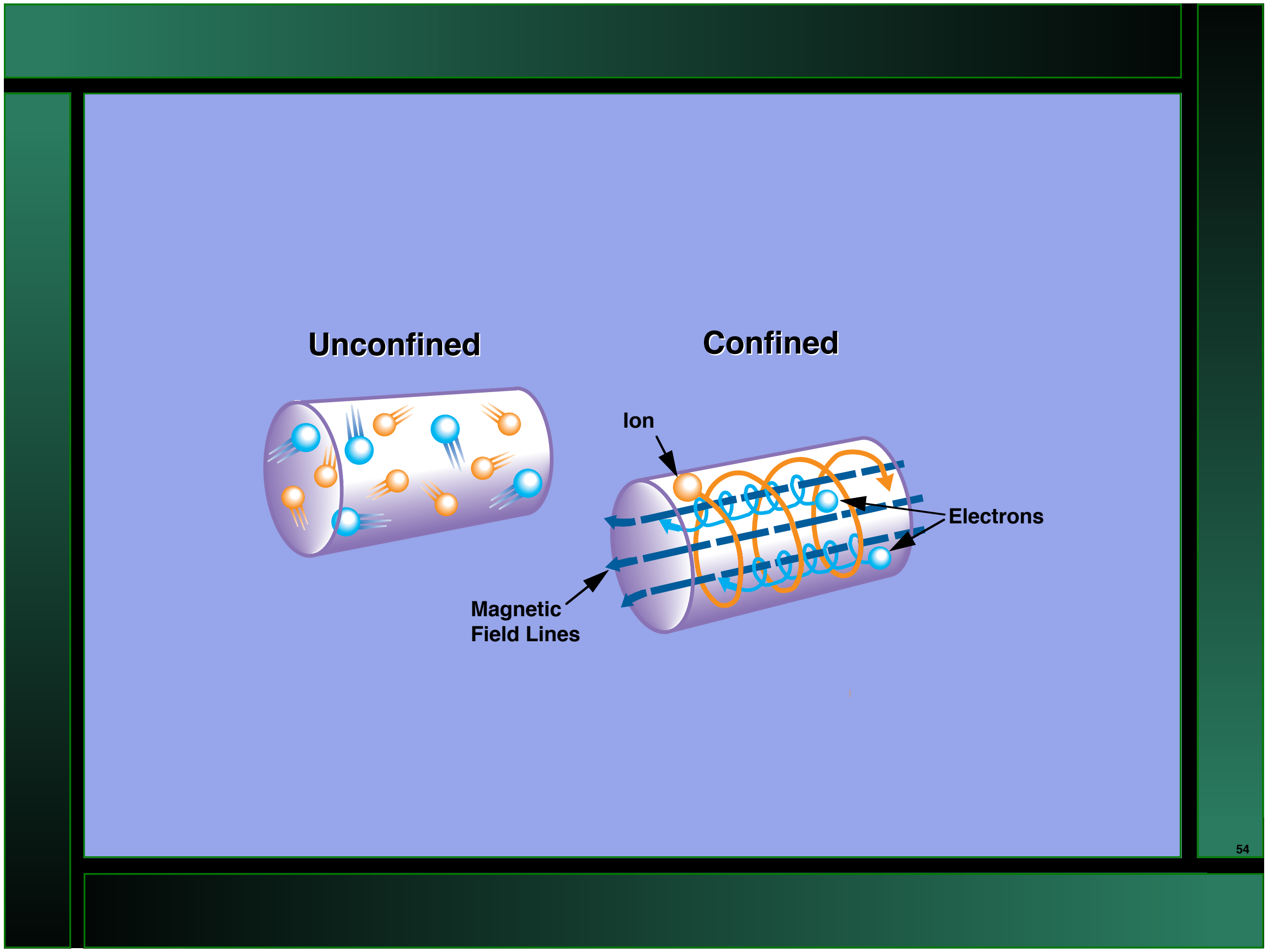
- Collisions between particles enable diffusion across magnetic field
  - Pitch angle scattering
- Kills trapping in
  - fusion devices
  - Planetary magnetospheres
  - Coronal flux tubes
- Collisions tend to dissipate highly ordered motions in wave propagation

START



Confinement means that particles leisurely linger => interact collectively

magnetic confinement here is one example



Handy formulary

- Debye length
  - λ_D(cm) = 740 $[T_e(\text{eV})/n(\text{cm}^{-3})]^{1/2}$
- plasma frequency
  - f_{pe}(Hz) = 9000 $n_e^{1/2}(\text{cm}^{-3})$ electrons
  - f_{pi}(Hz) = 210 $n_e^{1/2}(\text{cm}^{-3})$ ions
- gyro radius
  - r_{gi}(cm) = 100 $T^{1/2}(\text{eV})/B(\text{Gauss})$ ions
  - r_{ge}(cm) = 2.4 $T^{1/2}(\text{eV})/B(\text{Gauss})$ electrons
- Thermal speed
  - $v_{e,th}$(cm/sec) = $4.2\text{x}10^7\ T_e^{1/2}(\text{eV})$
  - $v_{i,th}$(cm/sec) = $10^6\ T_i^{1/2}(\text{eV})$

Introduction to Plasma Dynamo, Reconnection and Shocks

T. Intrator

P-24 Plasma Physics

2012 July 03

P-24 Plasma Physics Summer School

Center for Non Linear Studies conference room

Wednesdays 1pm-2pm

Abstract

In our plasma universe, most of what we can observe is composed of ionized gas, or plasma. This plasma is a conducting fluid, which advects magnetic fields when it flows. Magnetic structure occurs from the smallest planetary to the largest cosmic scales. We introduce at a basic level some interesting features of non linear magnetohydrodynamics (MHD). For example, in our plasma universe, dynamo creates magnetic fields from gravitationally driven flow energy in an electrically conducting medium, and conversely magnetic reconnection annihilates magnetic field and accelerates particles. Shocks occur when flows move faster than the local velocity (sonic or Alfven speed) for the propagation of information. Both reconnection and shocks can accelerate particles, perhaps to gigantic energies, for example as observed with 10^{20} eV cosmic rays.

Outline

- Our magnetic universe
- dynamo
- reconnection
- Shocks
- Summary

Universal factoids

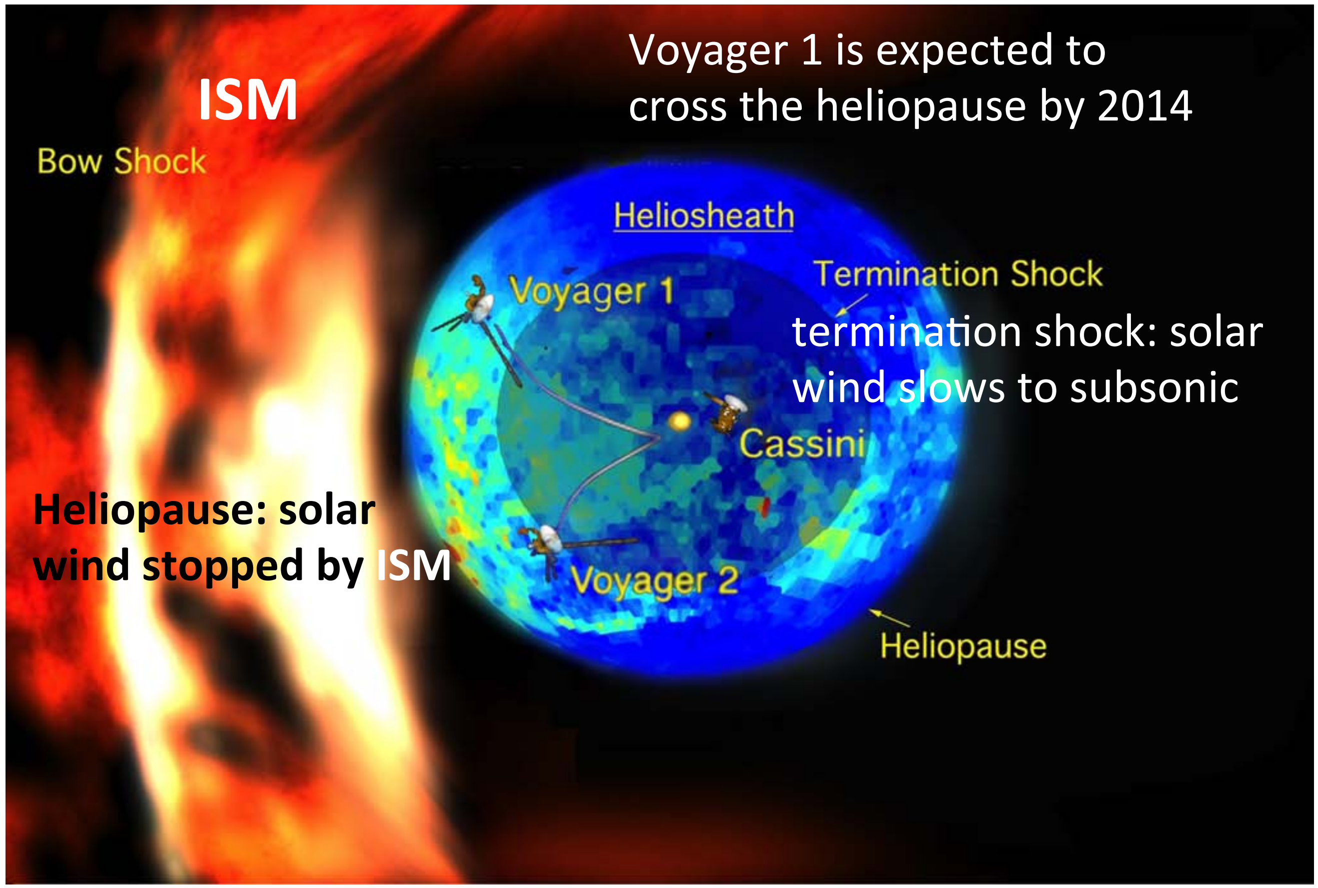
- Radius ≈ 46×10^9 LY ≈ 46×10^{25}m
- Volume ≈ 3.5×10^{80} m^3
- Mass ≈ 10^{52} kg (includes 73% dark energy, 23% dark matter, 4% baryonic matter)
- Mass-energy = $m_{baryon}c^2$ ≈ 4×10^{67} Joule
- Intracluster B field ≈ 0.1-10μGauss (Kronberg, PoP2003)
- Magnetic energy (1μG) = $B^2/2\mu_0$ x Vol ≈ 2.5%

Light Year = 9.46×10^{15}m, parsec = 3.26LY

Solar system confronts interstellar medium

magnetic bubbles

- Earth
- solar system



http://www.nasa.gov/images/content/462848main_helioshere_946-710.jpg

IBEX data

- **Description English:** In this illustration, the multicolored (blue and green) bubble represents the new measurements of the emission of particles known as energetic neutral atoms. The energetic neutral atoms were streaming in from the thick boundary known as the heliosheath. The heliosheath is the region between the heliosphere, the region of our sun's influence, and the interstellar medium, the matter between stars in our galaxy. Areas in red indicate the hottest, most high-pressure regions and purple the coolest, lowest-pressure regions.
- The yellow circle is our sun. The two Voyager spacecraft, illustrated with lines showing their path, are currently traveling through the heliosheath. In the heliosheath, the solar wind slows down and heats up as it interacts with the interstellar medium. The image also shows Cassini, which is still inside our solar system, orbiting Saturn. The spacecraft sizes are not to scale.
- The dark inner circle represents the volume bounded by the termination shock, formed where supersonic solar wind streaming out from our sun suddenly slows down. The outer circle, known as the heliopause, the outer boundary of the heliosheath, is the place where the interstellar medium and the solar wind are balanced. To the left of this bubble is the curve of the putative bow shock, where the interstellar medium, traveling in the opposite direction against the heliosheath, slows down as it collides with the heliosphere. The bow shock resembles a wave formed in a stream as it flows around a rock.
- **Date** 20 November 2009
- **Source** http://photojournal.jpl.nasa.gov/catalog/PIA12375 , http://en.wikipedia.org/wiki/File:PIA12375.jpg
- **Author** NASA/JPL/JHUAPL

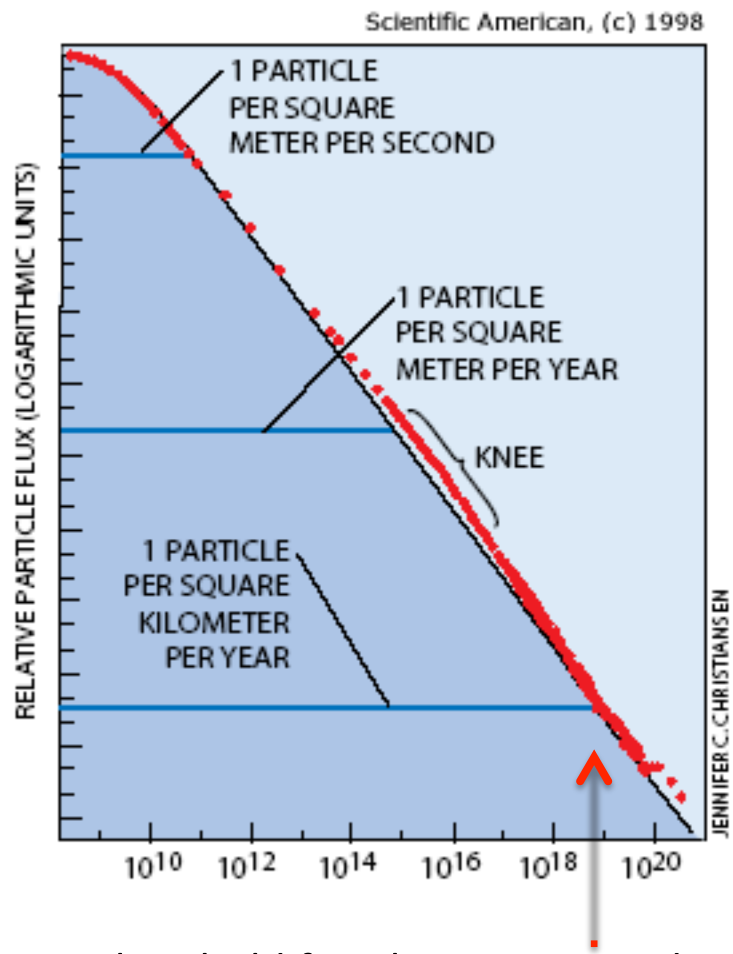
Heliospheric current sheet in 3D

http://en.wikipedia.org/wiki/Heliosphere

Magnetic field sourced by rotating sun, heliospheric current sheet out to the orbit of Jupiter - “Ballerina skirt”

High Energy Cosmic Ray Physics

- The flux of cosmic ray nuclei has to a first approximation, a rapidly falling power law in energy, $dN/dE \propto E^{-\alpha}$, with overall index $\alpha \approx 2.8$.
- The spectrum structure ("knee", "ankle") cannot be explained
- Unknown but probably magnetic particle acceleration mechanisms to such incredible energies



ankle GZK threshold for photo pion production

http://auger.physics.wisc.edu

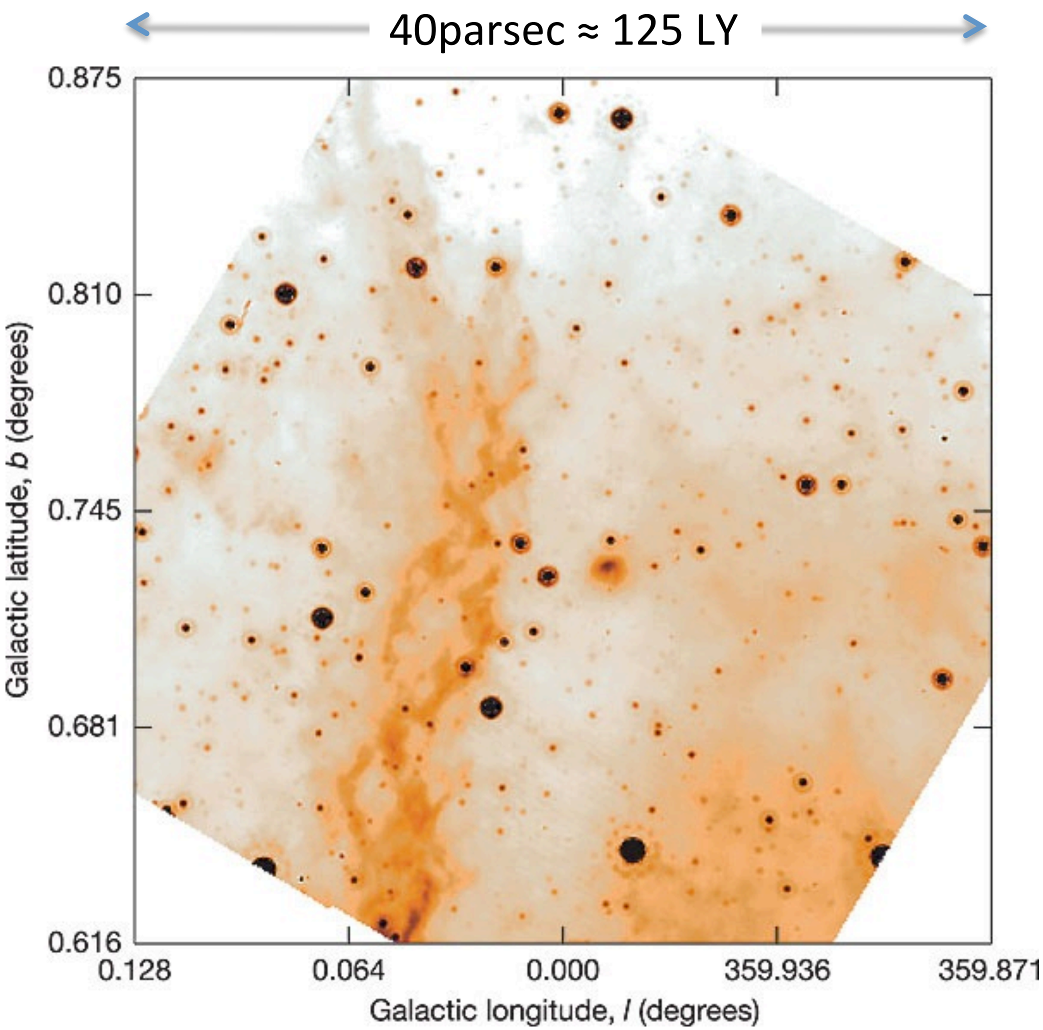
Magneto Hydro Dynamic approximation

- If a magnetized plasma has disturbances that are slow enough and large enough spatial scale
  - Mobile electrons can prevent the buildup of any electric field that moves with the plasma
    - Fields are "stuck" in plasma reference frame - *frozen flux* theorem
  - Higher frequency modes of oscillation and wave propagation do not play a role
  - Behavior simplifies substantially
- Magneto Hydro Dynamic (MHD) approximation

Magnetohydrodynamics: MHD

- MHD model presumes a single fluid with mass density
  - $\rho = n_i m_i + n_e m_e \approx n(m_i + m_e) \approx n\, m_i$
- Charge density
  - $\sigma = (n_i - n_e)e$
- Mass velocity
  - $\mathbf{U} = (n_i m_i \mathbf{u}_i + n_e m_e \mathbf{u}_e)/\rho \approx \mathbf{u}_i + (m_e/m_i)\, \mathbf{u}_e$
- Current density
  - $J = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e) \approx ne(\mathbf{u}_i - \mathbf{u}_e)$
- Single fluid equation of motion
  - $\rho\, du/dt = \{\partial u/\partial t + u \bullet \nabla u\} = \sigma E + j x B - \nabla p$
- Resistive MHD includes finite resistivity
  - $\mathbf{E + uxB = \eta J + \{j\ x\ B - \nabla p\}\ /en}$

Magnetic double helix



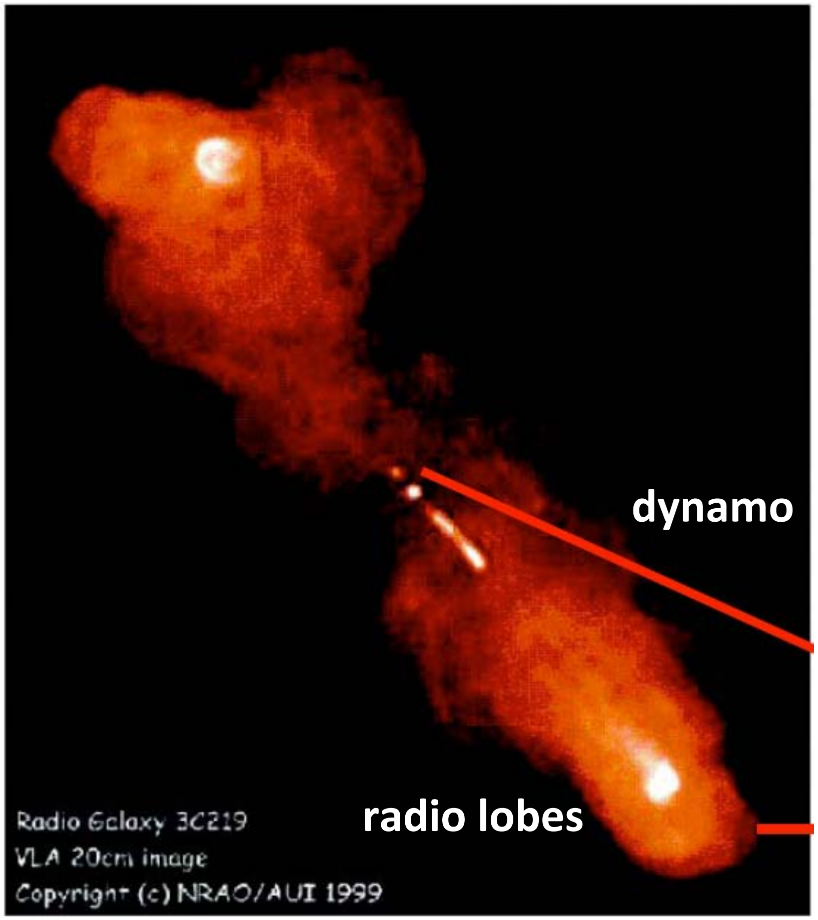
- Double helix nebula
- λ=24μm, MIPS camera, Spitzer Space Telescope
- Near galactic center (8 kpc away)
  - ★ 6 arcsec resolution
  - ★ 1 arcmin ⇔2.5 pc
  - ★ Parsec = 3.24 LY

A magnetic torsional wave near the Galactic Centre traced by a 'double helix' nebula, M Morris, et al, Vol 440|16 March 2006|doi:10.1038/nature04554

Dynamo

- The creation of magnetic field from flow in a conducting medium
  - Large scale coherent flows
  - Random and turbulent motions
- Twist up field lines that are trapped in flow
- Creation of B field from scratch (no one knows how this happens)

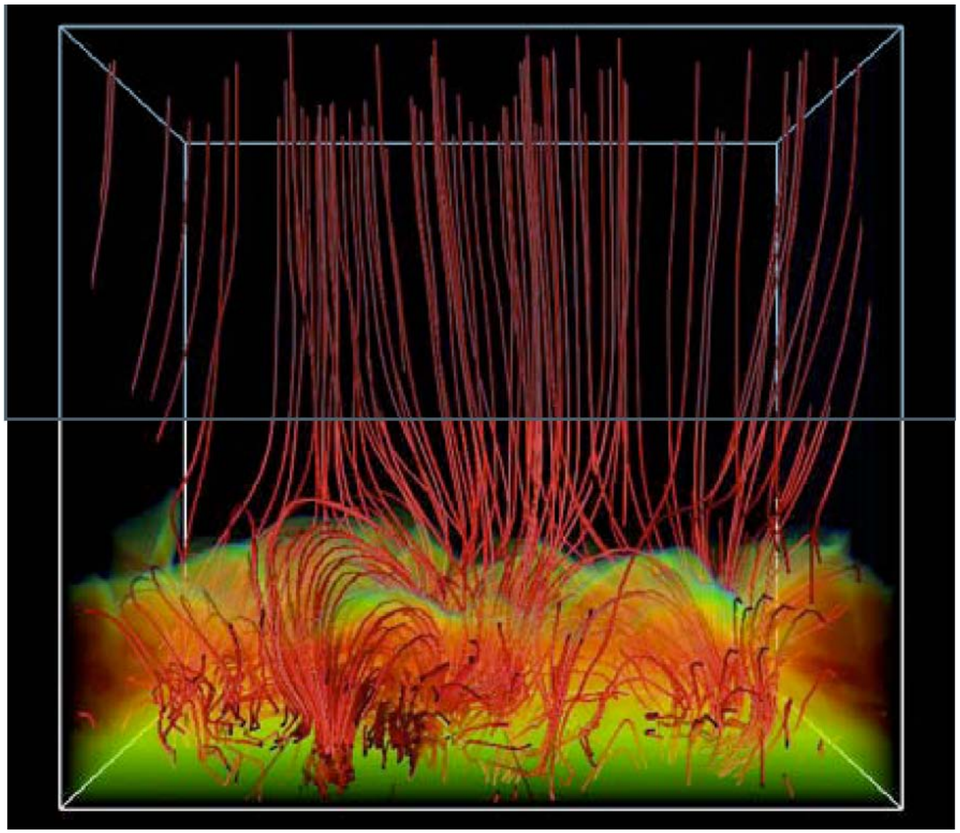
Astrophysical Dynamo: The Origin of Large Scale, Coherent Magnetic Fields



S. Colgate

- 3C219 Radio Lobe,
  - ~ 1% of MBH c^2 in 10^8 years.
- Not clear if it is the random motions + fluid turbulence, or large scale coherent motions
- Massive black hole accretion disk dynamo is small (≈ $30r_{Gi} \approx 10^{15}$cm) $\approx 10^{-10}$ of the radio lobe (~3 x 10^{24} cm) Frequency 10^{10} Hz
- $L_{lobe} \sim 10^{38}$-10^{39} Watt

Astrophysical Dynamo: The Origin of Large Scale, Coherent Magnetic Fields



- B fields on the sun
- simulation, 10^4 x granulation scale
- Convective plumes base of convective zone

$\alpha - \Omega$ dynamo

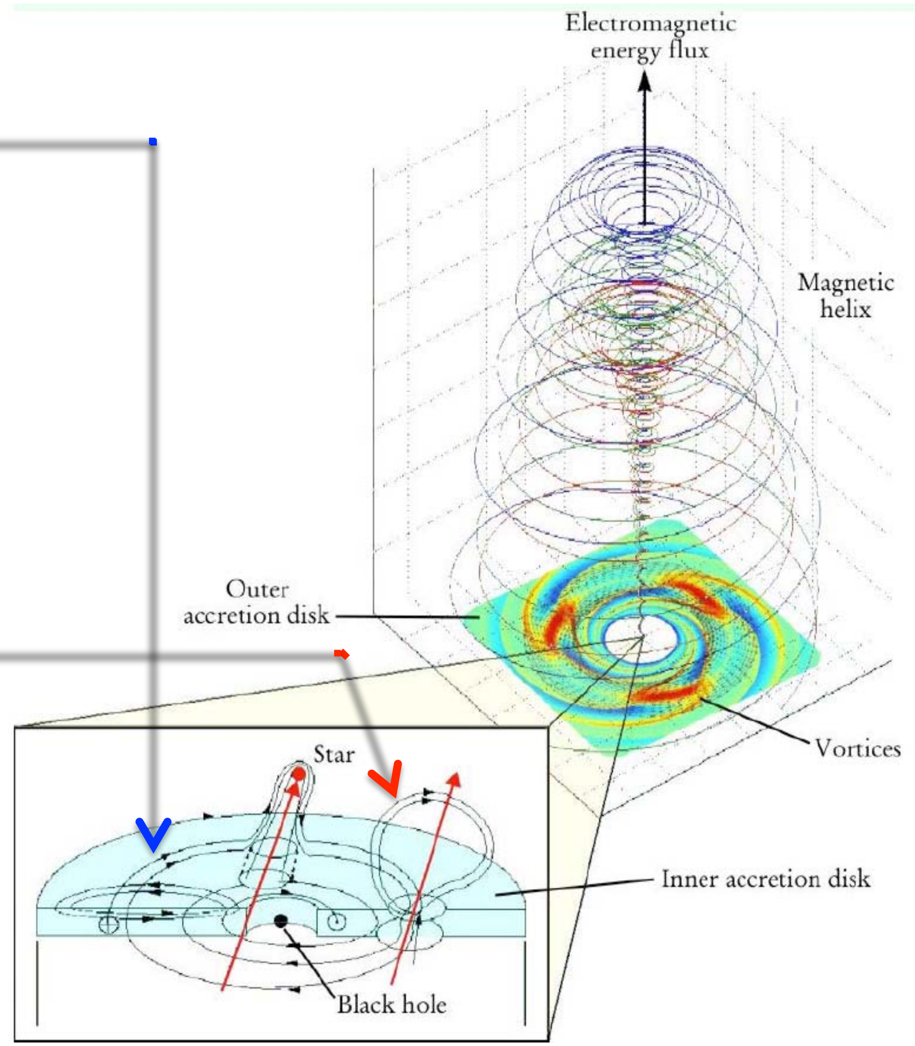
- Keplerian sheared galaxy rotation (the Ω part)
  - Balance centripetal with gravitational force
  - $V_\phi^2/r = GM/r^2$
  - $V_\phi(r) = (GM/r)^{1/2}$

Sheared motion winds up magnetic B_z lines (α shaped) outside the accretion disk.

⇒ Toroidal B field

⇒ force-free helix

⇒ magnetic flux of the universe??



Accretion disc, black hole rotating, shock emission

Integral Maxwell equations

Maxwell's Equations

Equation	Law	Meaning
$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o}$	Gauss' Laws	Relates ***E*** to charges. (Coulomb's law)
$\oint \vec{B} \cdot d\vec{A} = 0$	Gauss' Laws	***B*** is continuous. No magnetic monopoles.
$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	Faraday's Law	Changing ***B*** produces ***E***
$\oint \vec{B} \cdot d\vec{l} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$	Ampère-Maxwell Law	***I*** or Changing ***E*** produces ***B***

These equations describe all electric & magnetic phenomena (in the absence of dielectric or magnetic materials).

B. Bauer, U Nevada

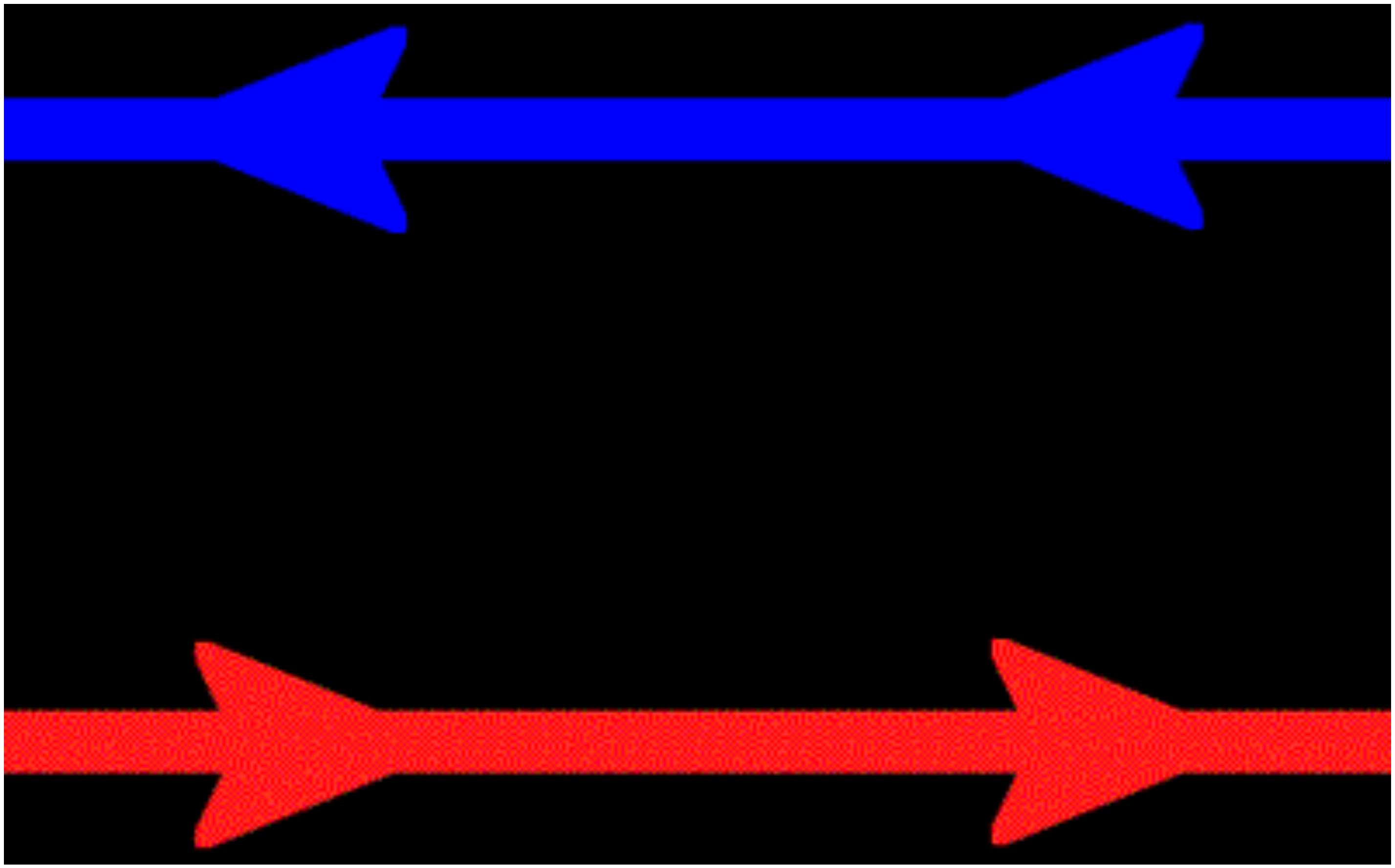
Magnetic field from fluctuations

- Newton's law F=ma↔momentum balance
  - $\partial(\rho v)/\partial t = E + vxB - \nabla p_e + JxB$
- Ohms law includes collisions (resistivity η)
  - $E + vxB = JxB/(en) - \nabla p_e/(en) + \eta J$
- Recall from Maxwell equations $\nabla xE=-\partial B/\partial t$
- Suppose fields have mean + fluctuations
  - $B = B_0 + \delta B$, $E = E_0 + \delta E$, $v = v_0 + \delta v$, … etc
  - $\nabla x\langle\delta E\rangle = -\nabla \times \langle\delta v \times \delta B\rangle + \nabla \times \langle\delta J \times \delta B\rangle + \ldots$
- → δB grows from correlated fluctuations

Magnetic reconnection

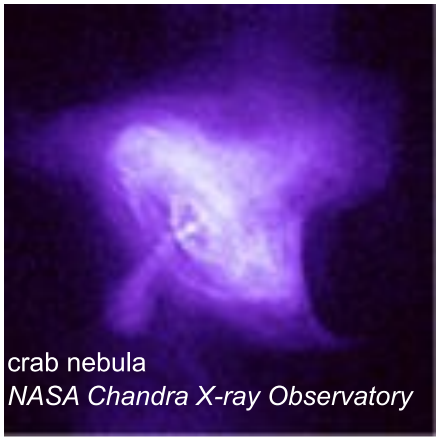
- Magnetic field lines and fluxes are “frozen” into fluid flow ($E + vxB = 0$)
- Fluxes are advected towards each other by colliding flows
- Oppositely directed magnetic fields smash together and annihilate (beyond ideal MHD)
- Magnetic field annihilation energy energizes particle velocities

Reconnection animation

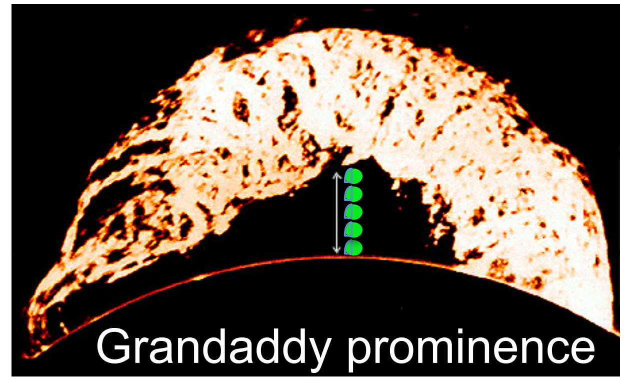


NASA, CLUSTER team

Filamented Plasma Structure in the Universe

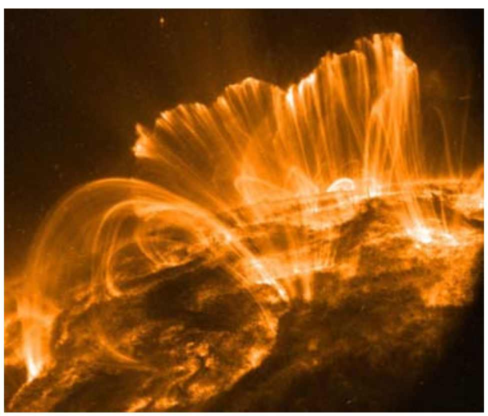


Crab Pulsar. This image combines optical data from Hubble (in red) and X-ray images from Chandra X-ray Observatory (in blue).



Arrow is 64,000km, 5 earths high, earth diameter = 12756km

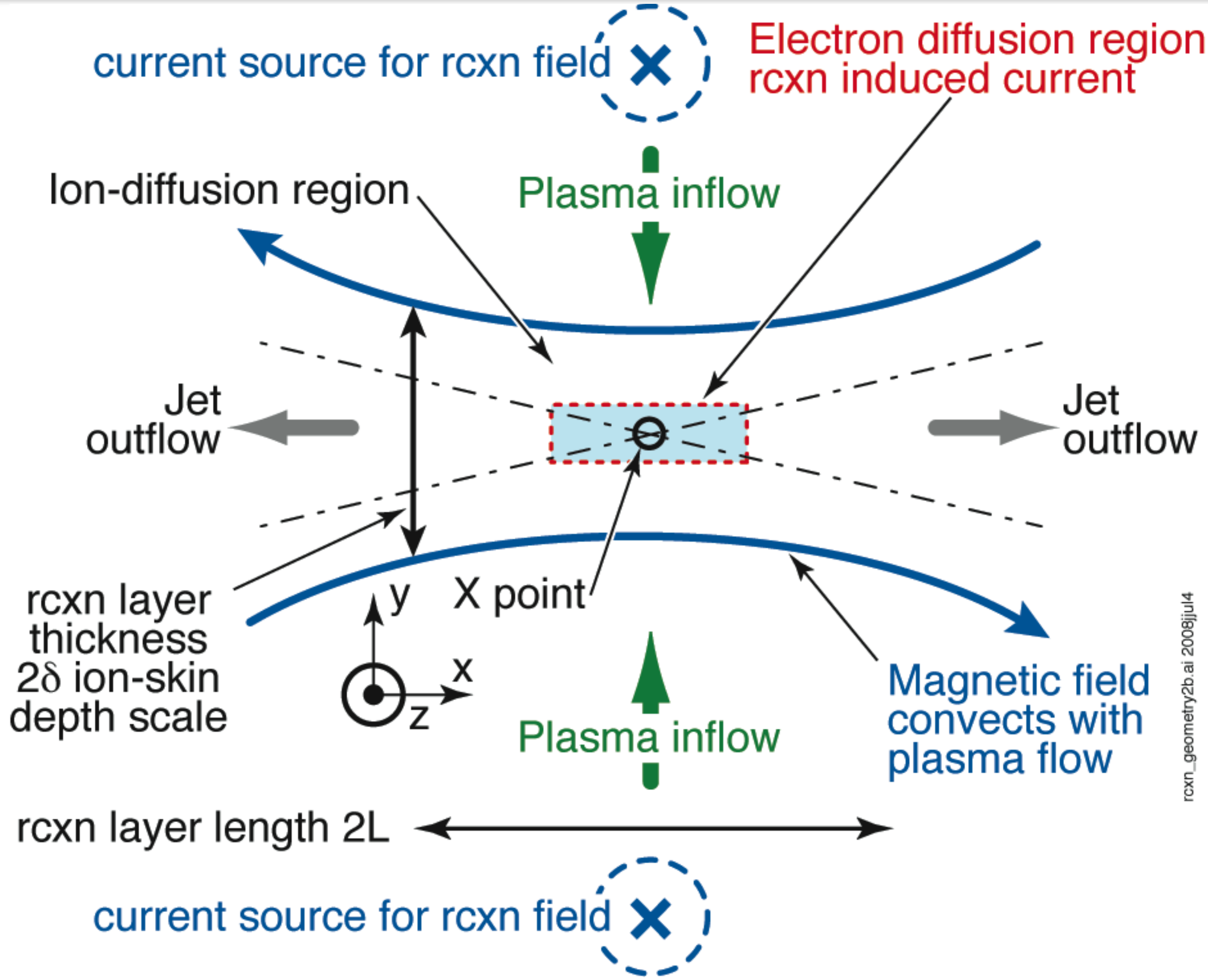
June 4, 1946 High Altitude Observatory…
http://solar-center.stanford.edu/compare



TRACE satellite VUV images

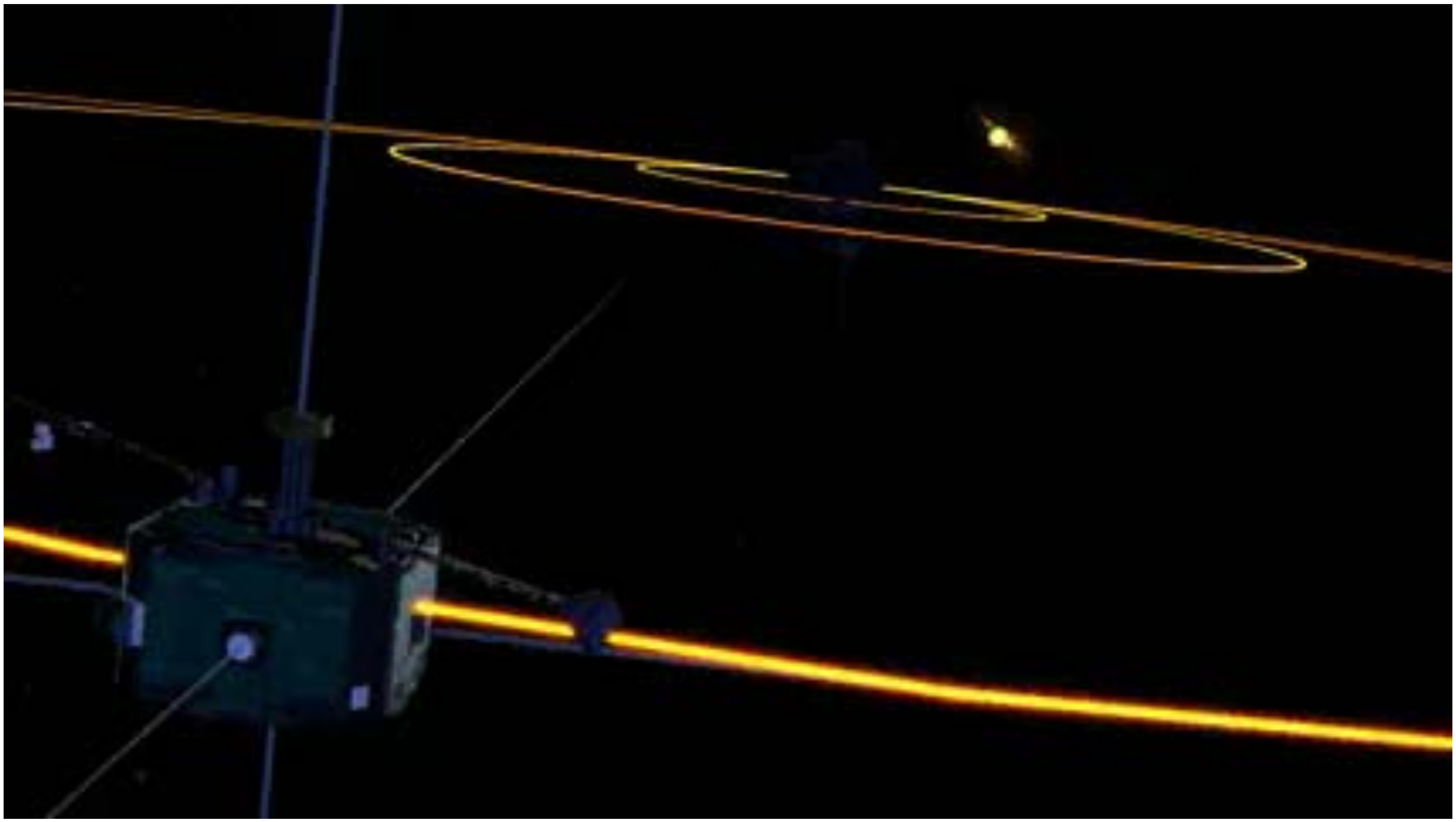
Coronal arches, magnetic structures

Fields and currents coexist: 3D relaxation

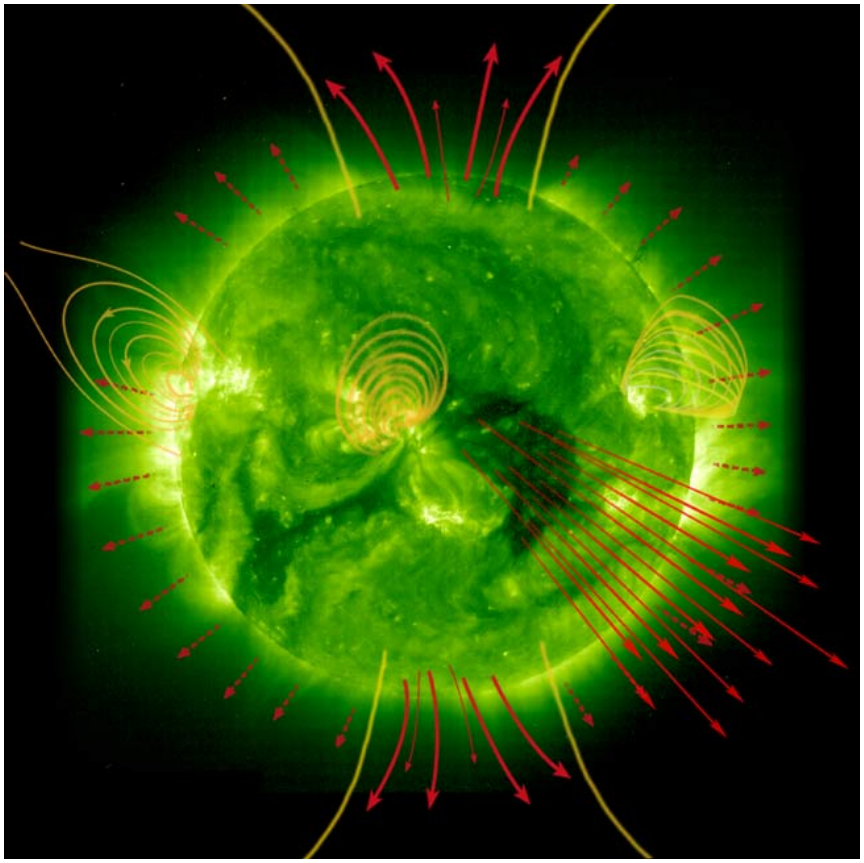


- RSX experiment:
  - Collisional plasmas, S~10-40
  - High guide field, B_z/B_{rcxn}=2-40

THEMIS view: reconnection & aurora



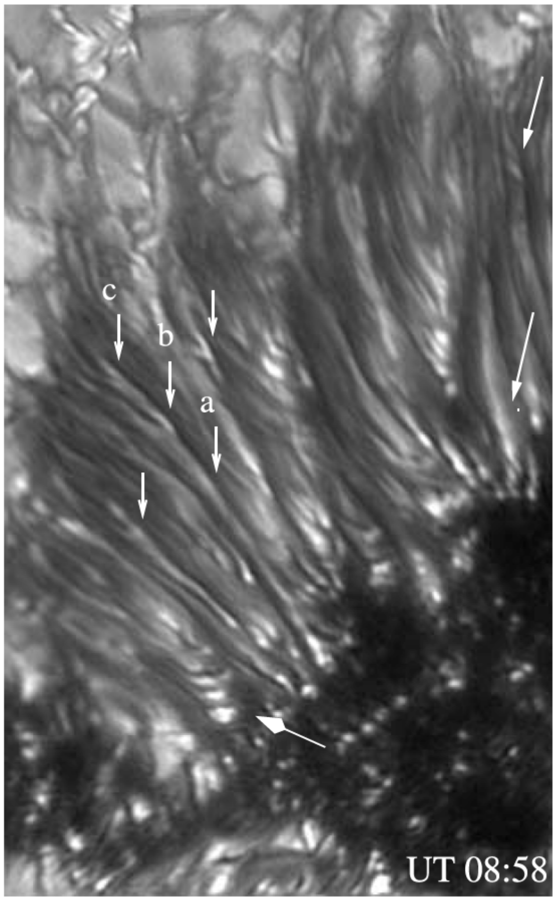
Fast solar wind from coronal holes

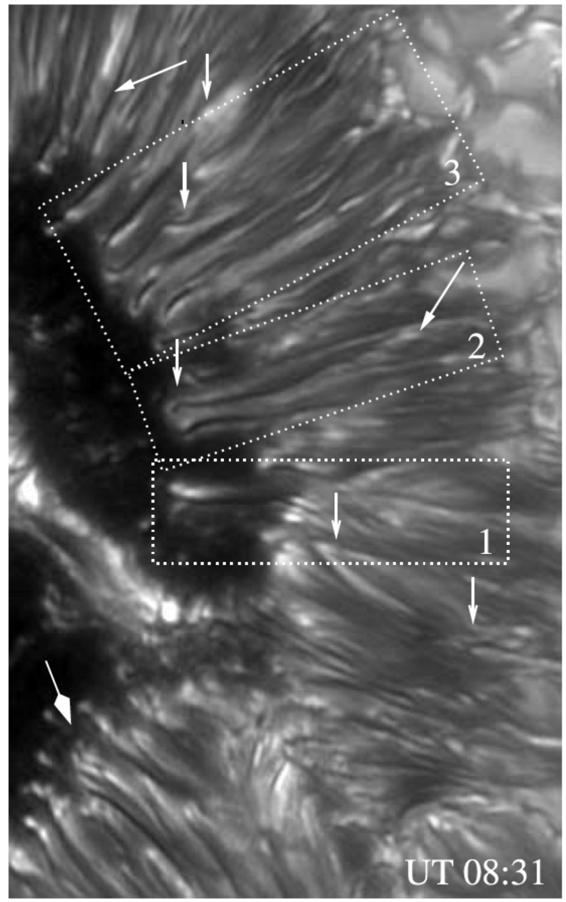


NASA STEREO EUV image

- Red arrows → direction of fast solar wind streams leaving the sun.
- Yellow → magnetic fields
- Fast solar wind anchored at coronal hole, but the other end winds out to Earth, Mars, Jupiter (magnetic clouds)
- Current + field => Flux ropes!

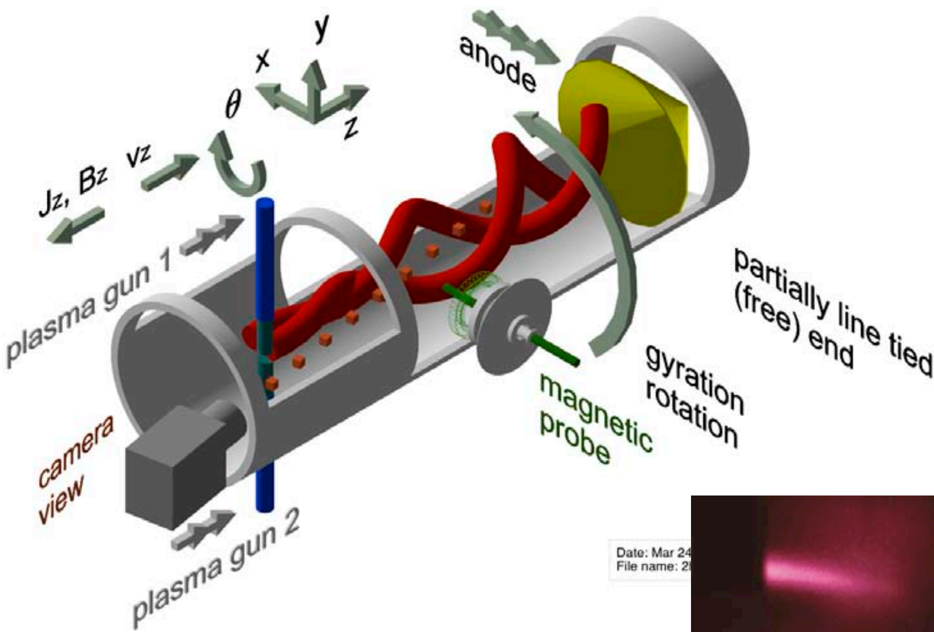
Zoom in on coronal holes: flux ropes

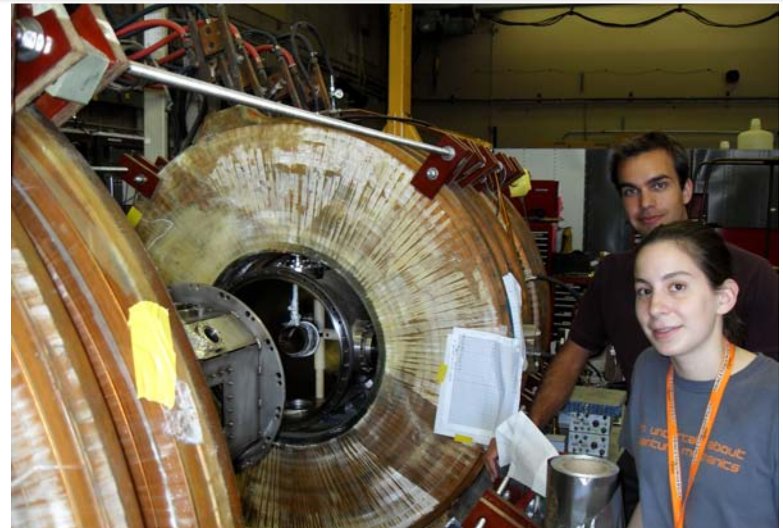


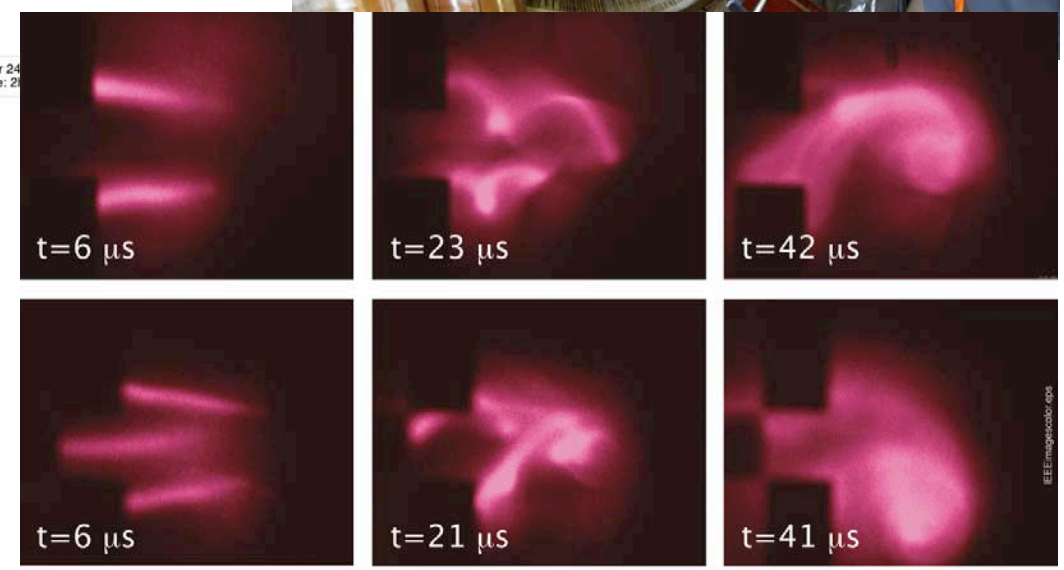


ryutova_fig1a_sunspot_ropes_ApJ2008

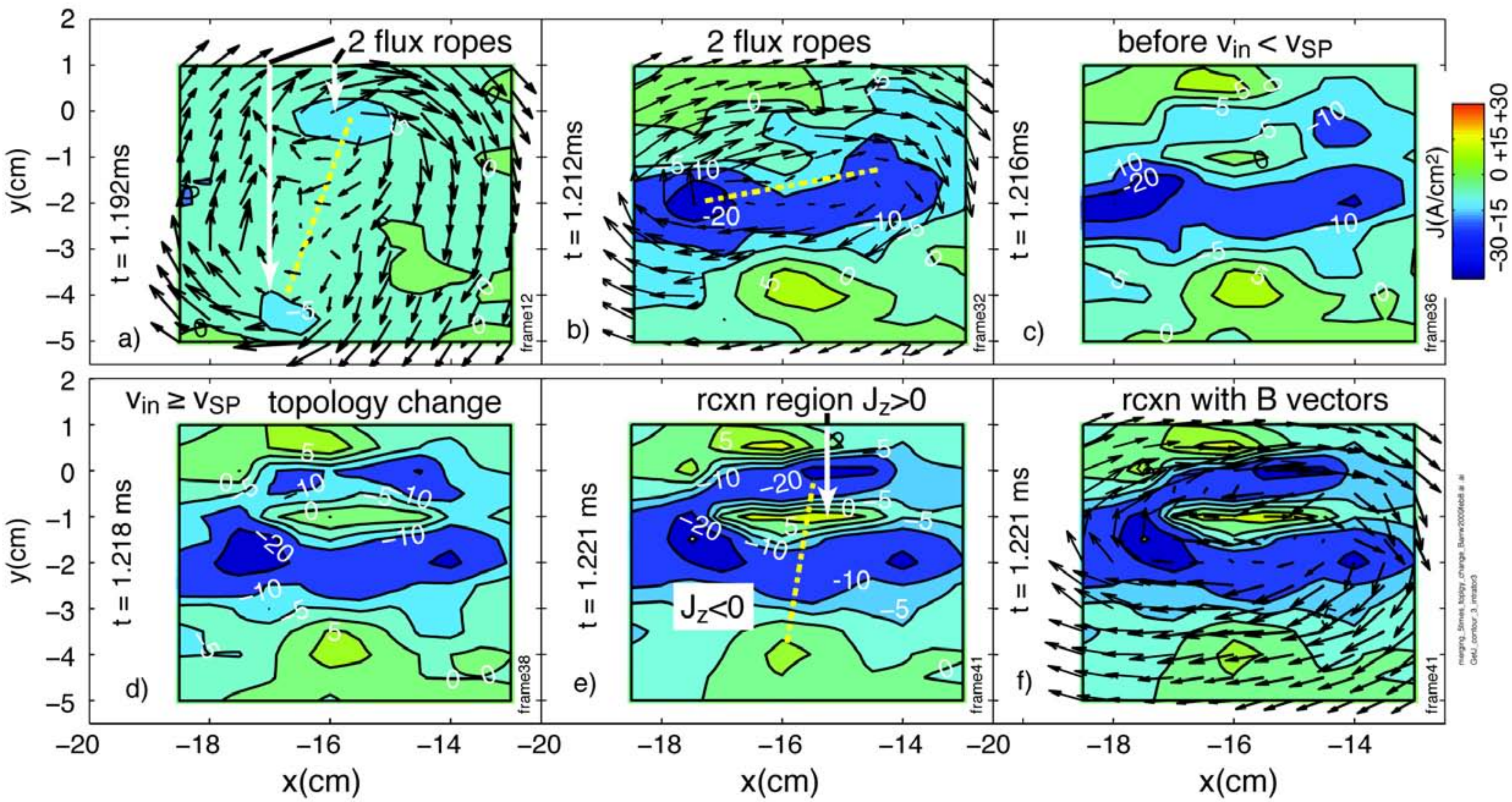
Current ropes modeled in the laboratory







Reconnection and pileup
when v(inflow) > v(Sweet Parker)



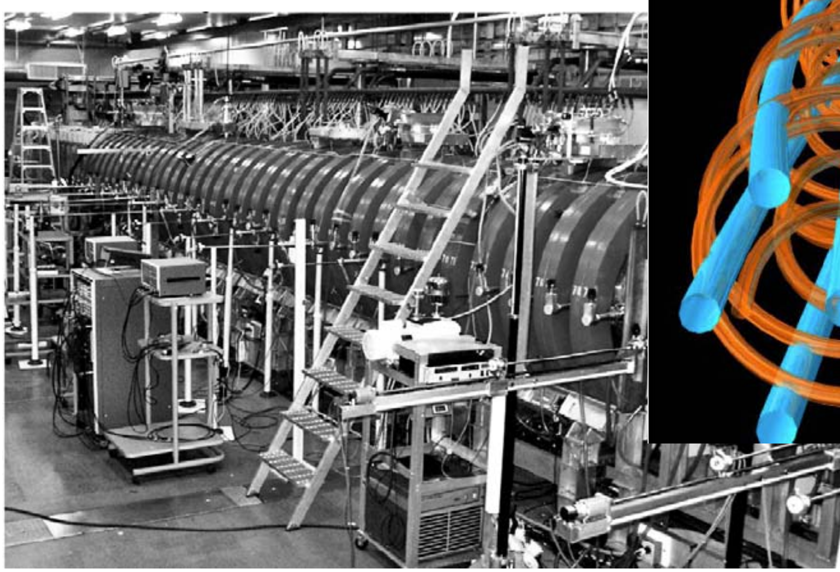
$$\frac{B(guide)}{B(rcxn)} = B_z / B_\theta \approx 10$$

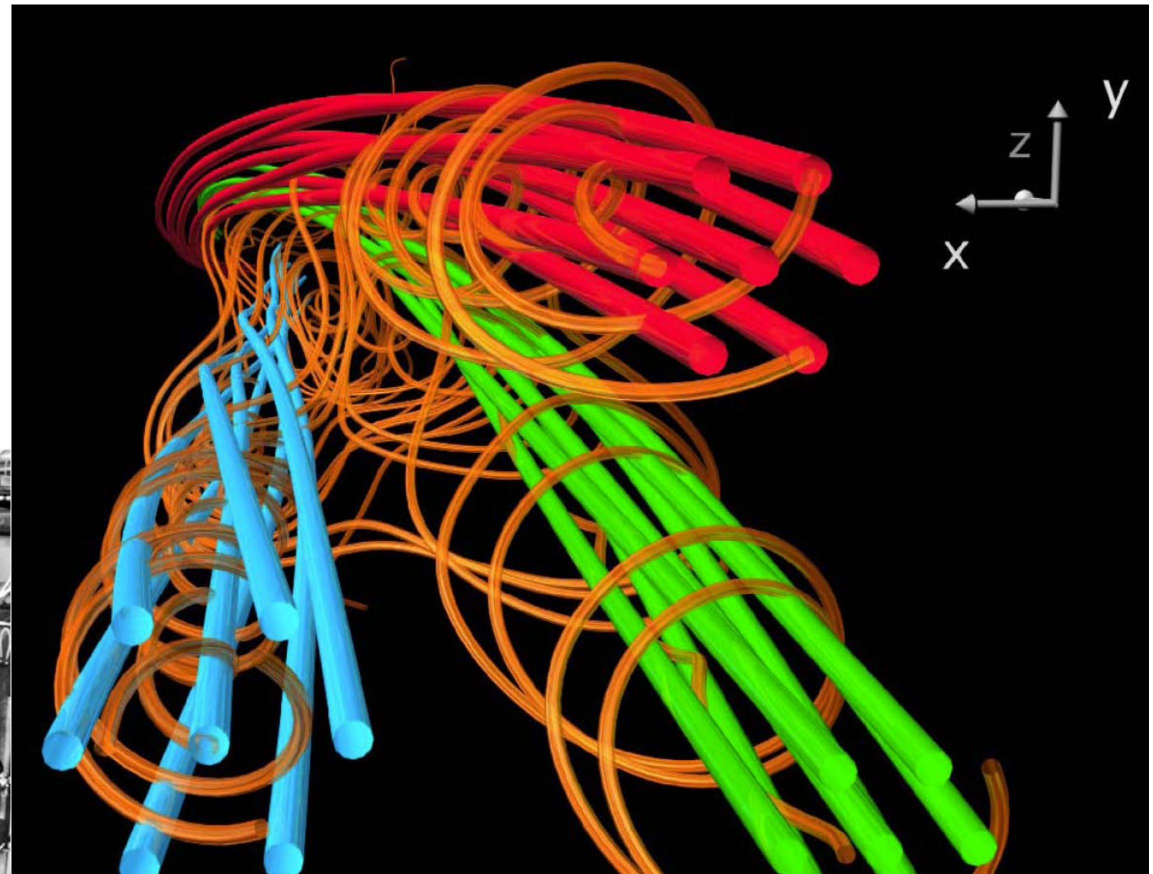
Vector B(x,y) & Jz contours

B = 100 Gauss, V_bias = 120volt

3 flux ropes in LAPD

Large Plasma Device
UCLA
W. Gekelman





- B lines (RGB)
- J lines (orange)
- Gekelman et al, ApJ2012

3D Reconnection current sheet is intrinsically unstable to formation of islands, plasmoids, flux ropes

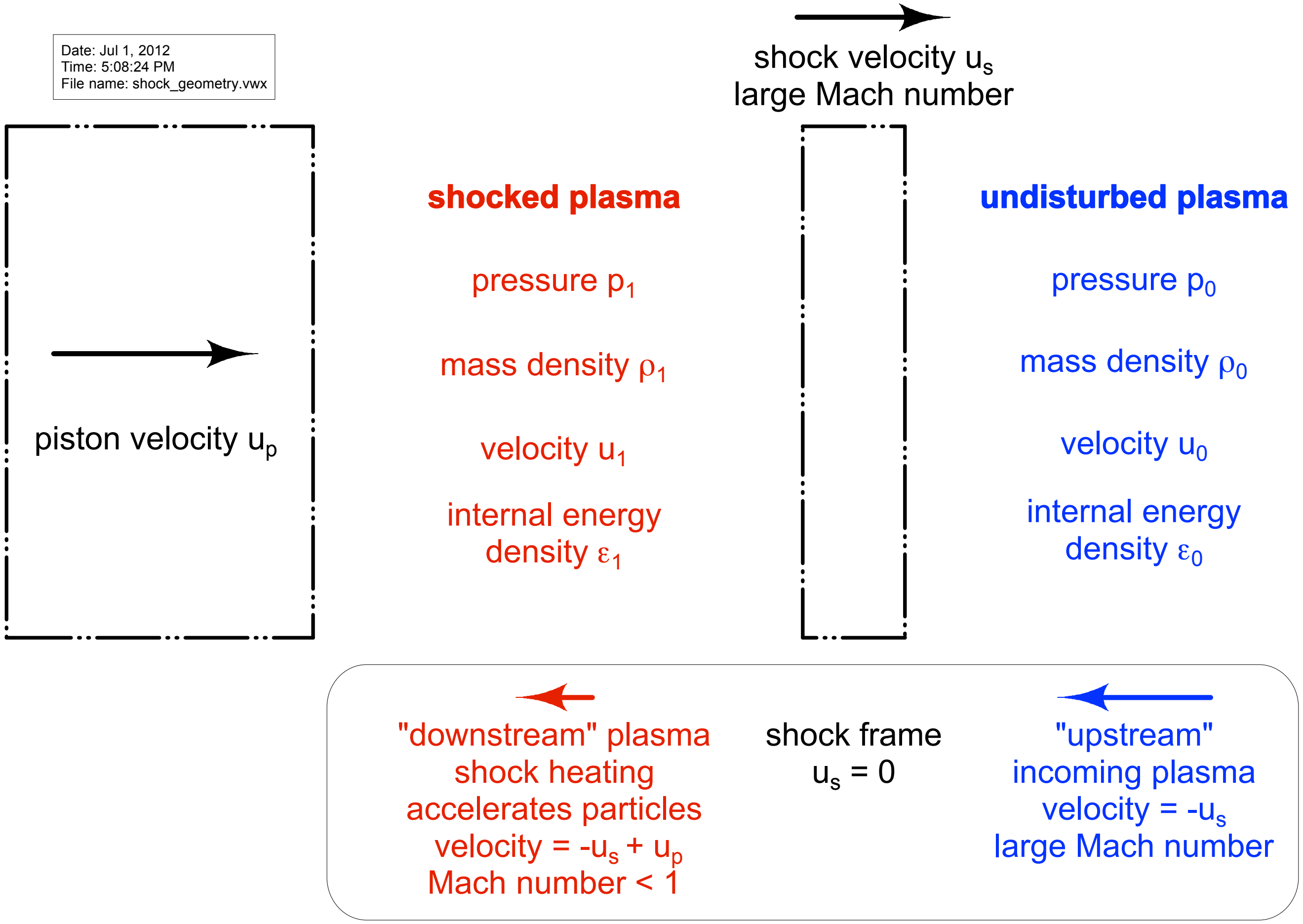
Shocks

- Shock moves faster than the local information speed
- Fluid near the disturbance cannot react before the disturbance arrives
- Jump in fluid properties (density, pressure, temperature, velocity, Mach number
- decelerates supersonic flow
- Sites of particle acceleration
- Thickness ~ mean free path
- in air: mean free path ~ micron
- On Earth, most shocks are mediated by collisions

A plane moving faster than the speed of sound creates shock waves that form a cone around the plane. This fighter plane is flying at supersonic speed at low altitude over an ocean in humid conditions. The compression caused by the conical shock waves around the plane causes the water vapour to condense forming the cloud over the rear of the plane, thereby making the effect of the shock waves visible. Cloud is moving with the plane.

Shock geometry

Laboratory vs shock reference frame

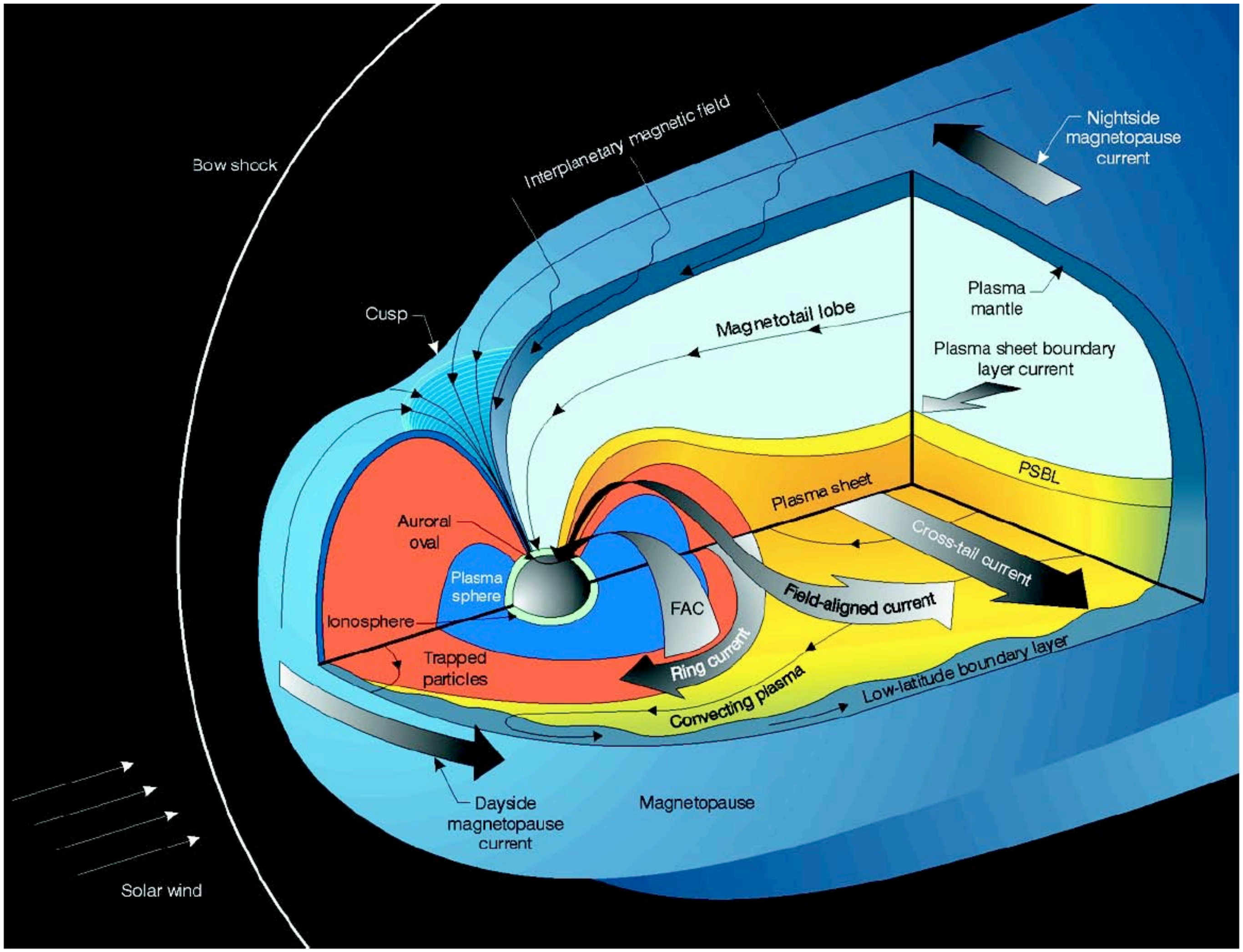


Physics of collisionless shocks

- Shocks are sites of particle acceleration
- Shock: sudden change in density, temperature, pressure that decelerates supersonic flow
- Thickness ~ mean free path
- Astrophysical settings: Mean free path to Coulomb collisions is enormous: 100pc in supernova remnants, ~Mpc in galaxy clusters
- *Mean free path > scales of interest*
- shocks must be mediated without direct collision, but through interaction with collective fields
- *collisionless shocks*

Earth + solar wind + dipole magnetic field

=> convecting field lines, bowshock, magnetotail, plasmoid



Supernova remnants show shock boundaries

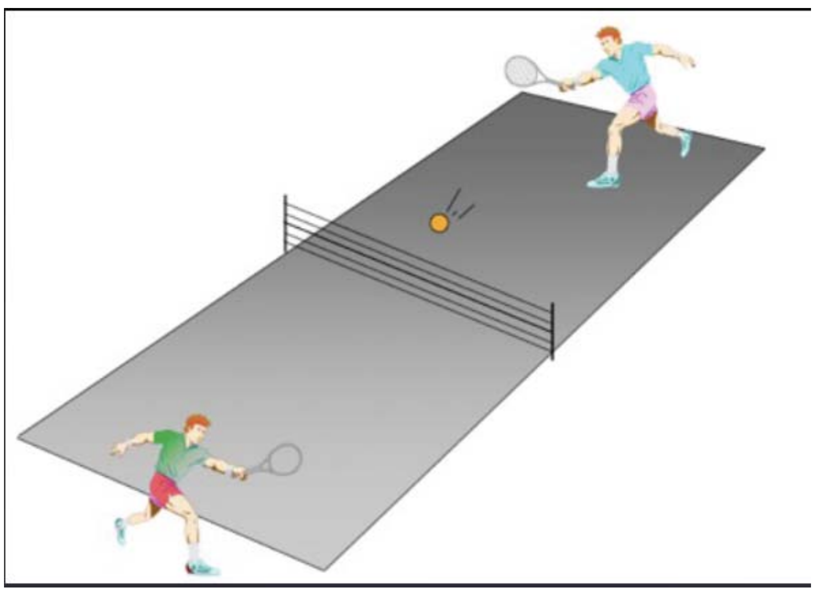
SNR results from the explosion of a star in a supernova, bounded by an expanding shock wave, and consists of ejected material expanding from the explosion, and the interstellar material it sweeps up and shocks along the way

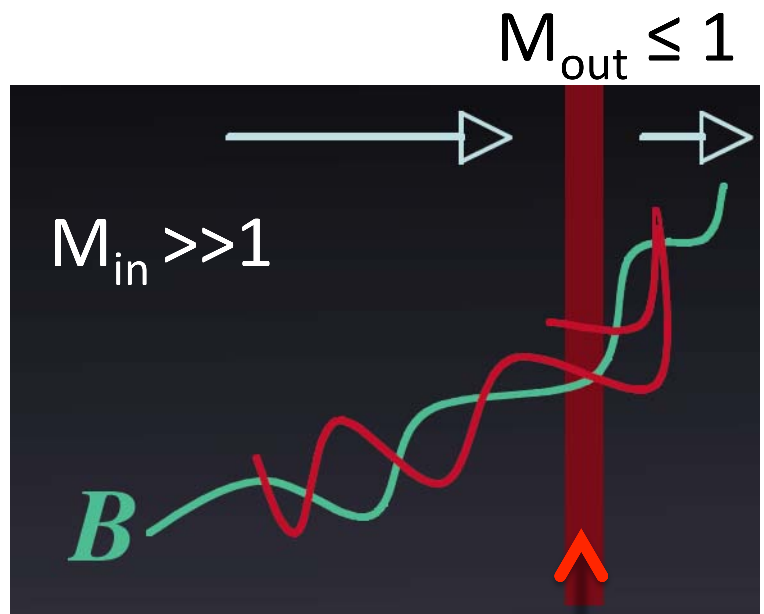
Ejecta velocities < 10% the speed of light, highly supersonic, M_S number initially > 1000. Strong shock wave forms ahead of the ejecta, that heats the upstream plasma up to temperatures well above millions of K.

Multiwavelength composite image of the supernova remnant N49 in the Large Magellanic Cloud.

Astrophysical collisonless shocks

- accelerate particles
- amplify magnetic fields (or create them)
- exchange energy between electrons and ions





Particles rattle back and forth

Mach number
$M_s = v/c_s$
$M_A = v/v_A$

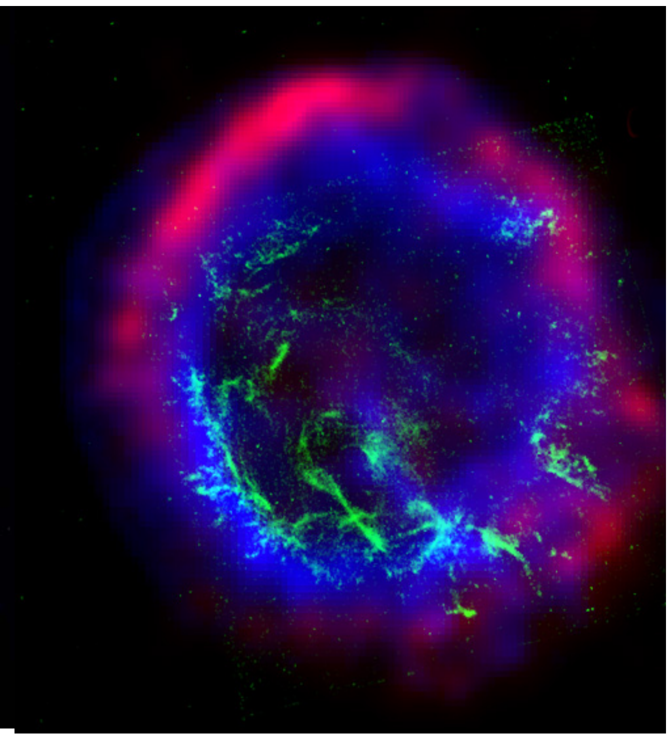
Shock front

Supernova Remnant (SNR)

Krauss-Varban, p6

Supernova Remnants

Kepler. Blue: highest energy X-rays ←→ shock



e0102-723. blue: Chandra X-ray, million-K gas; red: radio, electrons

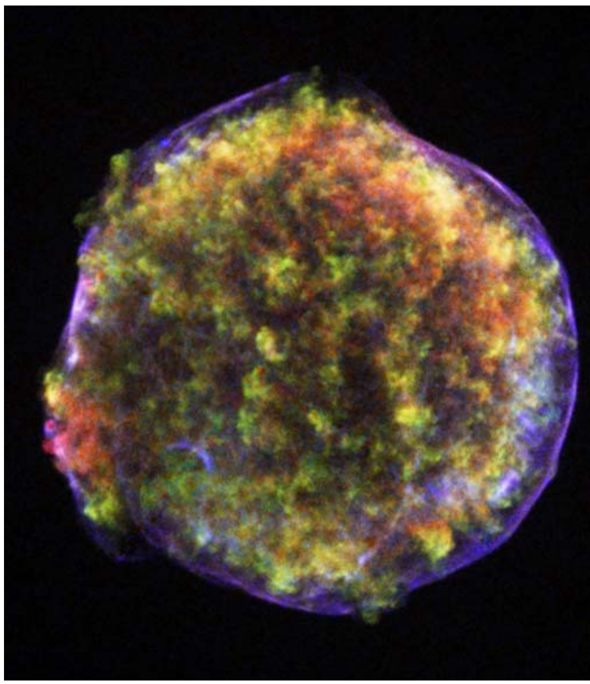
Rankine Hugoniot jump conditions

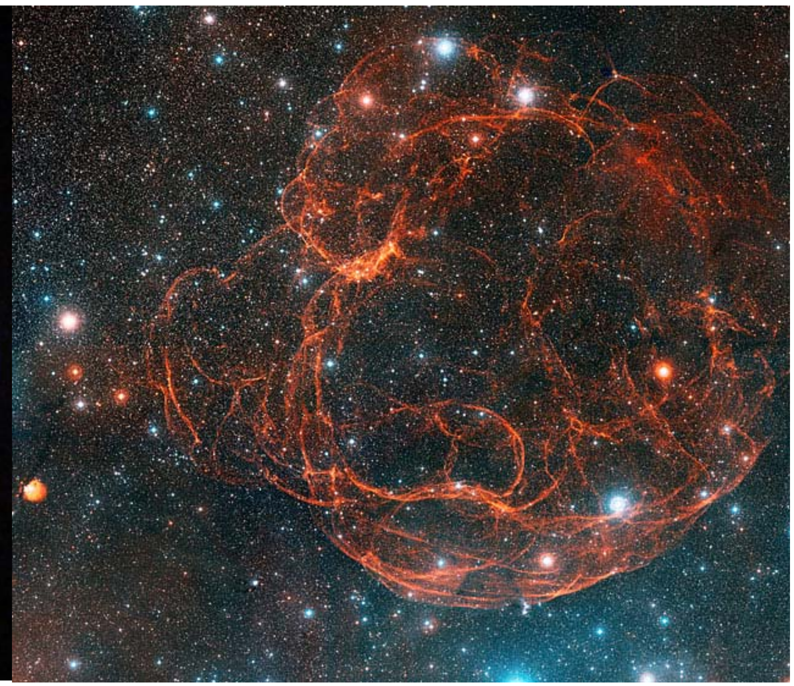
- Adiabatic gas law
  - $pV^\gamma \Rightarrow p/\rho^\gamma$ = constant, p=pressure, ρ=mass density
- conservation of mass flow (in shock frame)
  - $\rho_0 v_{n0} = \rho_1 v_{n1}$, v=velocity
- conservation of momentum flux
  - $\rho_0 v_{n0}^2 + p_0 = \rho_1 v_{n1}^2 + p_1$
- Jump in energy flux
  - $\varepsilon_0 + v_0^2/2 + p_0/\rho_0 = \varepsilon_1 + v_1^2/2 + p_1/\rho_1$

Supernova remnants show shock boundaries

Supernova Remnants

Krauss-Varban, p7



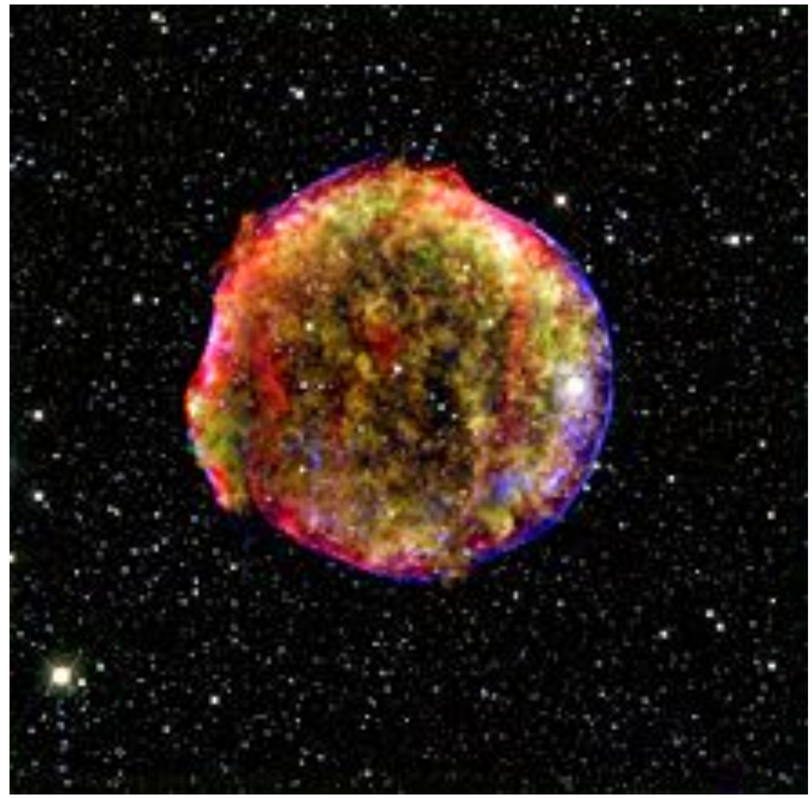


Tycho. Green, red: multimillion degree debris; blue: high-energy electrons. Nuclei

Simeis 147. 3° on the sky ←→ 160 light-years wide @ 3,000 light-years distance

More SNR

- Multiwavelength composite image of the remnant of Tycho's supernova, SN 1572.



Summary

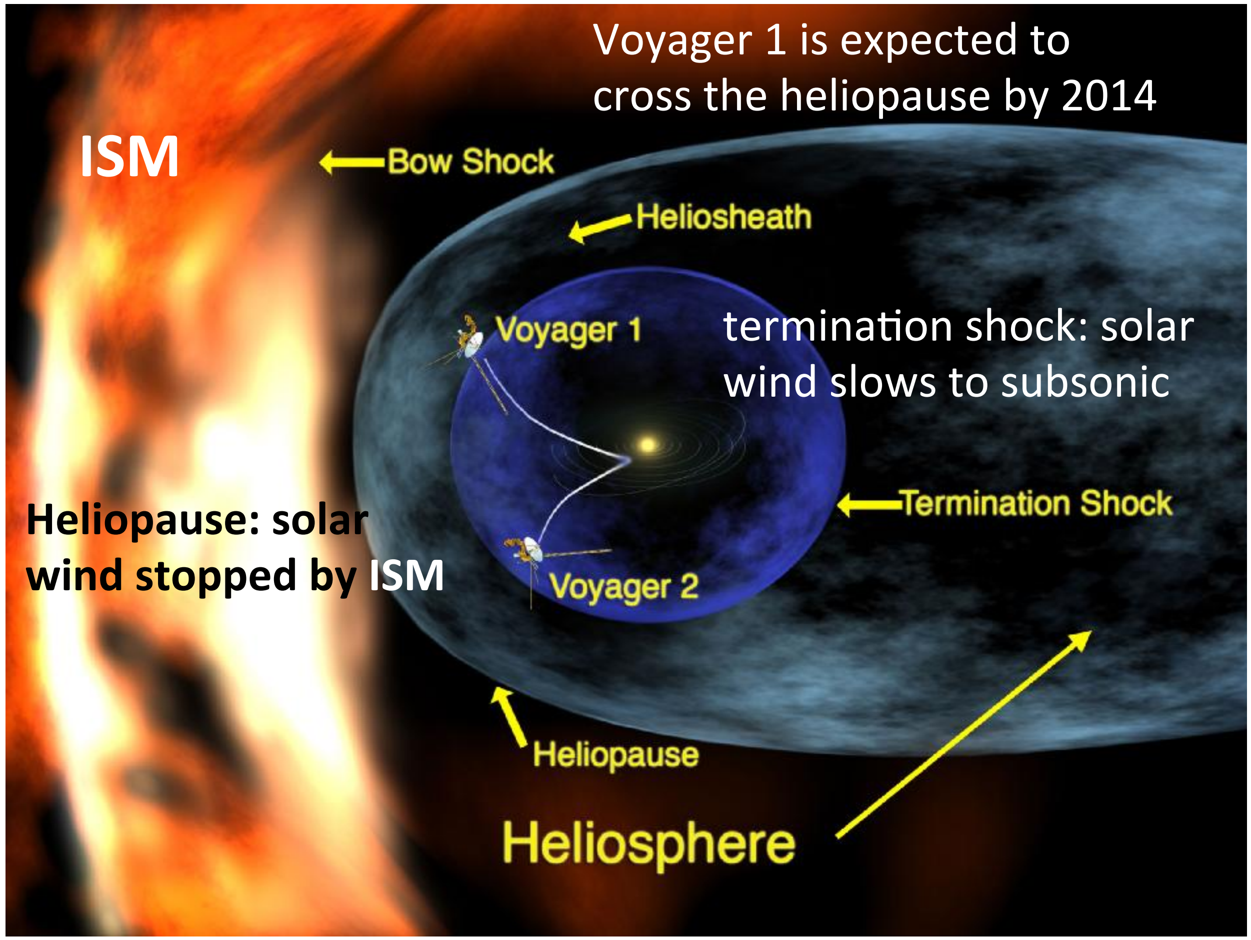
- Magnetic structure occurs from the smallest planetary to the largest cosmic scales.
- Introduce basic description some interesting features of non linear magnetohydrodynamics (MHD).
- dynamo creates magnetic field from flow
- reconnection annihilates magnetic field, accelerates particles.
- Shocks occur when flows move faster than the local velocity (sonic or Alfven speed) for the propagation of information.
- Reconnection, shocks, and ??? can accelerate particles.

Extra slides

Solar system confronts interstellar medium

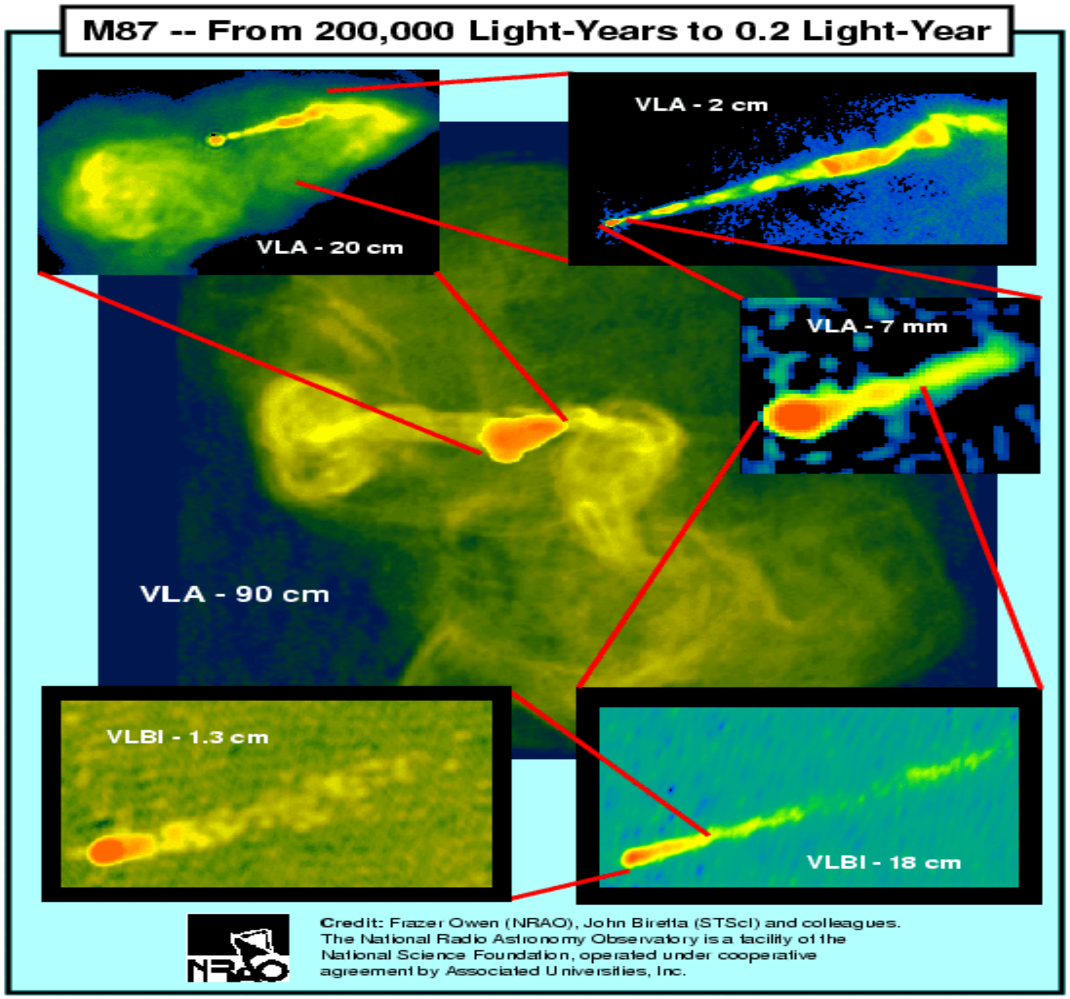
magnetic bubbles

- Earth
- solar system



http://www.nasa.gov/images/content/462848main_helioshere_946-710.jpg

Galactic jets - fluid flow and radiation powered by annihilation of magnetic fields (reconnection)?



Maxwell's equations (SI units)

describe electromagnetic fields

Faraday's Law	$\nabla \times E = -\frac{\partial B}{\partial t}$
Loop currents and magnetic field	$\nabla \bullet B = 0$
Poisson's eqn, charge density	$\nabla \bullet \varepsilon_0 E = e(n_i - n_e) = \rho$
Ampere's law	$\nabla \times B = \mu_0 J + \frac{1}{c^2}\frac{\partial E}{\partial t}$
Current conservation	$\partial(en)/\partial t + \nabla \bullet J=0$
Lorentz force equation	F = q(E + vxB)

Derivation of Plasma Fluid Equations

Plasma Physics Summer School

Bruno Bauer

UNR Physics Department

Center for Nonlinear Studies, LANL

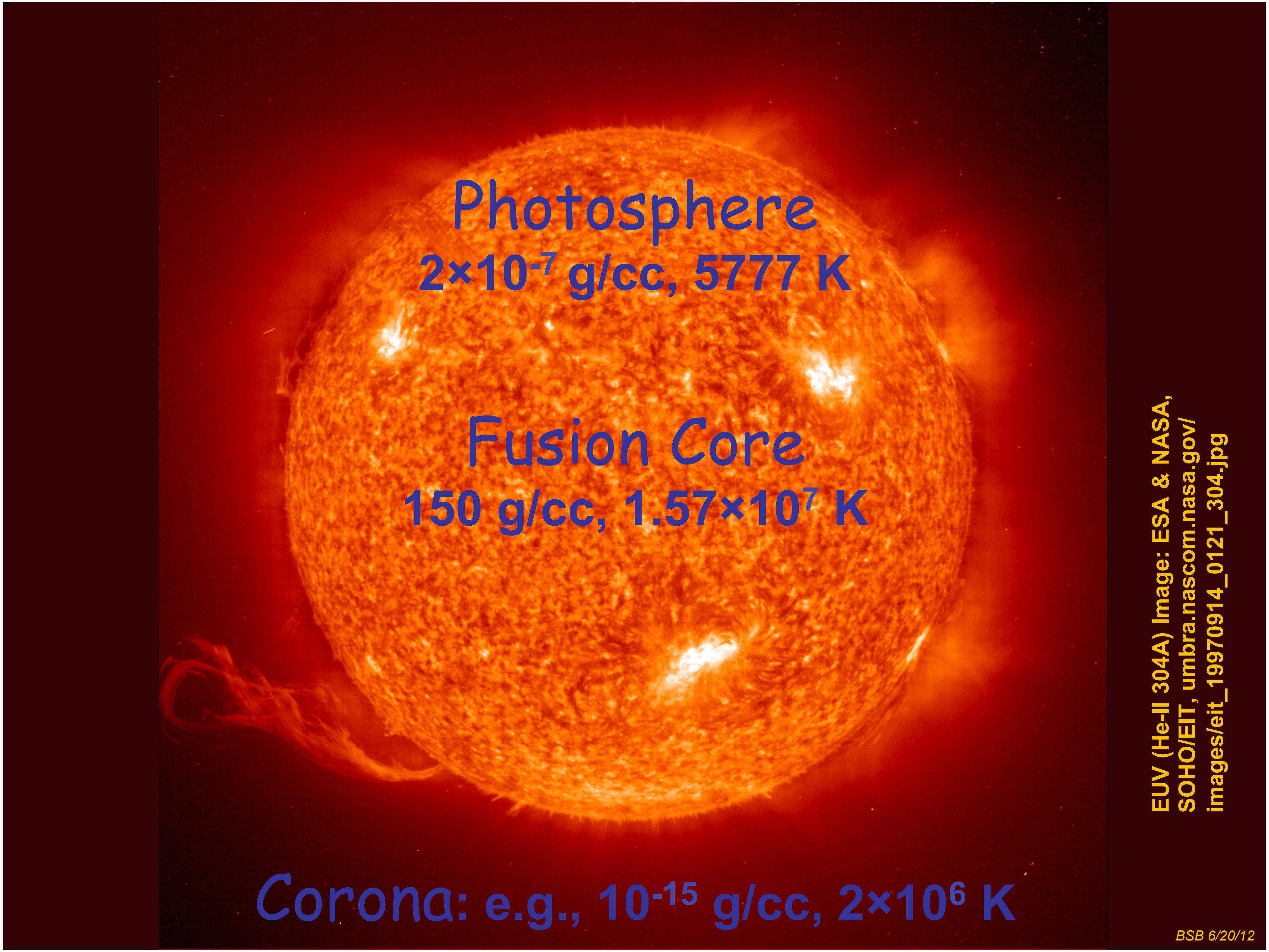
June 20, 2012

Abstract

Plasma physics is of great importance for science and technology. All plasma follows a common set of principles, whether it is the tenuous plasma of interstellar space or the ultradense plasma created in inertial confinement fusion experiments; or whether it is the cool, chemical plasma used in the processing of semiconductors or the hot, thermonuclear plasma of stars and fusion devices. This second lecture of the Plasma Physics Summer School continues a broad outline of plasma physics. Starting with particle motion, we build distribution functions and derive kinetic equations. Taking velocity moments of the Boltzmann kinetic equation we reduce the dimensionality of the plasma description but increase the number of partial differential equations. Through suitable approximations, we obtain and close fluid equations, including those of magnetohydrodynamics. At every stage in this cascade of derivations, we should pause to solve our equations and admire a new perspective on the wondrous world of plasma equilibria, waves, and instabilities.

Derivation of Plasma Fluid Eqns

1. The challenge of plasma physics
2. Kinetic equations
   - Klimontovich, Boltzmann, Vlasov
   - Distribution functions
3. Particle motion
4. Velocity moments of Boltzmann equation
   + approximations
   => fluid equations & magnetohydrodynamics
5. Plasma equilibria, waves, instabilities,
   self-organization


Photosphere
2×10^{-7} g/cc, 5777 K
Fusion Core
150 g/cc, 1.57×10^{7} K
Corona: e.g., 10^{-15} g/cc, 2×10^{6} K
EUV (He-II 304A) Image: ESA & NASA,
SOHO/EIT, umbra.nascom.nasa.gov/
images/eit_19970914_0121_304.jpg
BSB 6/20/12

Solar wind drives Earth magnetosphere

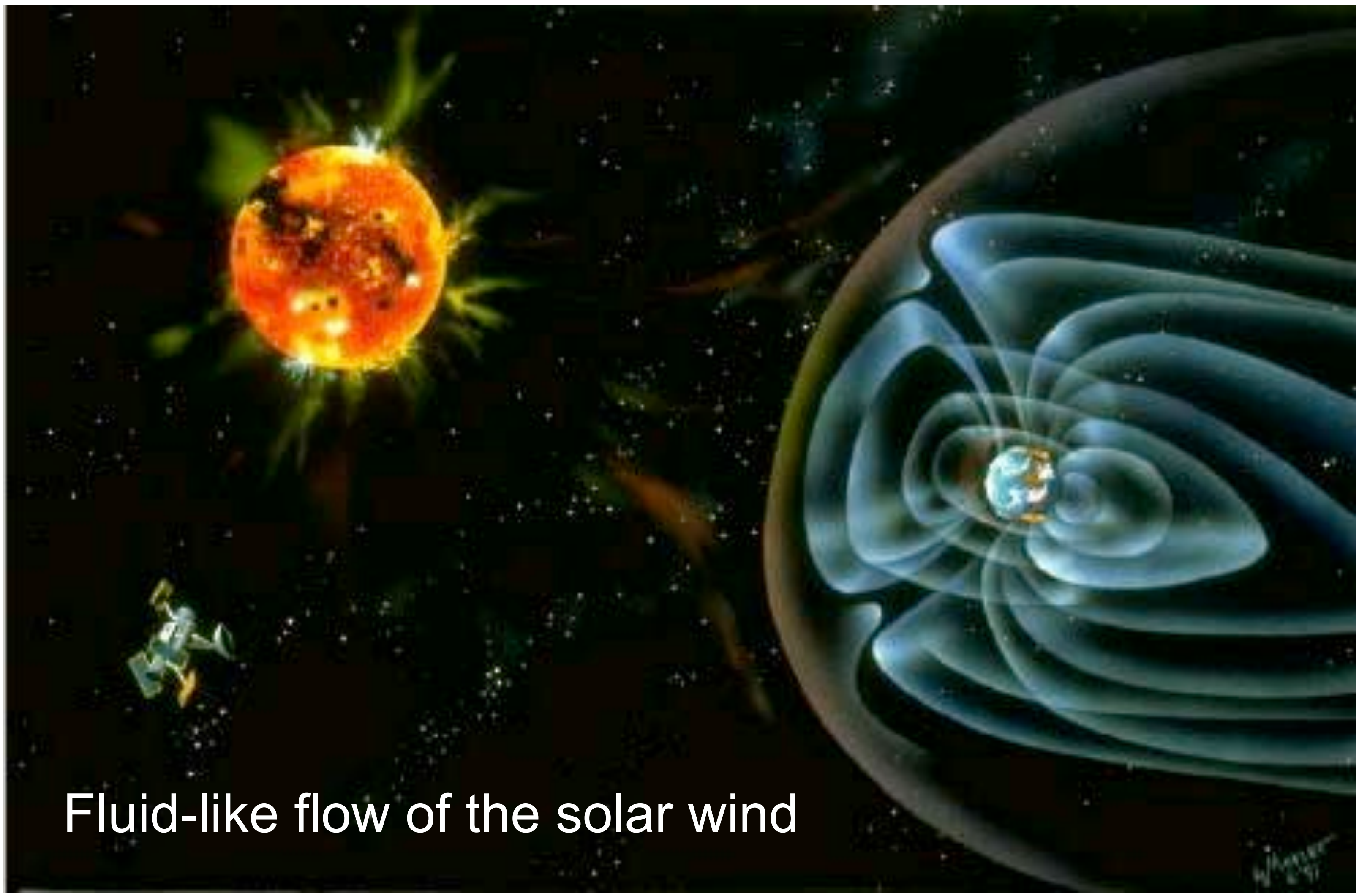


Image: Artist's Conception of the Sun-Earth System, Steve Mercer, Space Science Institute pwg.gsfc.nasa.gov/istp/news/0005/MercerMural40.jpg

The challenge of plasma physics

- **Explain macrostructure from microphysics**
- **Understand & control large collections of unbound charged particles & photons**

Classical plasma physics simplifying assumption:

- **Assume plasma parameters such that particles can be described with classical physics**

For $i = 1, 2, 3, \ldots, N$: $\{q_i, m_i, \vec{r}_i, \vec{v}_i, \vec{a}_i\}$

with Lorentz force $\vec{a}_i = \dfrac{q_i}{m_i}\left(\vec{E}_i + \vec{v}_i \times \vec{B}_i\right)$

where spiky fields are determined by the particles via Maxwell's equations

- **This is a complete, but intractable, set of equations**

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Laws

Relates ***E*** to charges. (Coulomb's law)

B is continuous. No magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

Changing ***B*** produces ***E***

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

Ampère-Maxwell Law

I or Changing ***E*** produces ***B***

These equations describe all electric & magnetic phenomena (in the absence of dielectric or magnetic materials).

Maxwell's equations for spiky fields

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

where $\rho(\vec{r},t) = \sum_{i=1}^{N} q_i \delta\left(\vec{r} - \vec{r}_i(t)\right) + \rho_{\text{ext}}(\vec{r},t)$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{j}(\vec{r},t) = \sum_{i=1}^{N} q_i \vec{v}_i(t)\delta\left(\vec{r} - \vec{r}_i(t)\right) + \vec{j}_{\text{ext}}(\vec{r},t)$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \varepsilon_0\frac{\partial \vec{E}}{\partial t}$$

Delta functions: $\int\delta(x)dx = 1$, $\delta(0) = \infty$, $\delta(x) = 0$ for $x \neq 0$

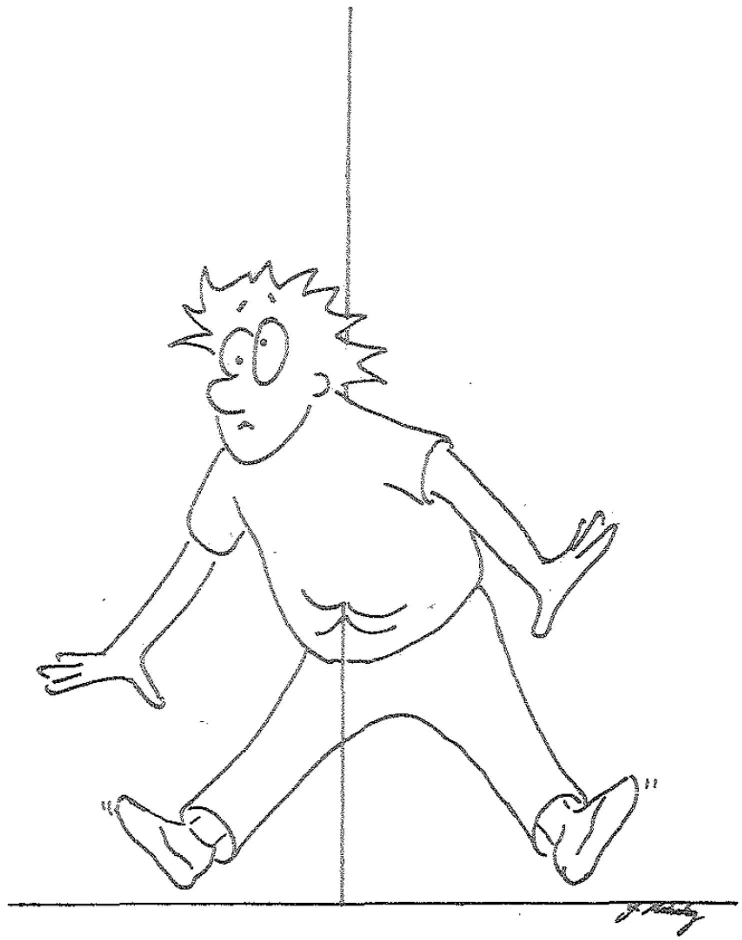


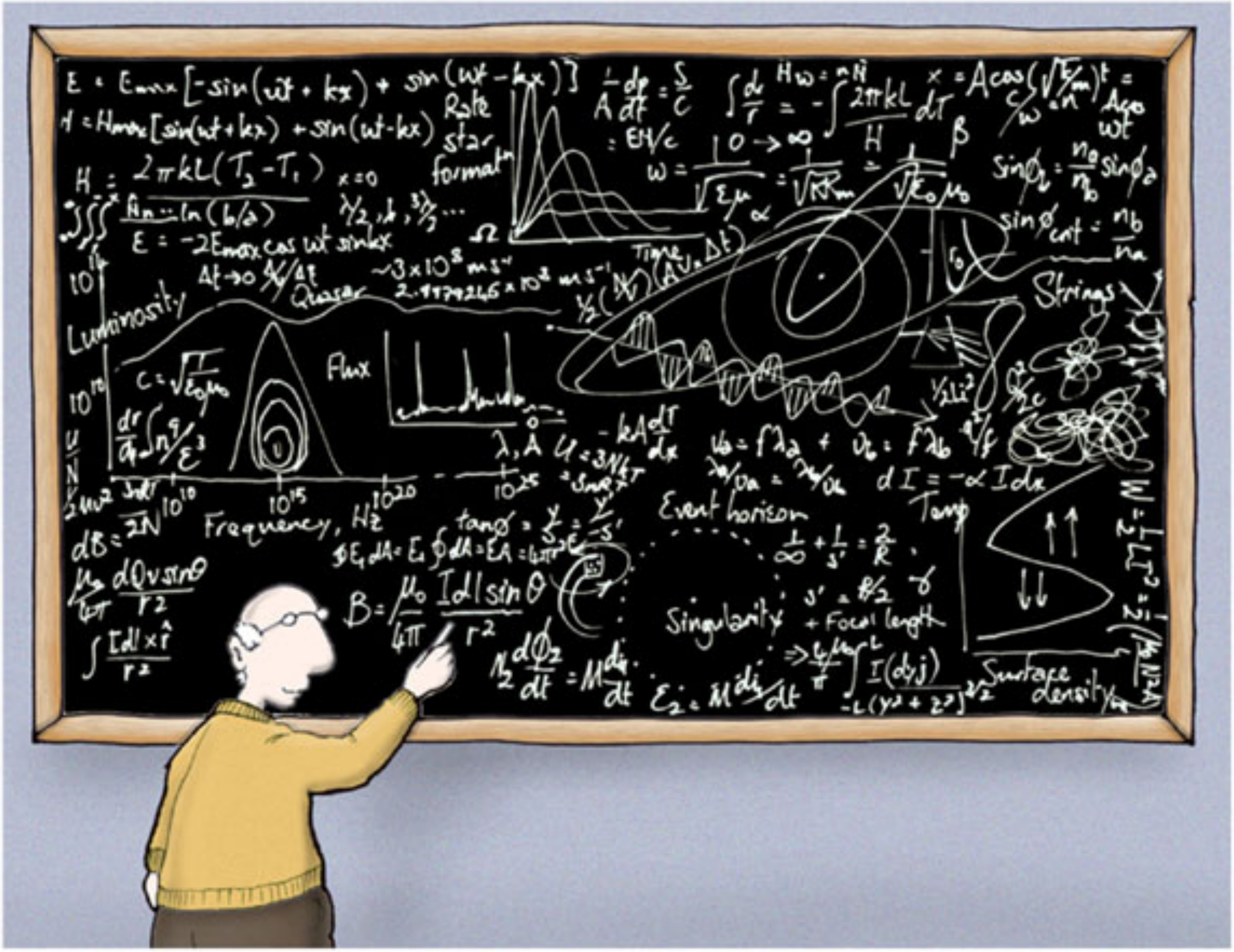
Figure P.1

Hapless Physicist Impaled on his own Delta Function
(Demonstrating the Perils of Insufficient Theoretical Rigor)

Cartoon: S.B. Cahn & B.E. Nadgorny,
A Guide to Physics Problems, Part 1,
Plenum Press, NY 1994

Derivation of Plasma Fluid Eqns

1. The challenge of plasma physics
2. Kinetic equations
   – Klimontovich, Boltzmann, Vlasov
   – Distribution functions
3. Particle motion
4. Velocity moments of Boltzmann equation
   + approximations
   => fluid equations & magnetohydrodynamics
5. Plasma equilibria, waves, instabilities,
   self-organization



Astrophysics made simple

Cartoon: Nick D. Kim, strange-matter.net

Take a statistical approach to tracking the large number of plasma particles

For each species of particle, α,
define the spiky microscopic phase-space distribution function of Klimontovich

$$f_{K\alpha}(\vec{r},\vec{v},t) = \sum_{i=1}^{N_\alpha} \delta(\vec{r} - \vec{r}_i(t))\delta(\vec{v} - \vec{v}_i(t))$$

Here N_α = # of particles of species α

Differentiating Klimontovich distribution => Klimontovich equation

$$f_{K\alpha}(\vec{r},\vec{v},t) = \sum_{i=1}^{N_\alpha} \delta(\vec{r}-\vec{r}_i(t))\,\delta(\vec{v}-\vec{v}_i(t))$$

$$\frac{\partial f_{K\alpha}}{\partial t} = -\frac{d\vec{r}_i}{dt}\cdot\vec{\nabla} f_{K\alpha} - \frac{d\vec{v}_i}{dt}\cdot\vec{\nabla}_v f_{K\alpha}$$

Let the delta functions do the localizing to particles:

$$\frac{\partial f_{K\alpha}}{\partial t} = -\vec{v}\cdot\vec{\nabla} f_{K\alpha} - \vec{a}\cdot\vec{\nabla}_v f_{K\alpha}$$

Inserting Lorentz force => Klimontovich equation:

$$\frac{\partial f_{K\alpha}}{\partial t} + \vec{v}\cdot\vec{\nabla} f_{K\alpha} + \frac{q}{m}\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_v f_{K\alpha} = 0$$

The Klimontovich equation conserves particle number and spikiness

$$f_{K\alpha}(\vec{r},\vec{v},t) = \sum_{i=1}^{N_\alpha} \delta(\vec{r}-\vec{r}_i(t))\delta(\vec{v}-\vec{v}_i(t))$$

$$\frac{\partial f_{K\alpha}}{\partial t} + \vec{v}\cdot\vec{\nabla} f_{K\alpha} + \frac{q}{m}\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_v f_{K\alpha} = 0$$

Recognizing convective derivative:

$$\frac{\mathrm{d}f_{K\alpha}}{\mathrm{d}t} = 0$$

Ensemble average to obtain smooth functions and the Boltzmann equation

$$f_{K\alpha}(\vec{r},\vec{v},t)=\sum_{i=1}^{N_\alpha}\delta(\vec{r}-\vec{r}_i(t))\,\delta(\vec{v}-\vec{v}_i(t))$$

$$\frac{\partial f_{K\alpha}}{\partial t}+\vec{v}\cdot\vec{\nabla} f_{K\alpha}+\frac{q}{m}\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_v f_{K\alpha}=0$$

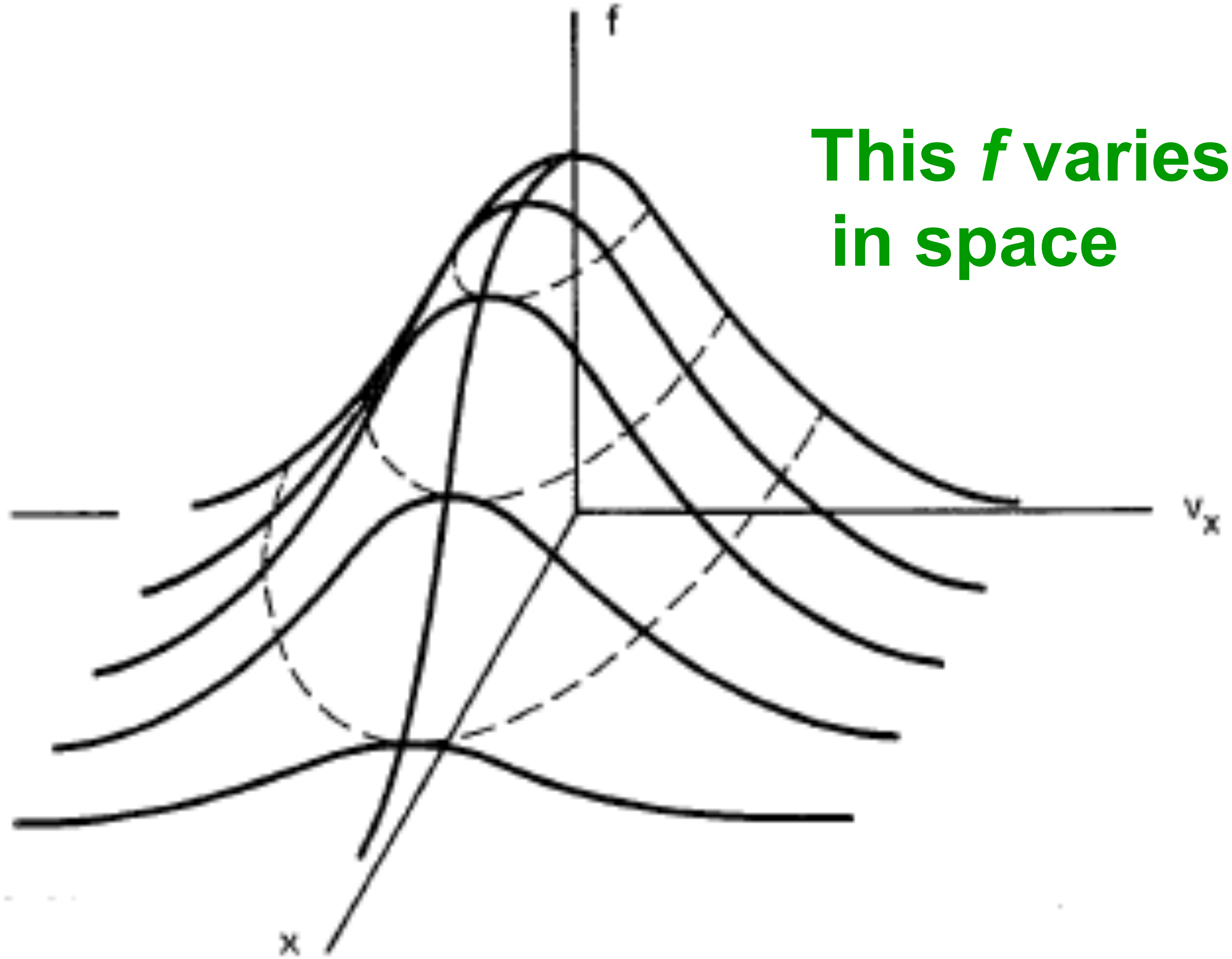
$$f_\alpha \equiv \langle f_{K\alpha}\rangle,$$
$$\delta f_\alpha \equiv f_{K\alpha}-f_\alpha,$$
$$\vec{E}\equiv\langle \vec{E}_K\rangle,\ \text{etc.}$$

=> the Boltzmann equation:

$$\frac{df_\alpha}{dt}=\frac{\partial f_\alpha}{\partial t}+\vec{v}\cdot\vec{\nabla} f_\alpha+\frac{q}{m}\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_v f_\alpha=\left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

Here the right-hand-side is short hand for a complex collision term.

A distribution function $f(x,y,z,v_x,v_y,v_z)$ is a density in phase space



This *f* varies in space

Figure: F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

Collisions drive the plasma *toward* maximum entropy

$$f_\alpha(\vec{r},\vec{v}) = n_\alpha|_{\phi=0} \exp\left[\frac{-q_\alpha\phi(\vec{r})}{k_\beta T_\alpha}\right] f_M(\vec{v})$$

$$f_M(\vec{v}) = \frac{1}{a^3\pi^{3/2}} e^{-v^2/a^2}, \quad a \equiv \sqrt{\frac{2k_\beta T_\alpha}{m_\alpha}}$$

Maxwellian distribution function

Plasma distribution functions are often non-Maxwellian

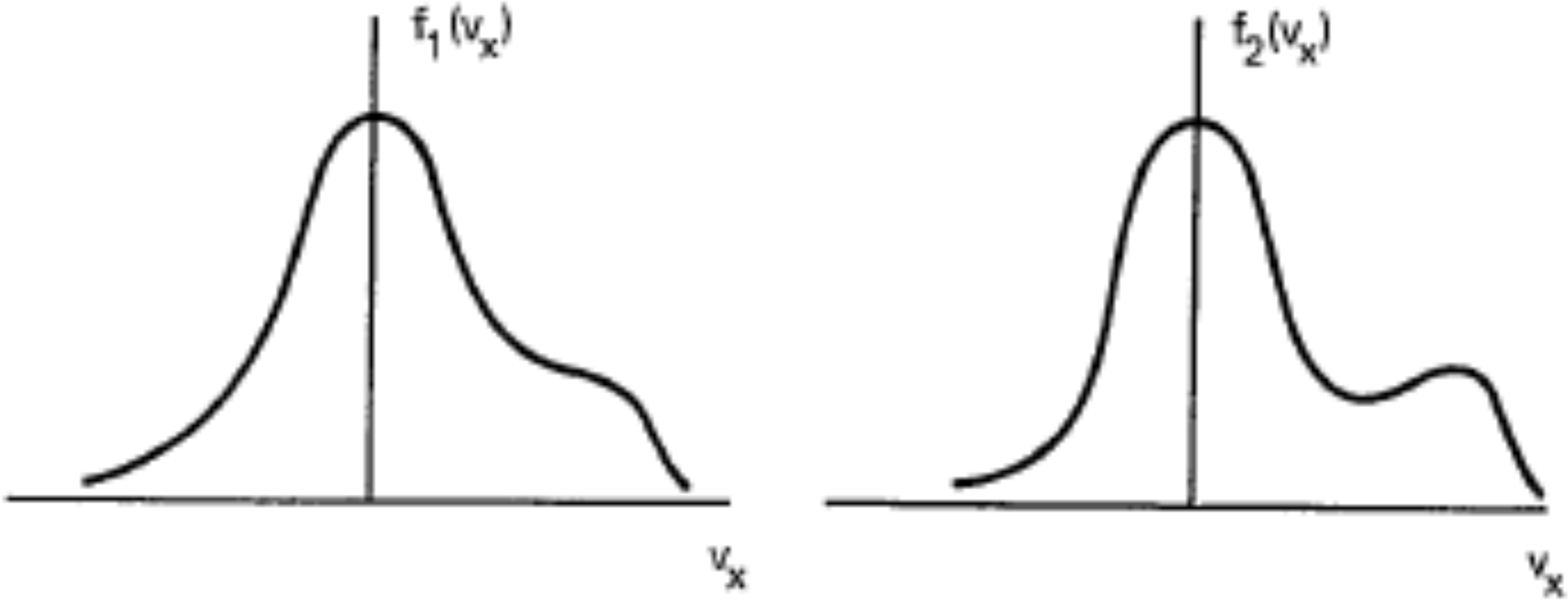


Figure: F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

Plasma distribution functions are often anisotropic

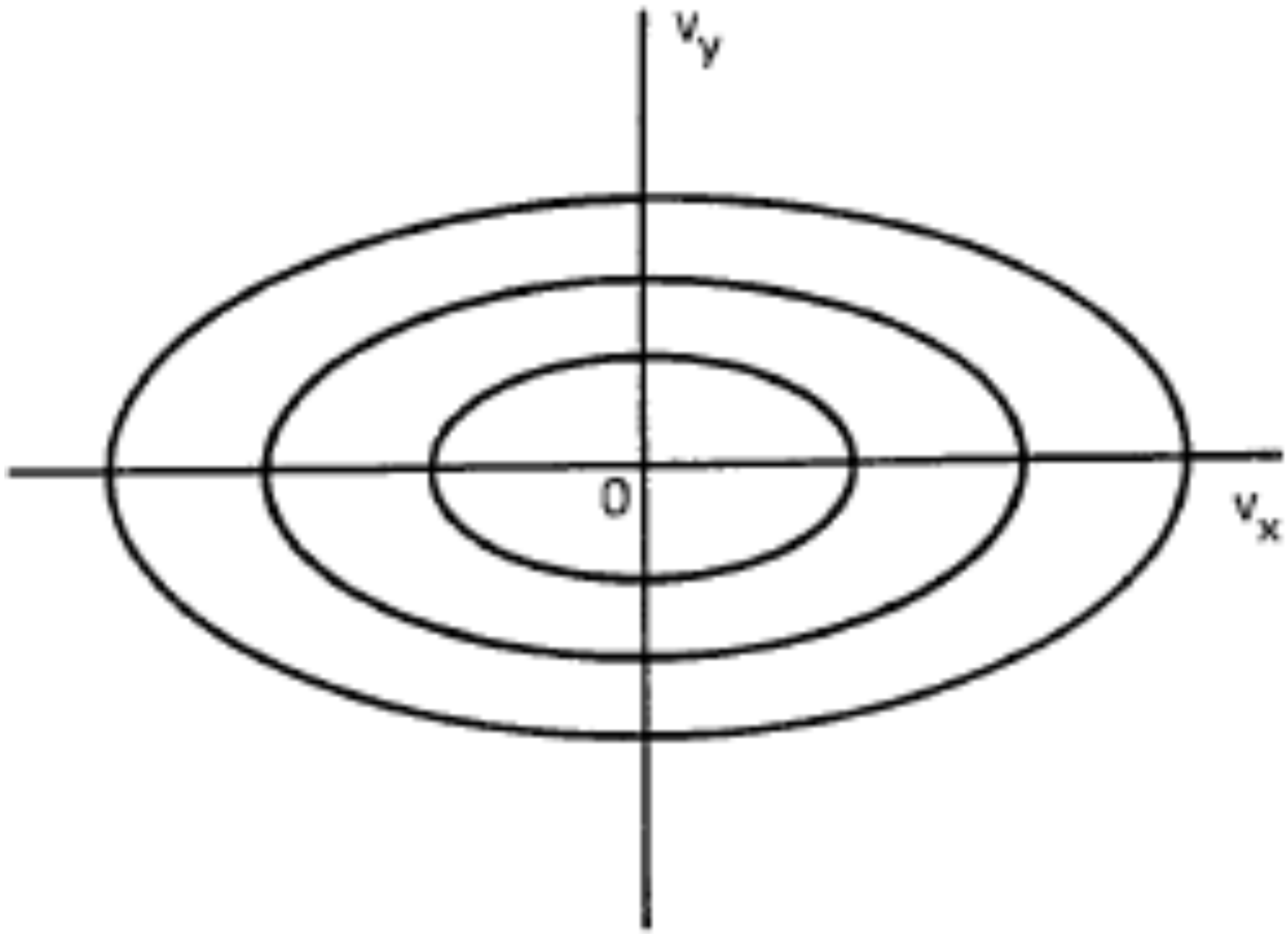


Figure: F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

Phase-space evolution can be complex and beautiful

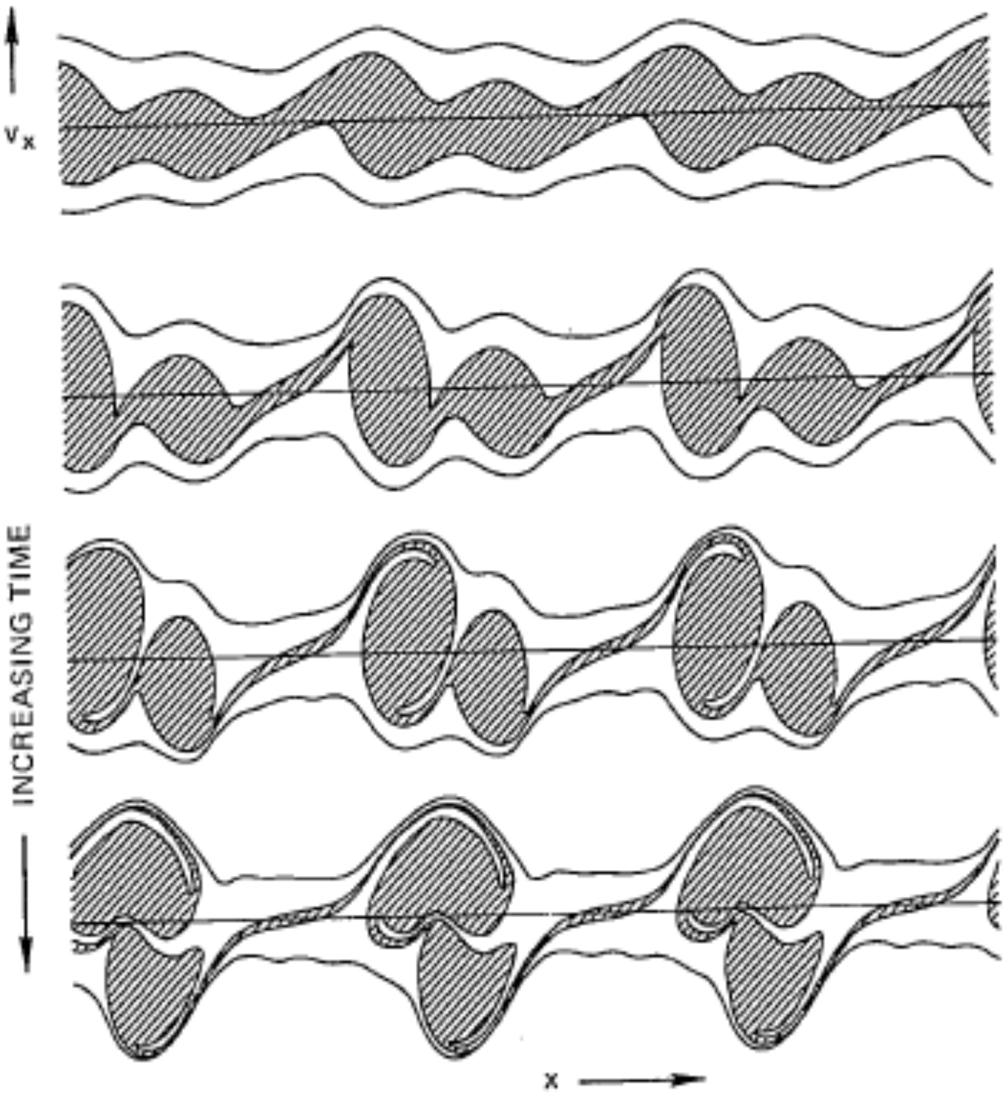


Figure: Electron 2-stream instability, H.L. Berk, C.E. Nielson, and K.V. Roberts, Phys. Fluids 13, 986 (1970). In F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

To better understand Boltzmann equation, follow particles in phase space & calculate collisions

$$\frac{\mathrm{d}f_\alpha}{\mathrm{d}t} = \frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha + \frac{q}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \vec{\nabla}_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

One Boltzmann equation per species of particle

- **Describes the evolution of the distribution of particles in phase-space**

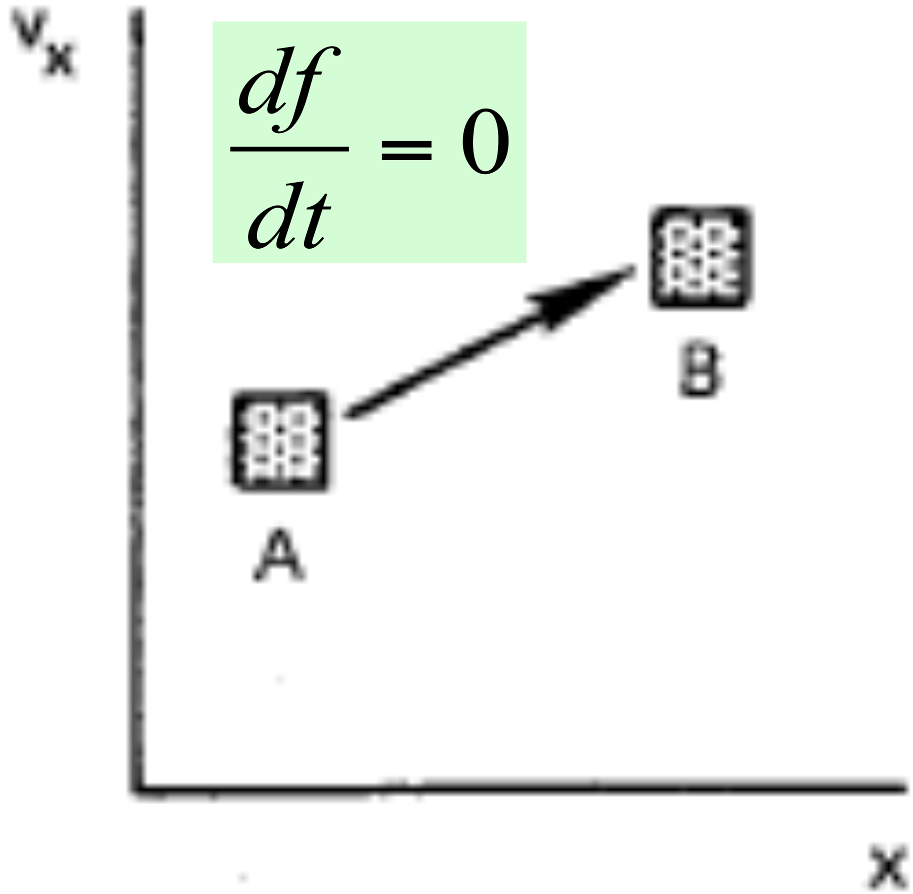
How do particles flow in phase space?

- **What is the effect of the collision term (on rhs)? How would *f* evolve without collisions?**

Neglect collisions => Vlasov equation

$$\frac{\mathrm{d}f_\alpha}{\mathrm{d}t} = \frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha + \frac{q}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \vec{\nabla}_v f_\alpha = 0$$

The Vlasov equation conserves phase-space density: incompressible flow



$$\frac{df}{dt} = 0$$

Figure: F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

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Charged Particles Curve in a Magnetic Field

Magnetic force on a moving charged particle:

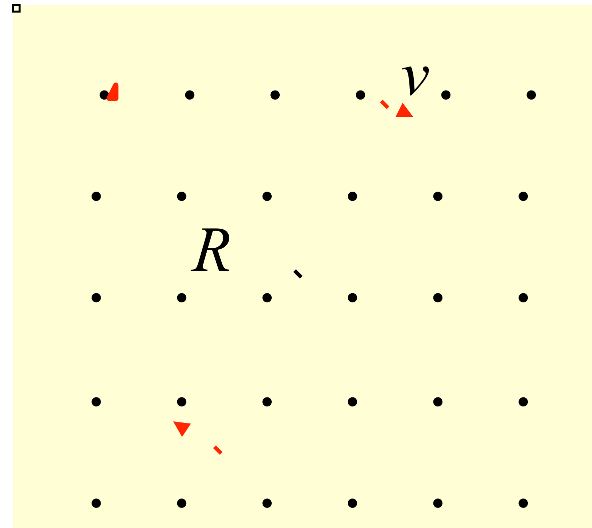
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}\right| = qvB\sin\theta$$

- **A magnetic field can change the velocity of a moving charged particle, but not its speed.**
- **A moving particle's direction will be changed by a magnetic field if it has a velocity component perpendicular to the field.**

Cyclotron Motion

If a moving charged particle's trajectory is entirely within a uniform transverse magnetic field, its orbit is closed and circular.



$\vec{B}$ out of plane

$$\frac{mv_{\perp}^{2}}{R} = F_{\perp} = qv_{\perp}B$$

Orbital Period T:

$$T = \frac{2\pi R}{v_{\perp}} = \frac{2\pi}{v_{\perp}}\frac{mv_{\perp}}{qB} = \frac{2\pi m}{qB}$$

Orbital Frequency f_c:

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

Cyclotron Frequency

Motion of a charged particle in an arbitrary direction relative to a magnetic field

- If $\vec{v}$ is neither perpendicular nor parallel to $\vec{B}$, a helical (spiral) motion about the direction of $\vec{B}$ results.

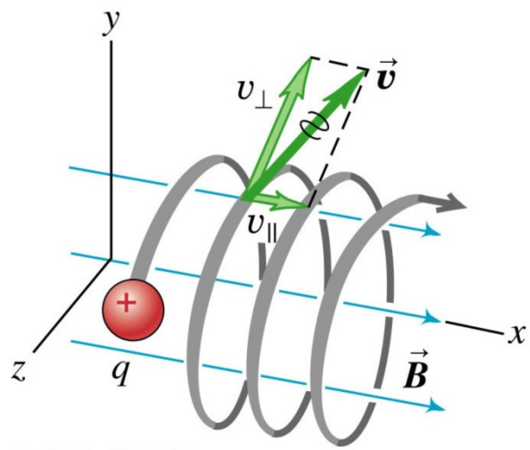


Fig.: H.D. Young and R.A. Freedman, *Sears and Zemansky's University Physics*, 11th ed., Pearson / Addison Wesley, San Francisco, CA, 2004.

- Such motion of positive ions from the solar wind ionizes the atmosphere and is responsible for the phenomenon of the **Aurora Borealis** (northern lights)

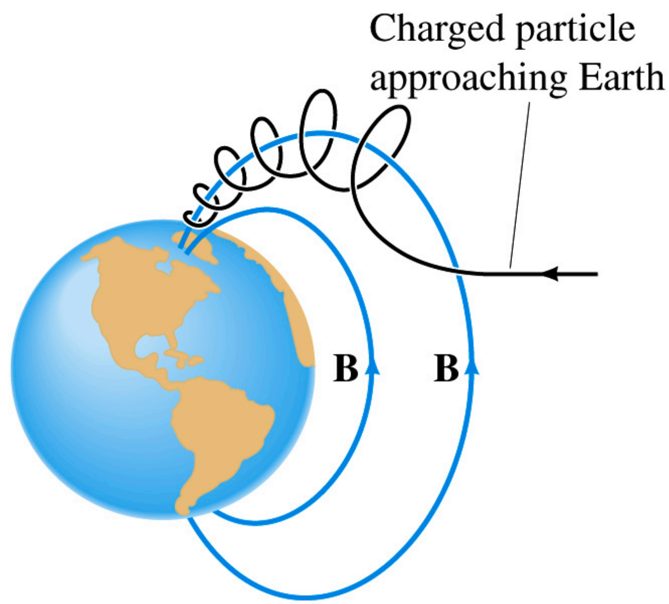


Figure: D.C. Giancoli, *Physics for Scientists & Engineers with Modern Physics*, 3rd ed., Prentice Hall, NJ, 2000.

Solar storms make the news

The Daily Show with Jon Stewart
Comedy Central, 2004

A “Magnetic Bottle” for Charged Particles

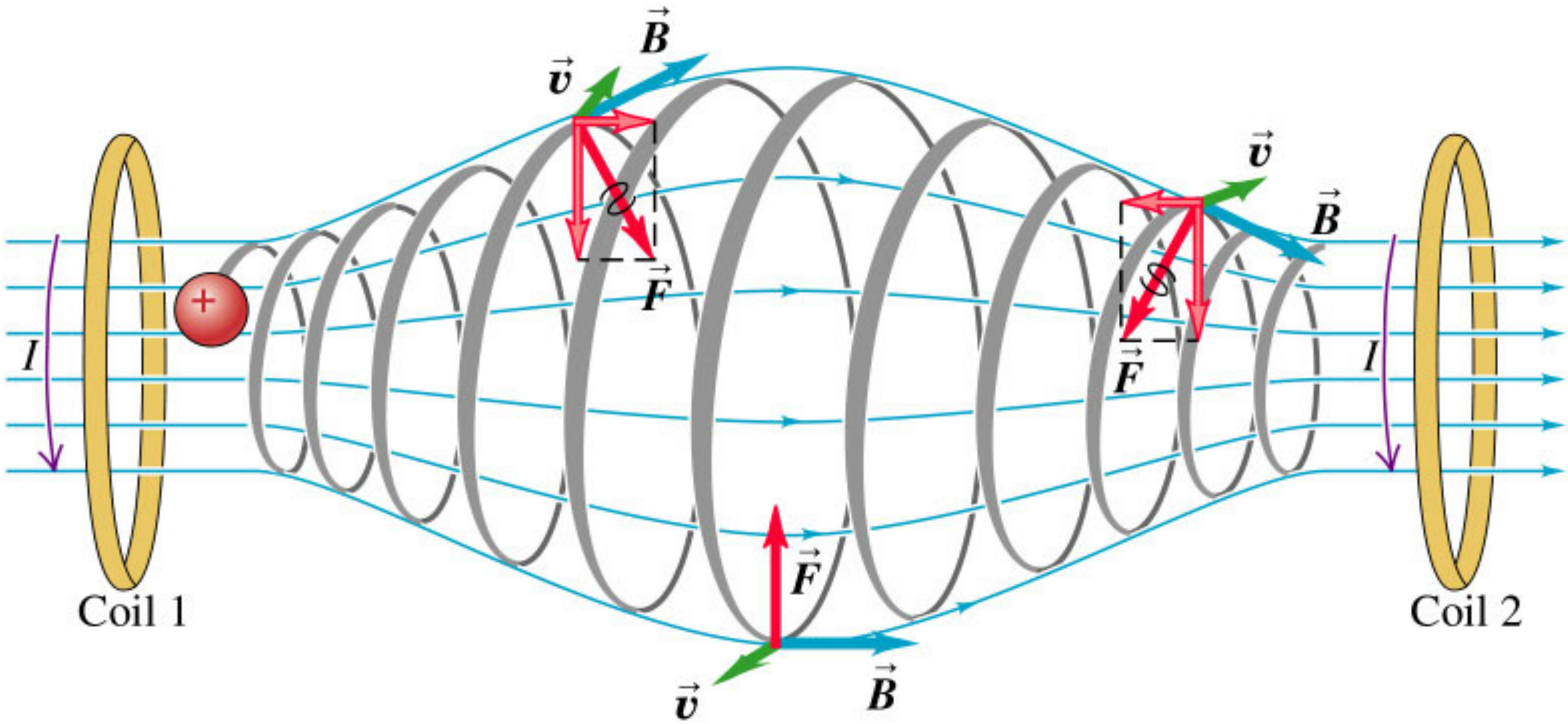
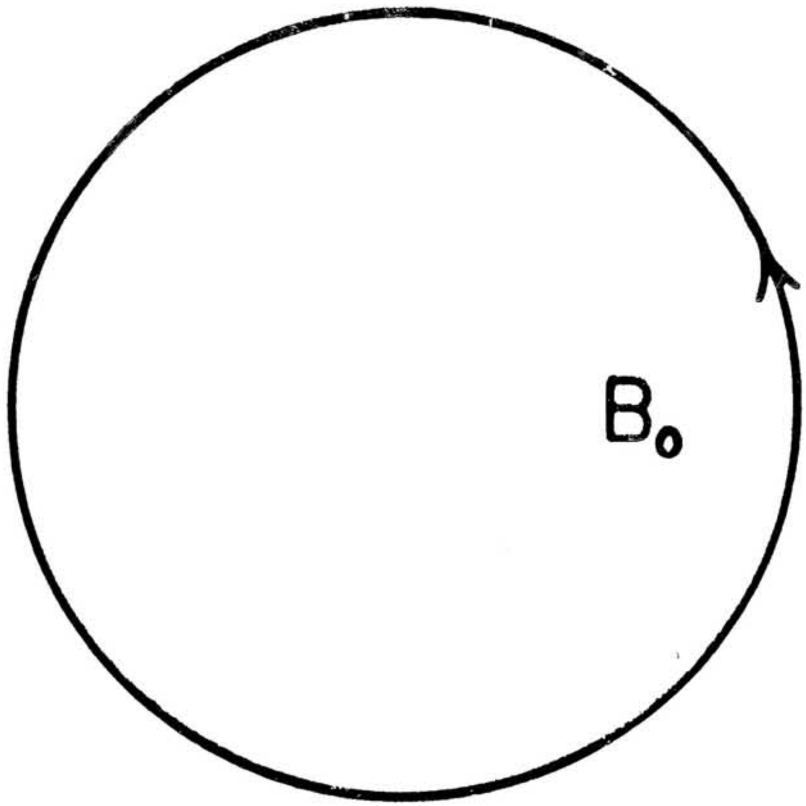


Fig.: H.D. Young and R.A. Freedman, *Sears and Zemansky's University Physics*, 11th ed., Pearson / Addison Wesley, San Francisco, CA, 2004.

Magnetic fields are used to confine the charged particles in hot ionized plasmas (e.g., in fusion energy research).

Magnetic Confinement Device



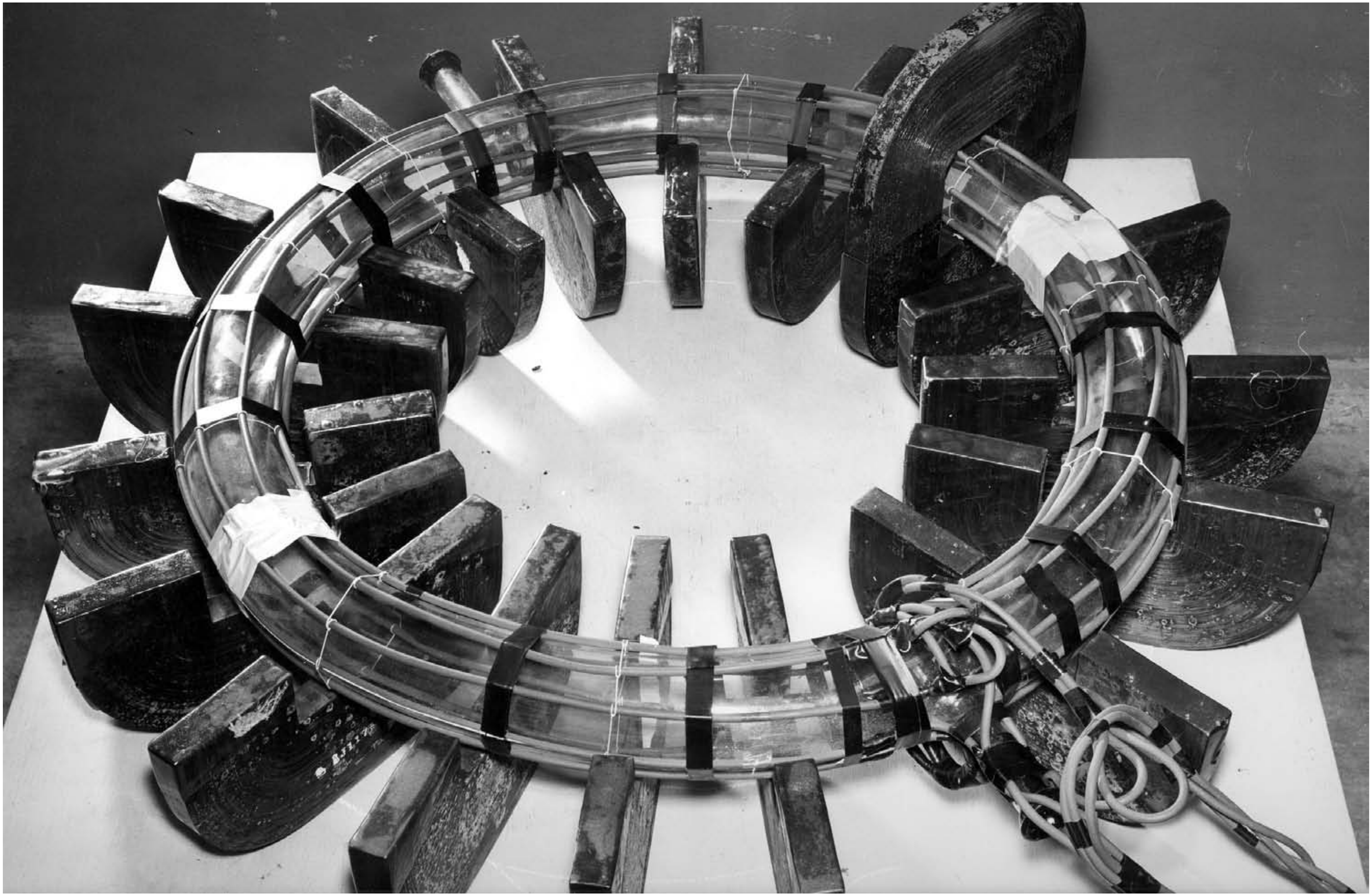
Cartoon:
ZT-H, LANL

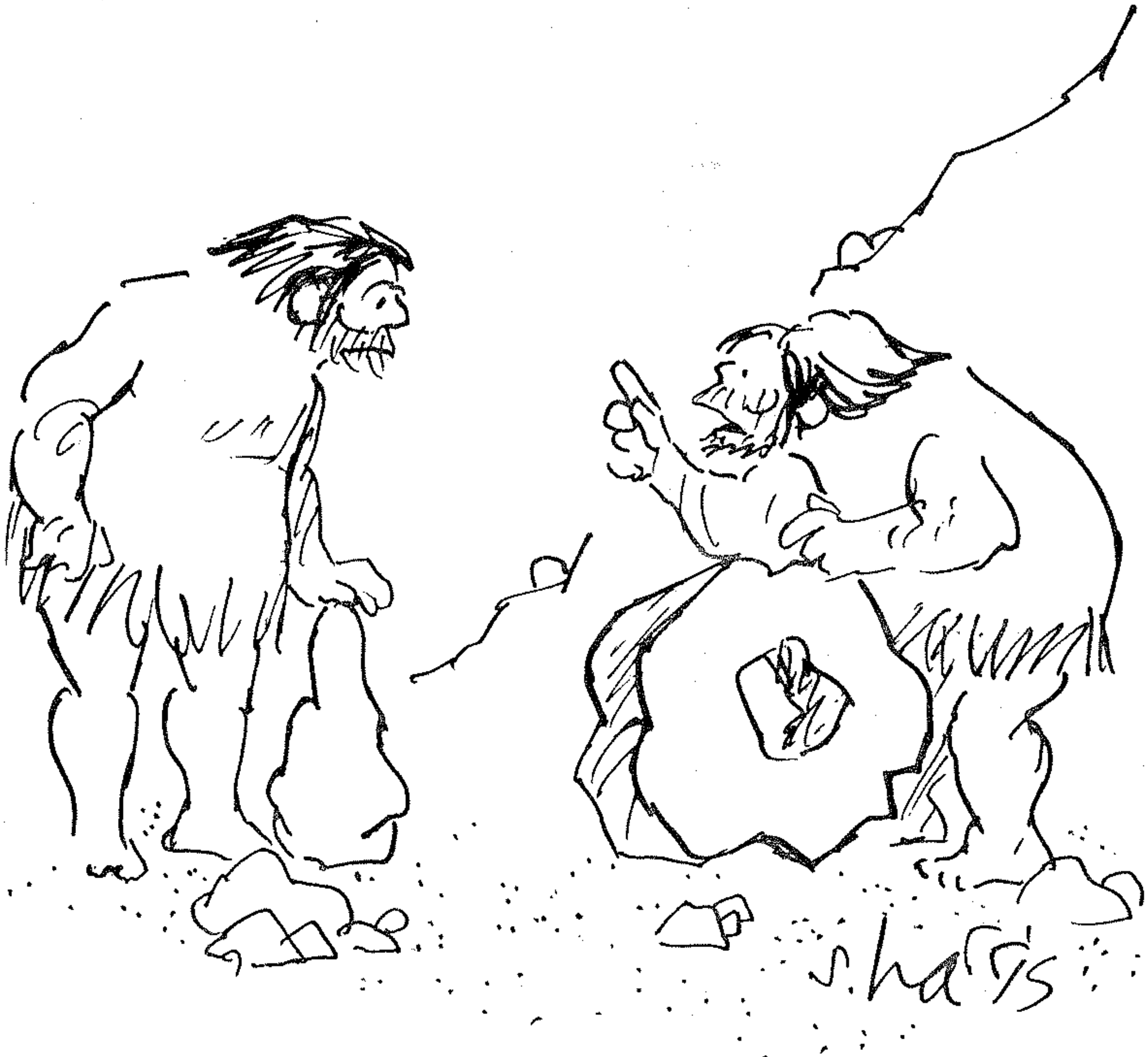
ZT–H 4MA Reversed Field Pinch

THEORICAL PHYSICSTS' DESIGN

(Lowest Order Only)

1953: Perhapsatron, LANL





"IT MAY NOT BE A PERFECT WHEEL, BUT IT'S A STATE-OF-THE-ART WHEEL."

Cartoon: S. Harris, *Einstein Simplified: Cartoons on Science*, Rutgers University Press, New Brunswick, NJ, 1989.

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Achieving a long τ_E~1s requires a large (10 m) plasma

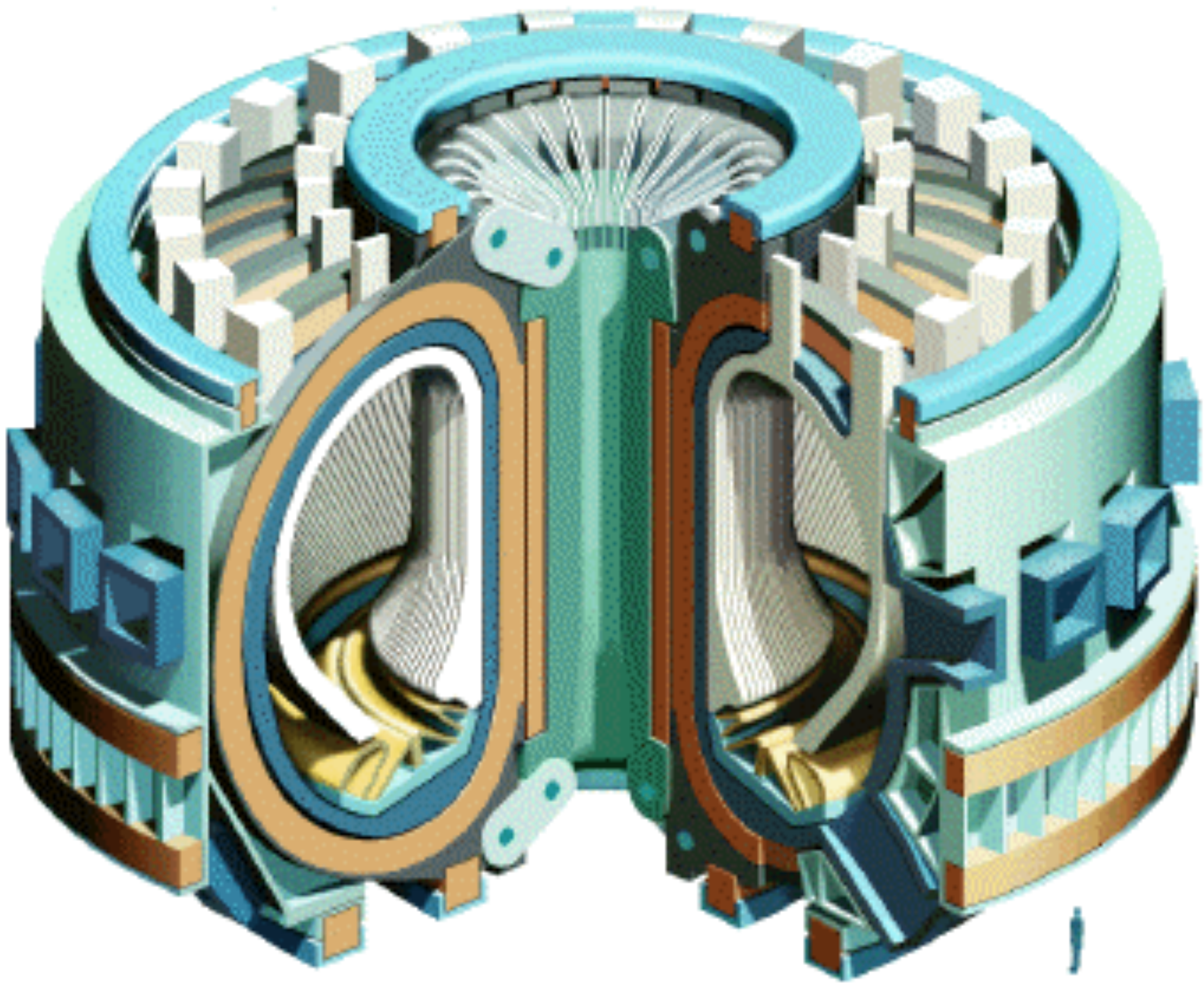


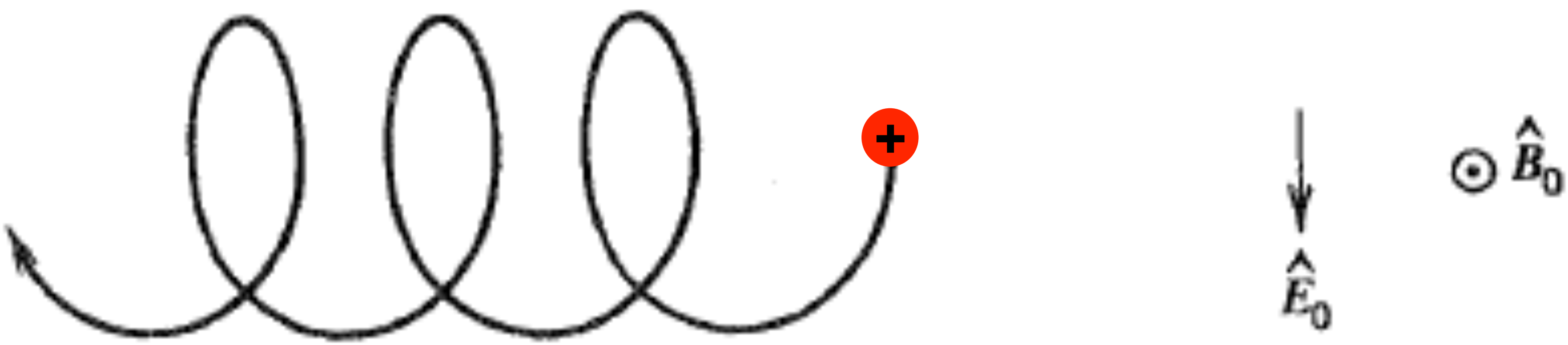
Figure: ITER, www.iter.org

International Thermonuclear Experimental Reactor (ITER)

500 MW
400 seconds
Gain Q>10

$21 billion
(July 2010)

E perpendicular to *B* makes a charged particle drift sideways

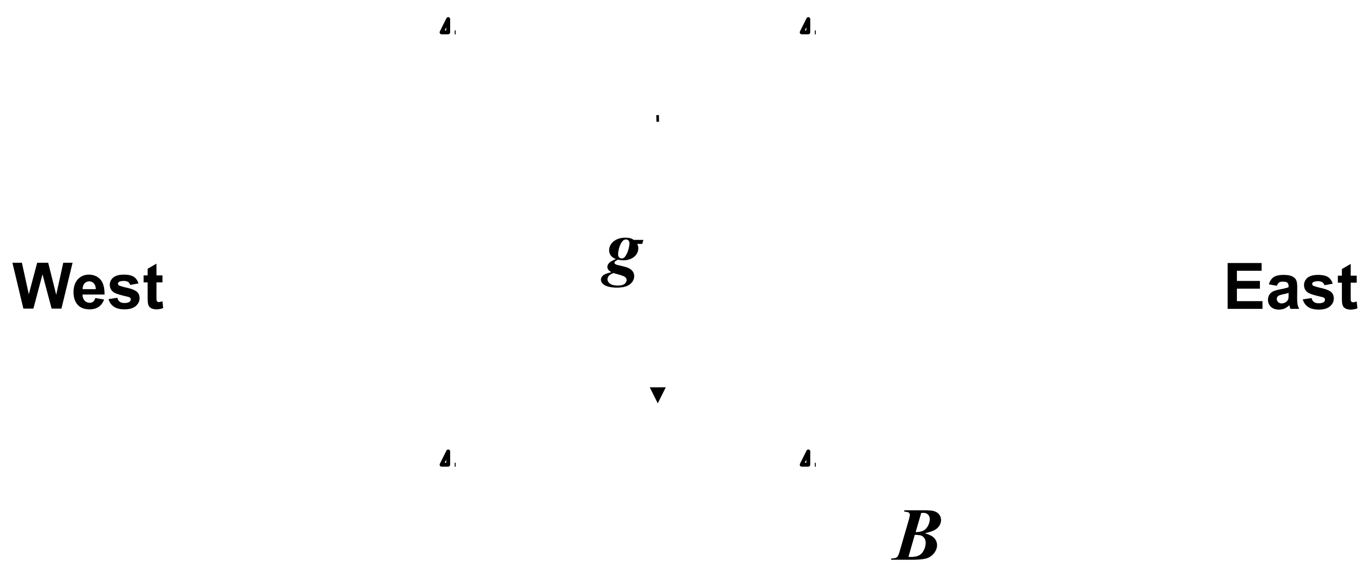


$$\vec{v}_E = \frac{\vec{E}\times\vec{B}}{B^2}, \quad \text{the "}\vec{E}\times\vec{B}\text{" drift}$$

Figure: D.R. Nicholson, *Introduction to Plasma Theory*, Wiley, NY, 1983.

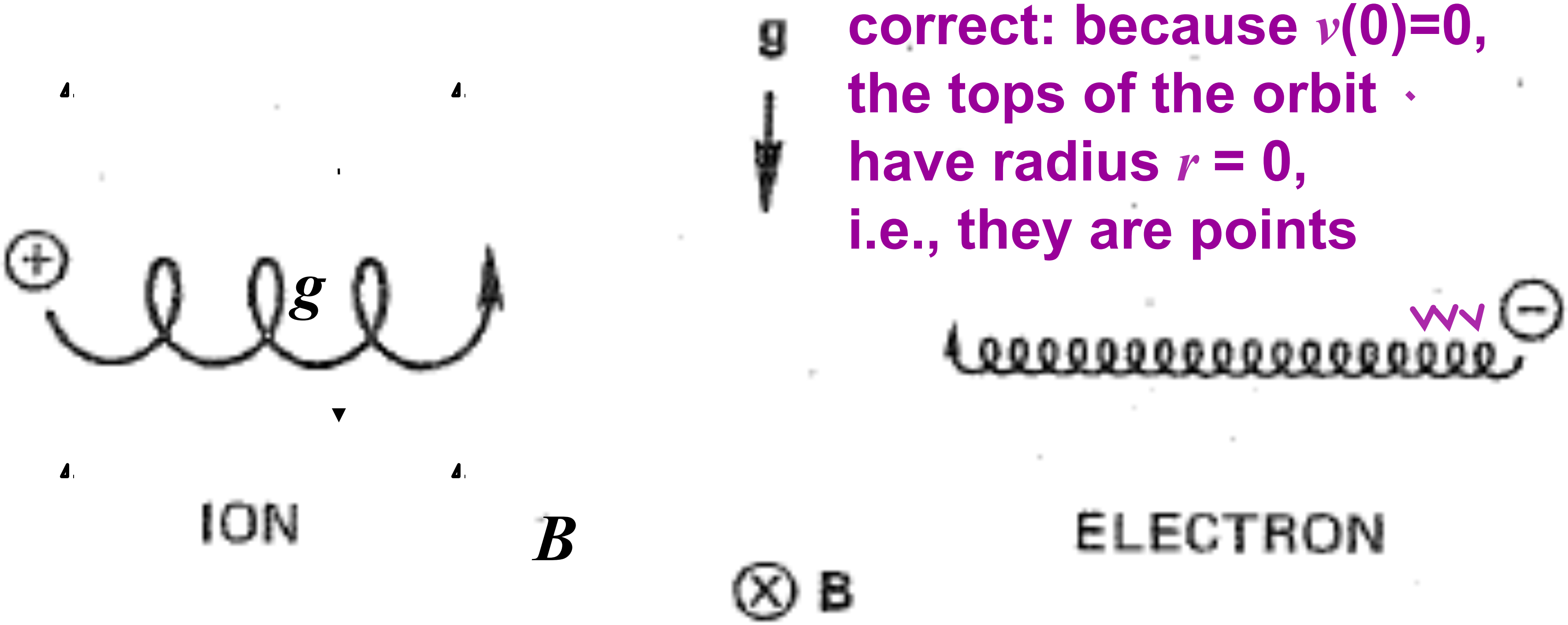
Bonus Quiz

An electron in vacuum is released from rest at a location near the Earth's surface where *E* = 0, *B* = 0.5 G north, and *g* = 9.8 m/s^2 down. Describe and sketch the motion of the electron.



The electron makes a cusp motion, falling then rising, drifting west

This image is not quite correct: because $v(0)=0$, the tops of the orbit have radius $r = 0$, i.e., they are points



West

East

Figure: F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

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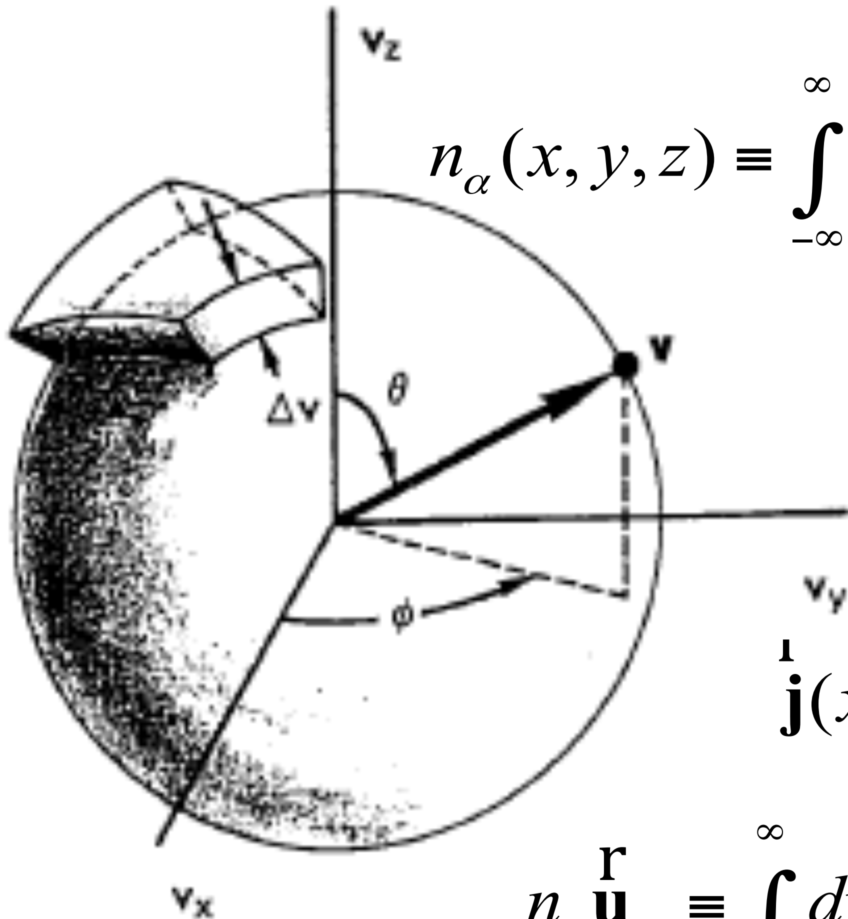
Integrate $f(x,y,z,v_x,v_y,v_z)$ over 3D velocity space to obtain density & other fluid quantities



$$n_\alpha(x,y,z) \equiv \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \; f_\alpha(x,y,z,v_x,v_y,v_z)$$

$$\rho(x,y,z) = \sum_\alpha q_\alpha n_\alpha(x,y,z)$$

$$\vec{\mathbf{j}}(x,y,z) = \sum_\alpha q_\alpha n_\alpha(x,y,z)\vec{\mathbf{u}}_\alpha(x,y,z)$$

$$n_\alpha \vec{\mathbf{u}}_\alpha \equiv \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \; \vec{v} \; f_\alpha(x,y,z,v_x,v_y,v_z)$$

Figure: F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1*, 2nd ed., Plenum Press, NY, 1984.

Integrate Boltzmann eqn to obtain fluid eqns that capture the main plasma collective properties

For each species α:

Mass conservation $$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla} \cdot (n_\alpha \vec{\mathbf{u}}_\alpha) \quad S_\alpha$$

Momentum conservation $$m_\alpha n_\alpha \frac{d\vec{\mathbf{u}}_\alpha}{dt} = n_\alpha q_\alpha \left[\vec{\mathbf{E}} + \vec{\mathbf{u}}_\alpha \times \vec{\mathbf{B}}\right] - \vec{\nabla} \cdot \mathbf{P}_\alpha - m_\alpha S_\alpha \vec{\mathbf{u}}_\alpha + \sum_\beta \vec{\mathbf{R}}_{\alpha\beta}$$

- **Convective derivative:** $$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{\mathbf{u}}_\alpha \cdot \vec{\nabla}$$
- **Resistive drag force density:** $$\vec{\mathbf{R}}_{\alpha\beta} = -m_\alpha n_\alpha \nu_{\alpha\beta} (\vec{\mathbf{u}}_\alpha - \vec{\mathbf{u}}_\beta) \quad \vec{\mathbf{R}}_{\beta\alpha}$$

Energy conservation: Heat equation
→ simplest to use Equation of State

$$\mathbf{P} = \mathbf{I}p$$

$$p = nT = Cn^\gamma$$

With further approximations, obtain single fluid magnetohydrodynamics (MHD), describing an electrically conducting fluid in the presence of magnetic field

Slow $\tau >> \frac{1}{\upsilon_{\text{collision}}}, \frac{L}{C}$ **Big** $L >> \lambda_{\text{mean free path}}, \lambda_{\text{De}}, r_{\text{L}}$

Quasi-neutral $n_e \approx n_i Z$ **Thermal equilibrium** $T_e \approx T_i$

Equations:

Mass Continuity $$\frac{d\rho}{dt} + \vec{\nabla}\cdot(\rho\vec{u}) = 0$$

Momentum Equation $$\rho\frac{d\vec{u}}{dt} = \vec{j}\times\vec{B} - \vec{\nabla}p + \rho\vec{g}$$

Equation of state $$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0, \quad p = \frac{\rho T}{M}(Z+1)$$

Ampere's Law $$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} \quad \Rightarrow \quad \vec{\nabla}\cdot\vec{j} = 0$$

Faraday Law $$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \quad \vec{\nabla}\cdot\vec{B} = 0$$

Ohm's Law $$\vec{E} + \vec{u}\times\vec{B} = \eta\vec{j}$$

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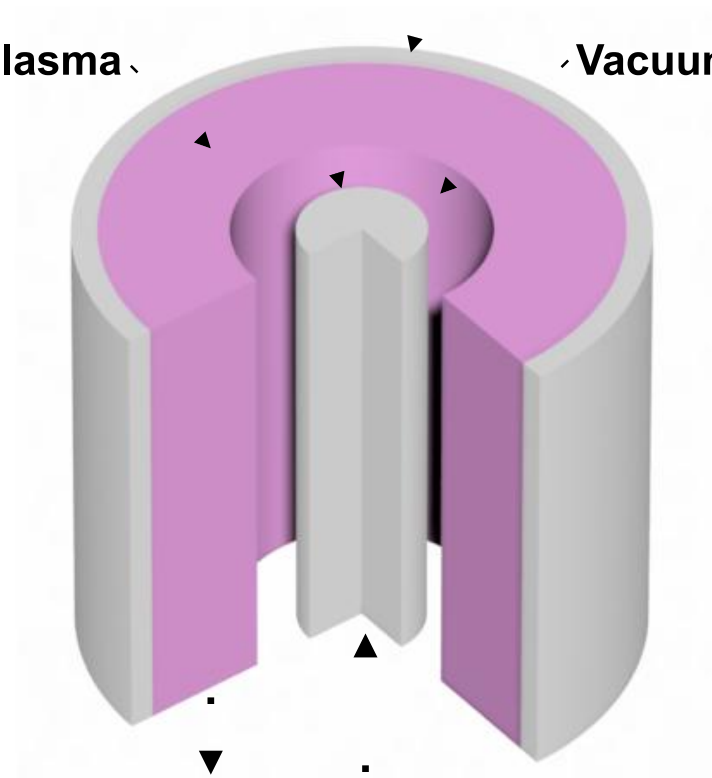
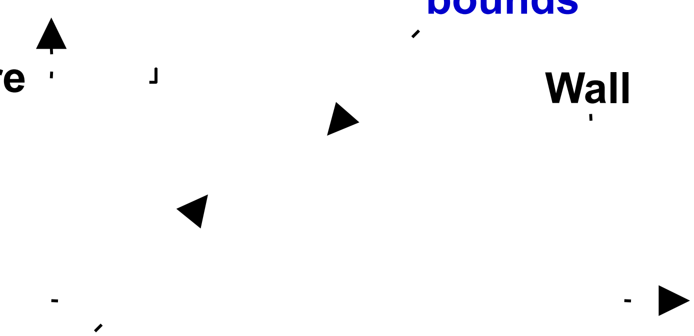
Plasmas are complex systems

- **Plasmas can exhibit self-organized behavior of high complexity**
- **Self-organization occurs in many areas**
  - **Space & astrophysics, biosystems, micro- and nano- components, protein folding**
  - ***Selective decay* processes, thermodynamics**
    - **Dissipation of some quantity on small scales (e.g., energy in eddies, turbulence)**
    - **Persistence of other quantities on larger spatial scales (e.g., helicity)**

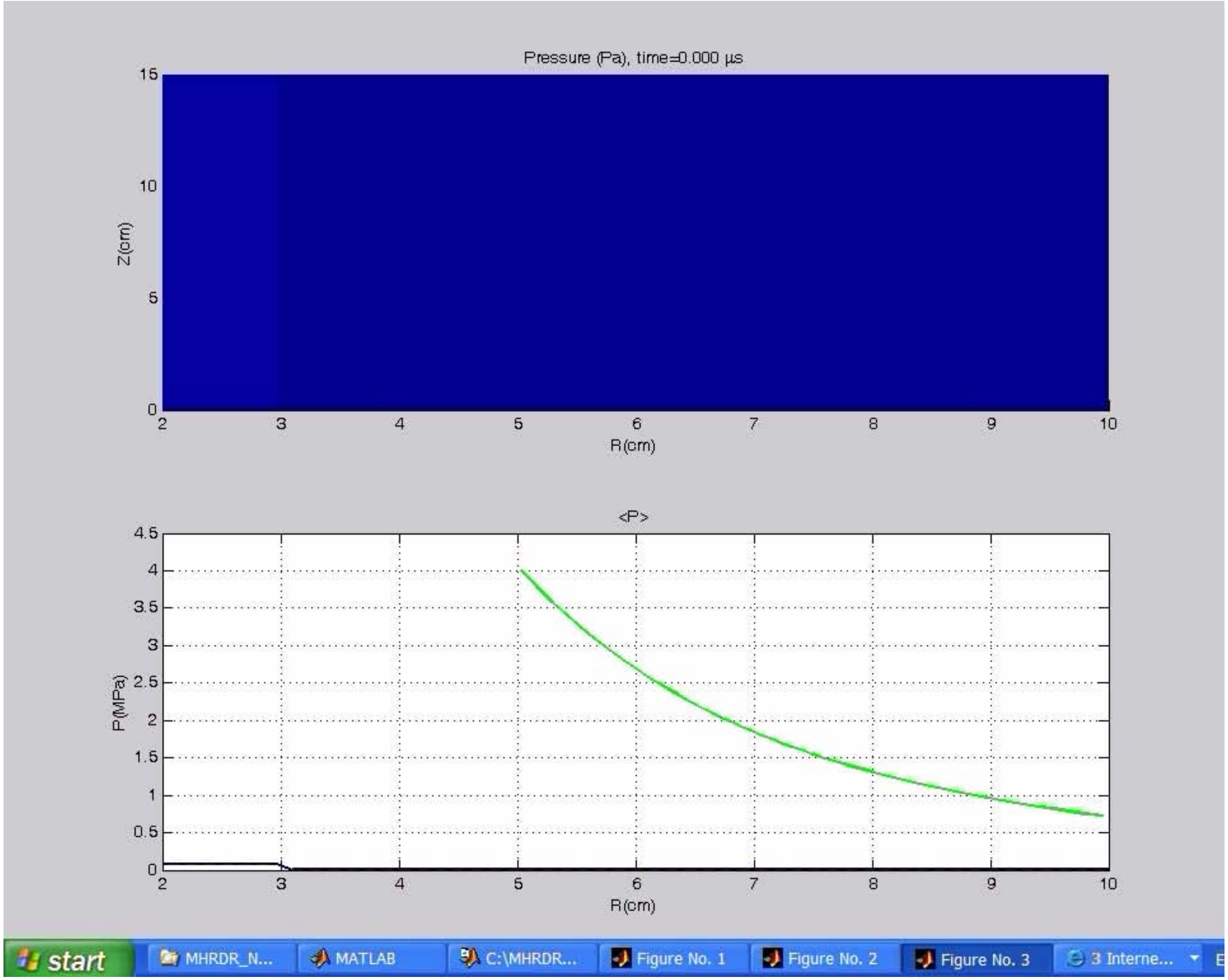
Example of self-organization (numerical simulation): reorganization of plasma pressure into stable profile



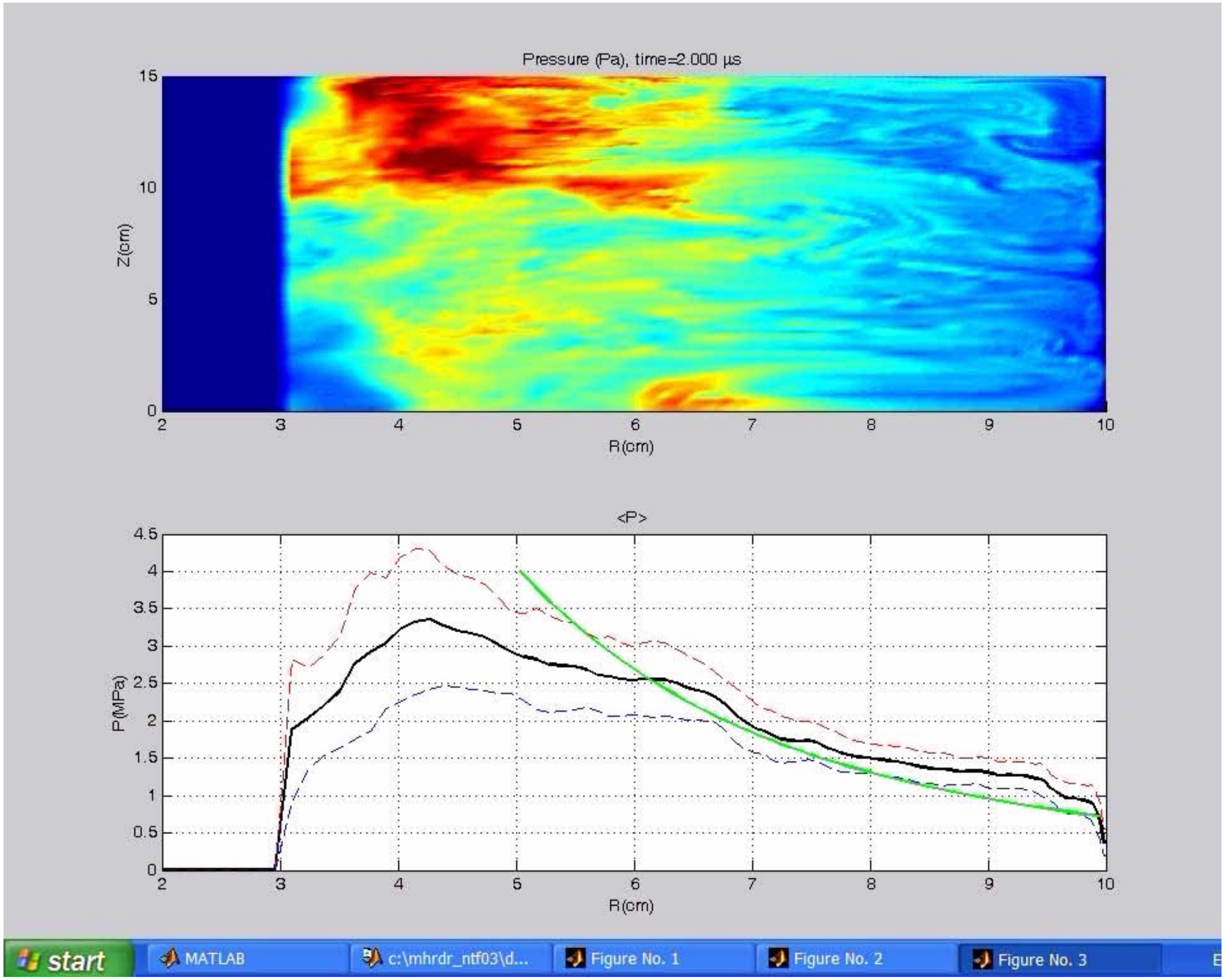


RS, SF, VM, BSB 6/20/12

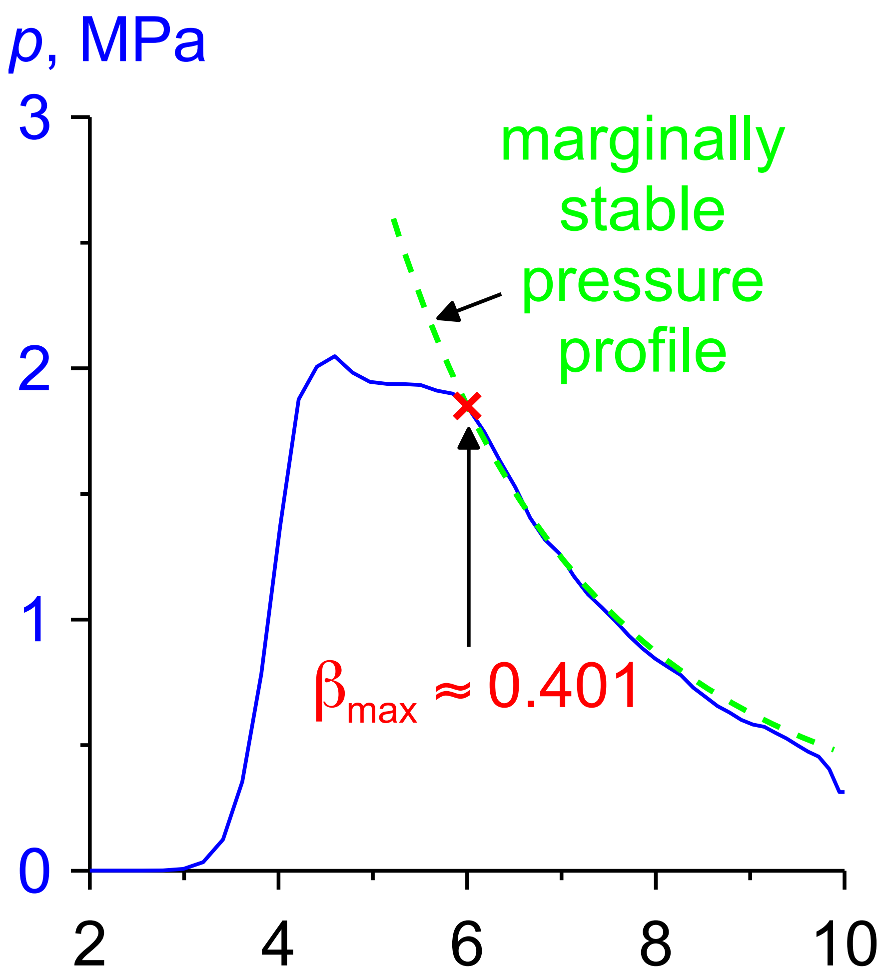
Movie of dynamic formation phase



Movie of equilibration phase



Simulated plasma finds Kadomtsev marginally stable profile

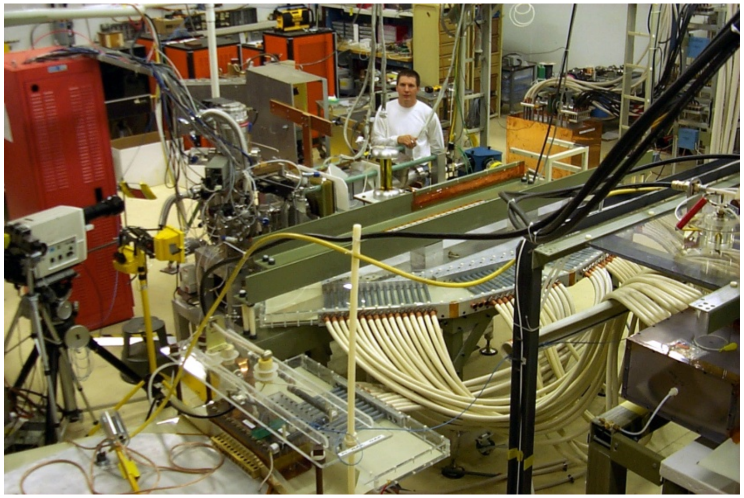


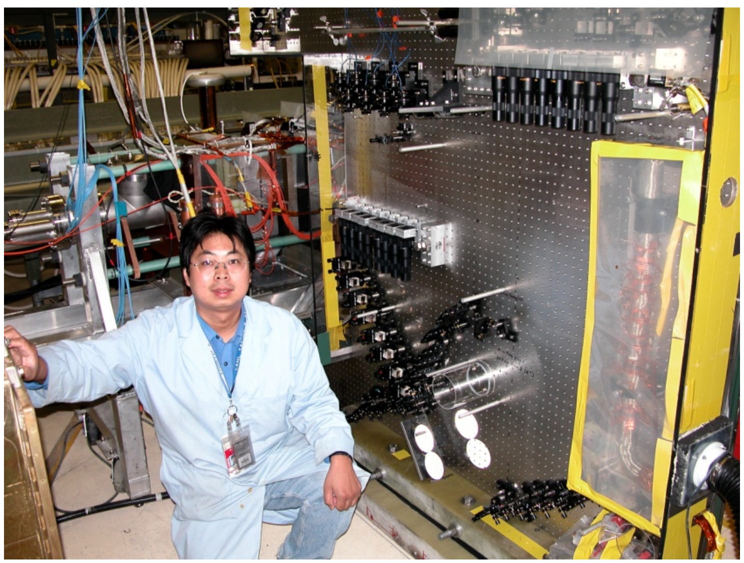
Point marked with "x" is where beta for simulated profile is matched to Kadomtsev beta

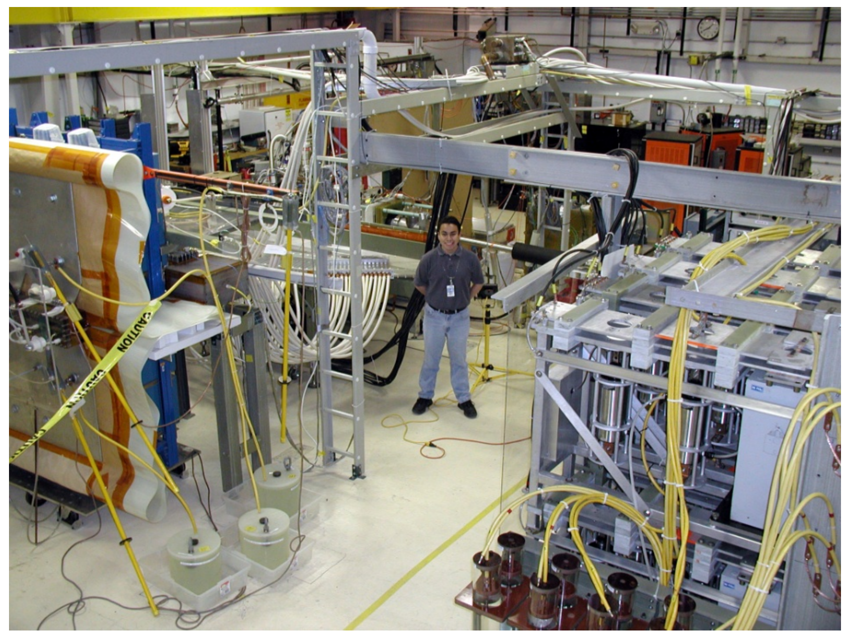
Through m=0 turbulence, the plasma organizes itself into the marginally stable Kadomtsev state

RS, VM, BSB 5/26/03

FRX-L: The Field Reversed Configuration (FRC) Plasma Injector for MTF



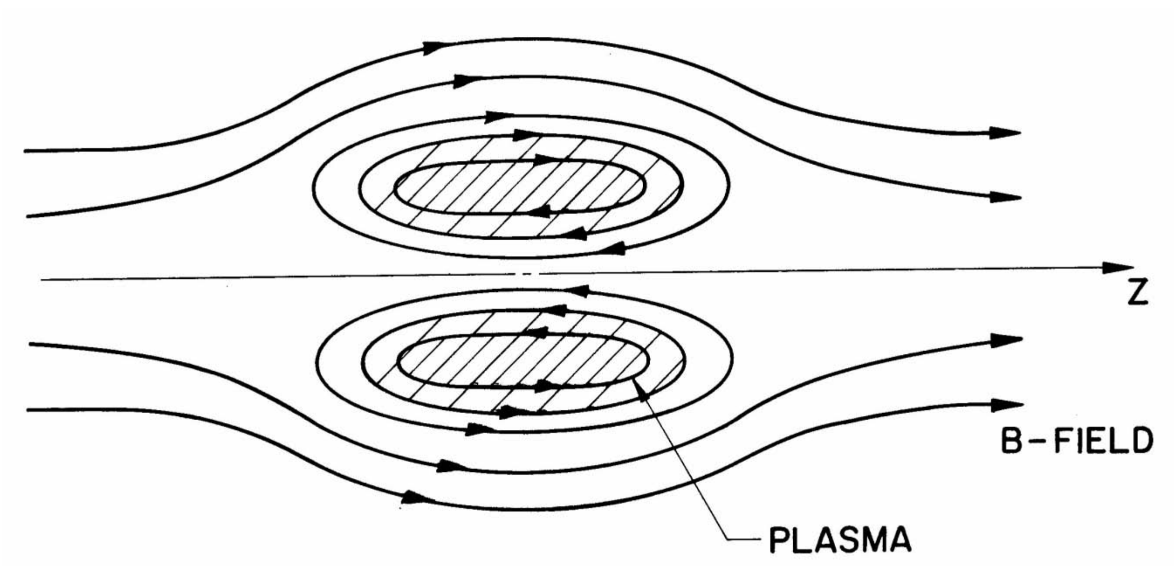




Los Alamos

GAW 2003

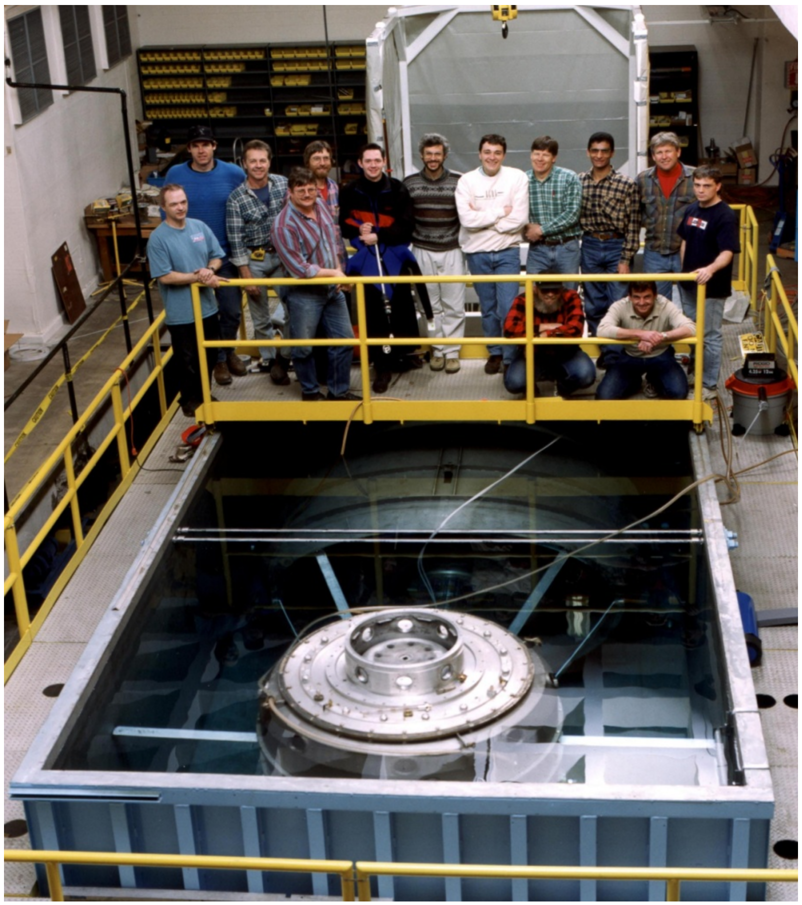
Field Reversed Configuration high-β self-organized plasma

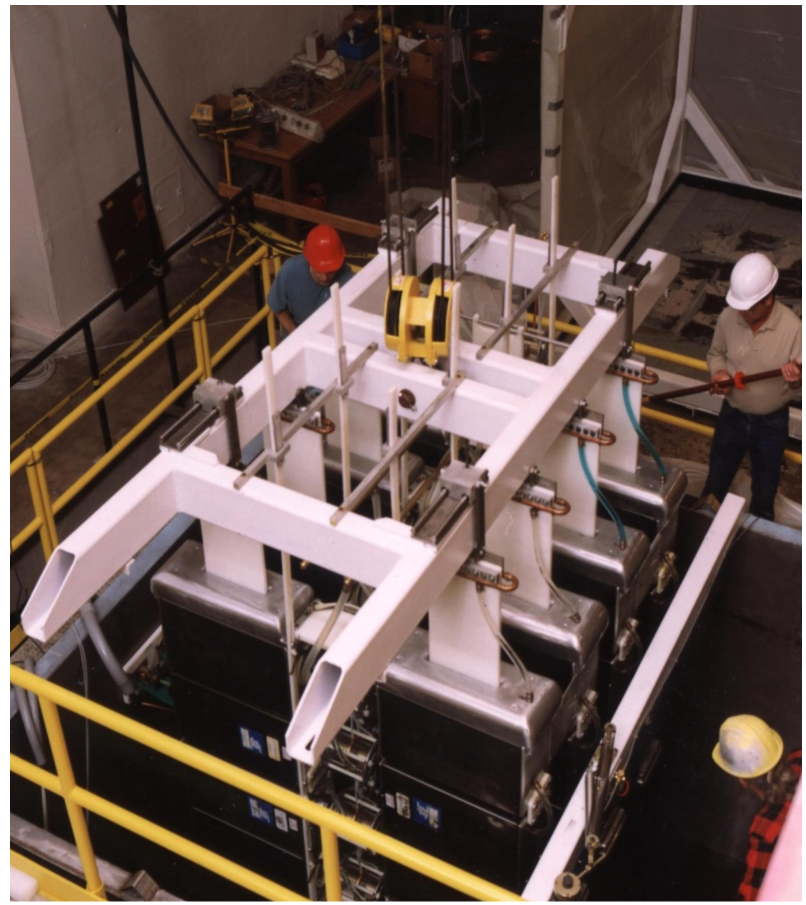


- $<\beta> \sim 1$
- compact torus like spheromak
- Can translate into liner

The LANL FRC has parameters orders of magnitude different than previous FRCs. How will FRC behave under compression? How will liner interact with FRC?

Experiment on 1-MA/100-ns Zebra (UNR) studies plasma formed by multi-MG field on aluminum





BSB, NLG 4/22/08

Zebra Megagauss Experiment

Movie: Stephan Fuelling, UNR, Oct 2006

Some good plasma references

- ✓ P.M. Bellan, *Fundamentals of Plasma Physics*, Cambridge University Press, Cambridge, United Kingdom, 2006.
- ✓ J.A. Bittencourt, *Fundamentals of Plasma Physics*, 3rd ed., Springer Science, New York, NY, 2004.
- ✓ F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion, Vol. 1: Plasma Physics*, 2nd ed., Plenum Press, New York, NY, 1984.
- ✓ R.P. Drake, *High-Energy-Density Physics: Fundamentals, Inertial Fusion and Experimental Astrophysics*, Springer Verlag, New York, NY, 2006.
- ✓ R.J. Goldston and P.H. Rutherford , *Introduction to Plasma Physics*, Institute of Physics Publishing, Philadelphia, PA, 1995.
- ✓ D.R. Nicholson, *Introduction to Plasma Theory*, John Wiley & Sons, New York, NY, 1983.

Photo: University of Nevada, Reno, www.unr.edu

Introduction to Fusion Energy and Reactor Design

Thomas Weber

Los Alamos Plasma Physics Summer School
July 18, 2012

Los Alamos
NATIONAL LABORATORY
EST. 1943

Exothermic nuclear reactions

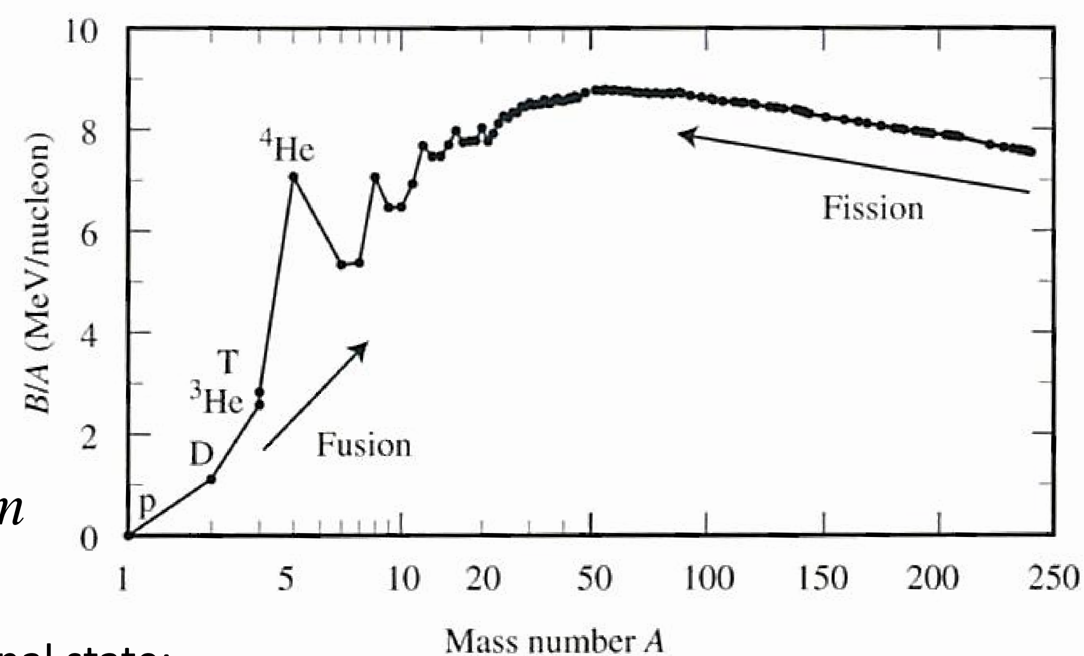
Mass-energy equivalence:

$$E = mc^2$$

Binding energy of an atomic nucleus:

$$B = \Delta mc^2$$

$$\Delta m = Zm_p + (A - Z)m_n - m$$



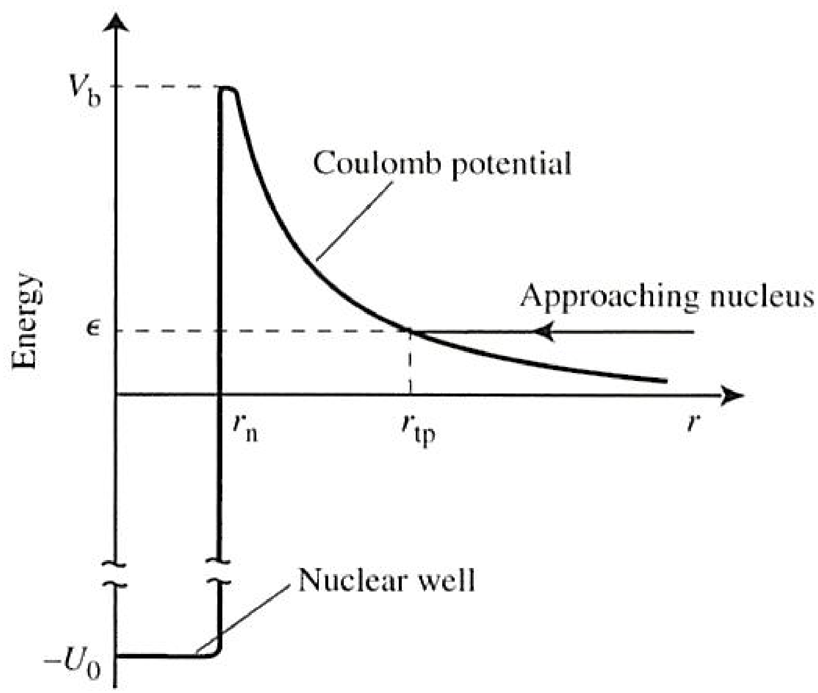
Audi and Wapstra (1995)

Energy released from initial to final state:

$$Q = \sum_f B_f - \sum_i B_i$$

Energy is released through fission or fusion of nuclei toward Iron (A ≈ 56, highest binding energy per nucleon).

Fusion



Classical Coulomb potential:

$$V_c(r) = \frac{Z_1 Z_2 e^2}{r}$$

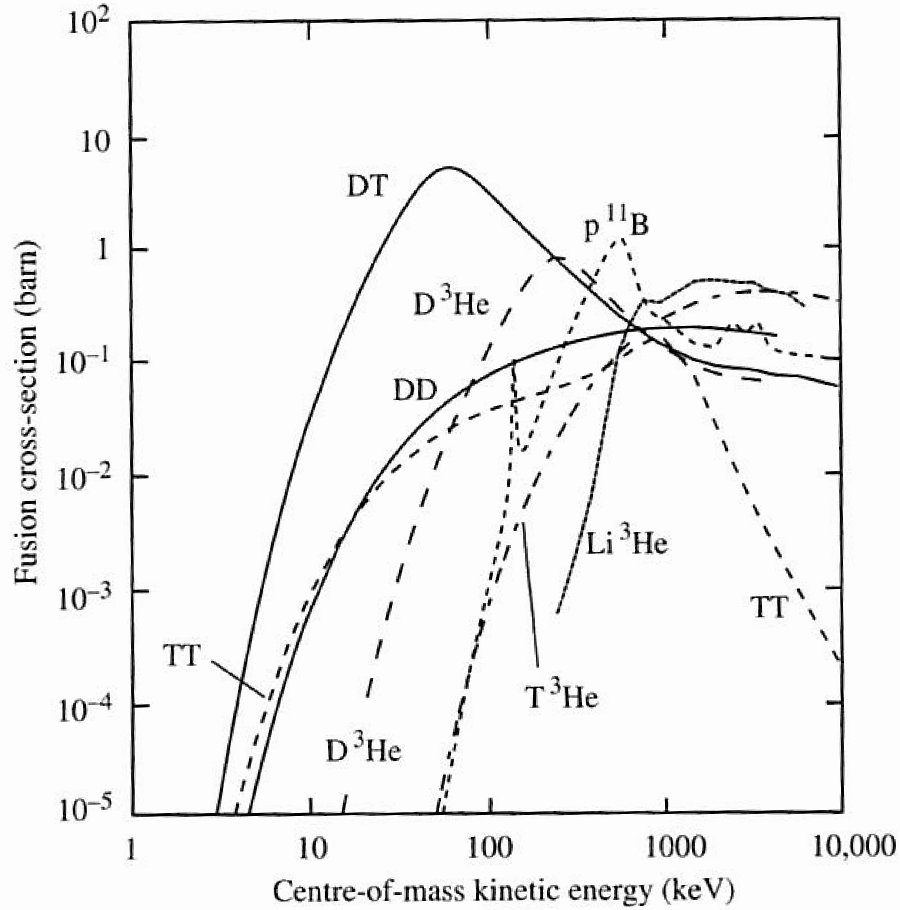
Nuclear radius:

$$r_n \approx 1.44 \times 10^{-13}(A_1^{1/3} + A_2^{1/3})[cm]$$

V_c is on the order of 1 MeV!!!

S. Atenzi, J. Meyer-ter-vehn, Int. Series of Monographs on Phys. - 125, 2004

Fusion Cross Section



Fortunately, Quantum Mechanics comes to the rescue!

$$\sigma \approx \sigma_{geom.} T_{barrier} R_{prob.}$$

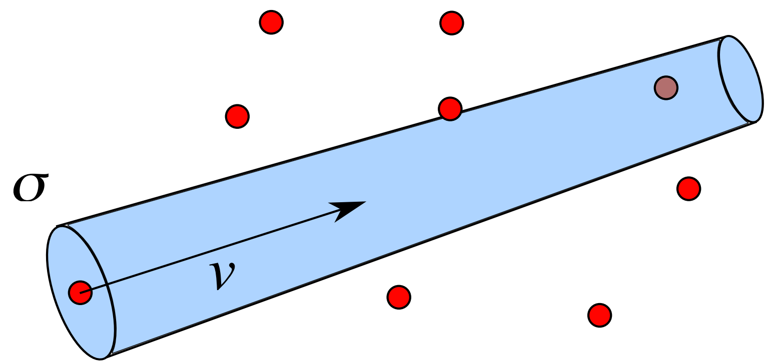
$$\sigma \approx \frac{\exp(-\sqrt{\varepsilon_G / \varepsilon})}{\varepsilon} S(\varepsilon)$$

S(ε) is the *astrophysical S factor.*

S usually varies more weakly as function of energy than the Gamow factor.

DT reaction has relatively large S(0) and relatively small Gamow energy.

Reactivity



Volume swept out per unit time:

$$v\sigma(v)$$

Inverse time to intersection (reaction rate) of 1 atom traveling at speed *v* through a target of density n:

$$nv\sigma(v)$$

Reactivity: probability of reaction per unit time per unit density = *v σ*

Averaged Reactivity: average over target velocities $\langle \sigma v \rangle = \int_0^\infty \sigma(v) f(v) v dv$

Reaction Rate: number of reactions per unit time, per unit volume.

$$R_{12} = \frac{n_1 n_2}{1+\delta_{12}} \langle \sigma v \rangle = \frac{f_1 f_2}{1+\delta_{12}} n^2 \langle \sigma v \rangle$$

Maxwell averaged reactivity

Average reactivity using two species:

$$\langle \sigma v \rangle = \iint d\vec{v}_1 d\vec{v}_2 \sigma_{1,2}(v) v f_1(v_1)$$

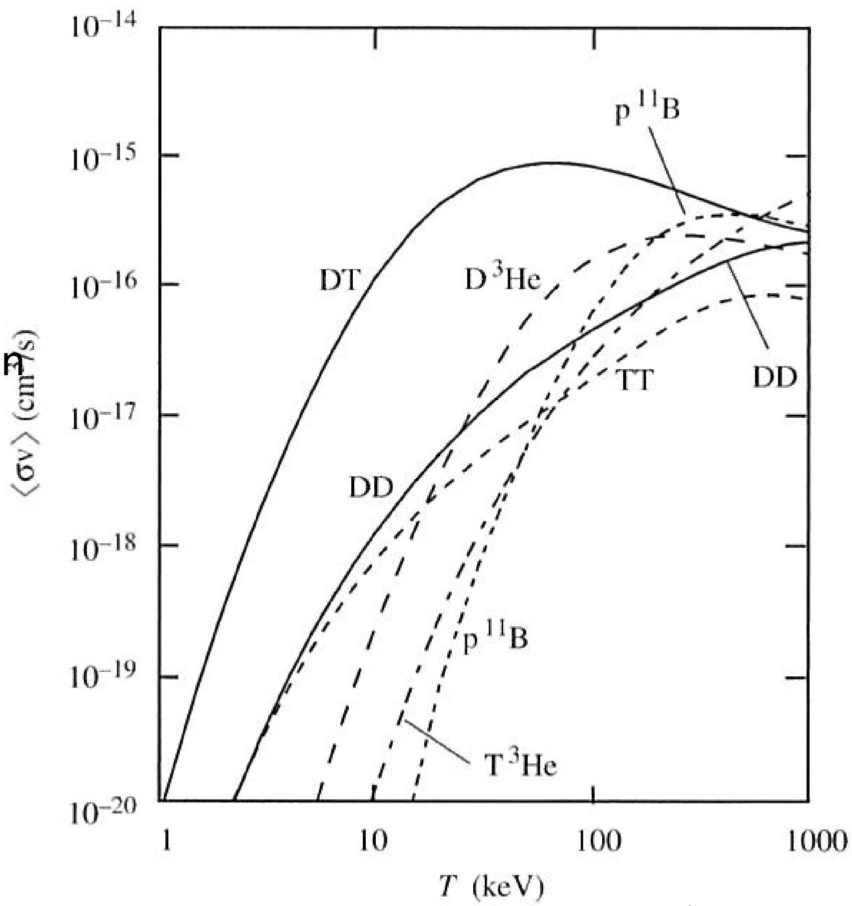
where $v = \left| \vec{v}_1 - \vec{v}_2 \right|$

Where f_i is the Maxwell-Boltzmann distribution function (thermal equilibrium).

$$f_j(v_j) = \left(\frac{m_j}{2\pi kT} \right)^{3/2} \exp\left(\frac{-m_j v_j^2}{2kT} \right)$$

Substituting in for the distribution function and using center of mass velocity and reduced mass:

$$\langle \sigma v \rangle = \left(\frac{m_r}{2\pi kT} \right)^{3/2} \int d\vec{v} \exp\left(\frac{-m_r}{2kT} \vec{v}^{\,2} \right) v\sigma(v)$$



Averaged reactivity vs. temperature of easiest fusion reactions.

p-p cycle

Dominant cycle in our sun.

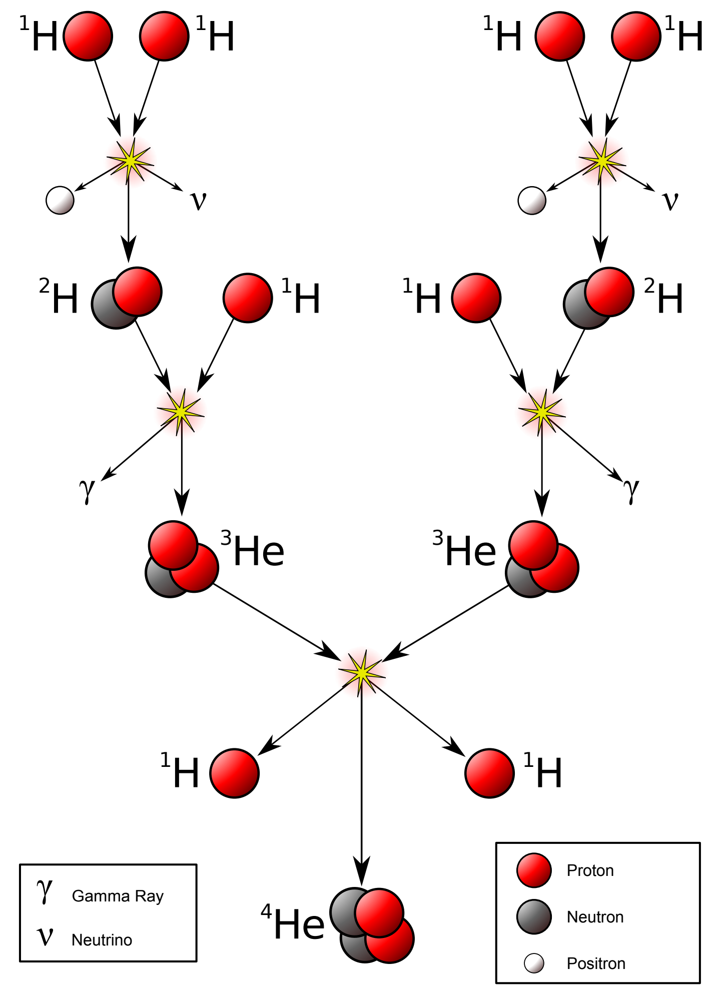
First stage of ^2He beta decaying into D is extremely rare. ^2He usually decays into 2H.

Branches: different outcomes of a reaction.

4 paths from ^3He to ^4He:

- pp I - $^3He + {}^3He$ (86% in sun)
- pp II - $^3He + {}^4He$ (14% in sun)
- pp III - $^3He + {}^4He$ (0.11% in sun)
- pp IV - ^3He + p (very rare in sun)

pp III cycle generates high energy neutrinos and is very important to the solar neutrino problem

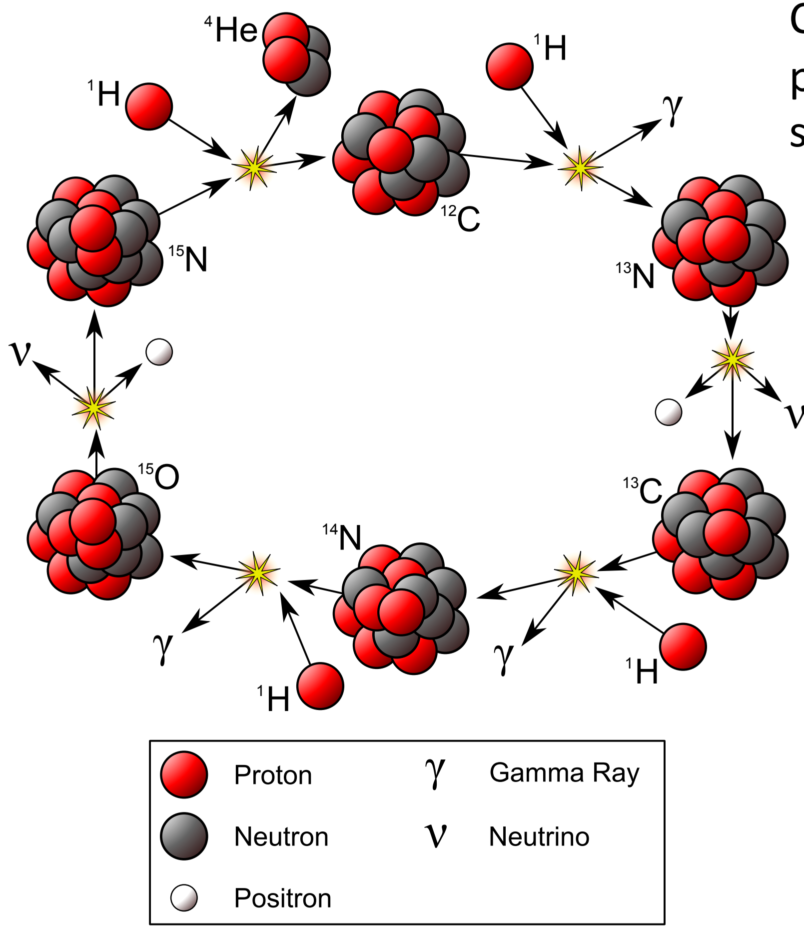


p-p cycle showing ppI branch (most probable)

CNO cycle

Catalytic reaction.

CNO dominates
kT > 1.5 keV
(below this temp.
p-p dominates).



Concentration of C, N, O place an upper limit on stable stellar mass.

Dominant in stars >1.3x mass of sun.

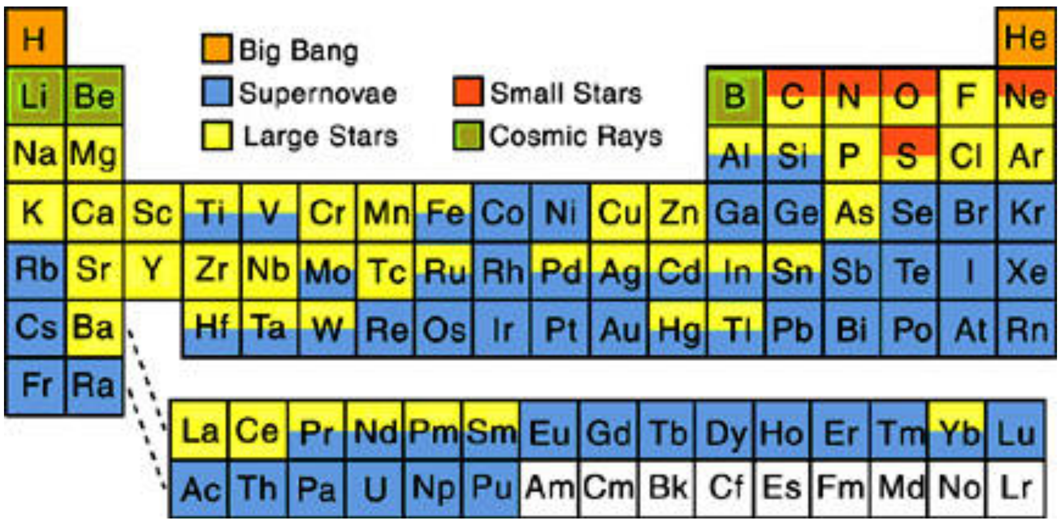
4 cold CNO cycles (stellar interiors)

- CNO-I
- CNO-II
- CNO-III
- CNO-IV

3 hot CNO cycles (novae, GRB)

- HCNO-I
- HCNO-II
- HCNO-III

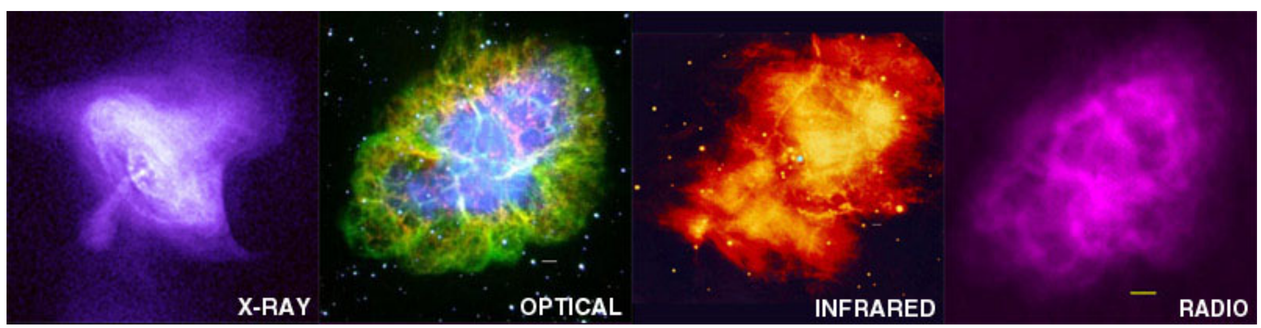
Cosmic nucleosynthesis



4 major sources:
- big bang
- stellar interiors
- supernovae
- cosmic ray spallation

Iron is lowest energy state (highest binding energy).

Higher energy species can be produced in exothermic reactions or in endothermic reaction in
high-energy systems.
- supernovae
- cosmic rays
- particle accelerators



Important terrestrial fusion reactions

$$D + T \rightarrow {}^3He + n$$

$$D + D \rightarrow \begin{cases} T + p \\ {}^3He + n \\ \alpha + \gamma \end{cases}$$

$$T + T \rightarrow \alpha + 2\,n$$

$$D + {}^3He \rightarrow \alpha + p$$

$${}^3He + {}^3He \rightarrow \alpha + 2\,p$$

$$p + {}^{11}B \rightarrow 3\,\alpha$$

Rare isotopes

Deuterium: stable isotope of Hydrogen
- mass = 2 amu
- naturally occurs at 0.03%
- can be harvested from ocean water
- constituent of "heavy water"

Helium-3: aka. Tralphium (stable)
- mass = 3 amu.
- naturally occurs at 0.000137% (earth)
- higher abundance on moon (1-50 bbp)
- proposed moon mining once fusion is "solved"

Tritium: radioactive isotope of Hydrogen
- mass = 3 amu
- half-life = 12.32 years.
- extremely rare (essentially man-made)
- very toxic
- Produced in fission, decay, high energy collisions.

Tritium is typically used in watches, gun sights, exit signs, etc. to excite phosphorescent materials that glow in the dark.

Lawson Criteria: considering losses

Conditions required for a fusion system to reach "ignition" (Lawson 1955).

Ignition: plasma self heating from fusion products > loss rate.

$$R_{DT} E_{ch} \geq P_{loss}$$

Reaction rate from earlier:

$$R_{DT} = n_D n_T \langle \sigma v \rangle = \frac{1}{4} n_e^2 \langle \sigma v \rangle$$

Energy confinement time: $\tau_E = \dfrac{W}{P_{loss}}$

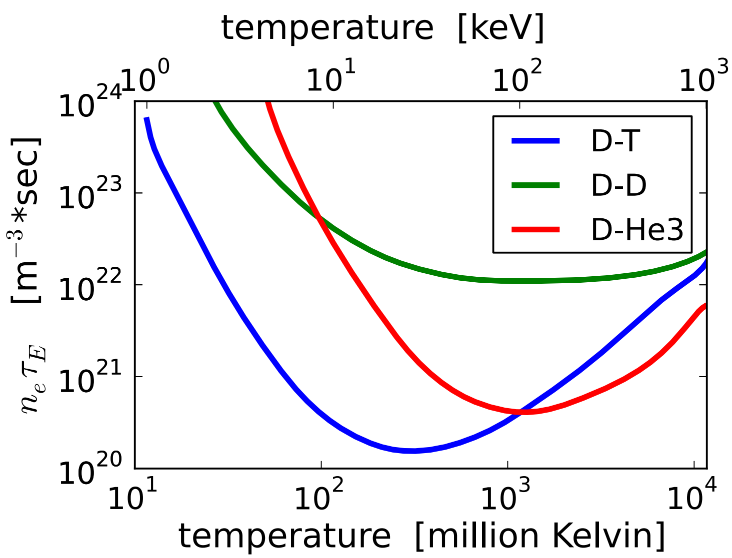
where $W = 3 n_e kT$

$$\frac{1}{4} n_e^2 \langle \sigma v \rangle E_{ch} \geq \frac{3 n_e kt}{\tau_E}$$

Lawson Criteria: $n_e \tau_E \geq \dfrac{12kT}{E_{ch} \langle \sigma v \rangle}$

Los Alamos NATIONAL LABORATORY EST. 1943

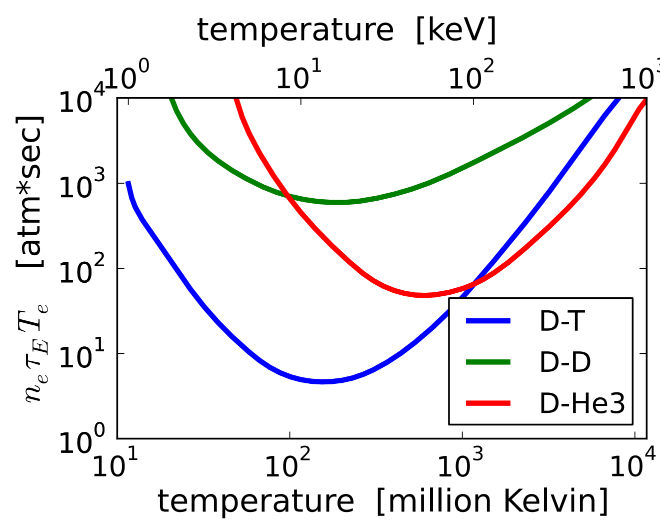
Inserting values



Required value of $n_e\tau_E$ depends on temperature.

Minimum for DT:

$$n_e \tau_E \geq 1.5 \times 10^{20} [s/m^3]$$



"Triple product" is typically a more useful figure of merit.

Minimum for DT:

$$n_e \tau_E T \geq 10^{21} [keV/m^3]$$

Energy Gain Factor (Q)

Ratio of fusion energy production to heating energy required.

$$Q = \frac{P_{fusion}}{P_{heatng}} = \left(\eta_{heat} f_{recirc.} \eta_{elect.} (1 - f_{ch})\right)^{-1}$$

where $$P_{heatng} = \eta_{heat} f_{recirc.} \eta_{elect.} (1 - f_{ch}) P_{fusion}$$

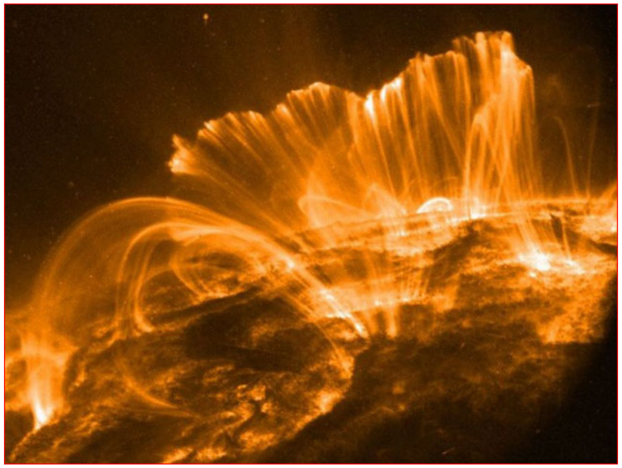
f_{ch} = 0.2 for DT
Assuming
$f_{recirc.}$ = 0.2
η_{heat} = 0.7
$\eta_{elect.}$ = 0.7

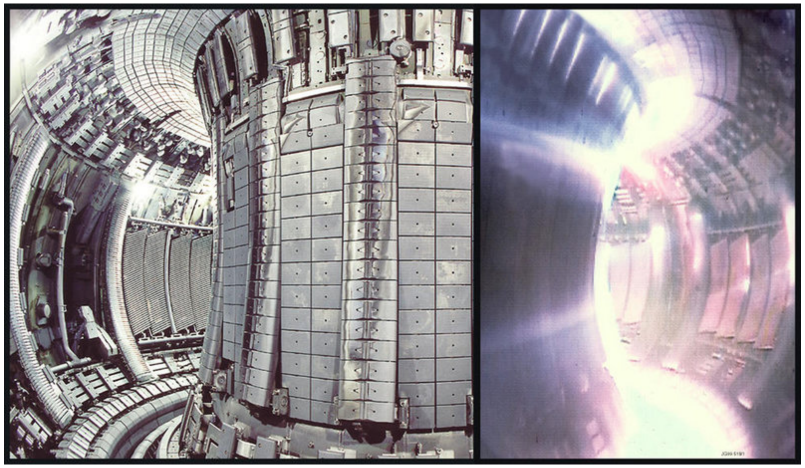
Q = 1 referred to as “*breakeven”.*

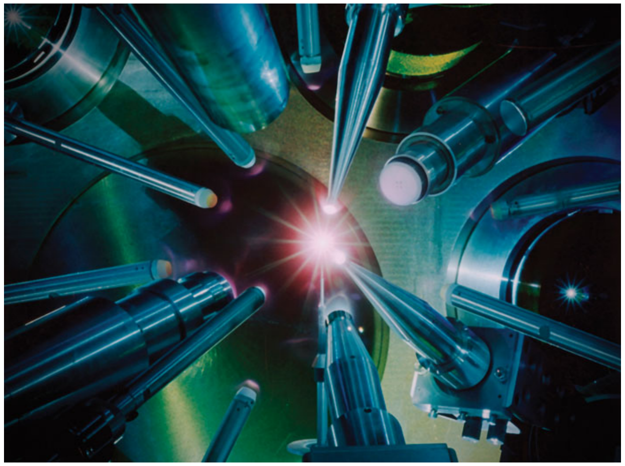
Ignition corresponds to Q = ∞

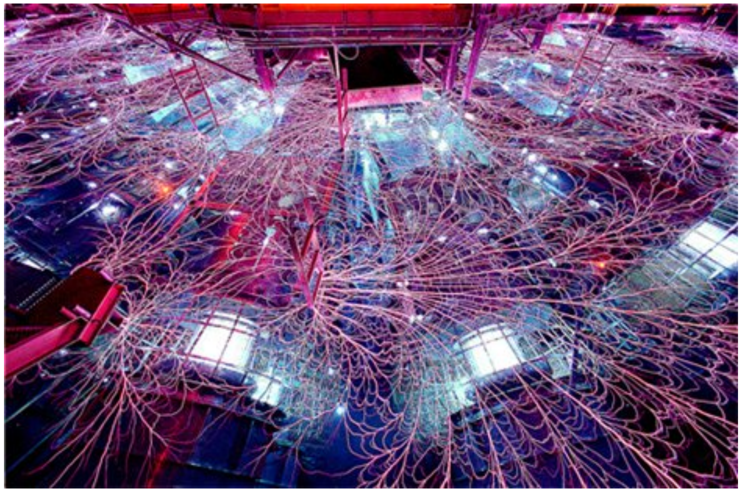
(ignition is not a requirement for a practical, energy producing reactor)

Confinement concepts



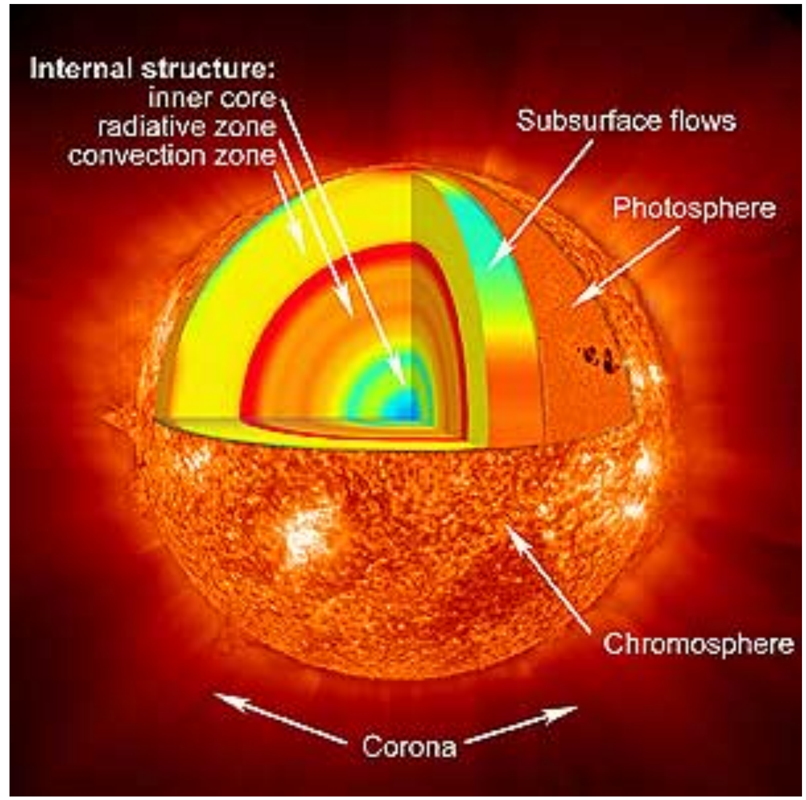






Gravitational confinement

- Exceptional confinement (billions of years)
- Capable of using common isotopes (p-p cycle)
- Naturally occurring (we don't have to do any work!)
- Poor scaling…



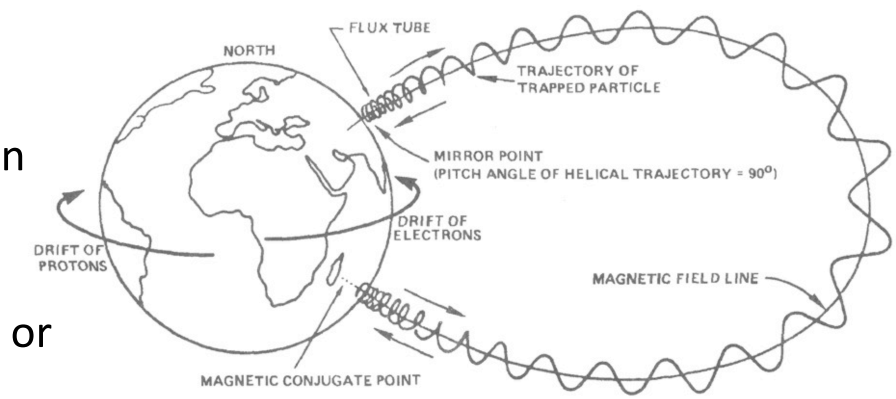
Magnetic confinement Fusion Energy (MFE)

• Use magnetic fields to confine the hot plasma.

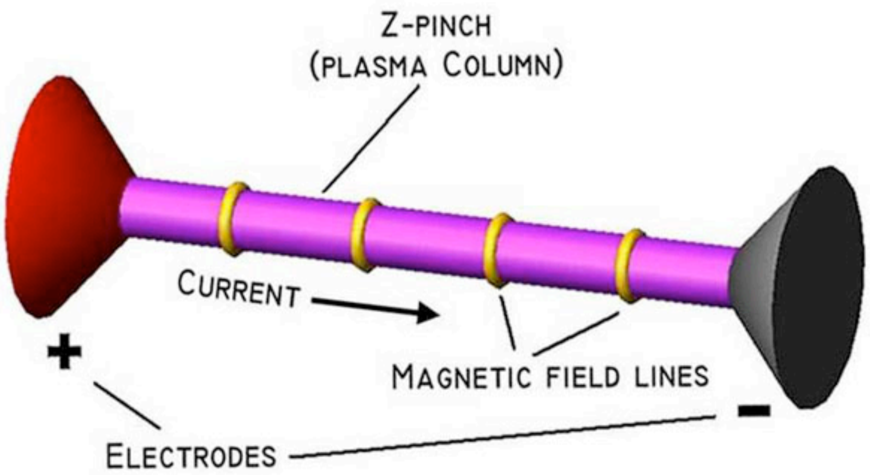
• Hopefully will have better scaling than gravitational confinement fusion.

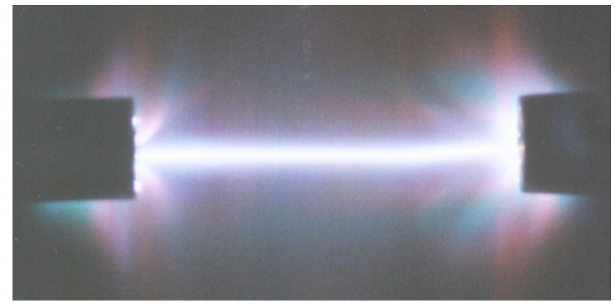
• Typically devices aimed at long-pulse or eventual steady-state operation.

• Pressure ranges from near vacuum to material strength limits (< 1kbar).

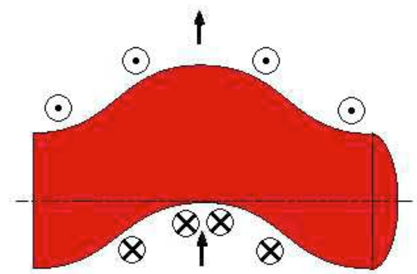


z-pinch

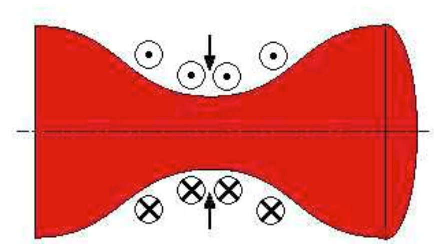




- Axial current, azimuthal magnetic field
- Material electrodes
- Pressure balance
- Notable variations:
    - plasma focus
    - cylindrical liner
    - wire array

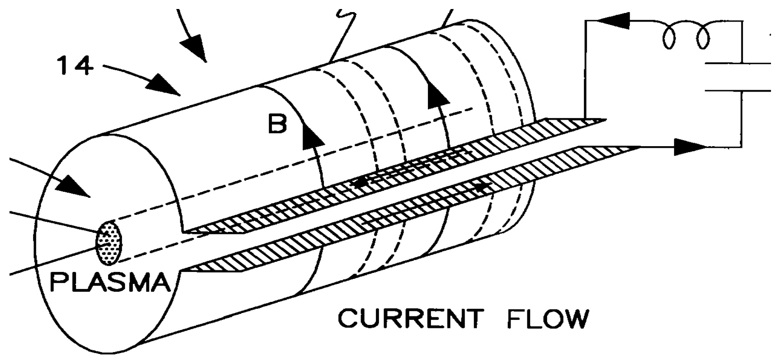


kink instability (m=1)



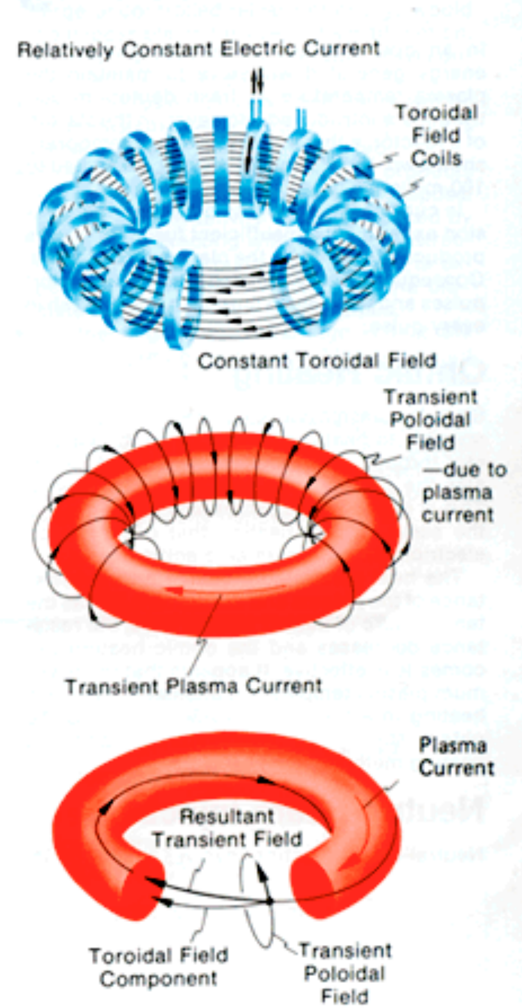
sausage instability (m=0)

θ-pinch

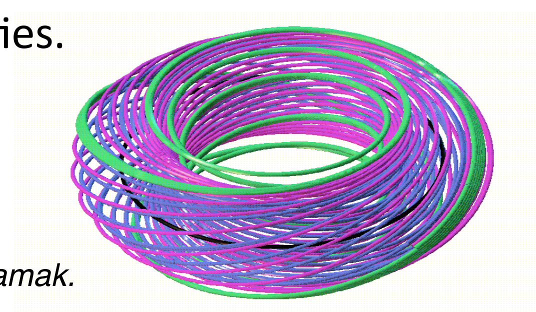


- Azimuthal current, axial magnetic field
- Magnetic coil driven
- Pressure balance
- Notable variations:
  - Screw pinch: combination θ and z pinchs
  - another way to crush a can or liner
  - possible way to form compact toroid

Tokamak

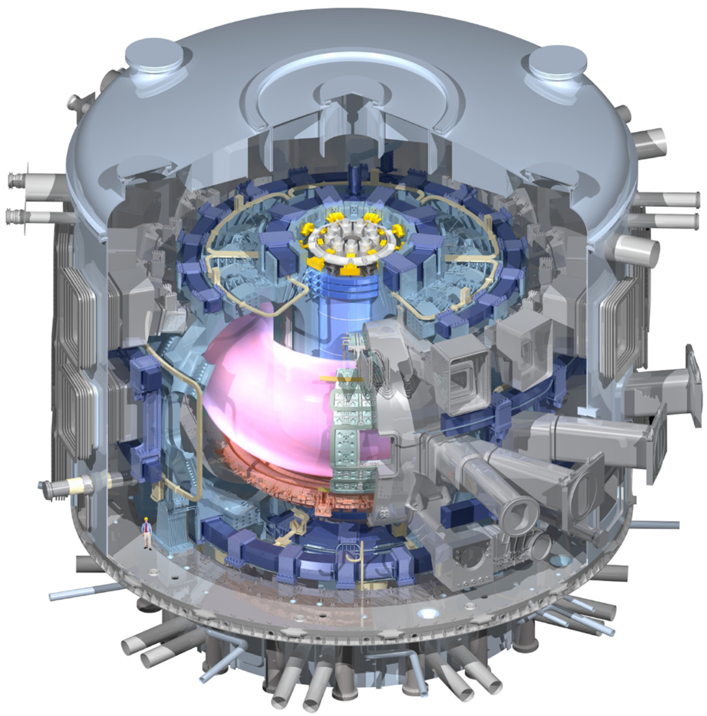


- Russian acronym for “toroidal chamber with magnetic coils”.
- Began in US as “perhapsatron”
- Primarily toroidal (axial) field with smaller poloidal (azimuthal) field.
- Needs current drive (goal of steady-state).
- Needs additional plasma heating.
- Confinement and instabilities.



Composite field in a tokamak.

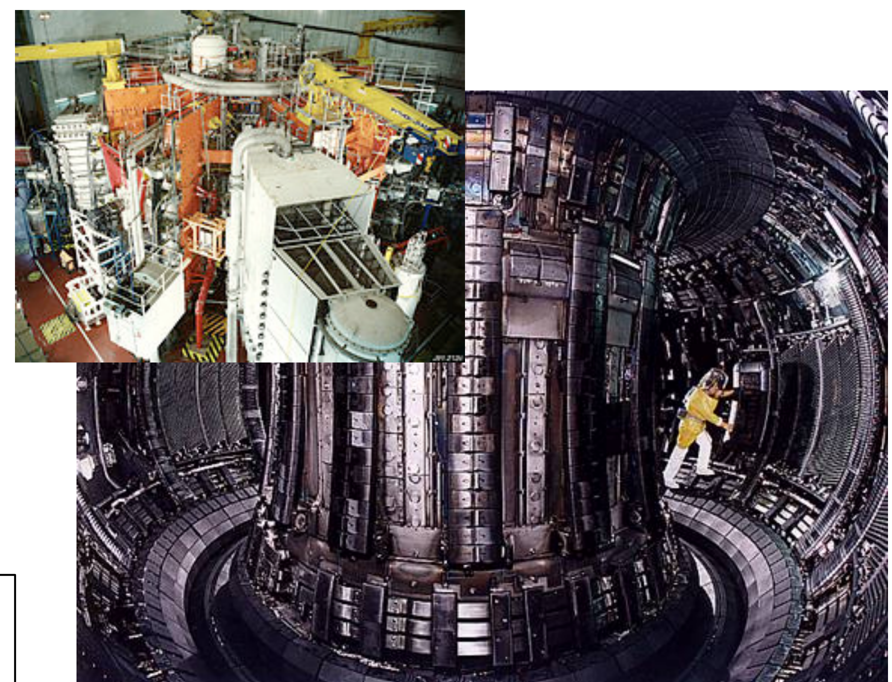
Currently the leading MFE concept.



ITER schematic, Cadarasche France

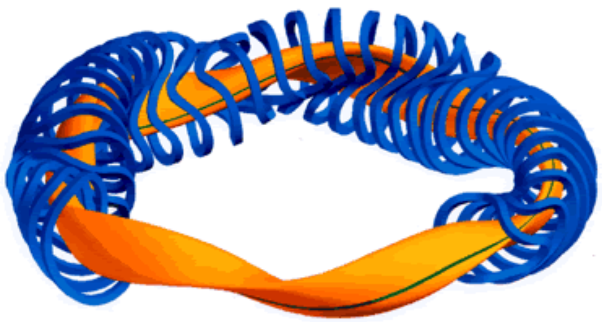
ITER participants: United States, European Union, India, Japan, China, S. Korea, (previously Canada).

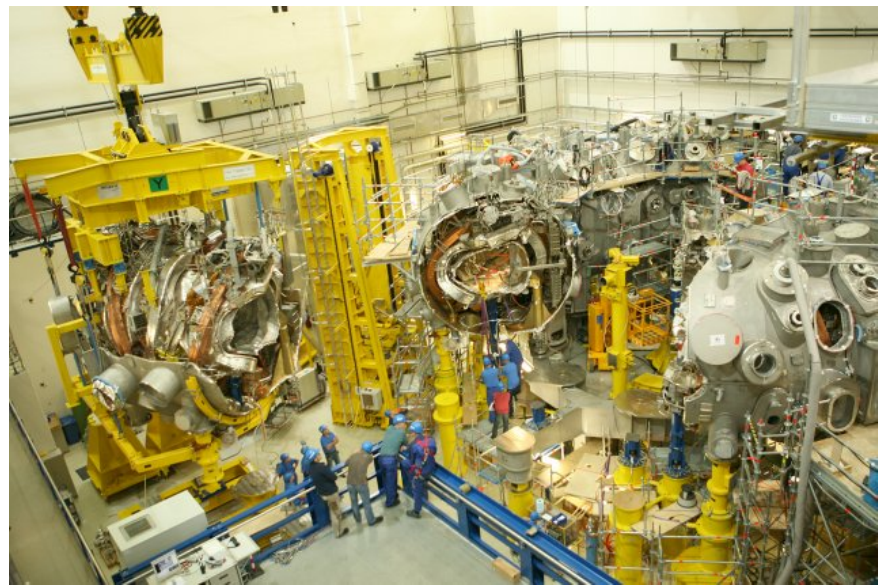
- Currently the leading MFE concept.
- Several large tokamaks around the world.



JET (Joint European Torus), Culham, Oxfordshire, UK.

Stellarator

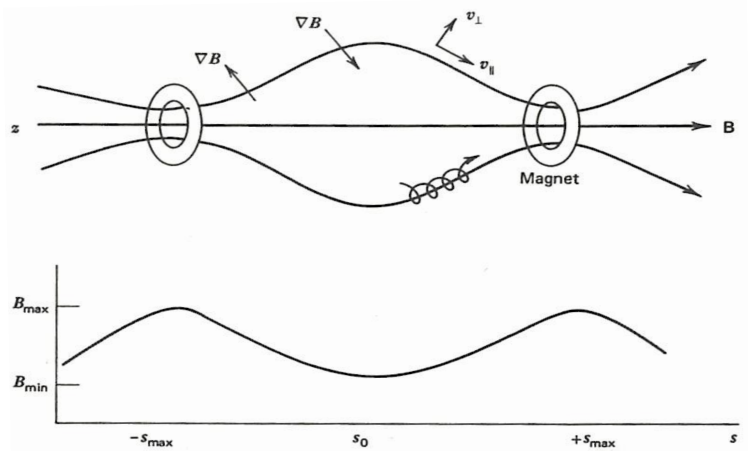




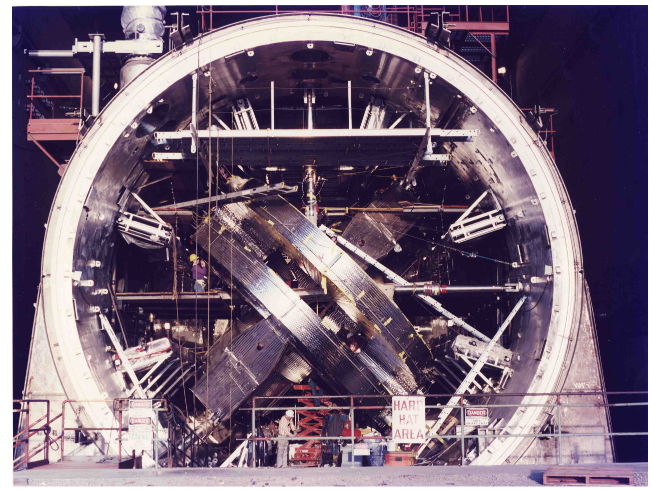
W7X stellarator Greifswald, Germany

- US initial concept (superseded by Tokamak)
- Helical symmetry
- steady state
- complex magnetic coils
- lower energy density than Tokamak
- Research producing continuously more advanced configurations.

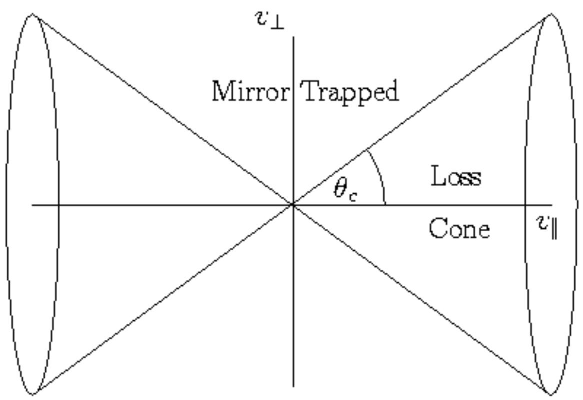
Mirror



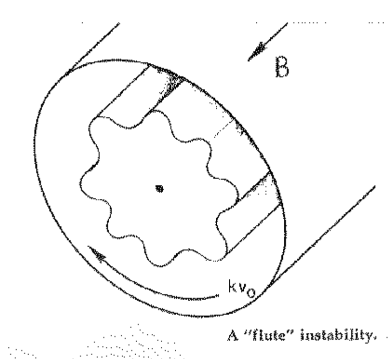
Magnetic geometry of mirror machine.



MFTF (mirror fusion test facility, Livermore CA 1977-1986



Loss cone



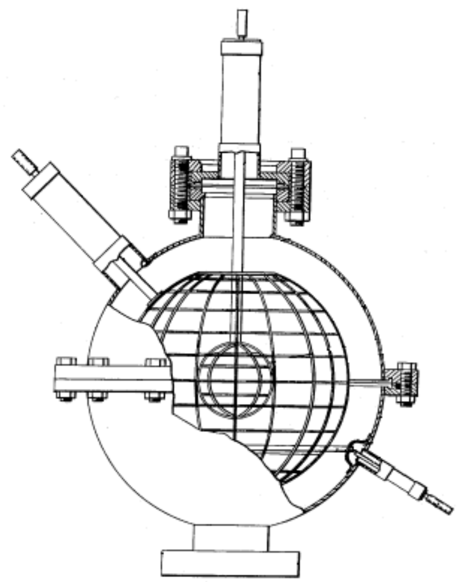
Flute instability

- Axial confinement via adiabatic invariant.
- Radial confinement due to reduced cross field transport.
- Heating via external mechanisms.
- End losses and instabilities an issue.

Others

- Elmo bumpy torus
- Compact tokamak
- Spherical tokamak
- Compact torous: FRC, spheromak
- Reversed field pinch

Inertial Electrostatic Confinement (IEC)



Hirsch–Meeks fusor From US patent 3,530,497

Farnsworth-Hirsch fusor
Polywell
•Space-charge limitation, bremsstralung, collision with electrodes removes energy, adds impurities.

•Beam instabilities quickly thermalize non-maxwellian distributions.
•Ions in tail escape potential well before fusing.

Areal density

Lawson criteria expressed in a different form for inertial confinement fusion.

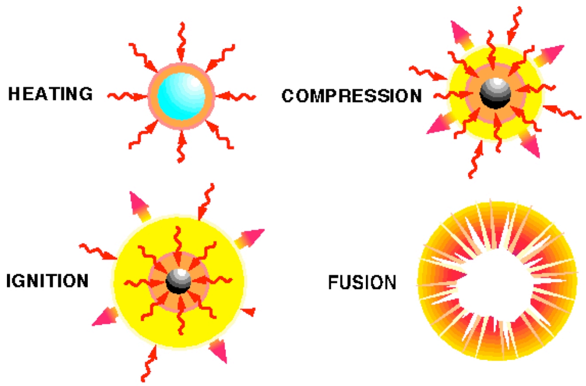
Energy confinement time (τ_E) in MFE systems is difficult to predict.
IFE energy confinement time is on the order of the radius over the sound speed

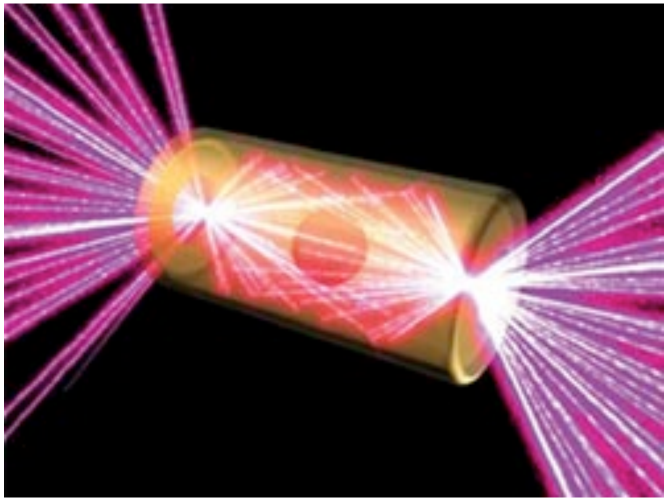
$$\tau_E \approx R/\sqrt{kT/m_i}$$

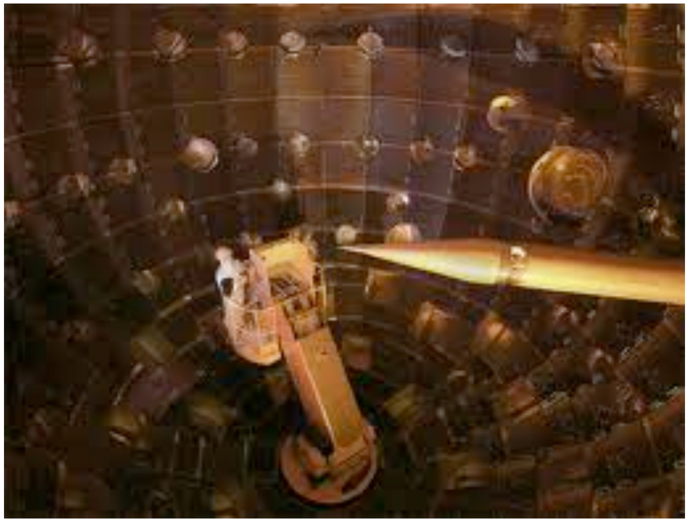
$$n\tau_E \approx nR/\sqrt{kT/m_i}$$

Leads to ρ R requirement for inertial confinement fusion schemes.

Inertial Confinement Fusion (ICF)

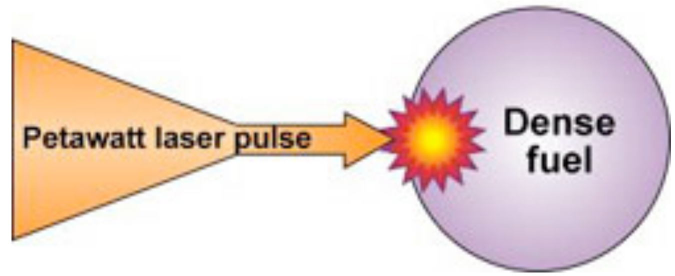






Alternative concepts for ICF

Fast Ignition

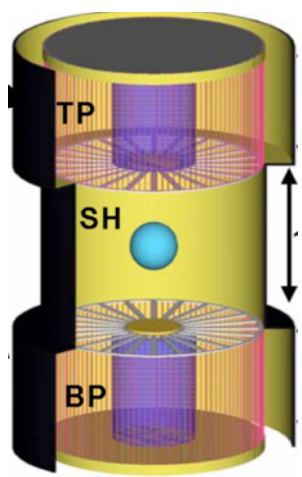


Heavy Ion Fusion

Fission-driven

Ivy Mike – First manmade fusion with gain >1

Z-pinch driven radiation drive



Magneto-inertial fusion (MIF)

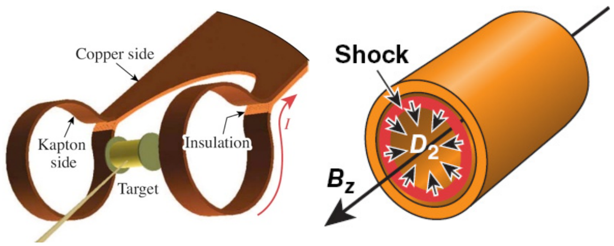
- MIF regime lies between magnetic and inertial confinement fusion approaches.
- 1-100 megabar pressures, multi-megagauss magnetic fields, ns-µs timescales.
- Compared to ICF:
    - Embedded magnetic field improves particle confinement, reduces energy transport, confines α-particles.
    - This reduces required ρR, implosion speed, and convergence for ignition.
    - Enables use of efficient (and inexpensive) pulsed power.
    - This relaxes the needed gain for break-even (e.g. ηG=0.5*10 for pulsed power vs. ηG=0.05*100 for lasers).

National Academy of Science IFE Review documents available at: http://fire.pppl.gov/icf_nas_review_2010.html
Scientific American, May 26, 2011

Wide range of driver/target combinations.

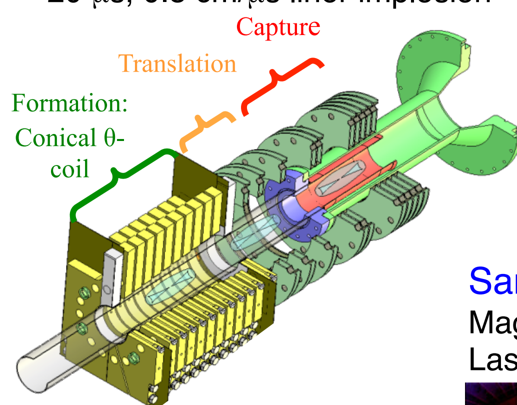
U. Rochester LLE

Magnetic transport reduction in ICF target.



Los Alamos / AFRL

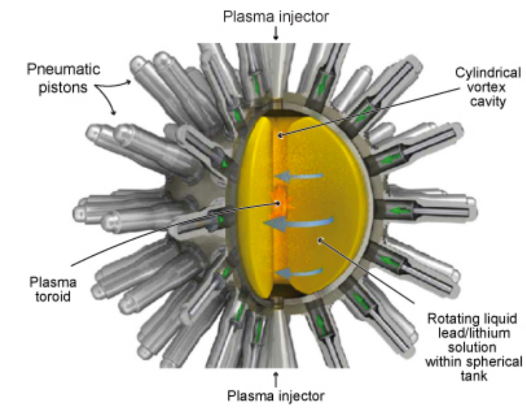
Magnetized Target Fusion (MTF)
FRX-L/FRCHX + Shiva Star
~20 μs, 0.5 cm/μs liner implosion



Taccetti, Intrator, Wurden *et al.*, *Rev. Sci, Instr.* **74**, 4314 (2003)

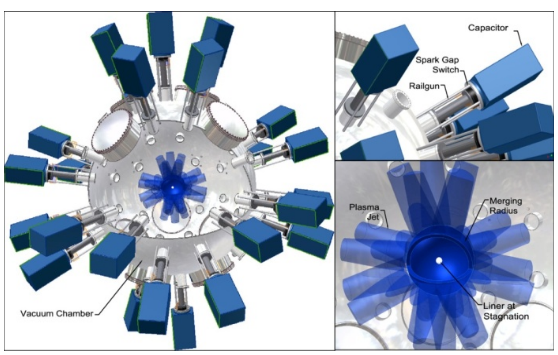
General Fusion

Spheromak merging and pneumatically driven compression with liquid liner/blanket.



Los Alamos / HyperV

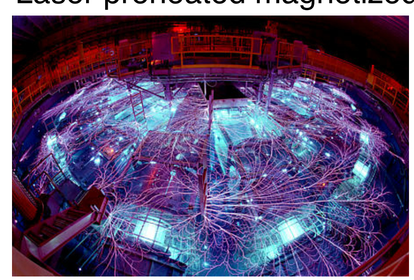
Plasma Liner Experiment (PLX)
Merging plasma jets for remote standoff.

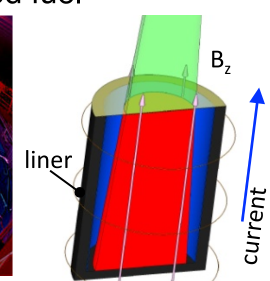


A. G. Lynn, et al, *Rev. Sci. Instr.* 81, 10E115 (2010)

Sandia National Laboratories

Magnetized Liner Inertial Fusion (MagLIF)
Laser preheated magnetized fuel

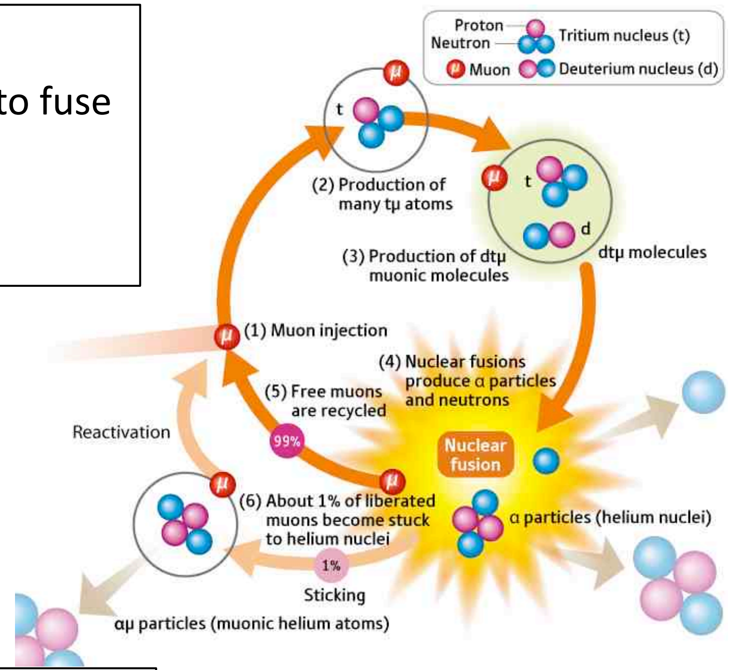




S. A. Slutz, et al., Phys. Plasmas 17, 056303 (2010)

Non-thermonuclear fusion

Muon Catalyzed Fusion
- muon replaces electron, "atoms" come close enough to fuse
- muons catalyze many reactions
- "alpha sticking problem"
- difficult to produce muons



Cold Fusion?
- Pons & Fleischmann 1989
- unknown process
- lack of evidence

Carnot Efficiency:

$$\eta \leq 1 - \frac{T_C}{T_H}$$

Pycnonuclear fusion
- Highly coupled crystalline strucure
- Lattice vibrations can exceed thermal energy
- high density, relatively cold
- responsible for energy release in carbon core of white dwarfs

Beam-target
- Inefficiencies make concept unsuitable for energy production.

Other engineering issues

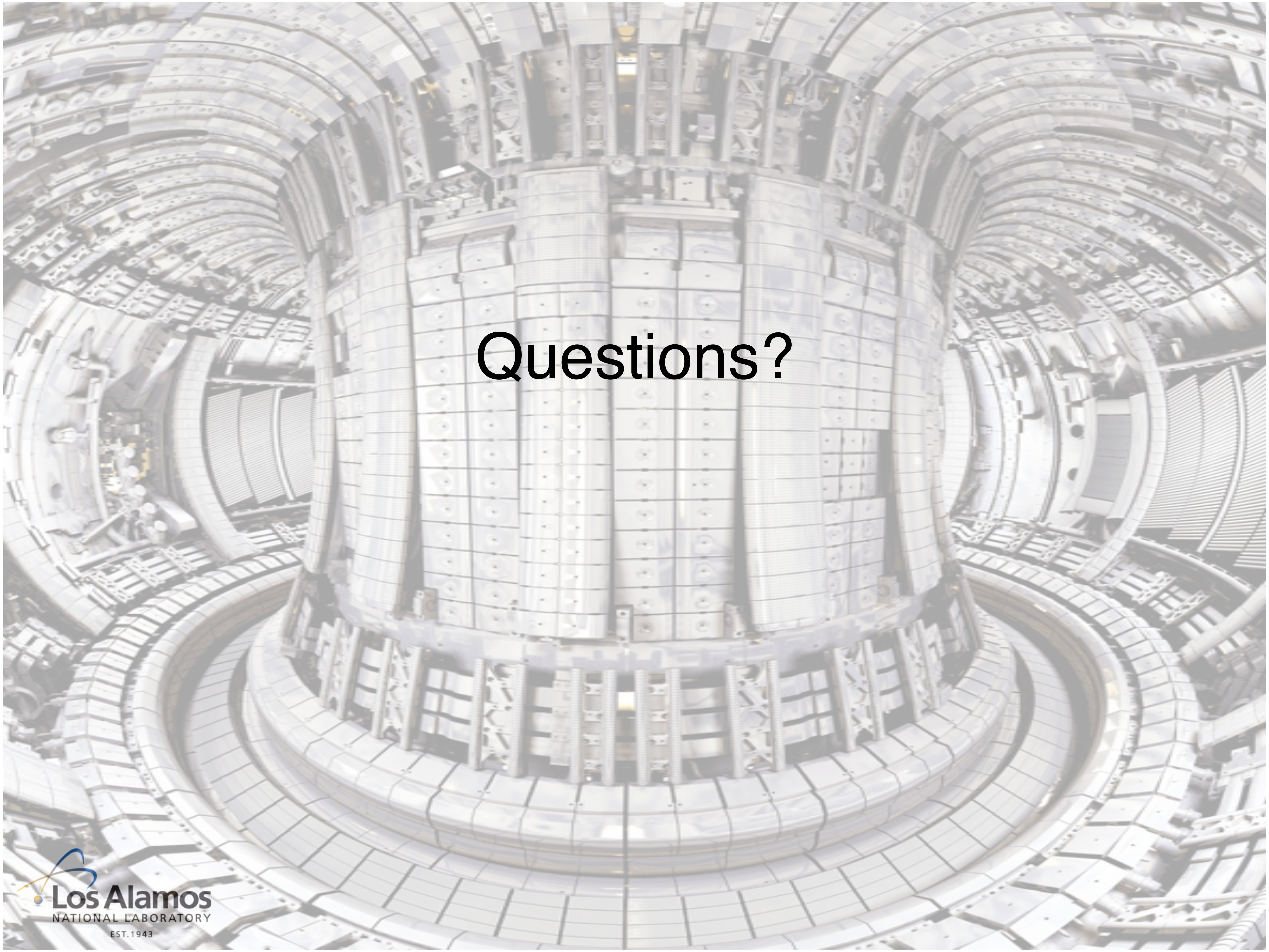
- Energy extraction – "Blanket"
- Tritium handling
- Tritium breeding
- Cooling
- Interaction of radiation with matter
- Superconducting magnets?
- Plasma-wall interactions
- Economic power generation.

Fission-Fusion Hybrid

- No need for Q > 1
- Waste transmutation
- Subcritical reactor

Useful References

- “Fusion” by Weston Stacey
- “The Physics of Inertial Fusion” by S. Atzeni and J. Meyer-ter-vehn



Questions?

A Gentle Introduction to Plasma Kinetic Simulation

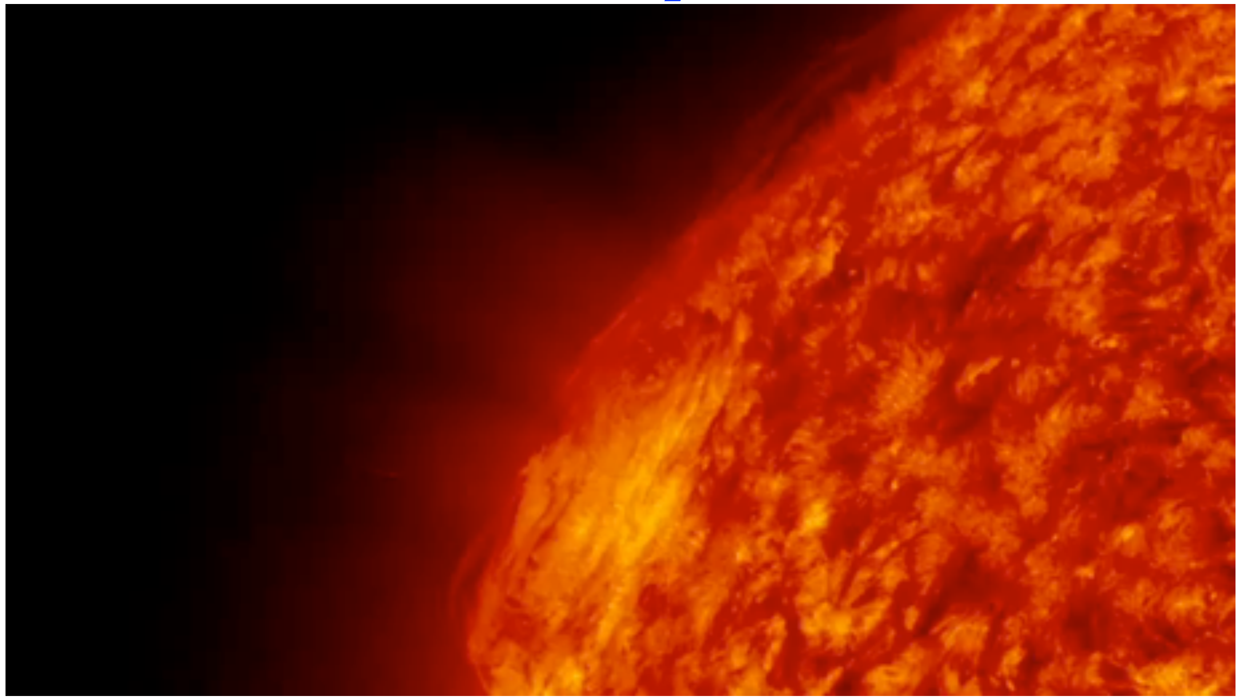
Bill Daughton

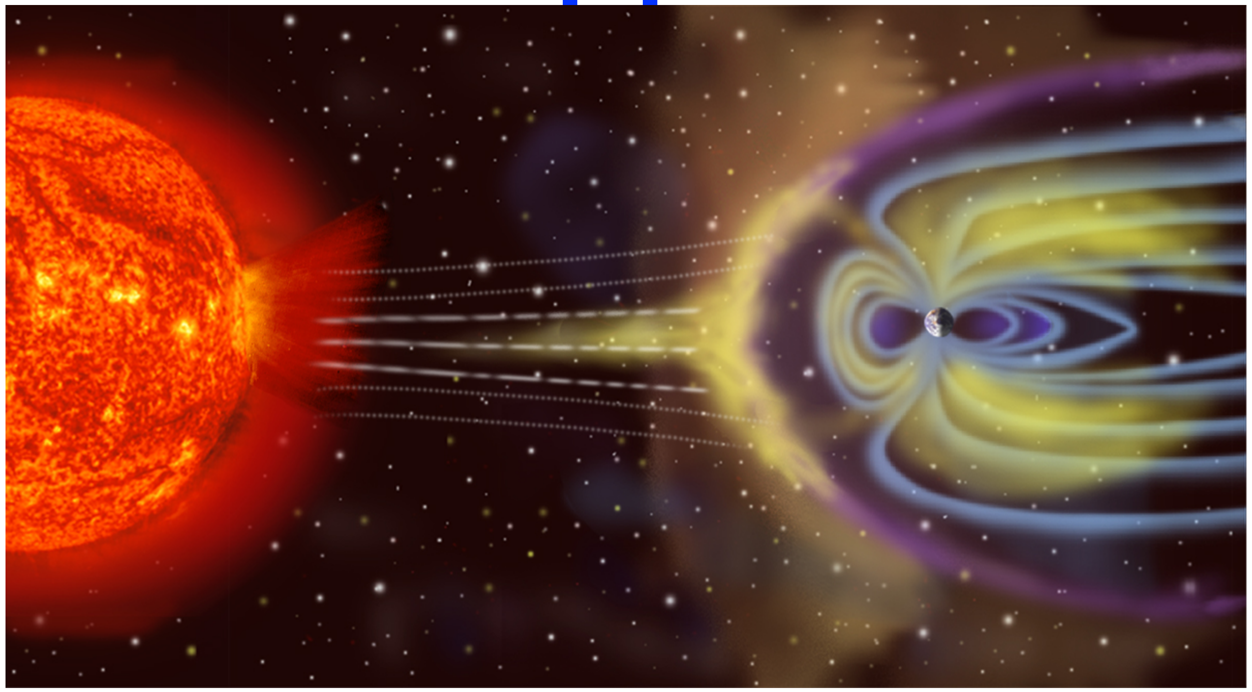
Plasma Theory & Applications, XCP-6

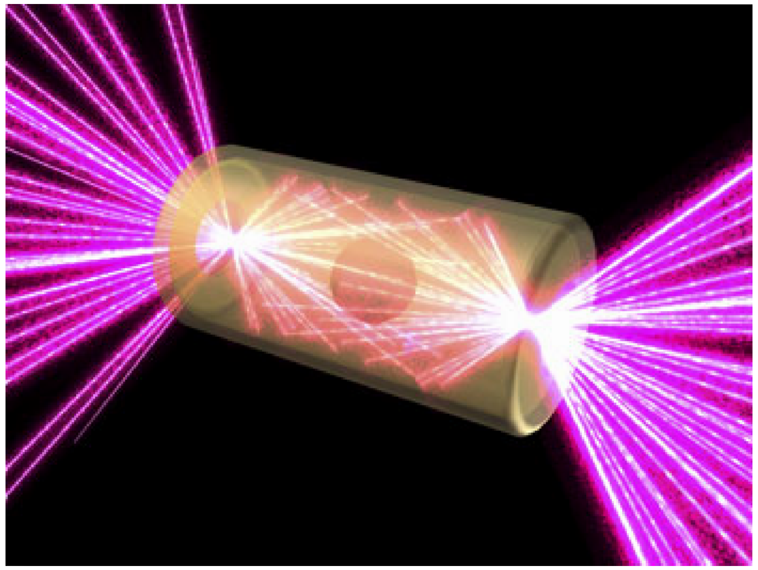
July 11, 2012
CNLS - Student Seminar

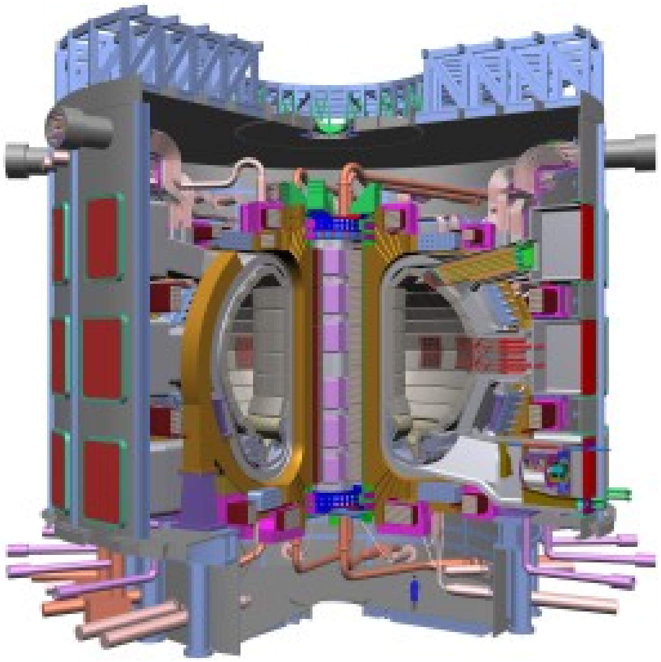
Los Alamos
NATIONAL LABORATORY
EST. 1943

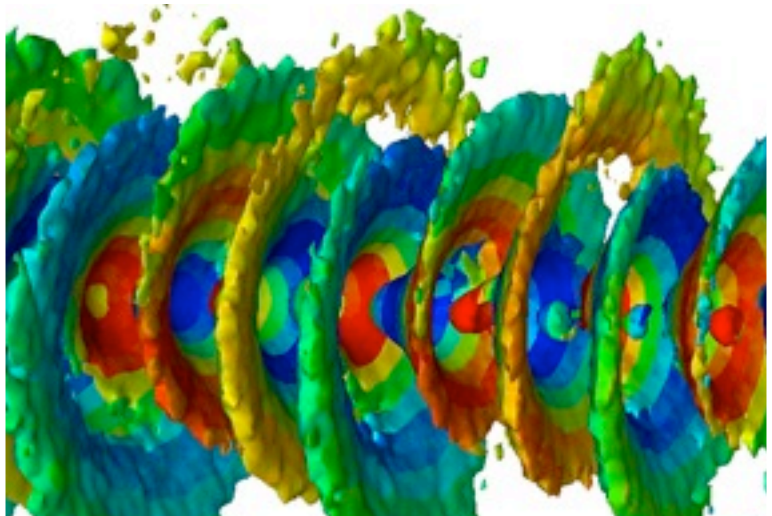
Plasma Physics is Rich with Applications

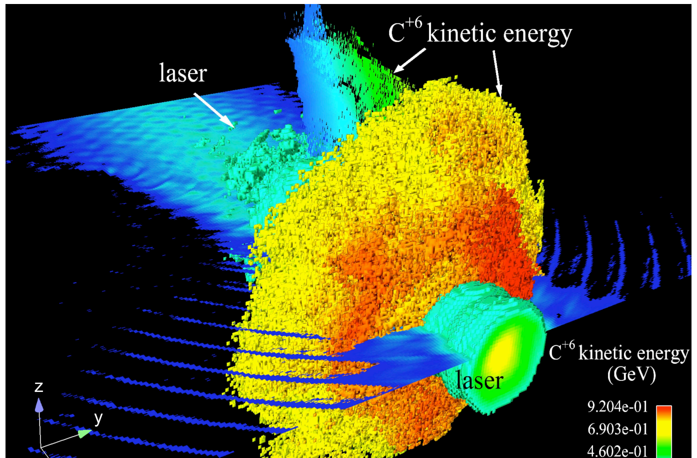


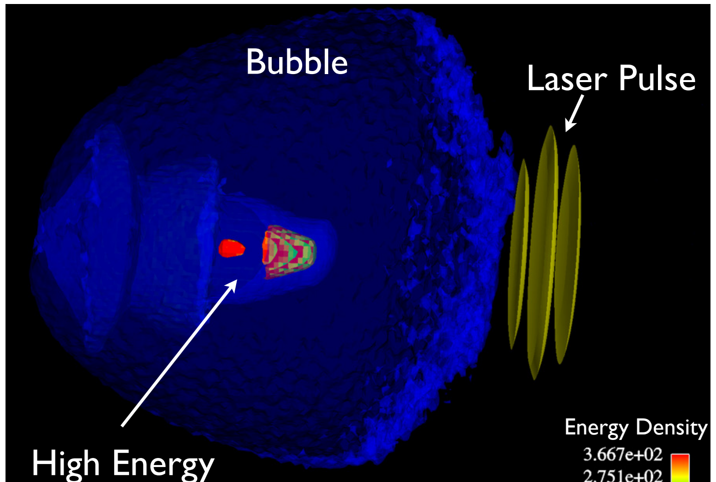












How do we describe these various plasmas?

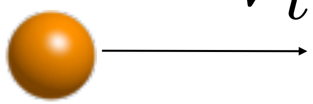
Kinetic ⟺ Fluid

Focus of this talk:

1. Theoretical framework for plasmas
2. Describe kinetic PIC method
3. Give some examples

Start with something familiar and consider air in this room

1. How fast do particles move?



$$V_{therm} = \left(\frac{k_B T}{m}\right)^{1/2}$$

2. How far does it travel before colliding with another atom?

$$L = \frac{1}{n\sigma}$$

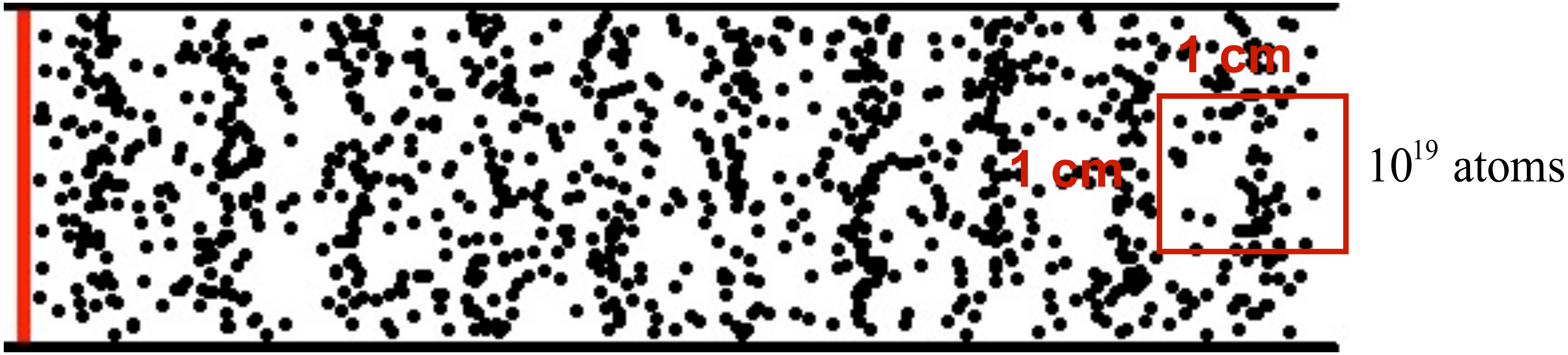
Density — Cross-section

$$\sigma \approx \pi R^2$$

For air in this room:

$$n \approx 2 \times 10^{19} \ \frac{\text{atoms}}{\text{cm}^3}$$

$$L \approx 0.002 \ \text{mm}$$



Conditions for sound waves to occur

Mean free path $L \ll \lambda$ **Wavelength**

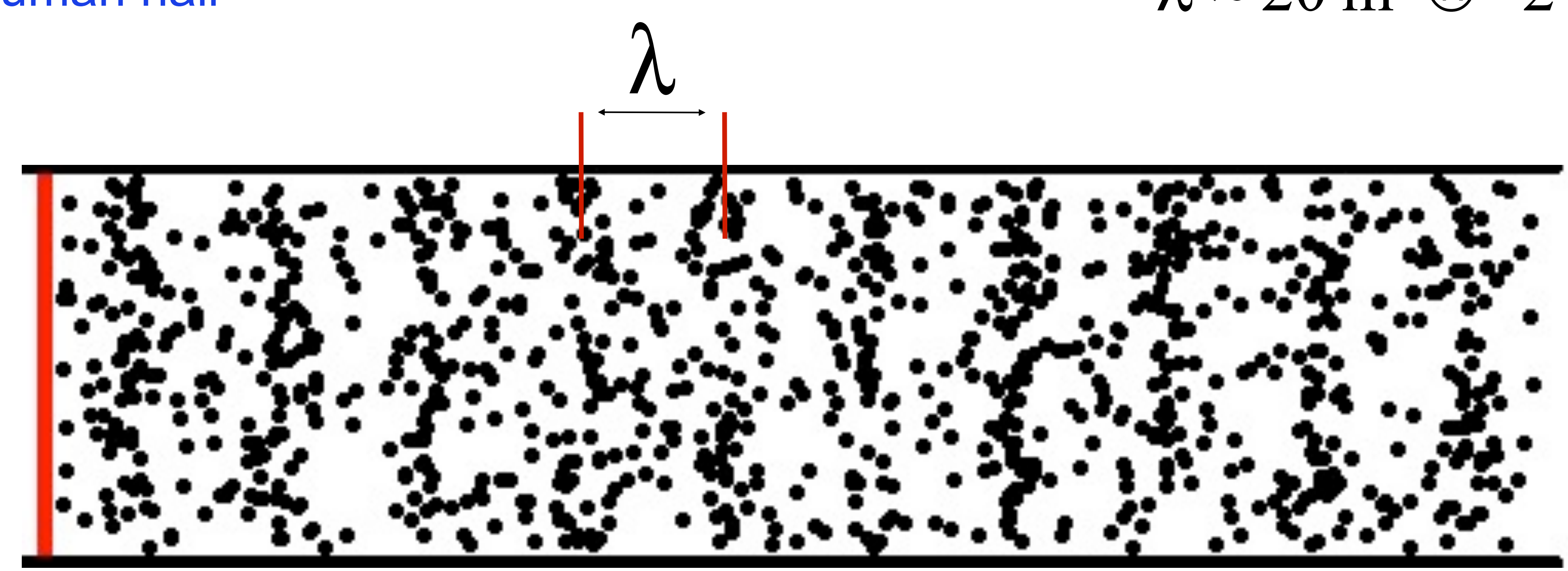
$L \approx 2\times10^{-4}\ \text{cm} = 0.002\ \text{mm}$

Width of Human hair 0.1 mm

Range of human hearing

$\nu \approx 20\ \text{Hz}$ ® $20{,}000\ \text{Hz}$

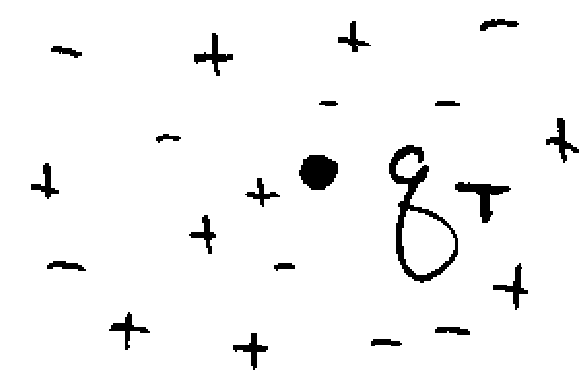
$\lambda \approx 20\ \text{m}$ ® $2\ \text{cm}$



What equations describe a plasmas?

First, we need to consider how charged particles interact

In a plasma - Coulomb interactions are screened



$$\nabla\cdot\vec{E} = -\nabla^2\phi = 4\pi e(n_i - n_e) + 4\pi q_T\,\delta(\vec{r})$$

$$n_e = n_o \exp\left(\frac{e\phi}{T_e}\right) \approx n_o\left[1 + \frac{e\phi}{T_e} + \dots\right]$$

$$n_i = n_o \exp\left(-\frac{e\phi}{T_i}\right) \approx n_o\left[1 - \frac{e\phi}{T_i} + \dots\right]$$

$$\phi = \frac{q_T}{r}\exp\left(-\frac{r}{\lambda_D}\right)$$

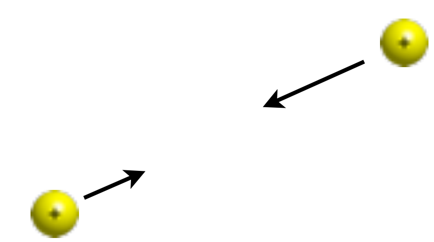
$$\frac{1}{\lambda_D^2} = 4\pi e^2 n_o\left(\frac{1}{T_e} + \frac{1}{T_i}\right)$$

Debye Length

Huge number of particles within Debye length

$$\Lambda \equiv n_o \lambda_D^3 \gg 1$$ Plasma Parameter

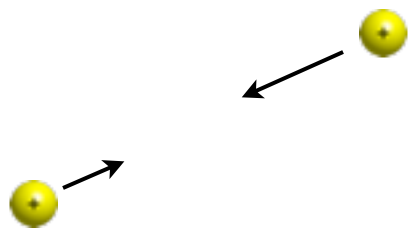
$$\Gamma = \frac{e\phi}{T_e} = \frac{e^2}{r\,KT} \propto \frac{1}{\Lambda_s^{2/3}}$$



$\Gamma \ll 1$ ⤳ weakly coupled plasma with many particles in a Debye cube

$\Gamma \gtrsim 1$ ⤳ strongly coupled plasma or sometimes called "Coulomb liquid" or "dense plasma"

Collisions in Plasmas?



Rutherford Cross section

$$\sigma \propto \frac{1}{v^4}$$

Collision Frequency

$$\nu = \langle \sigma v \rangle \propto \frac{n}{T^{3/2}}$$

Thermal Velocity

$$C_e = (2T/m)^{1/2}$$

Application	n_e (cm^{-3})	T (eV)	λ_{De} (cm)	$n_e\lambda_{De}^3$	ω_{pe}	ν_e	C_e (cm/sec)
Earth's Magnetotail	1	800	2.1×10^4	9.3×10^{12}	5.6×10^4	3.9×10^{-9}	1.2×10^9
Solar Corona	10^7	100	2.3	1.3×10^8	1.8×10^8	0.60	4.2×10^8
Tokamak	10^{14}	10^4	7.4×10^{-3}	4.1×10^7	5.6×10^{11}	5×10^3	4.2×10^9
Laser Plasma	10^{20}	500	1.7×10^{-6}	460	5.6×10^{14}	1.9×10^{11}	9.4×10^8

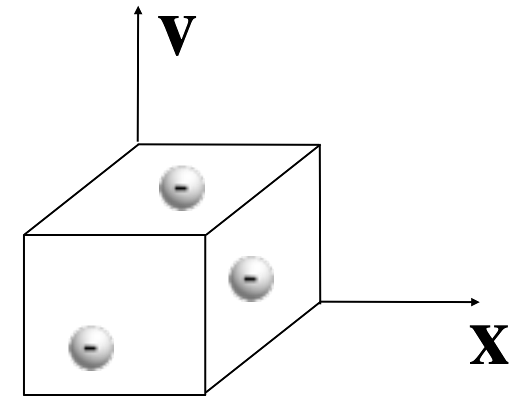
What equations describe a plasma?

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \bullet \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \bullet \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

Vlasov

$\varepsilon = 0$

$$f_s(\mathbf{x}, \mathbf{v}, t) \longrightarrow \frac{\text{Number of particles}}{\text{Unit volume of phase space}}$$



Small Parameter $\longrightarrow \quad \varepsilon = \frac{1}{n\lambda_D^3} \sim 10^{-6} \to 10^{-12}$

Maxwell's Equations

$$\rho = \sum_s q_s \int f_s \, d\mathbf{v}$$

$$\mathbf{J} = \sum_s q_s \int \mathbf{v} \, f_s \, d\mathbf{v}$$

$$\nabla \bullet \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \bullet \mathbf{E} = 4\pi\rho \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Vlasov-Maxwell Theory

Vlasov

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \bullet \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \bullet \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

Maxwell

$$\nabla \bullet \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \bullet \mathbf{E} = 4\pi\rho \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

- **Coupled by first 2 moments** $\rho = \sum_s q_s \int f_s \, d\mathbf{v}$ $\qquad \mathbf{J} = \sum_s q_s \int \mathbf{v} \, f_s \, d\mathbf{v}$

- **Complete description of collisionless plasma**

- **Very difficult to solve - 6D phase space!**

- **Fluid description is much easier**

Fluid Description of Plasma

Density ⟶ $n_s = \int f_s \, d\mathbf{v}$

Velocity ⟶ $\mathbf{U}_s = \int \mathbf{v} \, f_s \, d\mathbf{v}$

Pressure ⟶ $\mathbf{P}_s = m_s \int (\mathbf{v} - \mathbf{U}_s)(\mathbf{v} - \mathbf{U}_s) f_s \, d\mathbf{v}$

Take velocity space moments of the Vlasov Equation:

Mass conservation ⟶ $\frac{\partial n_s}{\partial t} + \nabla \bullet (n_s \mathbf{U}_s) = 0$

Momentum conservation ⟶ $m_s n_s \frac{d\mathbf{U}_s}{dt} = -\nabla \bullet \mathbf{P}_s + q_s n_s \left(\mathbf{E} + \frac{\mathbf{U}_s \times \mathbf{B}}{c} \right)$

Closure Problem - Each equation contains higher order moment!

MHD Model

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{U}) = 0$$ → **Mass conservation**

$$\rho \frac{d\mathbf{U}}{dt} = -\nabla \mathbf{P} + \frac{\mathbf{J} \times \mathbf{B}}{c}$$ → **Momentum conservation**

$$\mathbf{E} + \frac{\mathbf{U} \times \mathbf{B}}{c} = \eta \mathbf{J}$$ → **Ohm's Law**

$$\frac{d}{dt}\left(\frac{P}{\rho^{\gamma}}\right) = 0$$ → **Adiabatic equation of state**

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$ → **Ampere's Law**

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$ → **Faraday's Law**

$$\nabla \bullet \mathbf{B} = 0$$

Ideal MHD → $\eta = 0$ → $\mathbf{U}_{\perp} = c \frac{\mathbf{E} \times \mathbf{B}}{B^2}$

What problems need kinetic treatment?

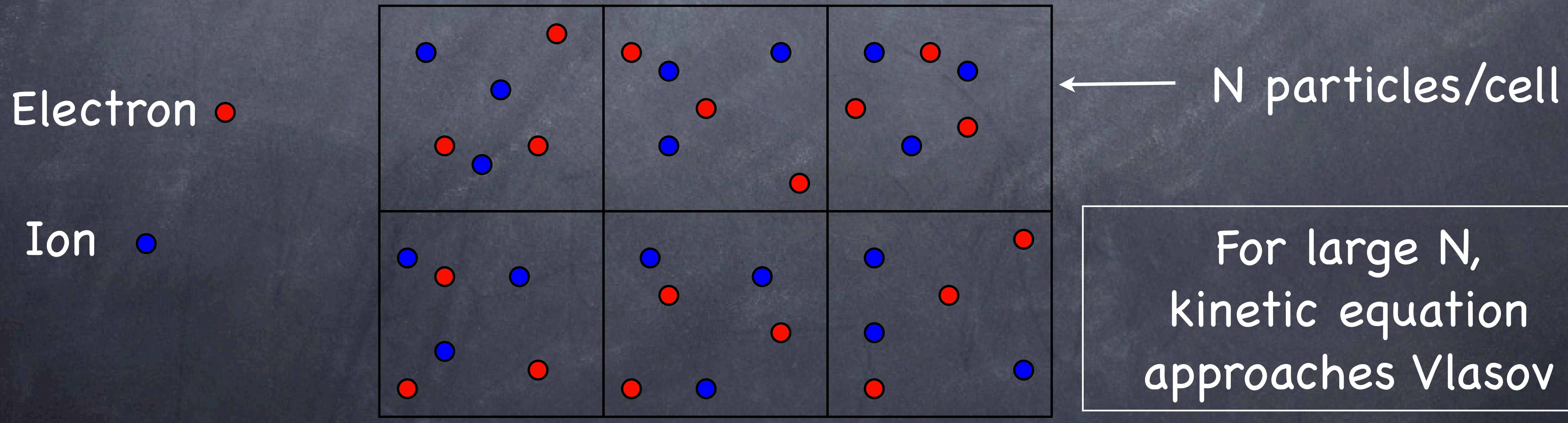
1. kinetic scales are important?
2. Resonant interactions ?
3. Don't have a reasonable fluid closure?
4. Need to predict particle acceleration?

How to solve kinetic equation?

1. Directly discretize kinetic equation
2. Monte-Carlo approach - i.e. Particle-in-cell

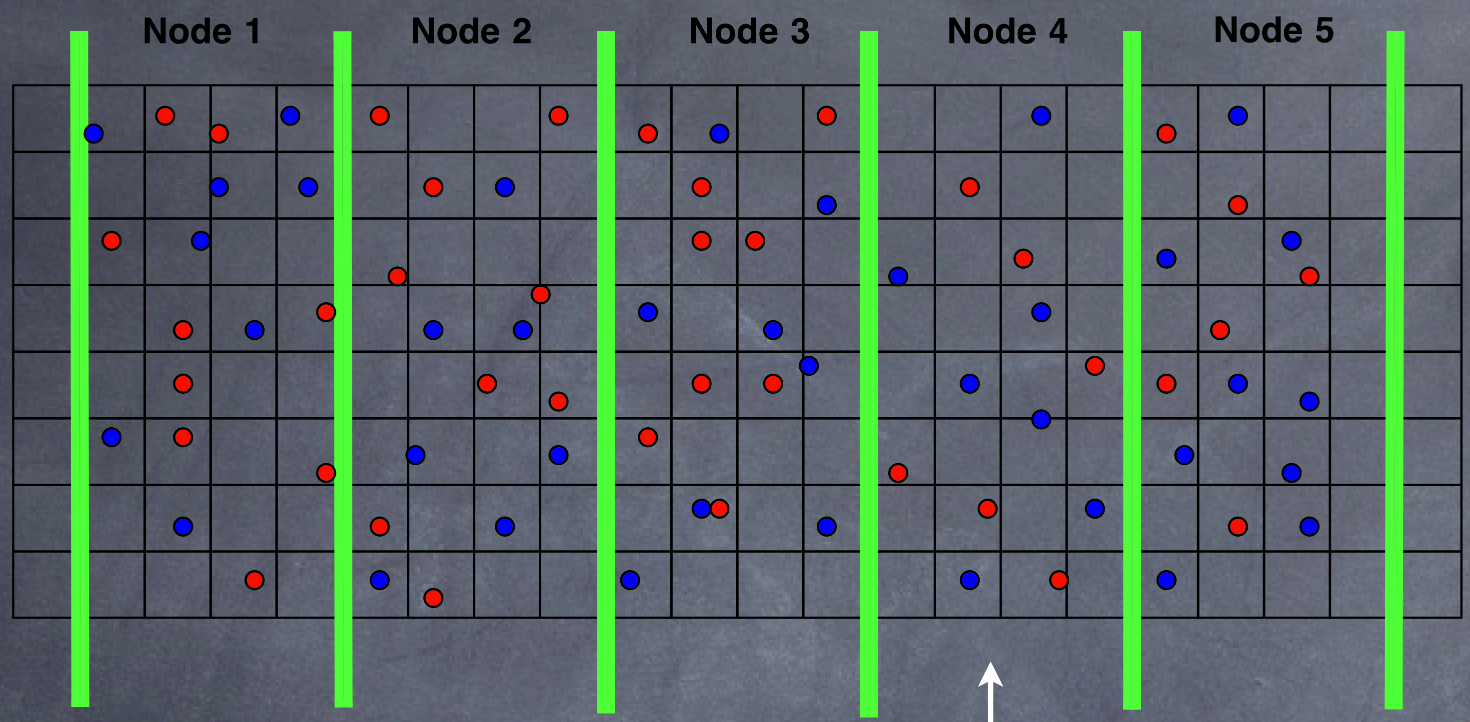
PIC = Particle-in-cell

- Introduce "super-particles" - Lagrangian tracers
- Create spatial grid (cells)
- Interpolate position and velocity of particle onto grid $\rho \quad \vec{J}$
- Compute resulting E and B fields
- Push particles using these self-consistent fields
- Evolution of this system obeys a kinetic equation



For large N, kinetic equation approaches Vlasov

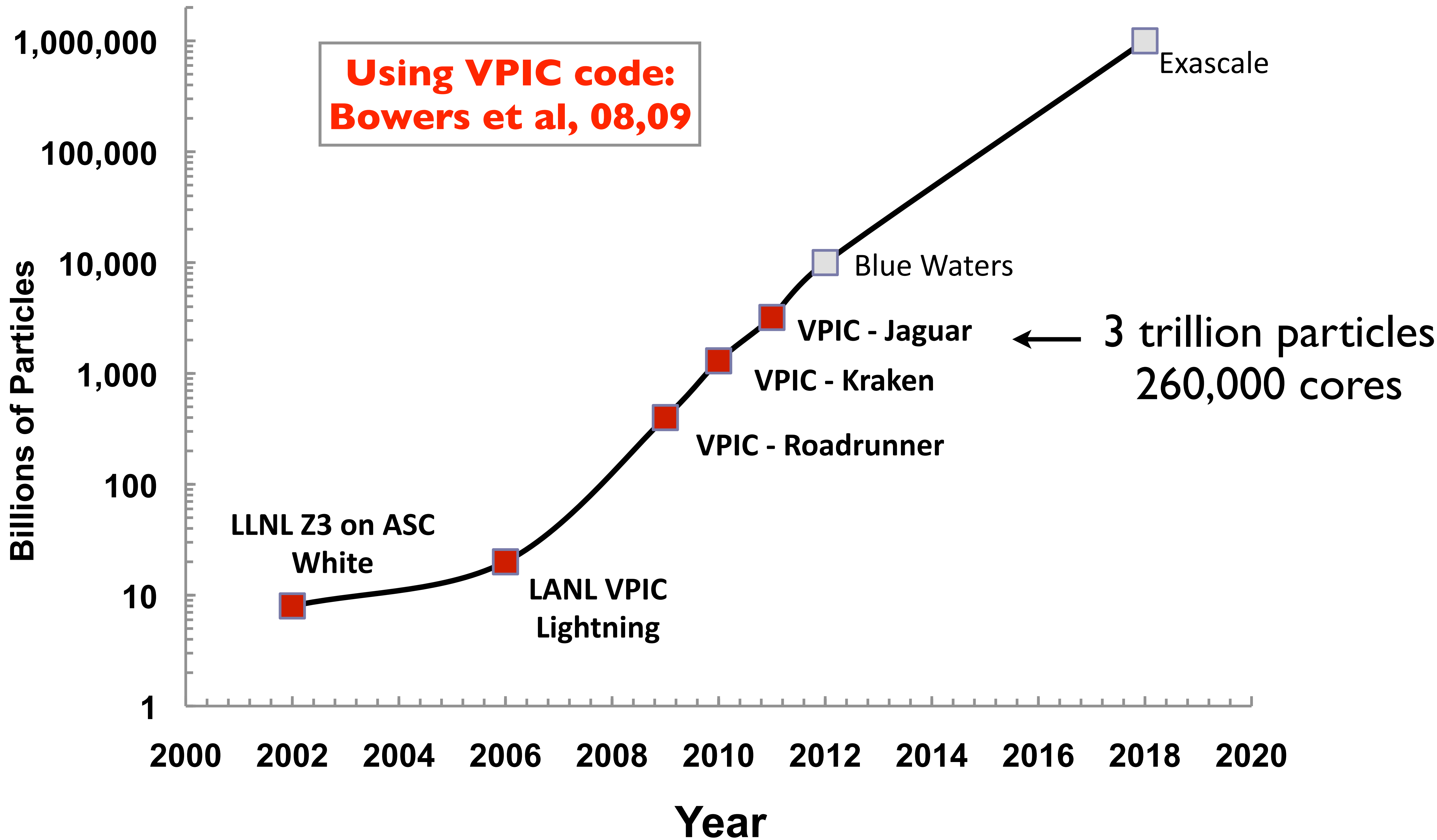
Easy to make PIC algorithm parallel



64 processor
Linux Cluster
in 209 VAN

33 - SMP Nodes
2 Opteron CPU's in each
5 GB RAM - 165 GB Total
4 TB Raid Storage
Gigabit Ethernet

Exponential increase in computing is permitting ever larger kinetic simulations



Even so, we usually can’t match all experimental parameters

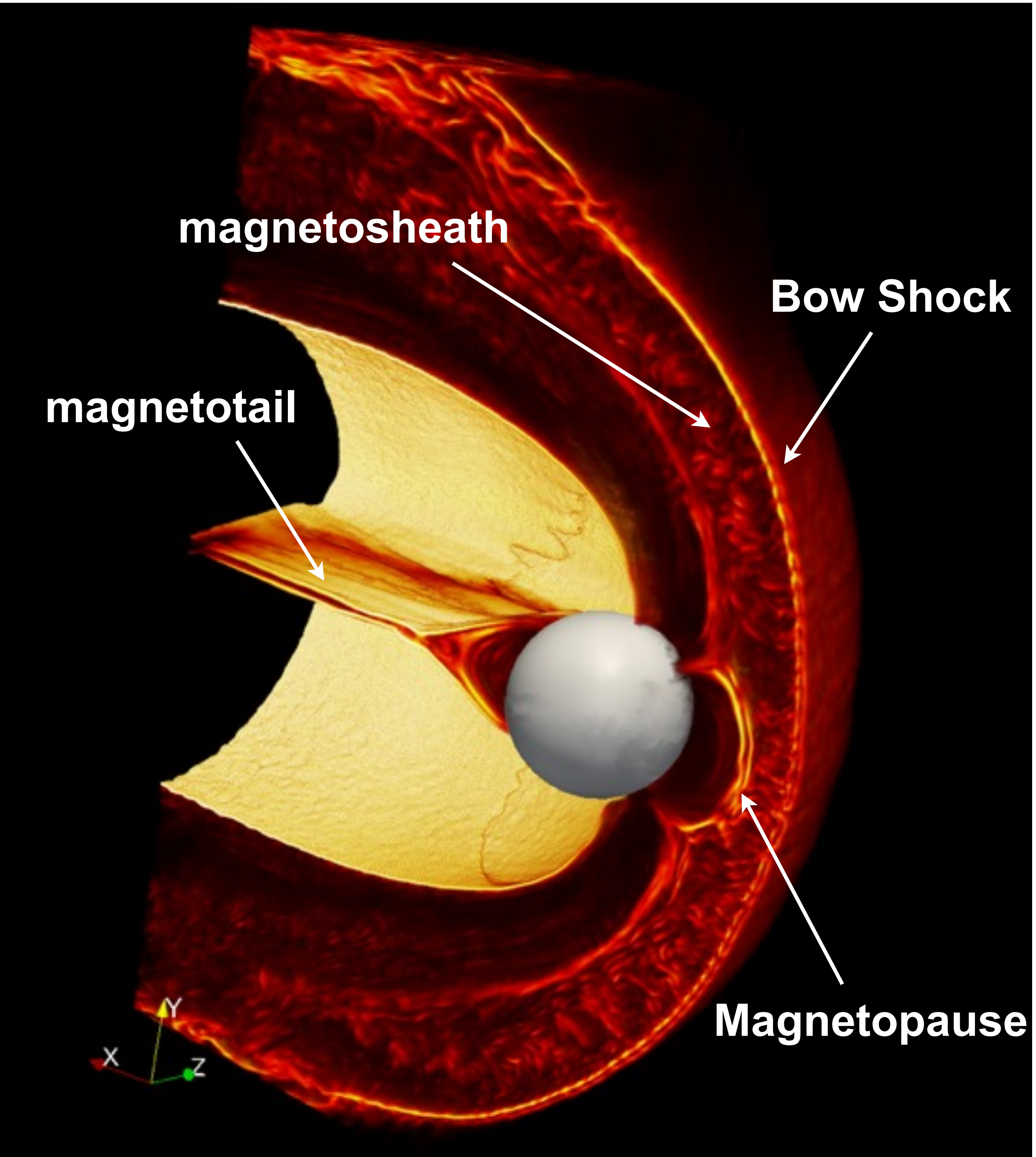
Depending on the problem - we often choose artificial values for

1. Ion to electron mass ratio
2. Plasma temperature
3. Other dimensionless ratios

Sample Problem #1

Magnetic Reconnection

At the petascale - global magnetospheric simulations are now feasible with hybrid codes



Hybrid $(2560)^3$ cells ~ 10^{12} ions

Hybrid offers good description of:

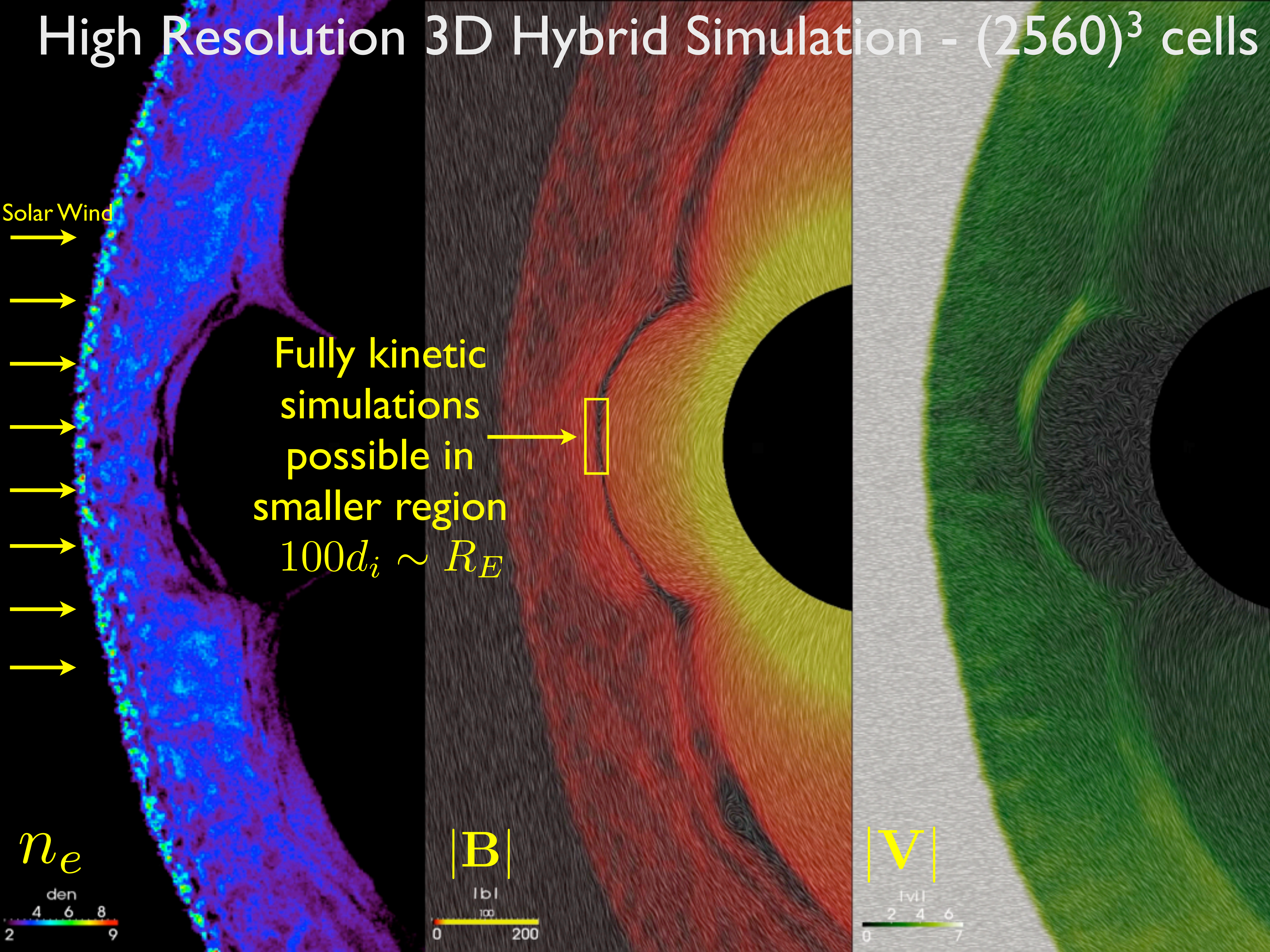
- Collisionless shocks
- Ion kinetic & FLR effects
- Temperature anisotropy instabilities
- Ion-scale boundary layers

Electrons are massless fluid - missing collisionless physics to break frozen-in constraint

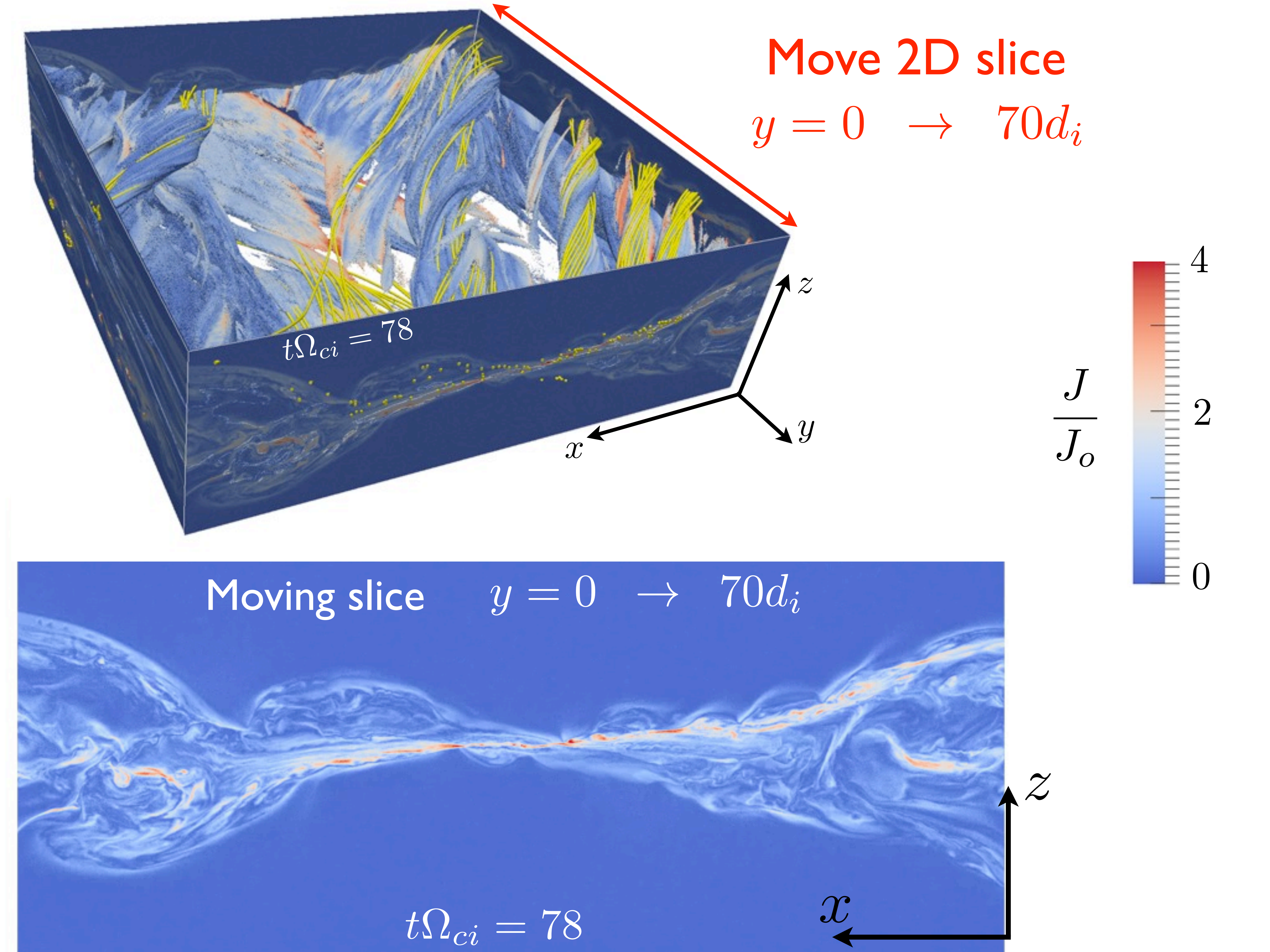
Influence of electron physics is the main science goal of NASA's upcoming MMS mission

MMS will launch in 2014 - orbits will be optimized for the magnetopause during first 1.5 years of operation

See - Moore, Burch, Daughton, et al, 2012



High Resolution 3D Hybrid Simulation - $(2560)^3$ cells
Solar Wind
Fully kinetic simulations possible in smaller region
$100 d_i \sim R_E$
n_e
den
4 6 8
2 9
$|\mathbf{B}|$
|b|
100
0 200
$|\mathbf{V}|$
|v|
2 4 6
0 7

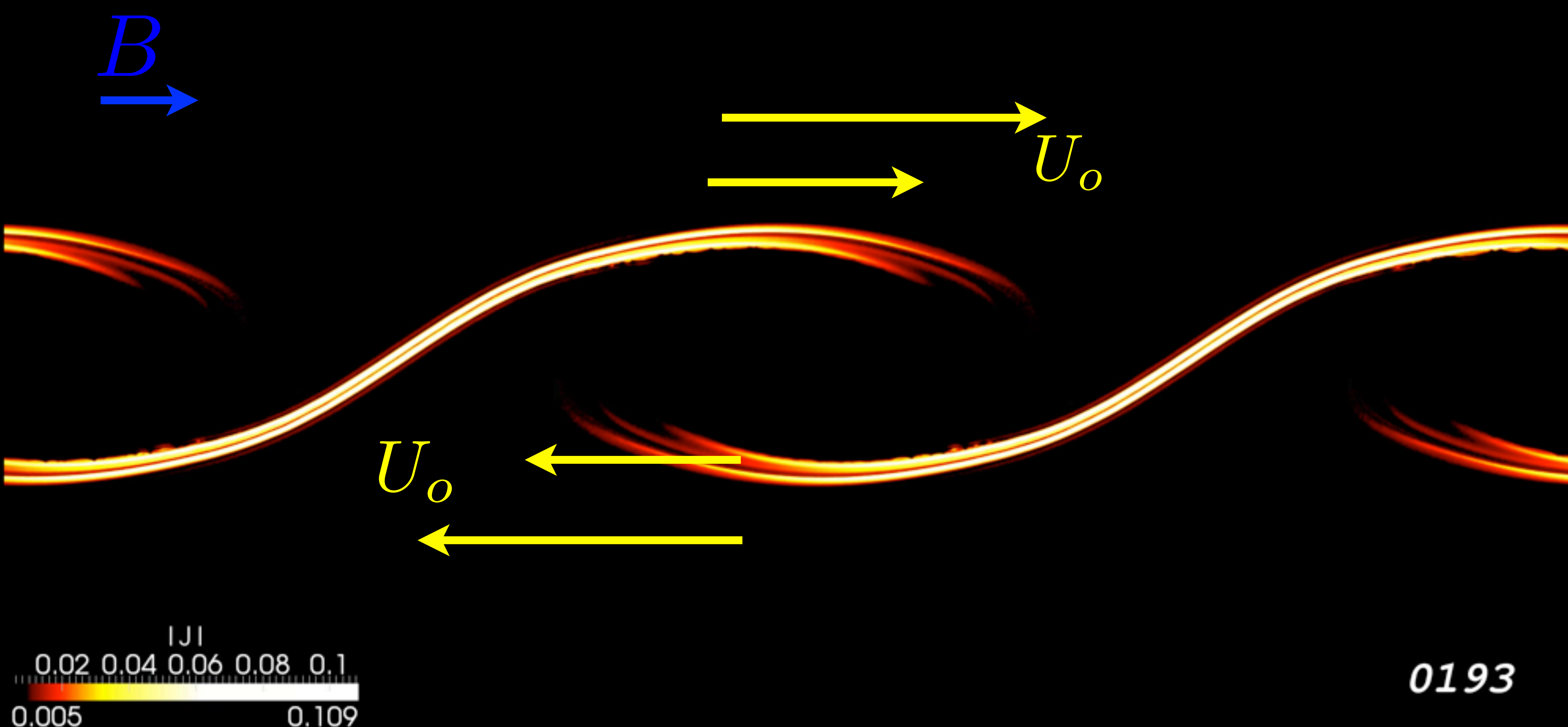


Move 2D slice
$y = 0 \rightarrow 70d_i$
$t\Omega_{ci} = 78$
z
x
y
$\frac{J}{J_o}$
4
2
0
Moving slice $y = 0 \rightarrow 70d_i$
z
x
$t\Omega_{ci} = 78$

Sample Problem #2

Flow Shear Turbulence

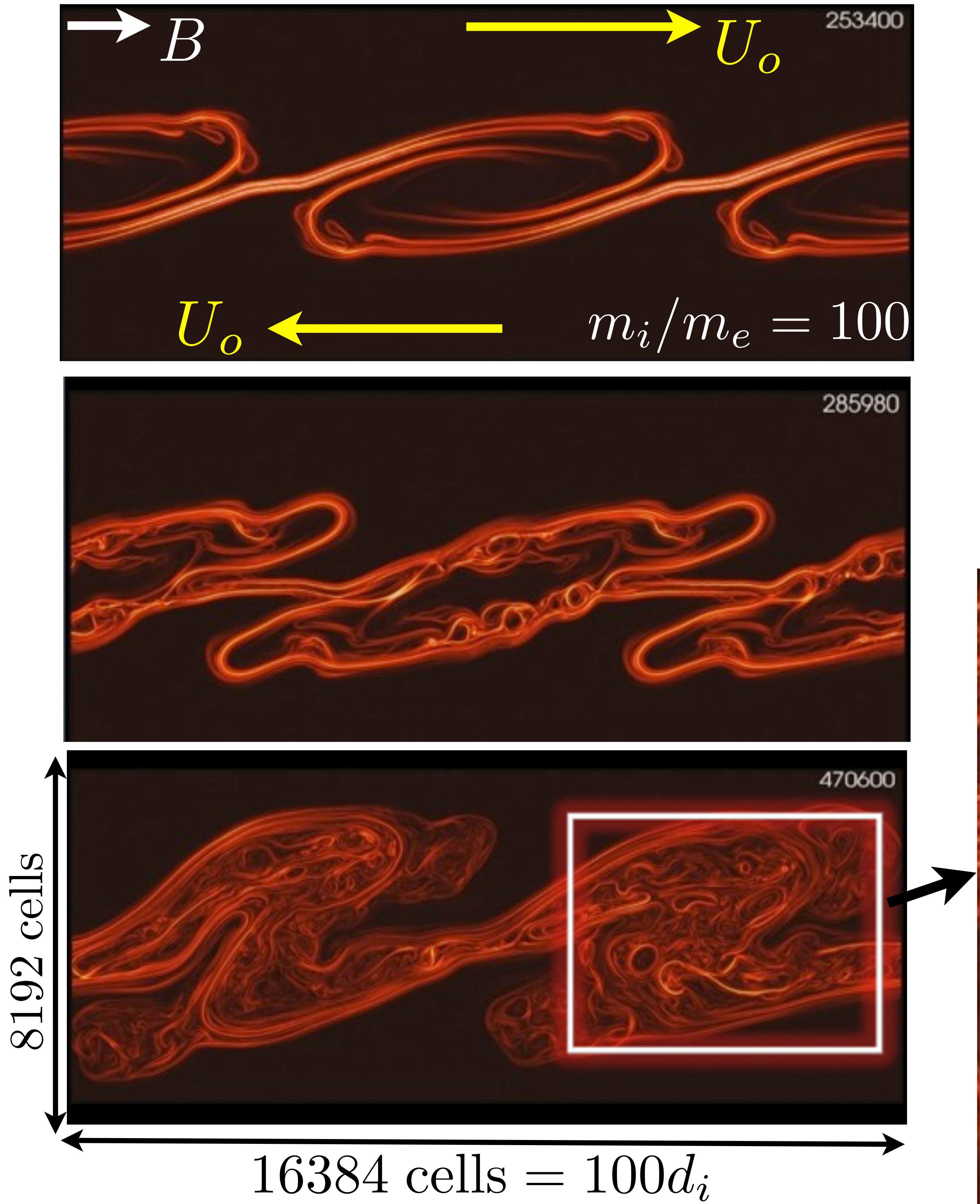
MHD scale vortices generate current sheets, flux ropes, reconnection & turbulence



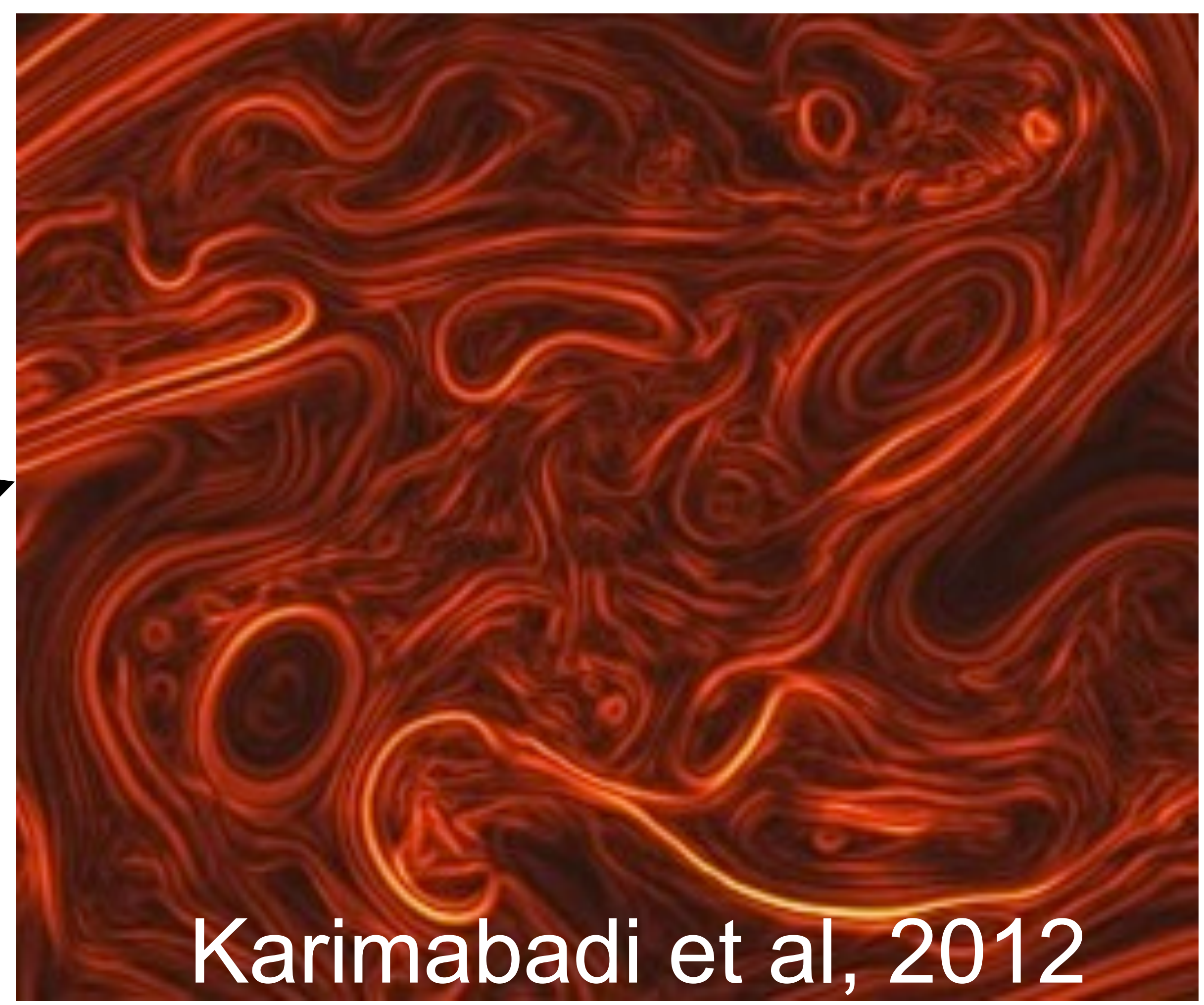
$U_o > V_A \quad \rightarrow \quad KH$

$m_i/m_e = 100$

KH turbulence drives reconnection

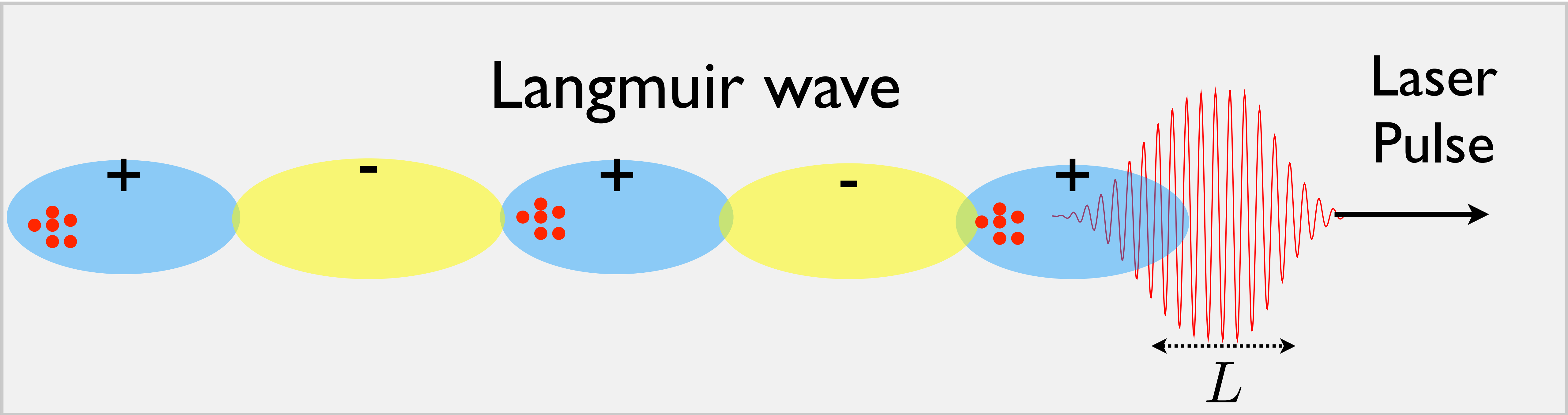


- Vortex scale $\sim 50 d_i$
- Kinetic scale layers
- Secondary tearing & KH
- Power law spectra
- Electron heating dominant
- In-plane B is crucial



Sample Problem #3
Laser Wakefield Accelerators

What is Laser Wakefield Acceleration ?



- Plasmas permit huge accelerating fields (0.1 - 10) TV/m
- Old idea - *Tajima & Dawson, 1979* - but rapid progress in recent years from experiments, theory & simulations
- Particularly efficient in the bubble or blowout regime

Two Requirements:

$$L < \lambda_p = \frac{2\pi c}{\omega_{pe}}$$

$$a_o \gg 1$$

Pukhov & Meyer-Ter-Vehn, 2002
Mangels et al, Nature, 2004
Geddes et al, Nature, 2004
Faure et al, Nature 2004

Example Simulation

Dimensionless Amplitude

$$a_o = \frac{eA_y}{m_e c^2} = 4$$

Laser Intensity

$$I = \frac{m_e^2 c^3}{8\pi e^2}(\omega_o a_o)^2$$

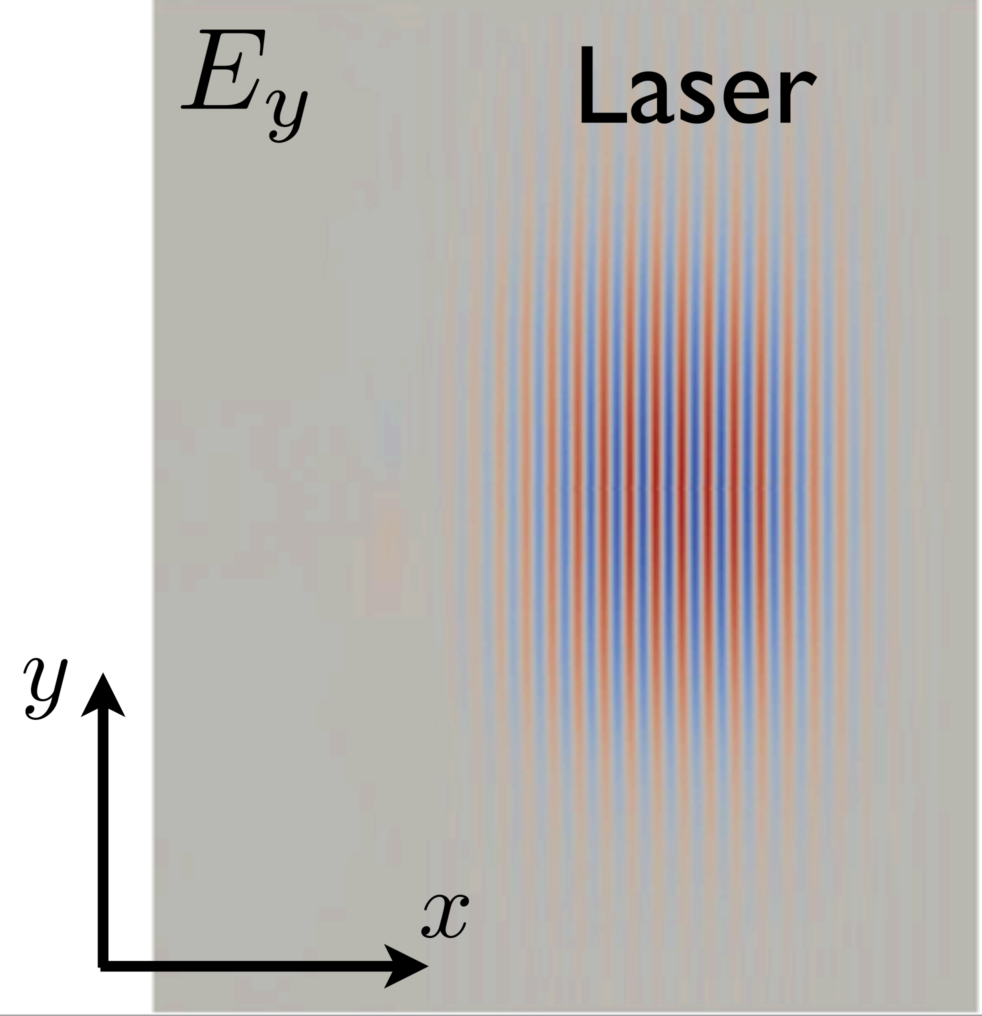
For 800nm laser:

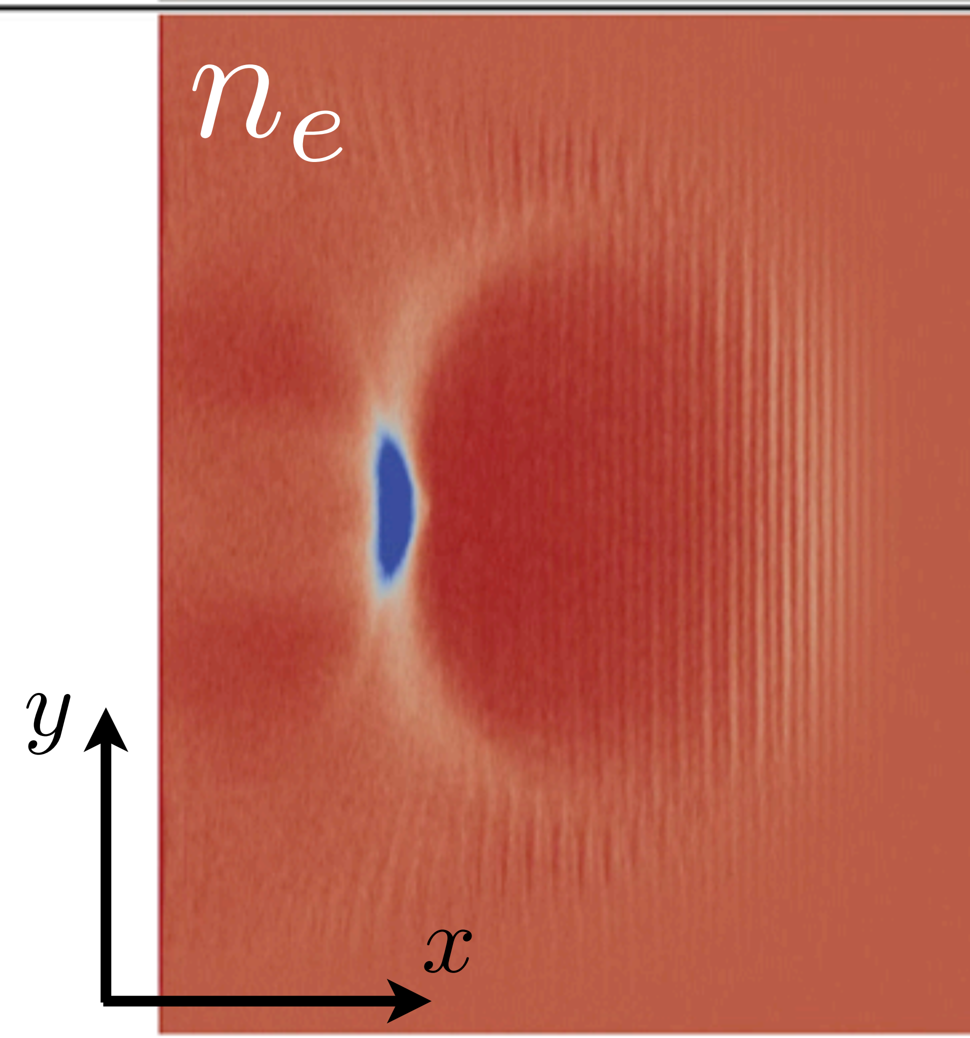
$$I \approx 2.2 \times 10^{18}\ a_o^2\ \ \mathrm{W/cm^2}$$

Simulation parameters

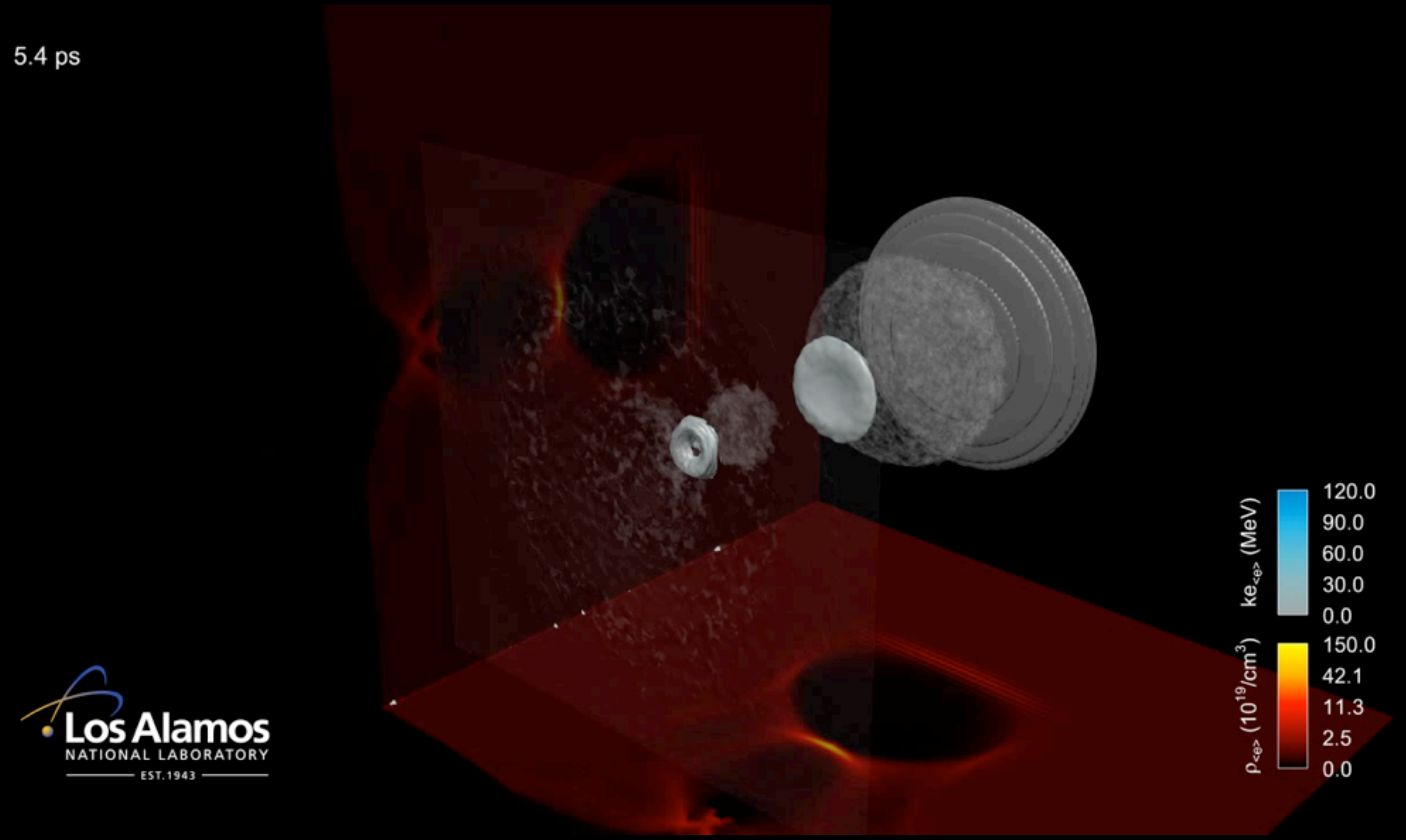
$$640 \times 512 \times 512\ \ \text{cells}$$

$$8\ \ \text{particles/cell}$$





Wakefield acceleration - in the Blowout Regime

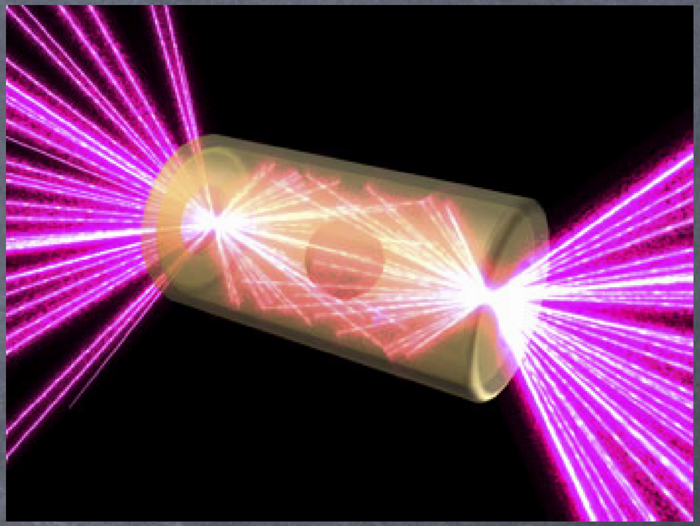


Sample Problem #4
Parametric Instabilities relevant to NIF

Many recent papers from Lin Yin - XCP-6

What is Inertial Confinement Fusion?

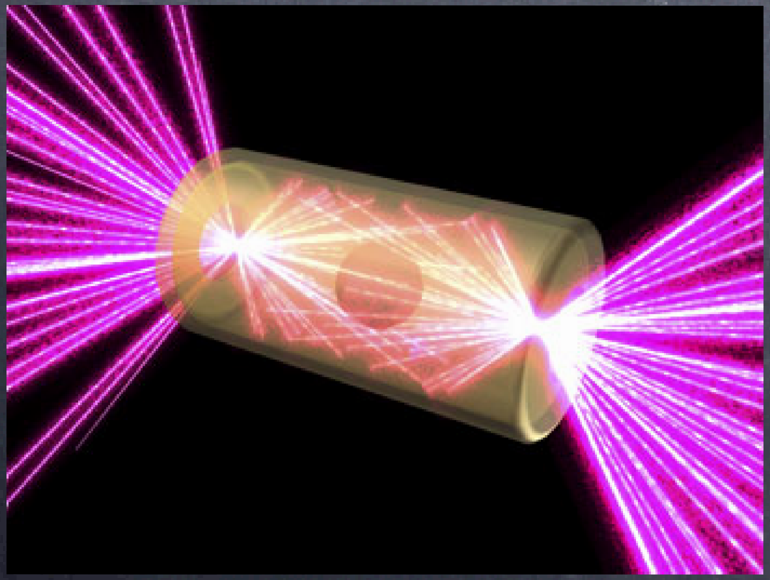
- Not really "confinement" at all
- Laser or ion beam to implode target
- Direct drive vs Indirect Drive →

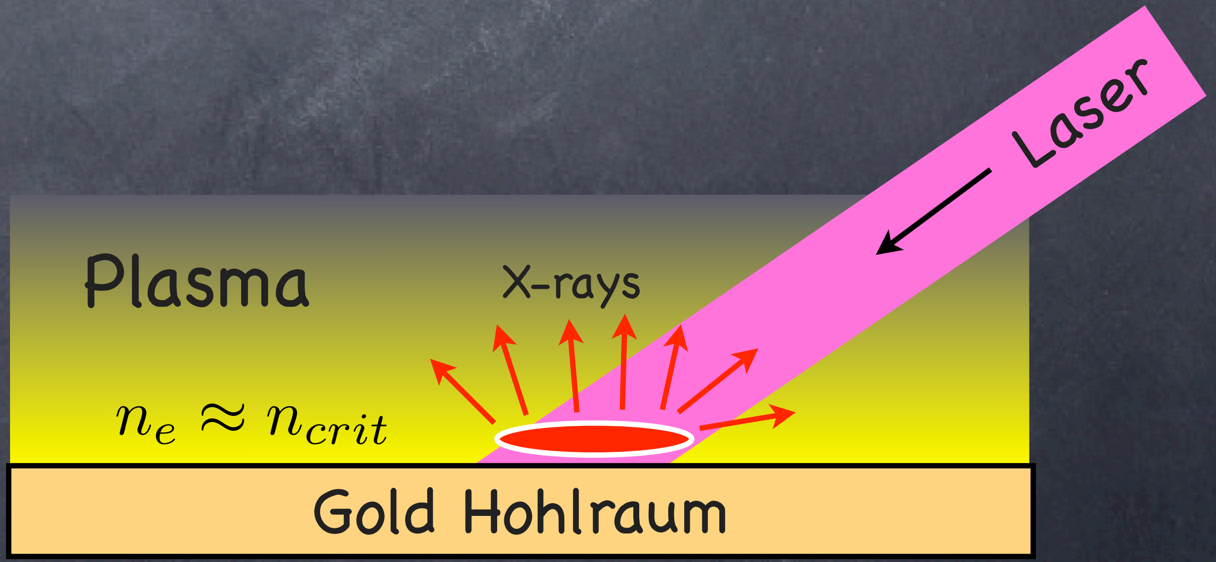


- Goal is to ignite DT fuel and burn significant fraction before capsule blows apart → High Gain
- ICF Ignition Experiments are being constructed in both U.S. and France

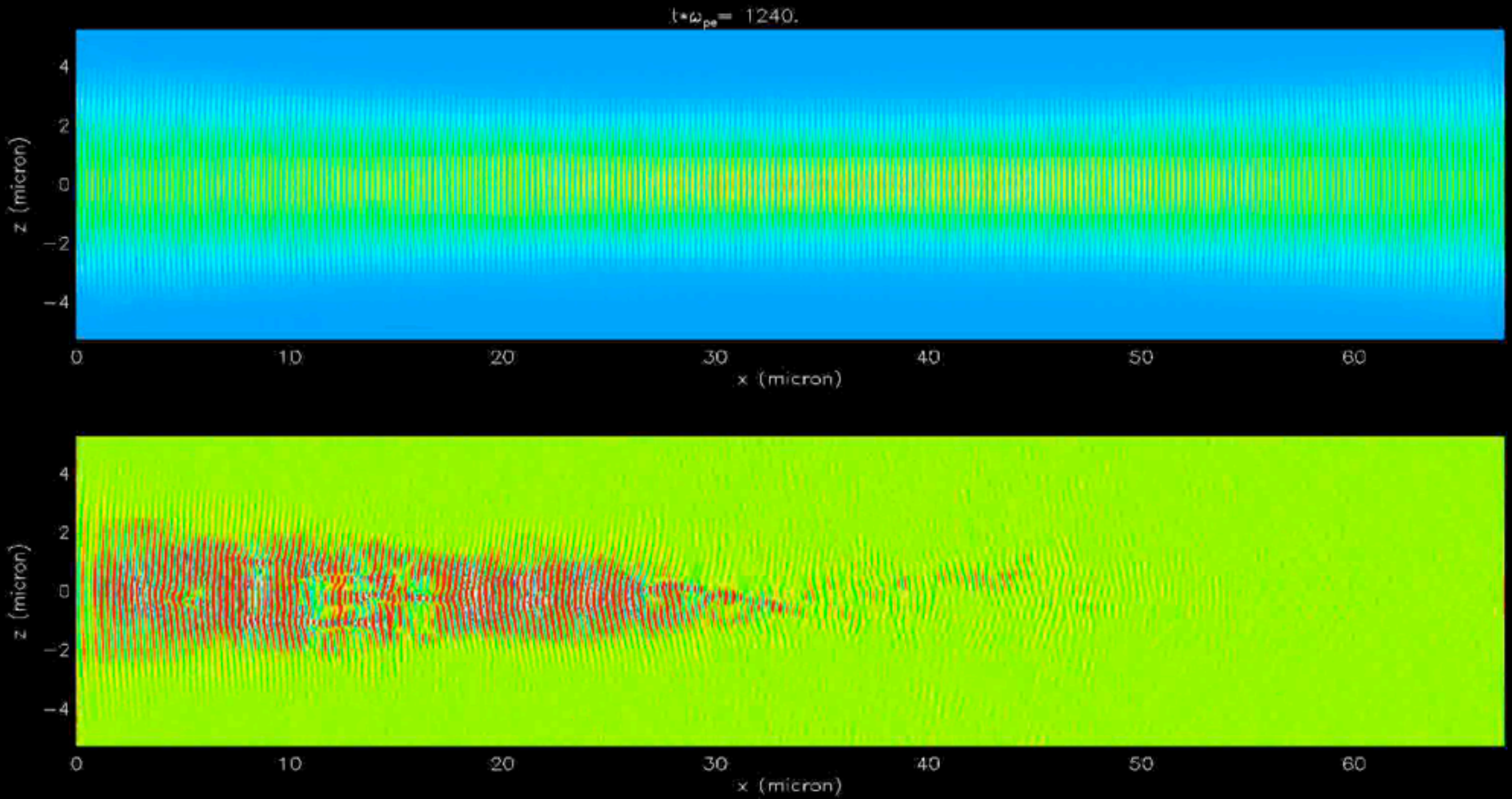
Significant Challenges Remain

- Factor of 40 larger in energy than NOVA
- Can it be built and operated?
- Fabrication of capsules, laser glass, etc.
- Can we deliver the energy where we want and when we want? - shock timing
- Laser Driven Parametric Instabilities





Poynting Flux of Laser

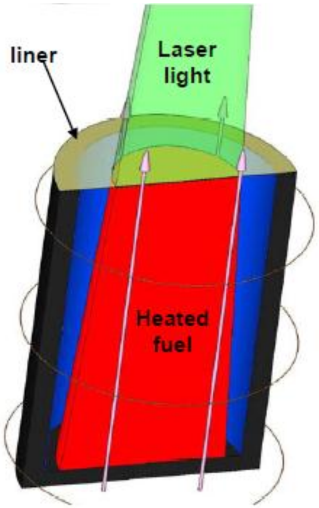


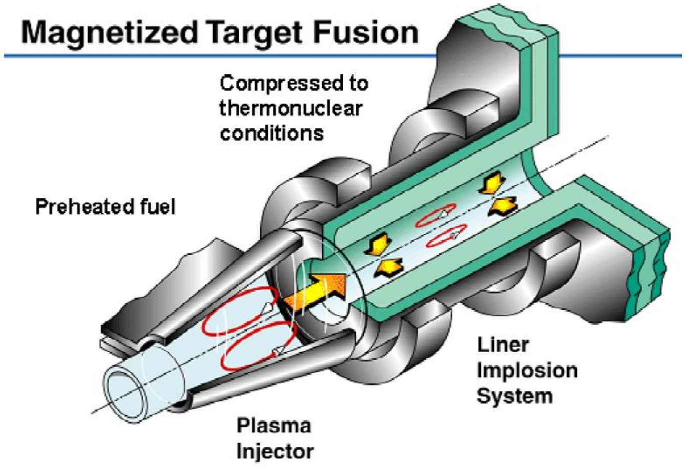
Longitudinal - Electrostatic Field

Summary

• Particle-in-cell simulations are conceptually simple, but very powerful method for simulating kinetic plasmas
• Method scales very well on new petascale computers and this is permitting a much wider range of 2D and 3D studies
• Good reference books on PIC simulations:

1. Birdsall & Langdon - Plasma Physics vis Computer Simulation
2. Hockney & Eastwood - Computer Simulation using Particles





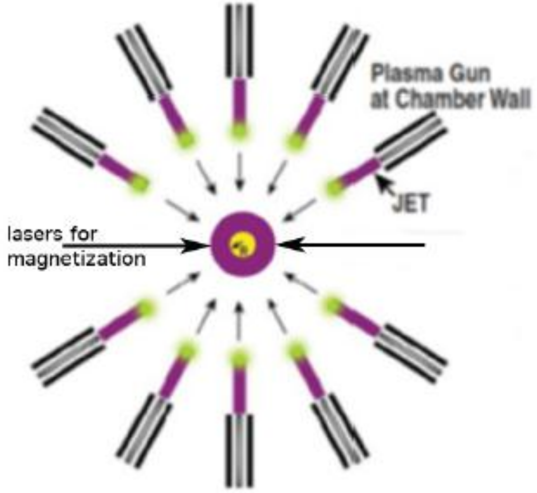
Magneto-Inertial Fusion: Basic Principles

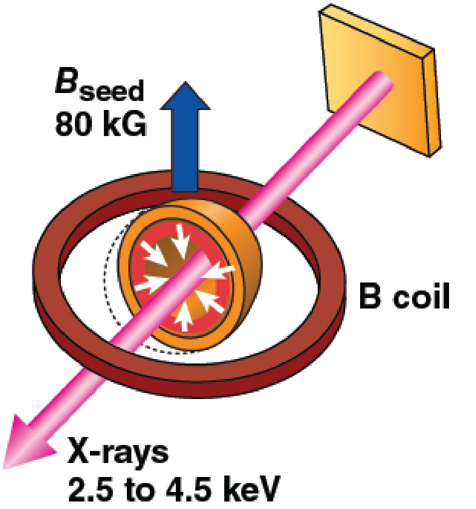
Thomas J. Awe
Sandia National Laboratories

Presented to the LANL-P24 Plasma Physics Summer School

August 8, 2012

SAND Number: 2012-6567 P





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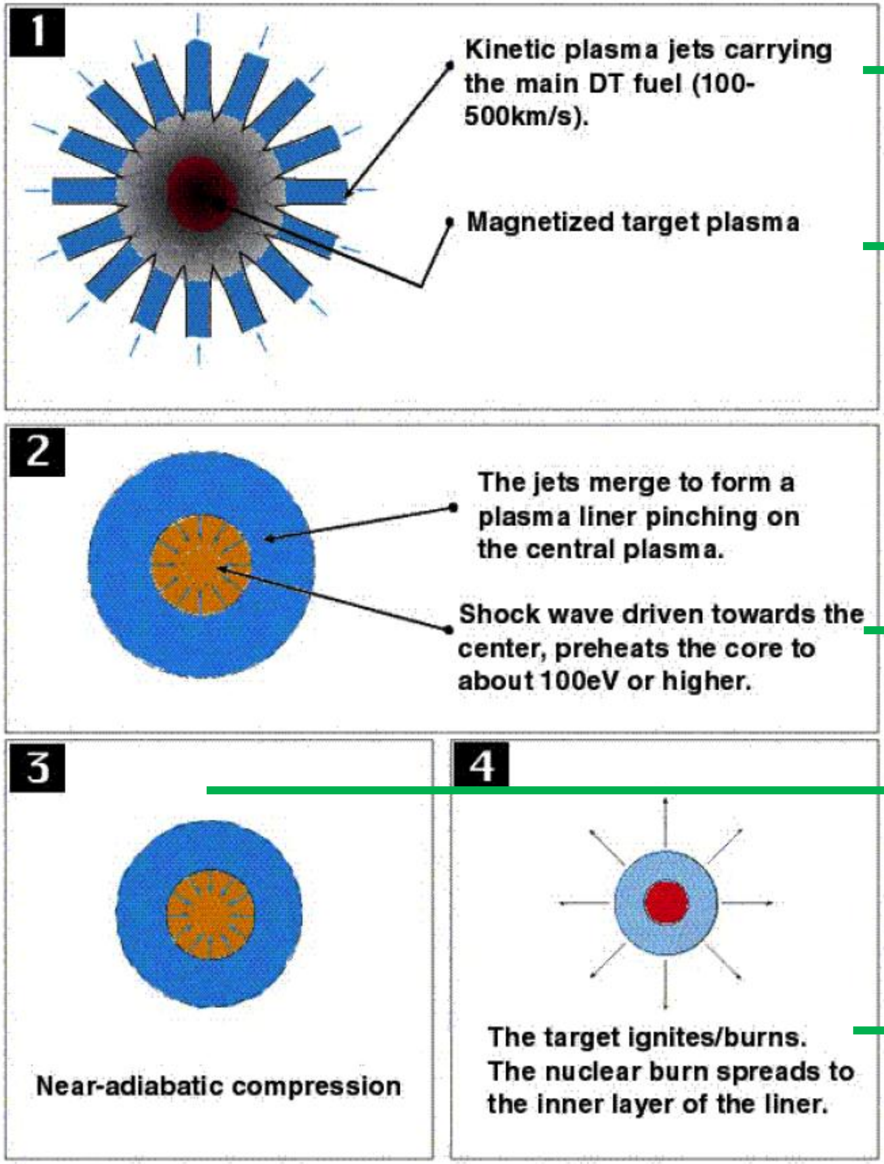
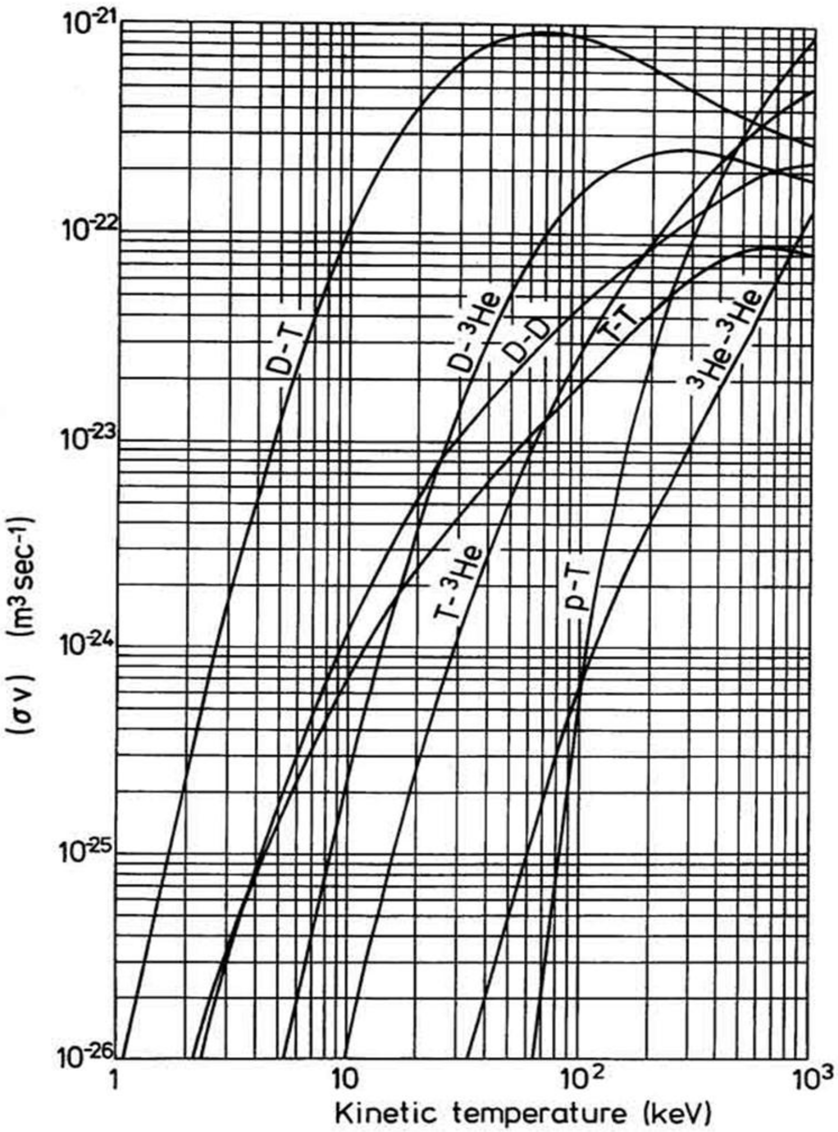
Magneto-inertial fusion: Inertially confined magnetized fuel



- Many "pusher" concepts exist
- Target is magnetized before compression; wide range of densities, field topologies, etc.
- Other systems aim towards isentropic compression; preheat can be delivered to the target in many ways
- Nearly adiabatic compression possible even for slow implosions due to magneto-thermal insulation
- High gain systems can include a cold fuel layer which surrounds the burning fuel

From Y.C.F. Thio et al., Magnetized Target Fusion in a Spheroidal Geometry with Standoff Drivers (1999)

The DT reactivity is 100 times larger than any other reaction for temperatures from 10 to 20 keV



σ is the reaction cross section [m²]

→Depends on species and temperature

$\langle\sigma v\rangle$ is the averaged reactivity [m³/s]

→Averaged over velocity distribution

→Depends on species and temperature

→Low Z nuclei are favorable due to reduced coulomb repulsion and increased probability of QM tunneling

→ $\langle\sigma v\rangle_{DT} \propto T^2$ for $T \in [8 \text{ to } 25 \text{ keV}]$

Reaction rate [1/ m³s] :

$$R_{DT} = n_D n_T \langle \sigma v \rangle$$

→Density dependent

Multiply reaction rate by the energy delivered to the fuel per reaction (some fraction of alpha energy) to determine self heating power

Image from: http://www.kayelaby.npl.co.uk

The "ideal ignition temperature" is that temperature where internal heating exceeds losses

Internal heating

D+T → **α (3.5 MeV)** + n (14.1 MeV)

→Requires high areal density (ρR)

→**Inclusion of a magnetic field enables delivery of a larger fraction of the alpha energy to the fuel**

Loss Mechanisms

Radiation loss→Dominated by (free-free) Bremsstrahlung for fully ionized plasma (what is opacity?)

Heat condution (electron or ion)→To surrounding cool fuel, or cool material pusher

→Heat conduction losses reduced if stagnation time is kept short

→Requires high density→ high implosion velocity→ expensive driver

→**OR, heat conduction can be suppressed by inclusion of a strong magnetic field**

Classical loss from radiation and thermal conduction can be evaluated in a straightforward way

$$Q_{loss} = C_{RAD} n_i^2 T^{1/2} - \nabla \bullet (K \nabla T)$$

Both radiation loss and fusion rates depend on n^2, thus temperature alone dictates the energy balance. The fusion rate exceeds the radiation rate for T>3 keV

$$Q_{TC} \approx -\frac{1}{V}\int \nabla \bullet (K\nabla T)dV = -\frac{1}{V}\oint_S K\nabla T \bullet d\bar{S} \approx -\frac{S}{V}K\nabla T \approx \frac{KT}{\gamma \alpha \mathrm{a}^2} \quad (11)$$

where $V = \varepsilon \mathrm{a}^3$, $\frac{V}{S} = \gamma \mathrm{a}$, $\nabla T \approx -\frac{T}{\alpha \mathrm{a}}$, a is a characteristic dimension, V the plasma volume, S the plasma surface area, ε and γ are geometric quantities (for example, for spheres, ε = 4π/3 and γ = 1/3), and α is a temperature-gradient scale factor that, as described later, depends on the form of K.

From Section III of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

Magnetized fuel: Reduced cross-field thermal conductivity

→Electrons gyrate around field lines: thermal conductivity is reduced perpendicular to field lines

→Hall parameter: $\chi_e=\omega_{ce}\tau_{ei}$ indicates the degree of magneto-thermal insulation

→**Thermal insulation when $\chi_e=\omega_{ce}\tau_{ei} > 1$ → $\tau_{ei} > 1/\omega_{ce}$ → $\tau_{ei} > \tau_{ce}$**

→ implies time between collisions exceeds time for full gyration, so electrons stay "frozen" to field line

→ Electron thermal conductivity written in terms of the Hall parameter is:

$$K_{ce} = \frac{3.16nkT}{m_e}\tau_e\left[\frac{1+0.39\chi_e^2}{1+3.9\chi_e^2+0.26\chi_e^4}\right]$$

$$\frac{Q}{L} = C_1 T^{7/2}\left[\frac{1+0.39C_2^2T^3(B/\rho)^2}{1+3.9C_2^2T^3(B/\rho)^2+0.26C_2^4T^6(B/\rho)^4}\right] \qquad [W/cm]$$

So, for large B/ρ… $\frac{Q}{L} \propto T^{1/2}\left(\frac{\rho}{B}\right)^2$ And for B=0… $\frac{Q}{L} \propto T^{7/2}$

See Section II of Slutz et al., Phys. Plasmas, **17**, 056303 (2010)

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where $V = \varepsilon a^3$, $\frac{V}{S} = \gamma a$, $\nabla T \approx -\frac{T}{\alpha a}$, a is a characteristic dimension, V the plasma volume, S the plasma surface area, ε and γ are geometric quantities (for example, for spheres, ε = 4π/3 and γ = 1/3), and α is a temperature-gradient scale factor that, as described later, depends on the form of K.

From Section III of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

Considering losses in simple geometric terms allows estimates of the required fuel volume for energy gain

$$Q_{TC} \approx \frac{KT}{\gamma \alpha a^2} \Rightarrow a^2 = \frac{KT}{\gamma\alpha}\frac{1}{\phi Q_{DT} - Q_{RAD}} \quad \text{where} \quad Q_{loss} = \frac{dE_{loss}}{dt} = \phi Q_{FUS}$$

Once the minimum size required to operate at or below a specified value of ϕ for a specified density-temperature (and magnetic field) is determined, important physical parameters such as the minimum fuel mass M and the minimum plasma thermal energy E_{PLAS} can be determined:

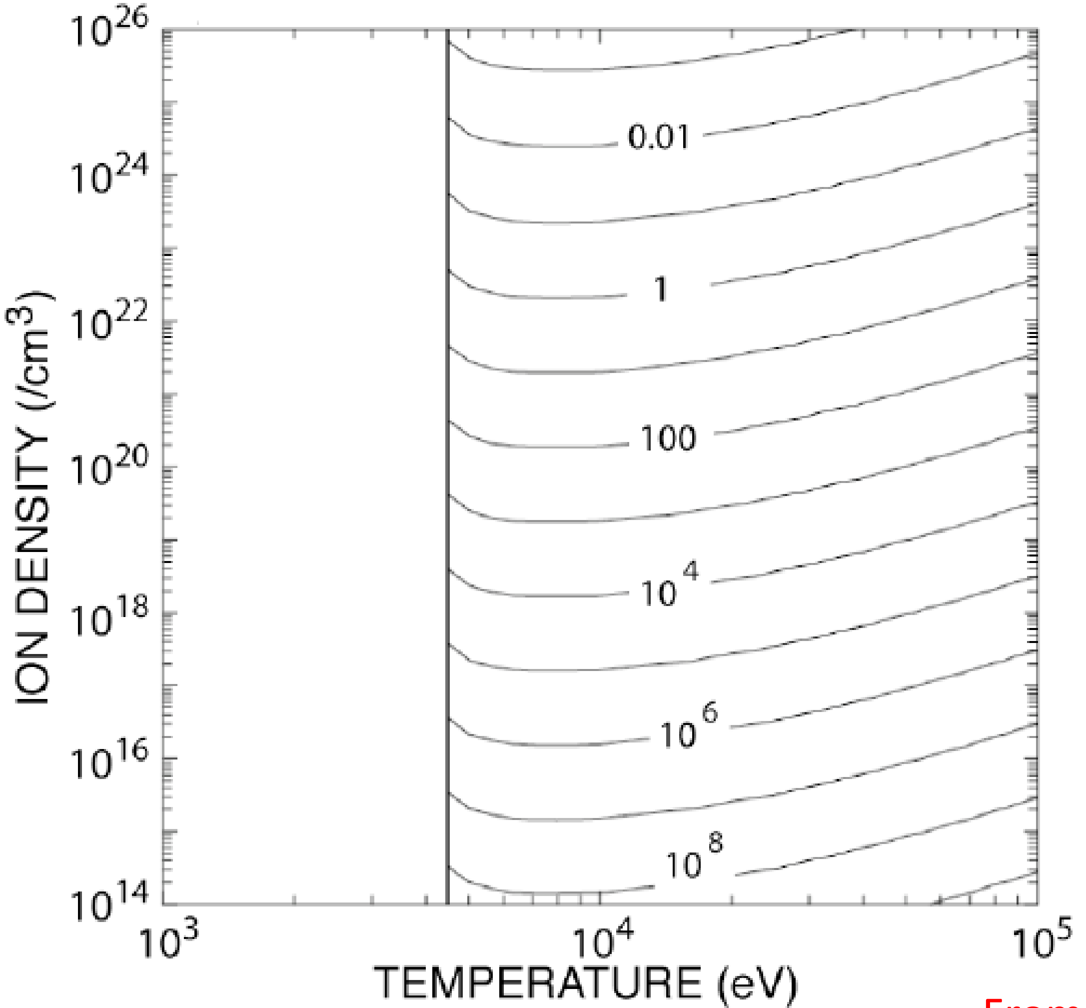
$$M = n_i(m_i + m_e)\varepsilon a^3, \qquad E_{PLAS} = 3n_i T\varepsilon a^3. \qquad (13)$$

The minimum required heating rate to sustain the plasma, and the corresponding heating intensity, can also be determined:

$$P_{HEAT} = Q_{TC} + Q_{RAD}, \quad I_{HEAT} = \frac{P_{HEAT}}{S}. \qquad (14)$$

From Section IV of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

The minimum system size is excessively large unless the density is very large



The minimum system size (cm) for unmagnetized fuel (spherical geometry) operating at ϕ=0.2.

From Section V of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

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Extreme density is also required to avoid impossibly high minimum energy and power

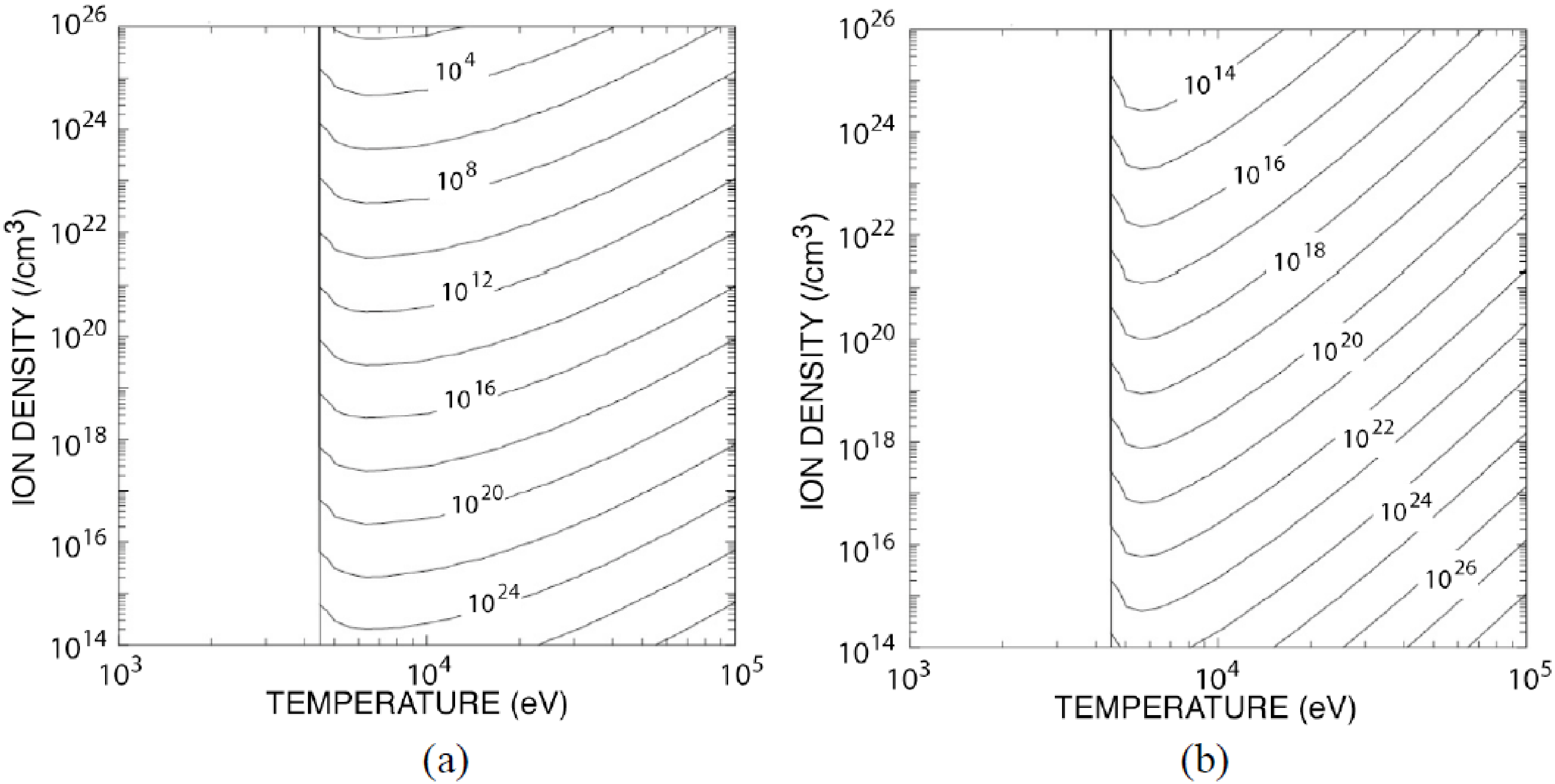

Figure 2. (a) The minimum energy (J), and (b) the minimum power (W) for unmagnetized fuel (spherical geometry) operating at $\phi \leq 0.2$.

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From Section V of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

For un-magnetized fuels, only extreme fuel configurations are interesting for energy production

See Atzeni & Meyer-Ter-Vehn, Ch2

High areal density, ρR ~3 g/cm^2 is required so that:

→Alphas deposit their energy in the fuel (self heating)

→The fractional burn up is high

BUT…

The fuel mass must be kept rather small (<10 mg) since, for obvious practical reasons, the energy released per fusion event must be kept to a few GJ

AND…

Mass, $M=\rho V=\rho(4/3)\pi R^3= (4/3)\pi (\rho R)^3/\rho^2$

SO…

For fixed/minimum ρR (for reasons above), to accommodate the small mass requirement, ρ must be extremely large:

$\rho^2=(4/3)\pi (\rho R)^3 /M \rightarrow \rho\sim 336$ g/cm^3

This is ~1500 times the density of cryogenic DT!!!

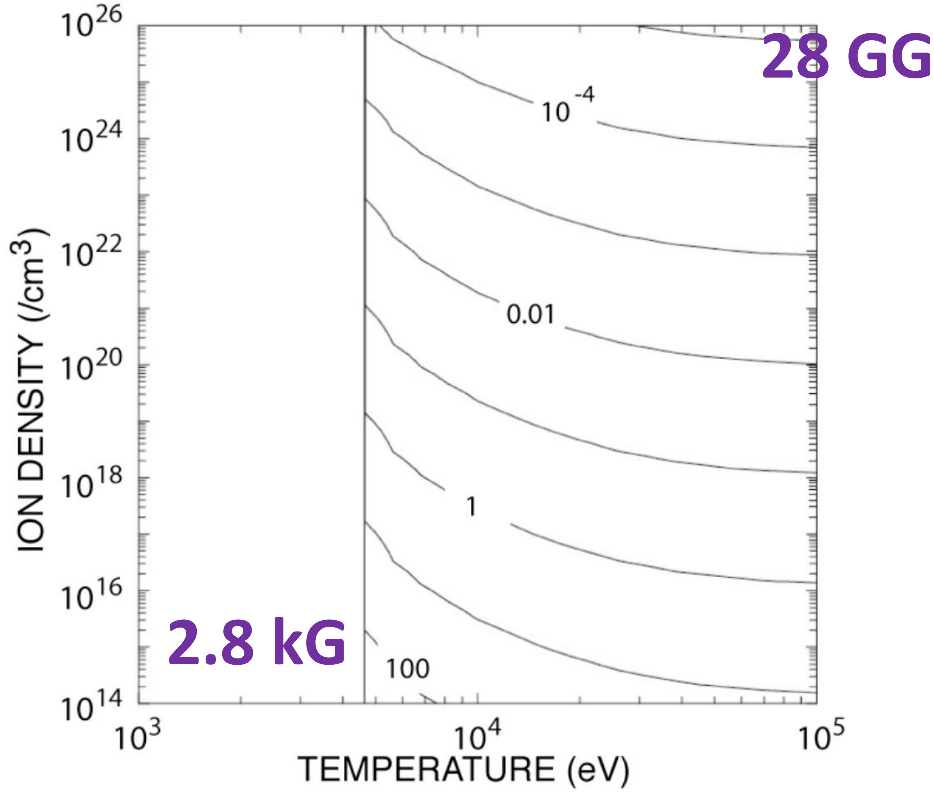
Inclusion of a strong magnetic field reduces the system size, energy and power requirements

Cross field thermal conductivity is reduced by a factor proportional to B^{-2}

In the following plots, β (ratio of plasma pressure to magnetic pressure is held equal to 1, so the field strength changes as a function of n, T as:

$$B = 2.8 \times 10^{-4} \left(n_i T_{keV} / \beta \right)^{1/2}$$

The minimum system size (cm) for **magnetized** fuel (spherical geometry) operating at ϕ<0.2.



From Section VI of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

Inclusion of a strong magnetic field reduces the system size, energy and power requirements

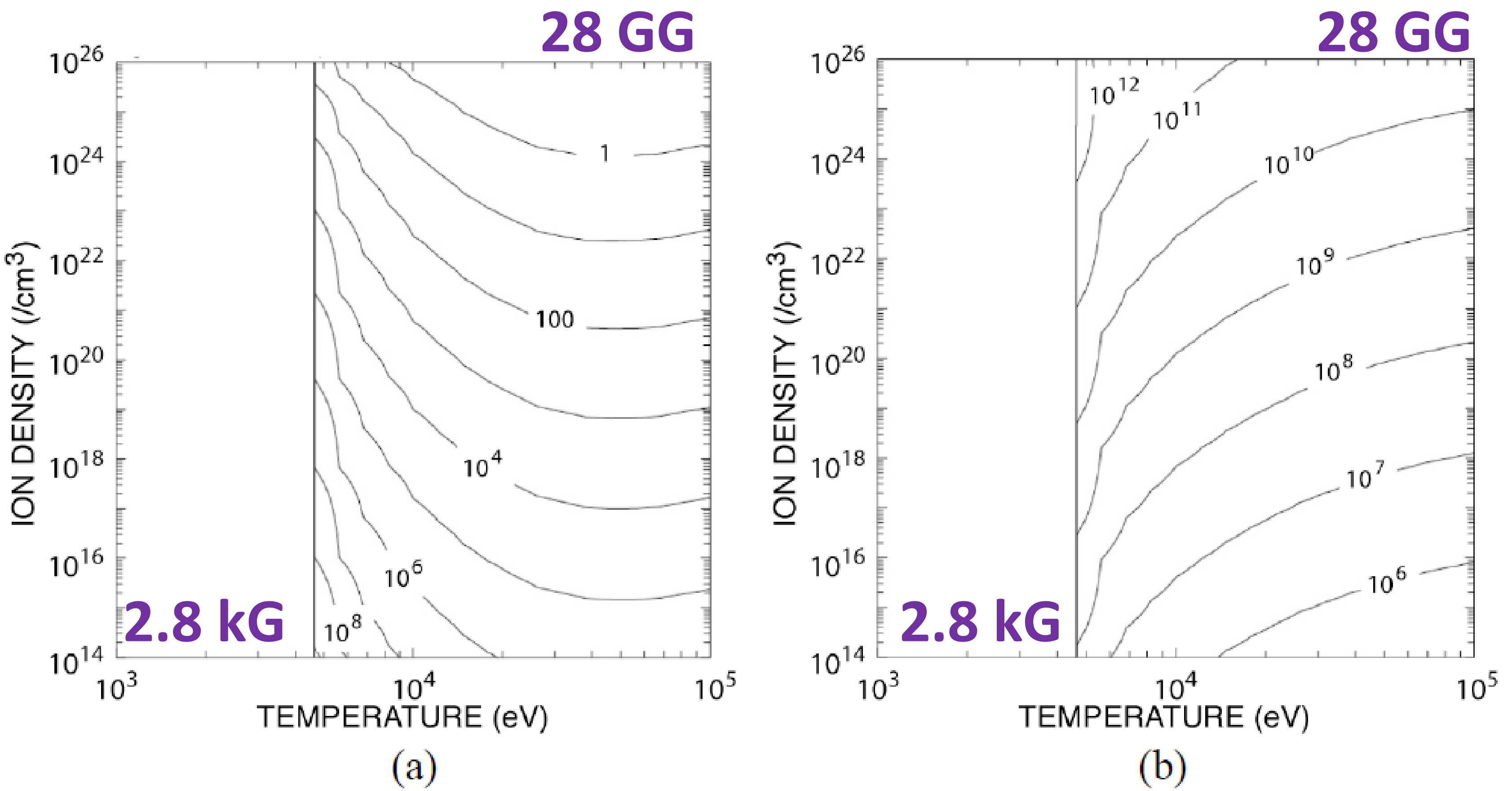


Figure 4. (a) The minimum energy (J), and (b) the minimum power (W) for magnetized fuel (toroidal geometry, $\beta = 1$) operating at $\phi \leq 0.2$.

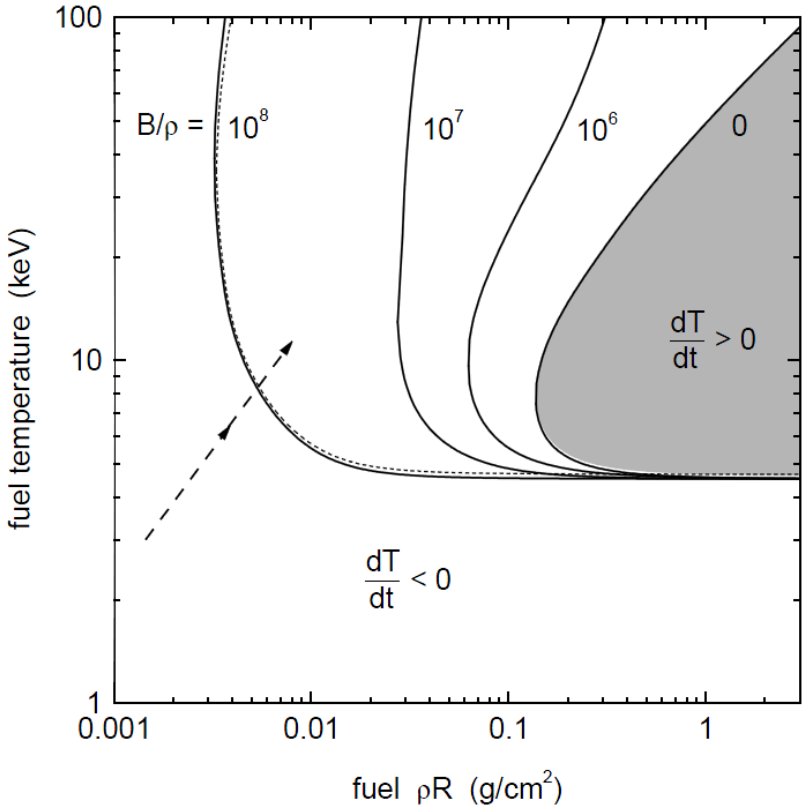
From Section VI of Lindemuth & Siemon, Am. J. Phys., Vol 77, No. 5, May 2009

Introduction of a strong magnetic field into the fuel opens up the ignition parameter space

Ignition conditions for magnetized target fusion in cylindrical geometry

M.M. Basko[a*], A.J. Kemp[b], J. Meyer-ter-Vehn[b]

Nuclear Fusion, Vol. 40, No. 1 (2000)



Recall, for large B/ρ:

$$\frac{Q}{L} \propto T^{1/2}\left(\frac{\rho}{B}\right)^2$$

Figure 3. Lindl–Widner diagram for magnetized DT cylinders at stagnation. Solid curves show a series of ignition boundaries in the $\rho R, T$ plane calculated for four fixed values of the parameter B/ρ, given near each curve in units of G cm^3/g. The shaded area is the pure ICF ignition domain at $B = 0$. The dotted curve illustrates the effect of synchrotron radiation losses at $\rho = 1$ g/cm^3, $B = 10^8$ G. Dashed arrows indicate how the fuel states advance towards the ignition boundary in the process of a quasi-adiabatic implosion.

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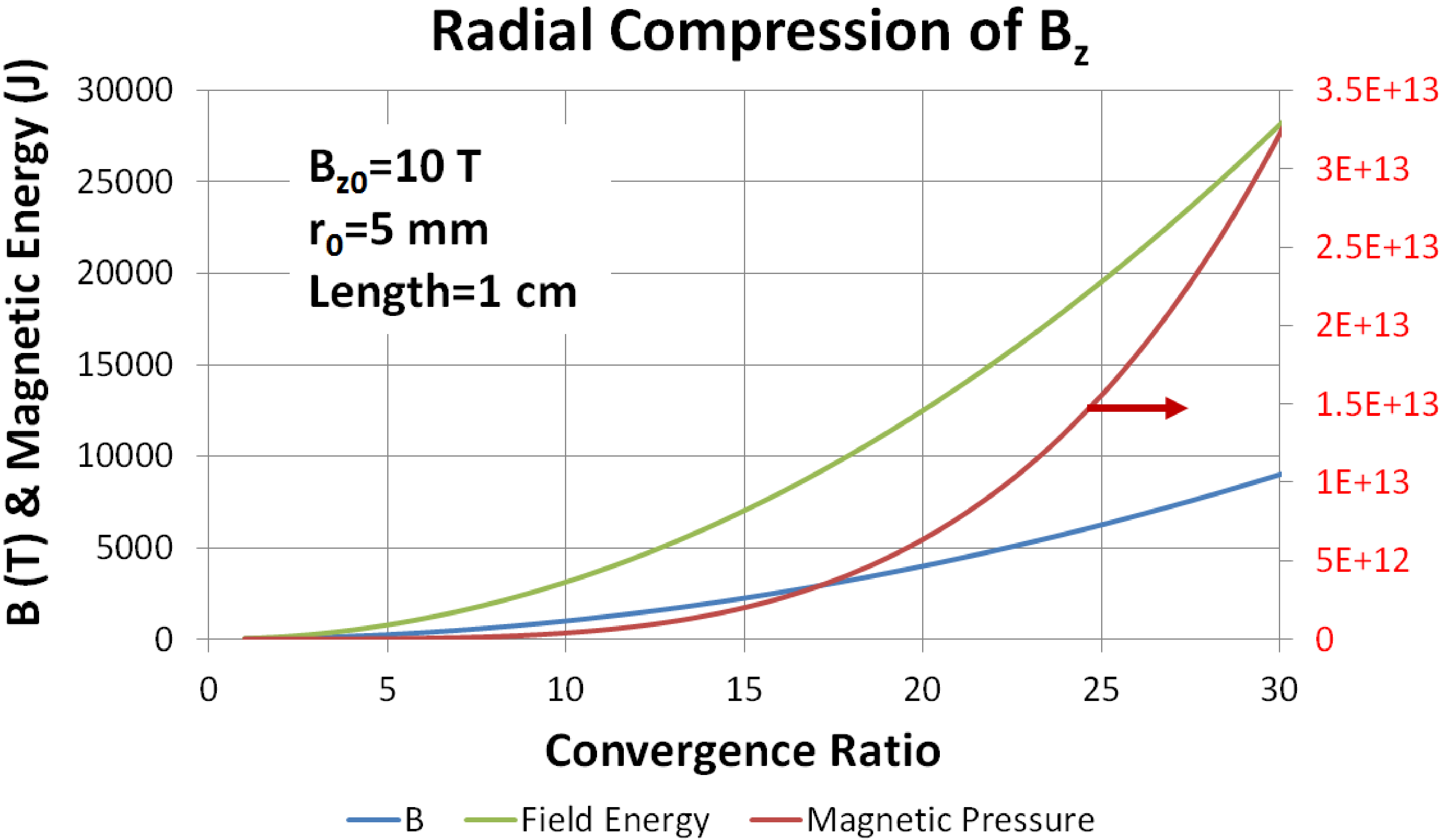
Embedded magnetic fields can be compressed to very high values with little effect on implosion dynamics



*****Magnetic pressure for an I=25 MA driver on a 300 micron radius liner is 10^{14} Pa**

The evolution of the liner inner surface is a major unknown physics issue for MIF

- **How & when does liner inner surface change state?**
- **How well does liner confine magnetic flux?**
- **How big does RT grow during liner deceleration?**
- **How much high-Z mixing with DT fuel?**
- **How accurate is numerical modeling?**

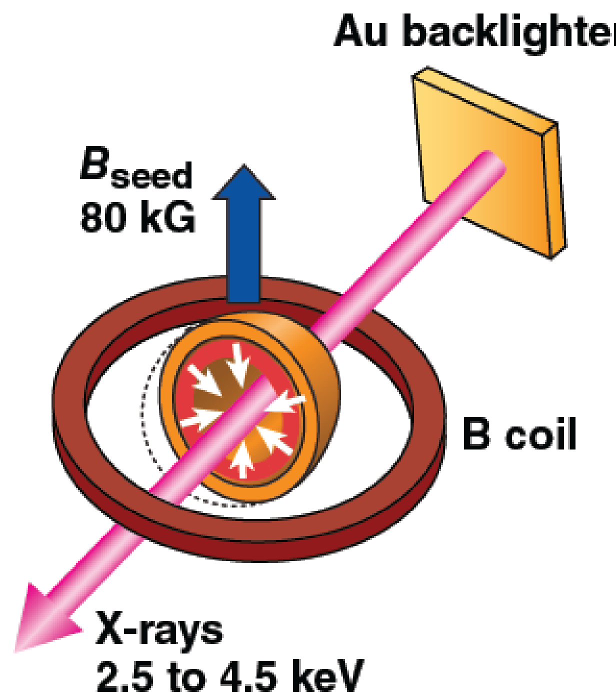
A variety of magneto-inertial fusion efforts leverage the apparent advantages of magnetized fuel

MIFEDS→ laser (direct drive) ICF with magnetized target
PLX→Plasma jets form spherical liner to compress magnetized target
FRCHX→FRC formed in theta coils—translated into and compressed by large imploding metallic liner
MagLIF→Fast (100 ns) metallic liner compresses a preheated and dense fuel with an axial magnetic field

Other MIF related concepts:
MAGO
LINUS
General Fusion

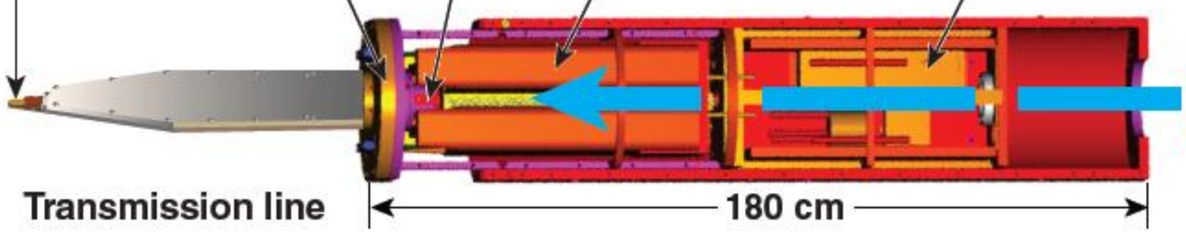
Single-coil B field was used for fusion-enhancement measurements in spherical geometry

- Single-coil provides stronger seed fields (80 kG), less interference with laser beam
- 40 beams (18 kJ/1 ns) were used for compression
- Implosion uniformity is diagnosed using x-ray BL radiography
- nTOF diagnostic was used for T_i and neutron-yield measurements

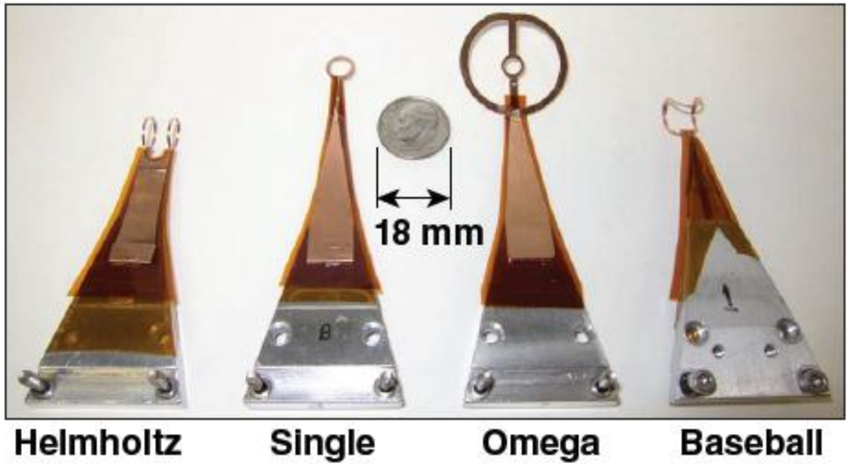


From G. Fiksel, 2012 MagLIF Workshop

The seed B field is created by a compact, self-contained magnetic field generator



- **MIFEDS–Magnetized Inertial Fusion Energy Delivery System**
- **Various coils were tested**
- **Seed fields up to 150 kG can be obtained (depends on the coil size and geometry)**

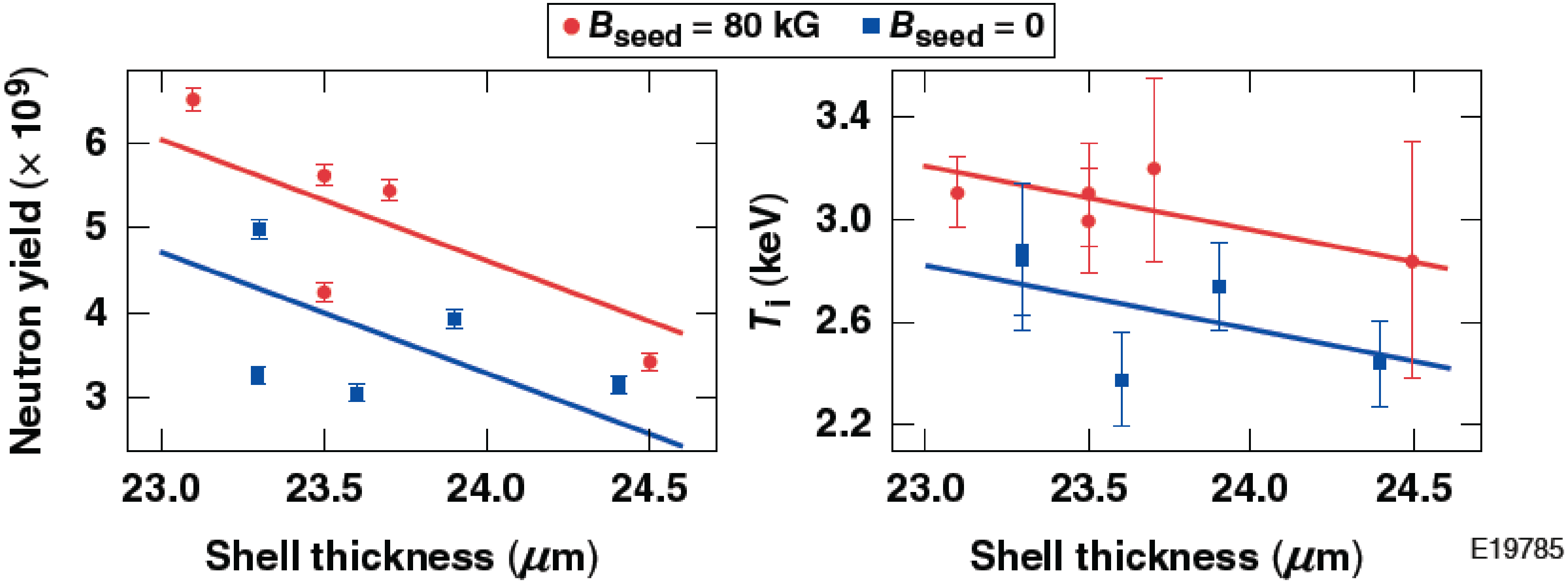


O. V. Gotchev *et al.*, Rev. Sci. Instrum. 80, 043504 (2009).

E19778

From G. Fiksel, 2012 MagLIF Workshop

We observe a (30±12)% neutron-yield enhancement and (15±4)% ion-temperature increase for magnetized targets

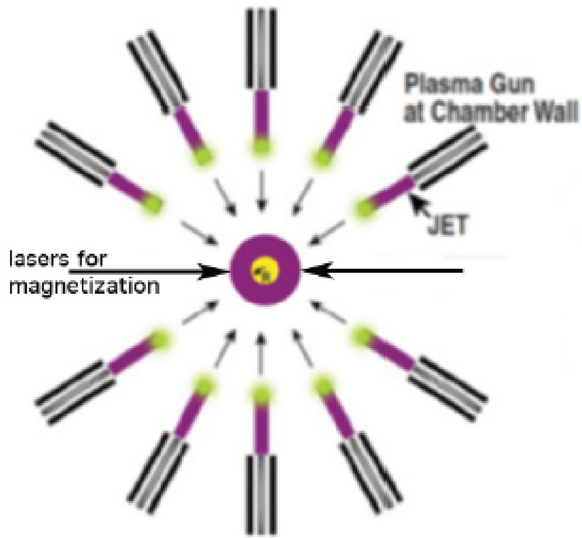


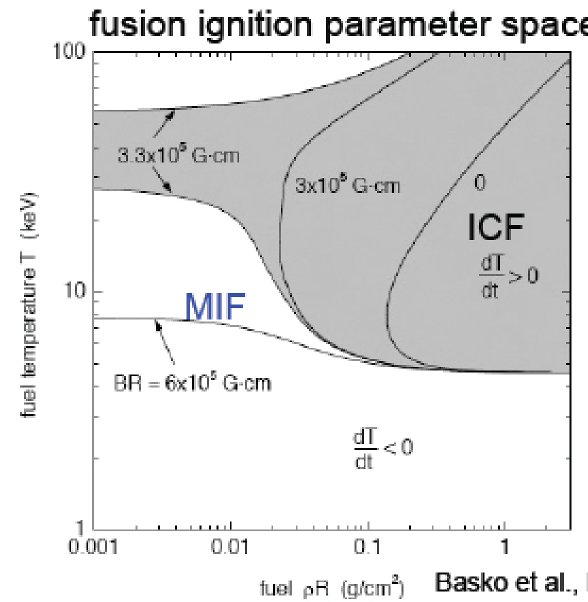
- Diagnostic—nTOF (neutron time-of flight)
- Fusion performance scales with shell thickness

Linear regression fit reveals clear enhancement of magnetized hot-spot performance.

From G. Fiksel, 2012 MagLIF Workshop

Motivation: Convergent plasma jets could be an attractive approach to inertial fusion energy (IFE) using magnetic fields



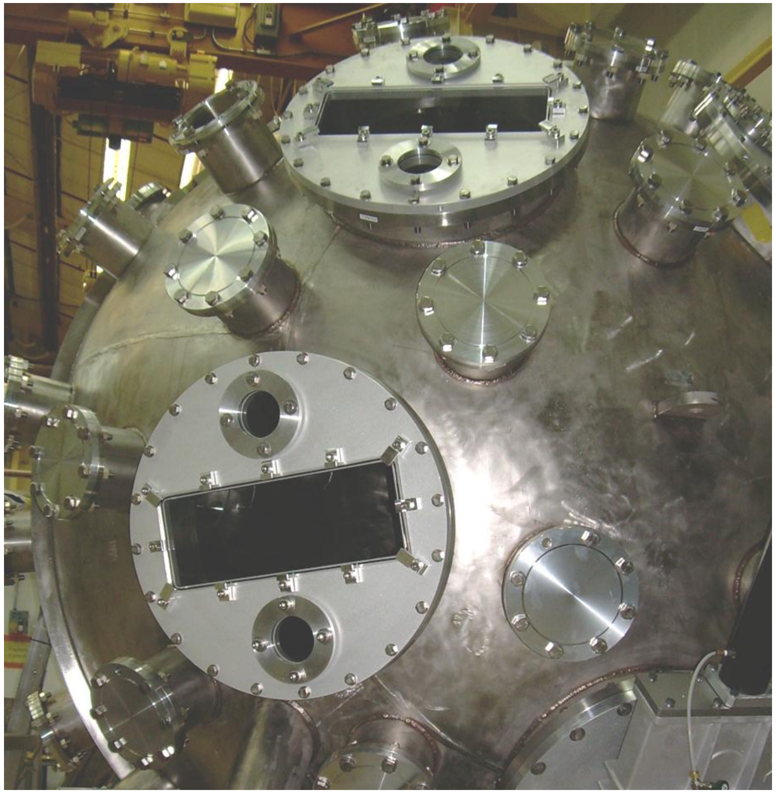


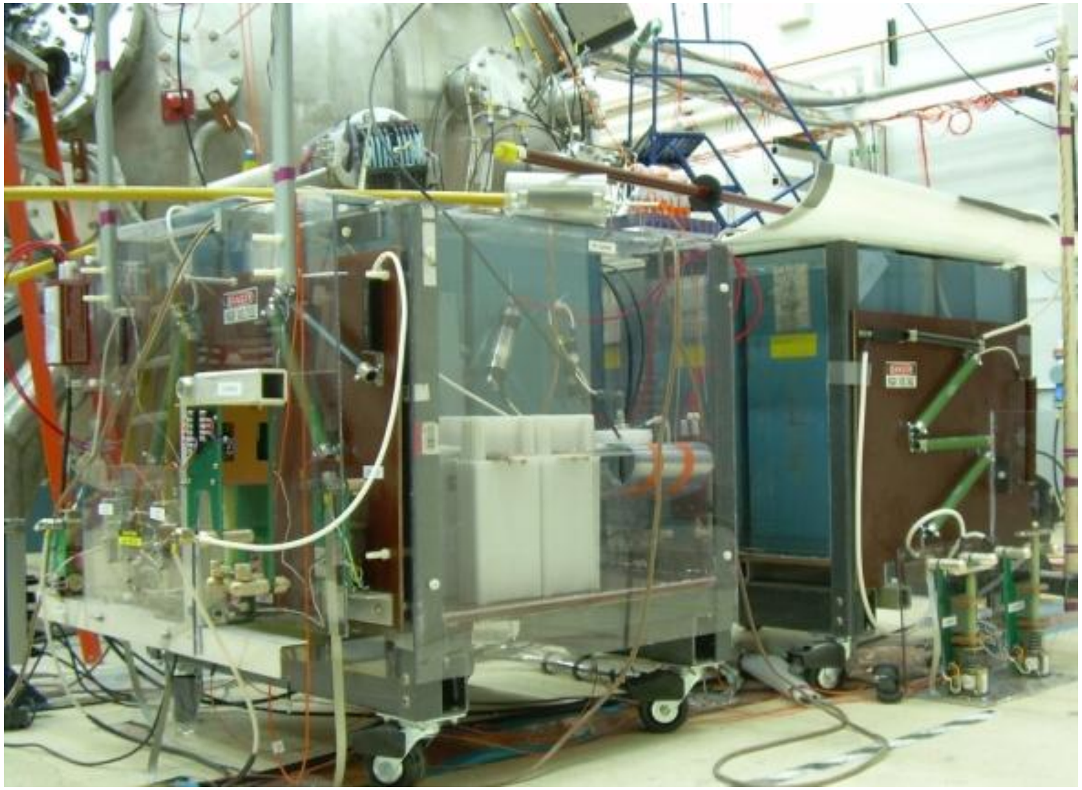
Basko et al., Nucl. Fusion (2000)

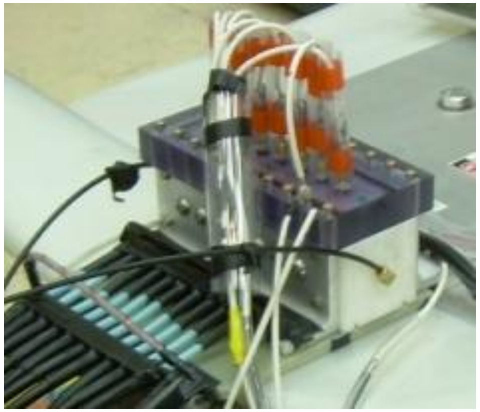
- **Composite DT (yellow) and high-Z (purple) jets are imploded to ρr_{DT}~0.01 g/cm^2**
- **DT is magnetized via lasers just before peak compression**
- **Implosion speed ~100 km/s**
- **Dwell time ~1 μs**
- **Batch burn with ~10% fuel burn-up**
- **Peak pressures ~50 Mbar**

From S.C. Hsu, 2010 APS-DPP

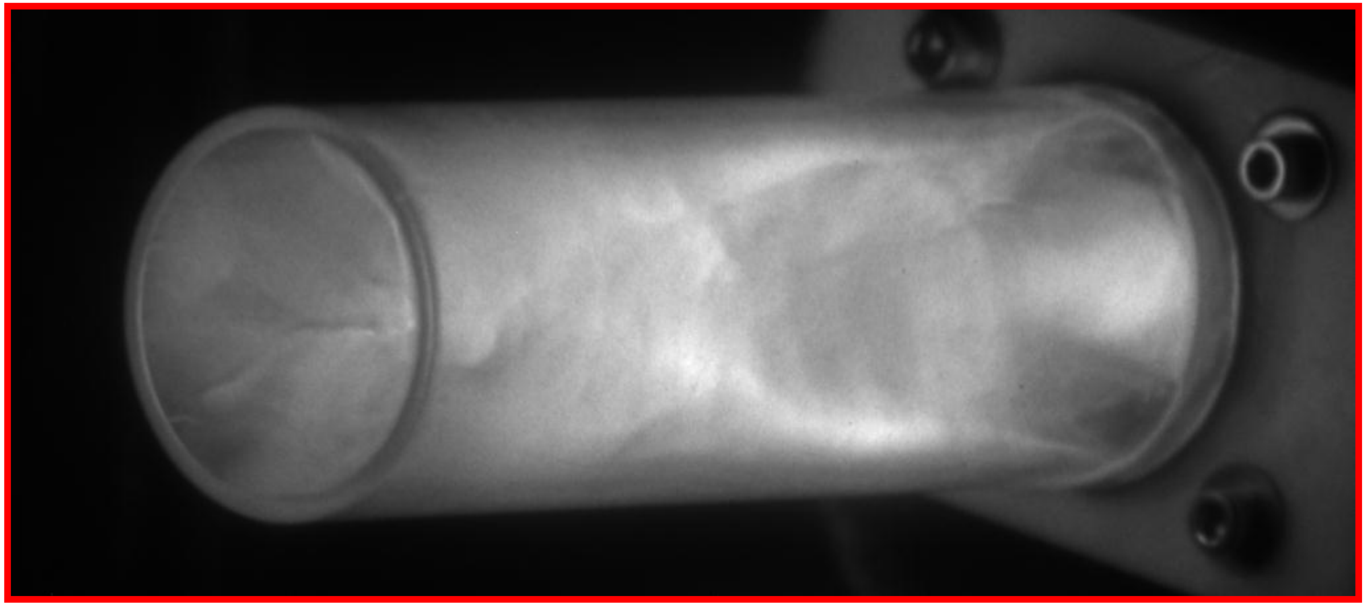
First plasma 9/13/11: Single-jet experiments are underway





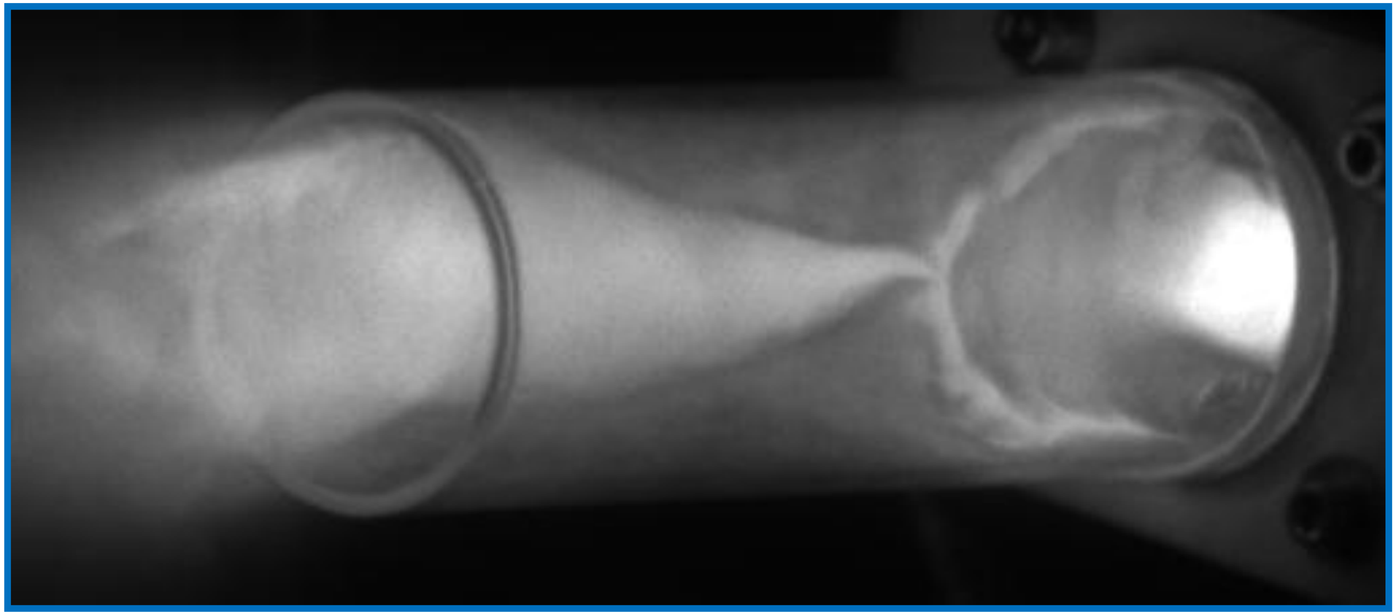


Complex structure exists as the jet exits the gun bore and propagates down the nozzle



Shot 76

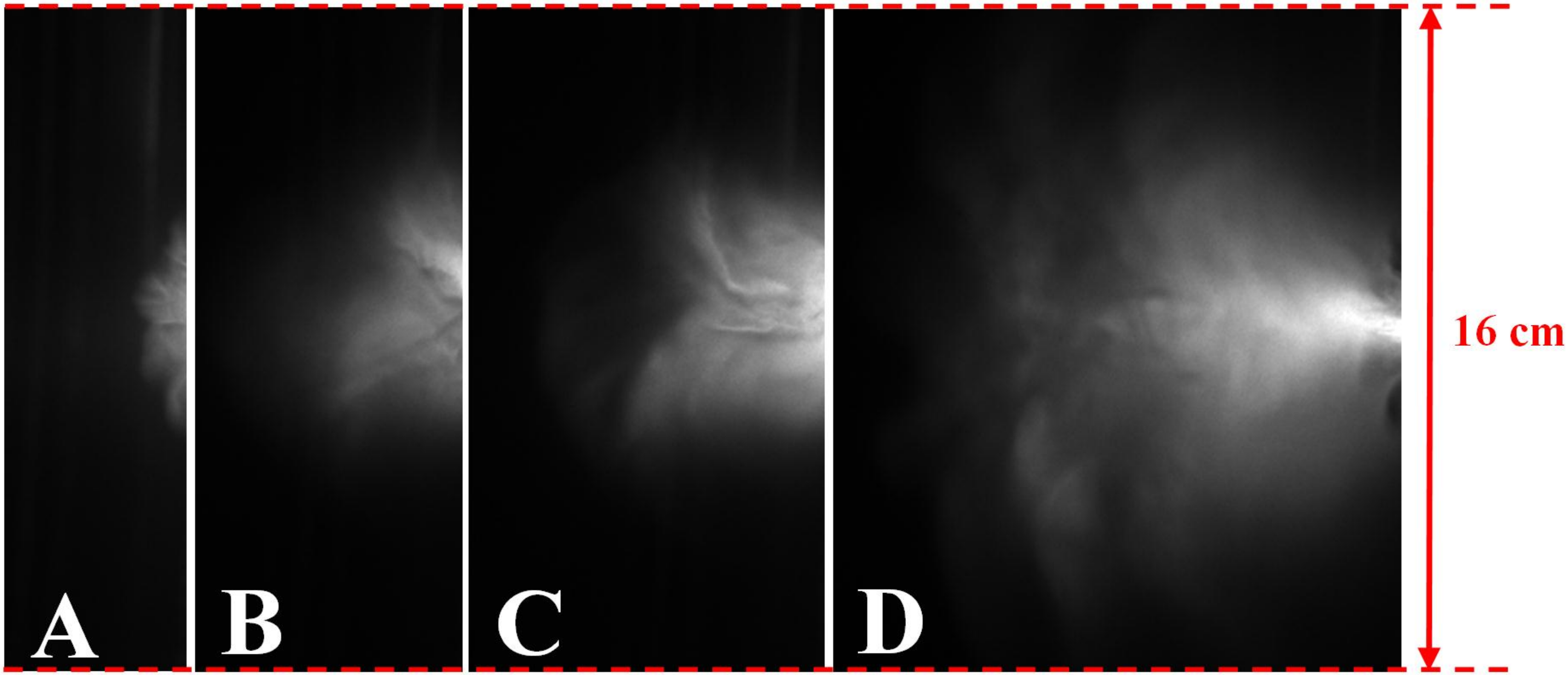
- Δt_{switch}=28.6 μs
- 3 ns exposure



Shot 72

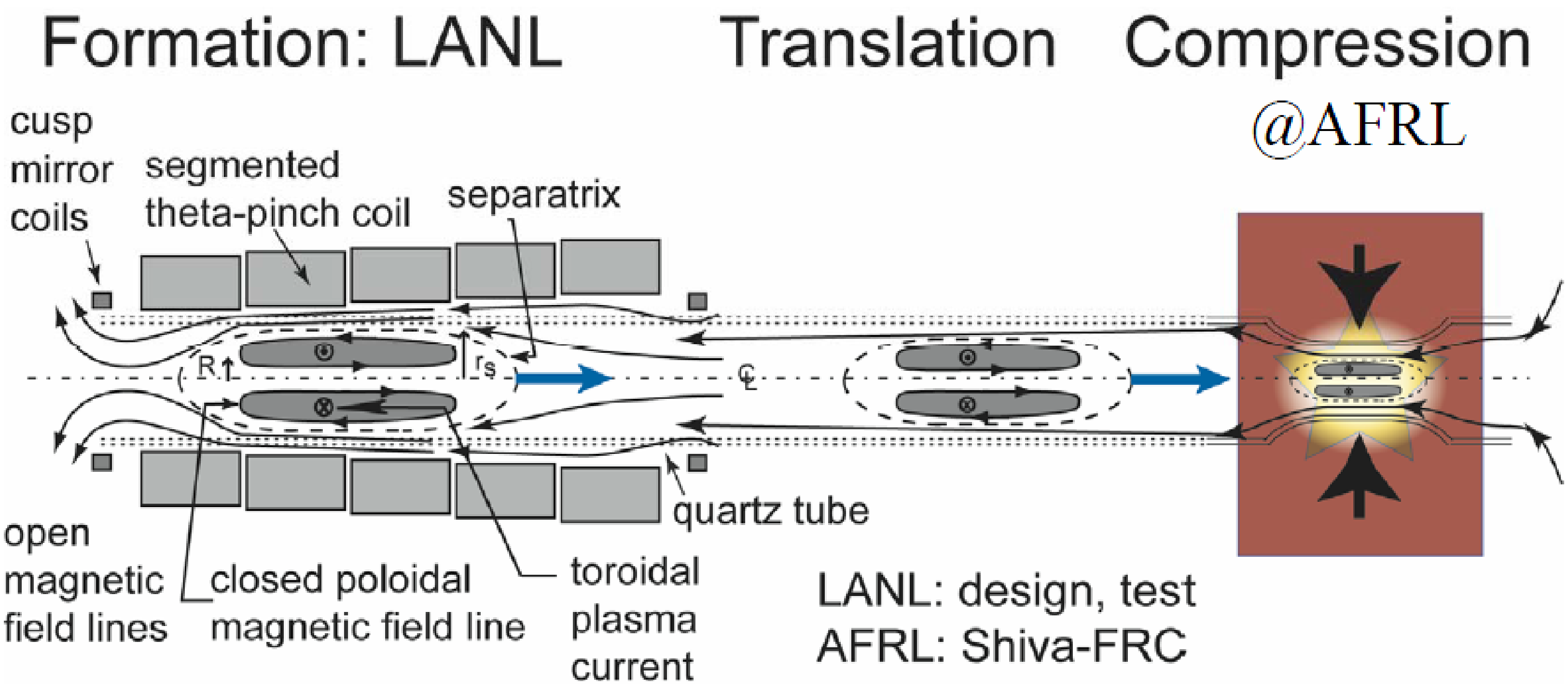
- Δt_{switch}=38.7 μs
- 3 ns exposure

Fine structure in plasma jet persists long after leaving the nozzle→Can MHD explain complex plasma dynamics?



A: Shot 65 (shutter A), 29 μs after switch closure, v_{jet}~11 km/s
B: Shot 57 (shutter A), 29 μs after switch closure, v_{jet}~19 km/s
C: Shot 59 (shutter A), 28.7 μs after switch closure, v_{jet}~21 km/s
D: Shot 65 (shutter A), 39 μs after switch closure, v_{jet}~11 km/s

Magnetized Target Fusion, liner compression of FRC, physics test



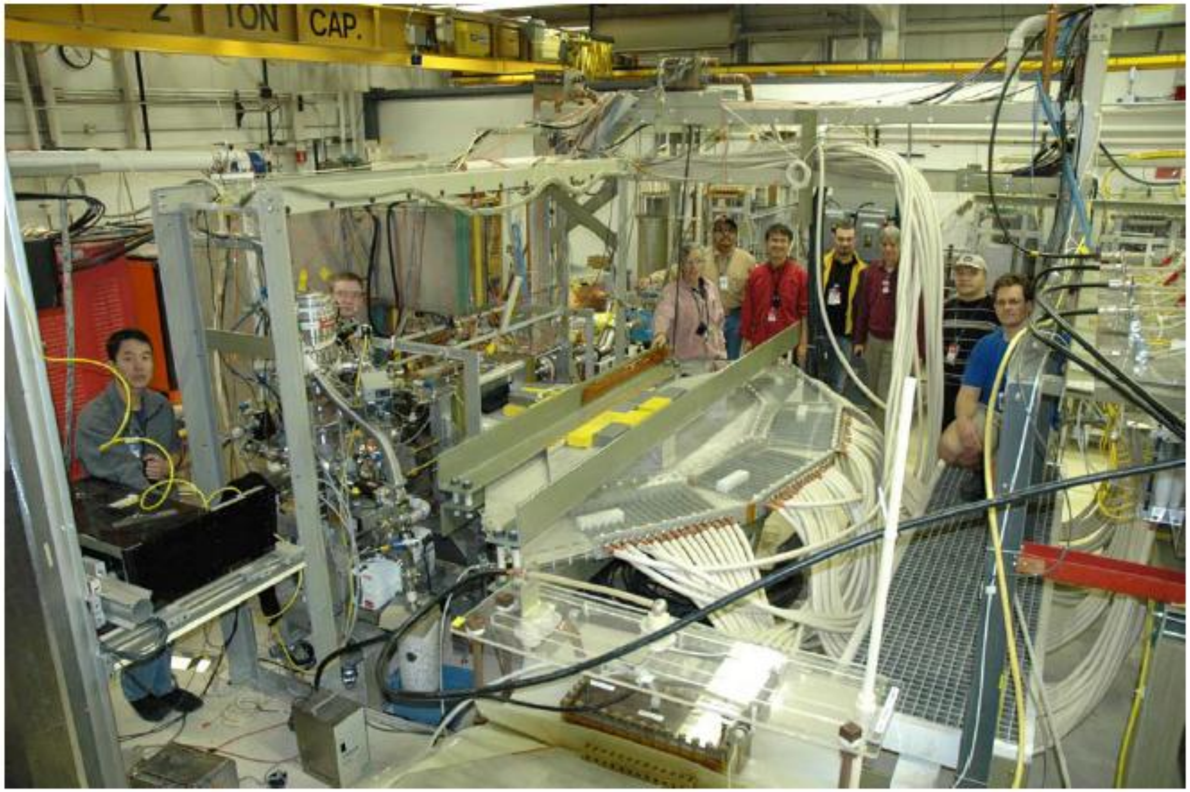
- Pulsed, high pressure approach to fusion
- Inertial + magnetic confinement
- Multi-keV fusion grade plasma

From G.A. Wurden, 2006 Fusion Power Associates Annual Symposium

LANL's FRX-L Field Reversed Configuration (FRC) for target plasma development

Project/Concept Description: Magnetized Target Fusion. Develop a suitable plasma injector using a high density FRC

GOAL: To make the first physics demonstration of MTF by imploding a field reversed configuration plasma with a metal liner



- The FRX-L experiment and team

From G.A. Wurden, 2006 Fusion Power Associates Annual Symposium

Shiva Star Facility at AFRL

Parameters for magnetic pressure implosions of cylindrical or spherical metal shells (solid liners)

- 80 to 90 kV, 1300 uF, 25 to 45 nH

- 11 to 16 Megamp, ~10 μsec risetime discharge implodes 10 cm diameter, 1 mm thick, 4 to 30 cm long Al liner in 15 to 24 μsec

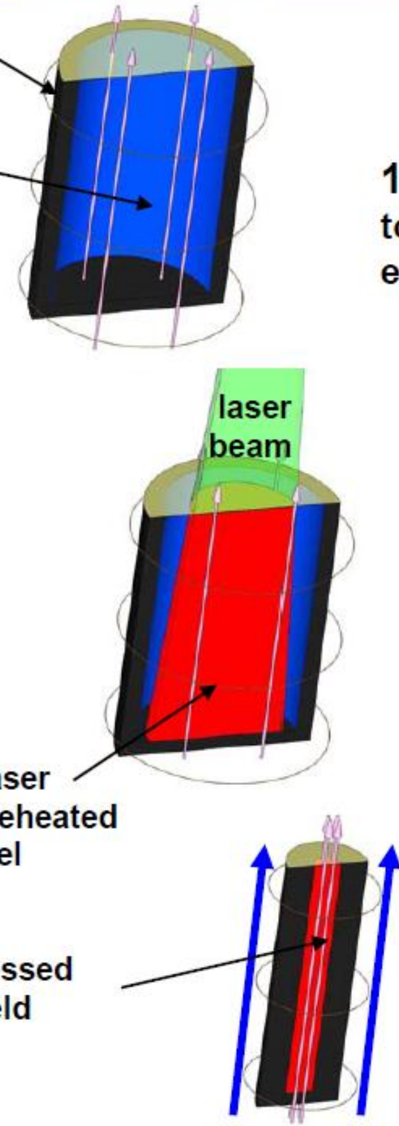
- e.g., 4.4 MJ energy storage gives 1.5 MJ in liner KE

Shiva Star Capacitor Bank (up to 9 Megajoules, 3 μsec) used for implosion - compression experiments

From G.A. Wurden, 2006 Fusion Power Associates Annual Symposium

The MagLIF concept is a 3 step process*

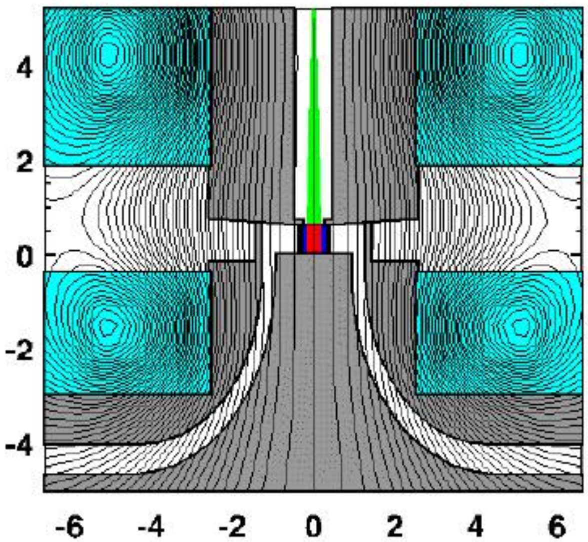
* *S.A. Slutz et al. Physics of Plasmas 17, 056303 (2010)*



1. An axial magnetic field is applied to inhibit thermal conduction and enhance alpha particle deposition

2. Z Beamlet preheats the fuel

3. The Z accelerator efficiently drives a z-pinch implosion



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From S.A. Slutz, 2012 MagLIF Workshop

For adiabatic compression, preheat reduces the require convergence ratio

For cylindrical system

$$V(t) = V_0 [r(t)/r_0]^2$$

$$V_f = V_0 [r_f/r_0]^2 = V_0 C_R^{-2}$$

Now, for adiabatic compression: $PV^\gamma = \text{constant}$ (assume γ = 5/3)

$$P_0 V_0 = NkT_0 \qquad P_f V_f = NkT_f$$

so,

$$\frac{T_f}{T_0} = \frac{P_f V_f}{P_0 V_0} = \frac{P_f V_f^{5/3} \cdot V_f^{-2/3}}{P_0 V_0^{5/3} \cdot V_0^{-2/3}} = \frac{V_0^{2/3}}{V_f^{2/3}}$$

and,

$$\left(\frac{V_0}{V_f}\right)^{2/3} = \left[\left(\frac{r_0}{r_f}\right)^2\right]^{2/3} = \left[\frac{r_0}{r_f}\right]^{4/3}$$

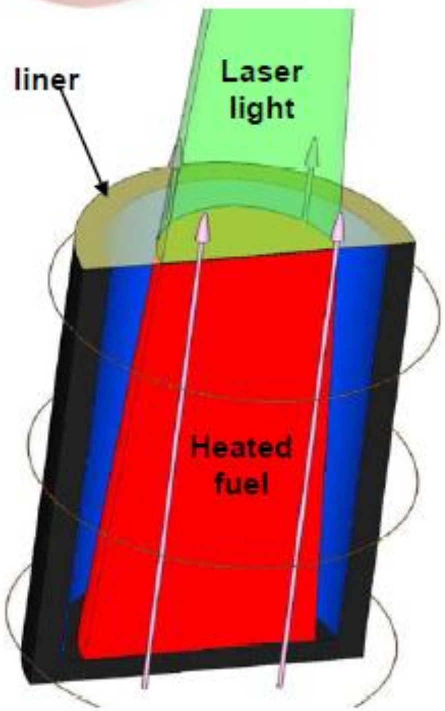
so,

$$\frac{T_f}{T_0} = \left[\frac{r_0}{r_f}\right]^{4/3} \Longleftrightarrow C_R = \left[\frac{T_f}{T_0}\right]^{3/4}$$

And, $T_f \geq T_{ignition} \sim 4.3$ keV

T_0 must be 100's of eV or C_R will be unobtainable

The Z-Beamlet laser could preheat the fuel for experiments on Z



Conditions for heating of DT gas

The critical density for green light is 17 mg/cc in DT

...initial gas density is 2-3 mg/cc implies absorption by inverse bremsstrahlung

The gas can be held in place by a thin plastic foil

...a 1 μ window will have less areal density than the gas

The total laser energy is modest

$$E_{laser} = \pi r^2 L \rho C_V \theta \qquad C_V \approx 1.2x10^8 J/g/keV$$

$$E_{laser} \approx 3.4 \left(\frac{r}{0.3cm}\right)^2 \left(\frac{L}{1cm}\right) \left(\frac{\rho_0}{1mg/cc}\right) \left(\frac{\theta}{0.1keV}\right) kJ$$

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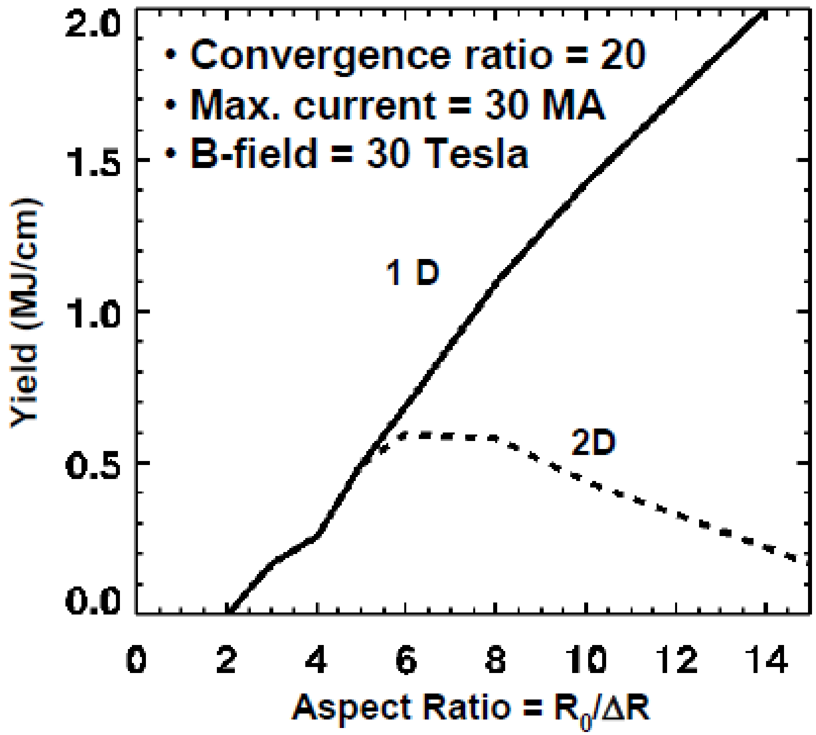
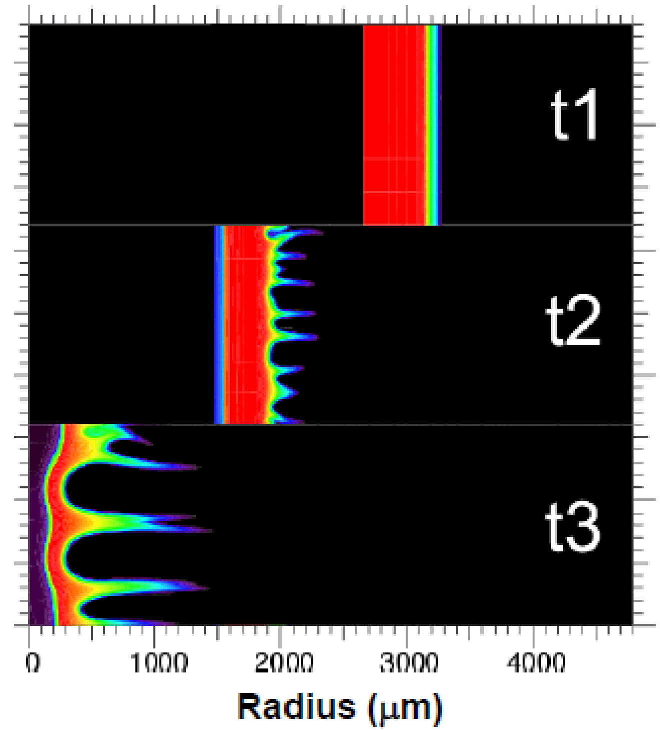
From S.A. Slutz, 2012 MagLIF Workshop

The MRT instability determines performance of MagLIF and other implosion systems





The Magneto-Rayleigh-Taylor instability degrades the yield as the aspect ratio is increased due to decreased liner ρr

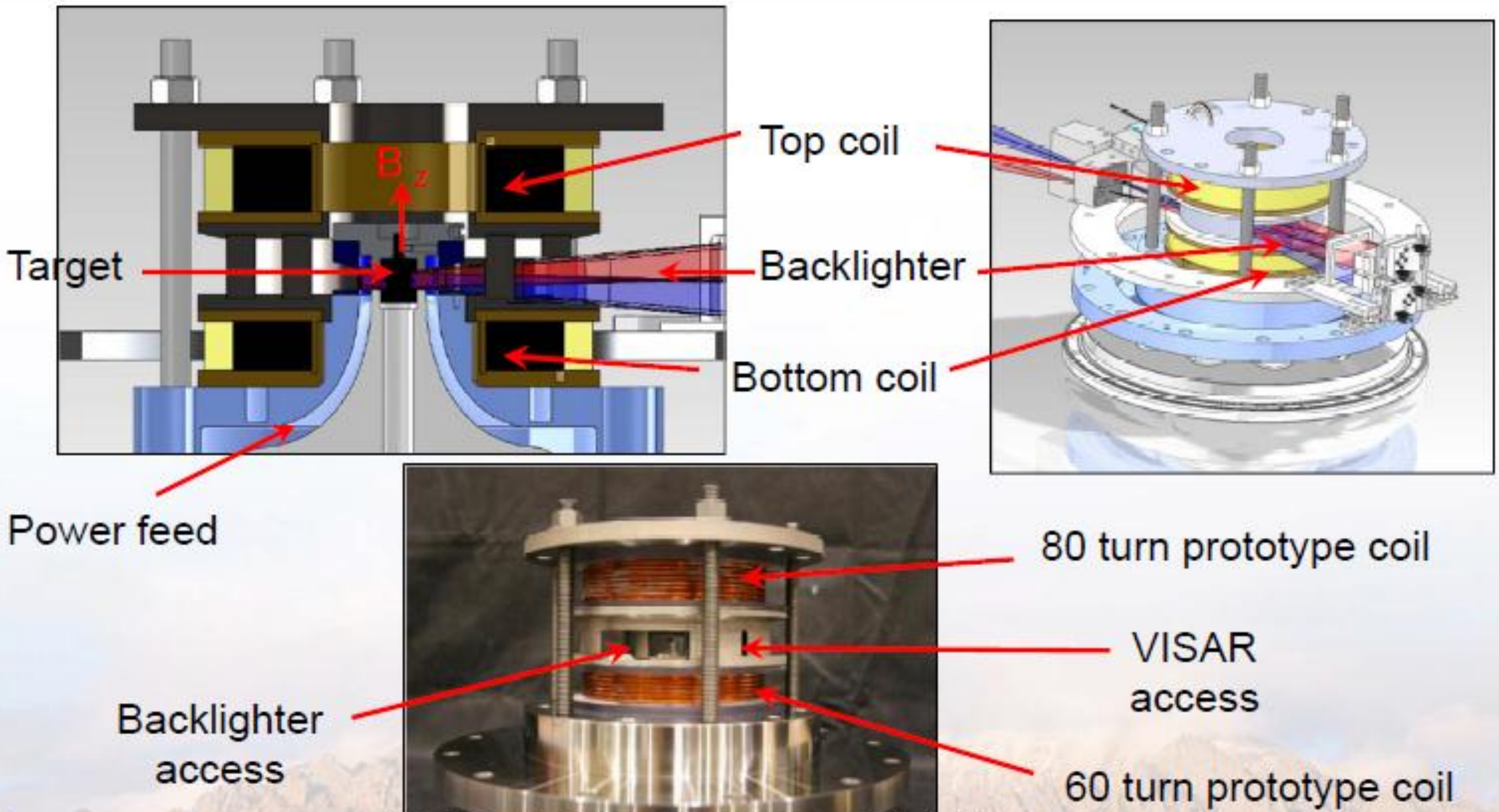
2D Lasnex Be liner simulations AR=6
- 60 nm surface roughness
- resolved wavelengths 200-1600 μm
- wavelengths of 300-600 μm near stagnation

Sandia National Laboratories

From S.A. Slutz, 2012 MagLIF Workshop

We have designed, fabricated, and tested a prototype 10-T MagLIF magnetic coil system

10 T MagLIF design with access for 2 frame backlighter

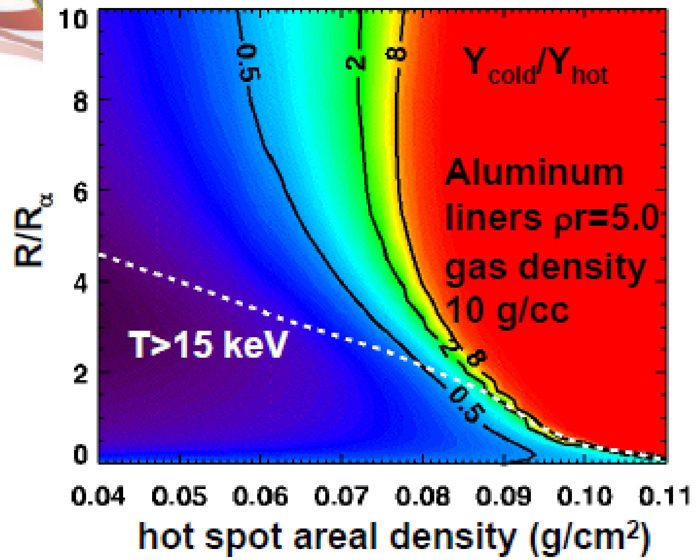


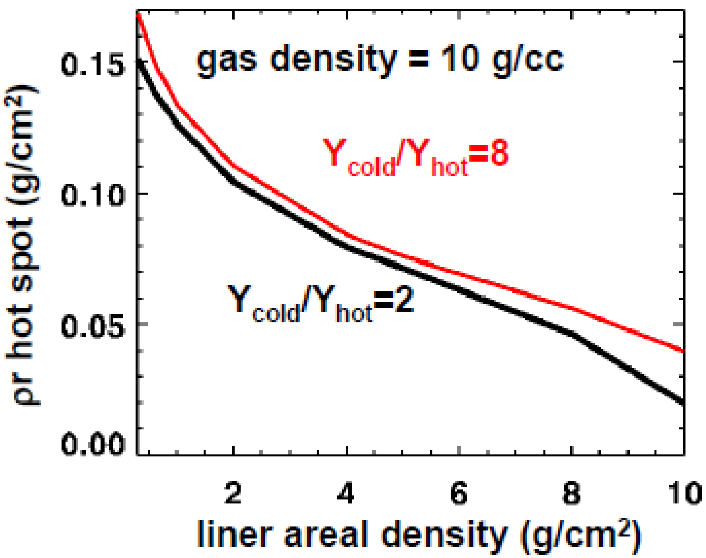
10-T MagLIF prototype assembly with test windings of coils

SAND2012-0884C

From Dean Rovang, 2012 MagLIF Workshop

There is an optimal magnetic field strength for radial propagation





- The inhibition of thermal conduction and alpha transport lowers the hot spot areal density required for for ignition
- Too large a B-field inhibits propagation into the cold fuel
- Minimum hot spot areal density is a weak function of gas density, but is significantly affected by the areal density of the liner
- More confinement time allows slower burn propagation!

From S.A. Slutz, 2012 MagLIF Workshop

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Electrical Discharge in Gases and LANL's Low-Temperature Plasma Research

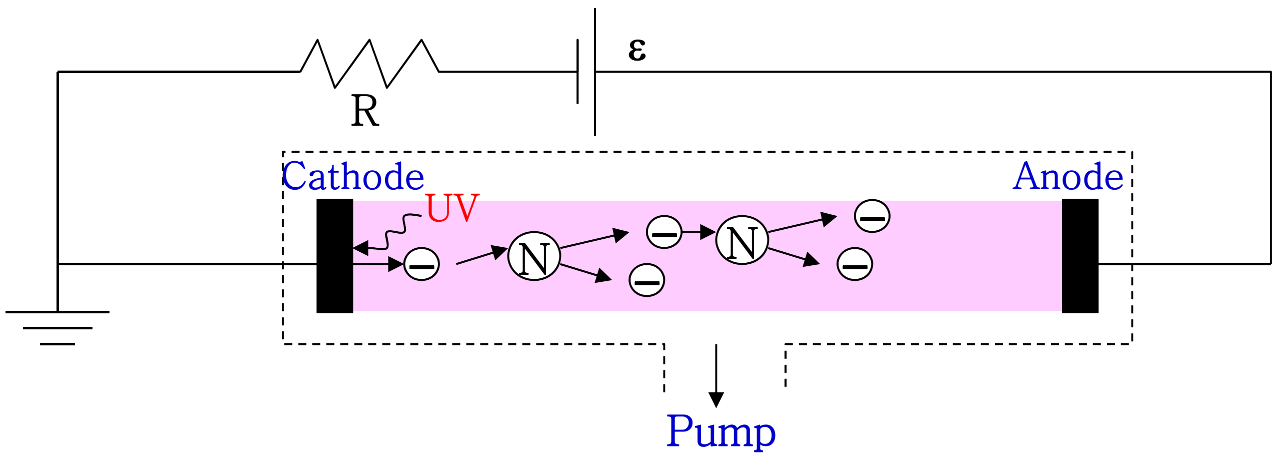
Yongho Kim, P-24

Aug. 15, 2012

Outline

- **Early work by Townsend and Paschen**
  - Electron avalanche (α-mode)
  - Breakdown condition (γ-mode)
  - Breakdown voltage
- **Low-temperature plasma properties**
  - Sheath potential, Vs
  - Plasma temperature
  - Dynamics of plasma parameters
- **LANL's atmospheric-pressure, non-thermal plasma projects**
  - Atmospheric pressure plasma jet
  - Dielectric barrier discharge

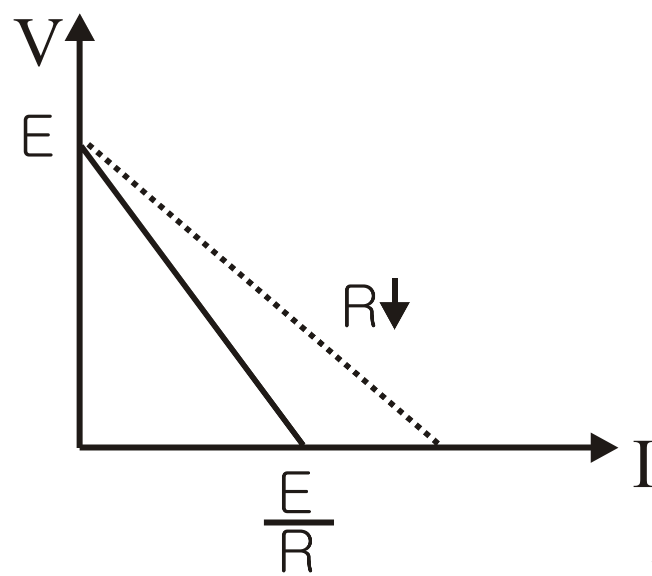
DC low-pressure discharge: the early work by Townsend and Paschen



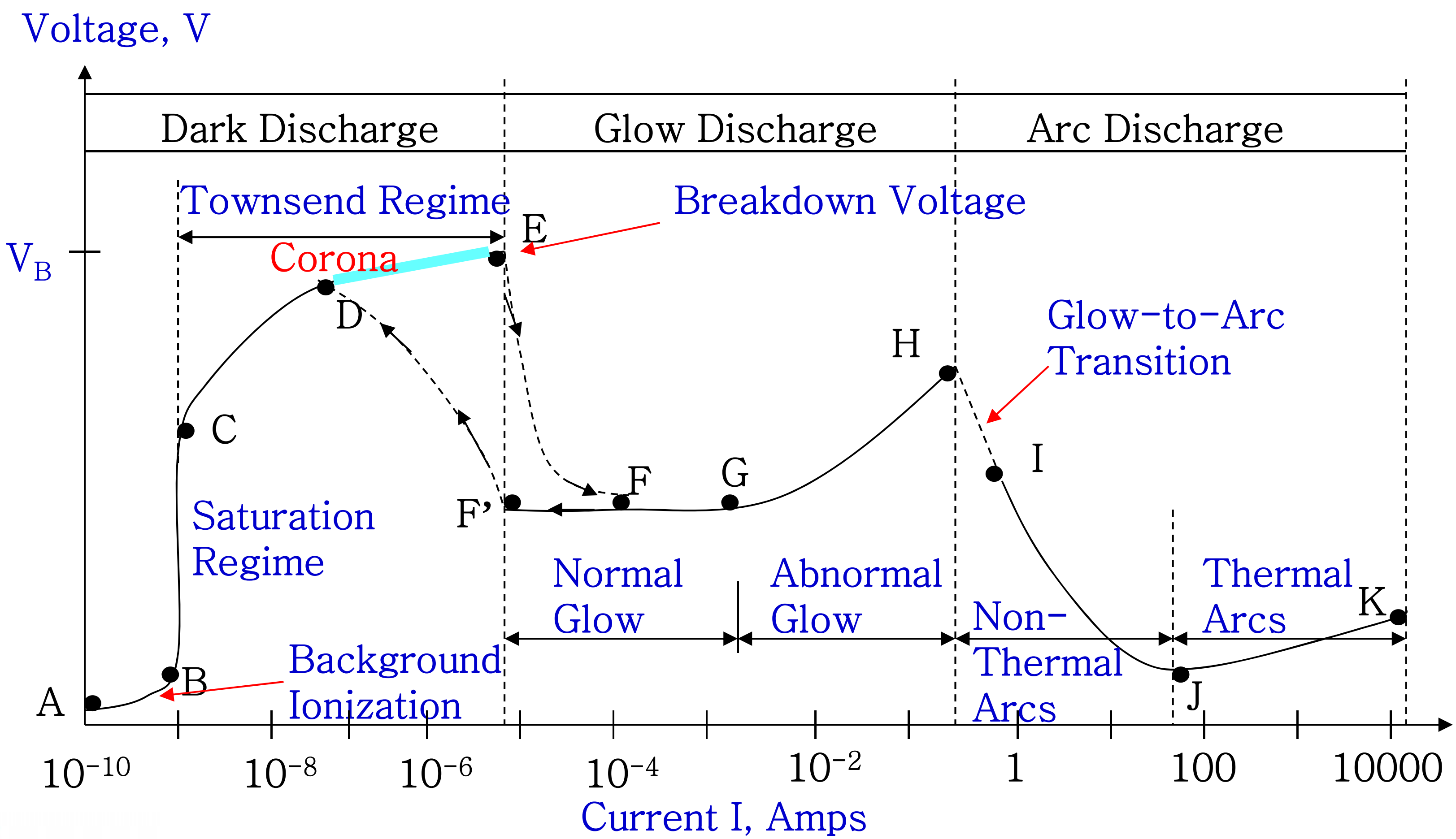
- Electric Circuit and Load Line

$$\varepsilon = IR + V$$

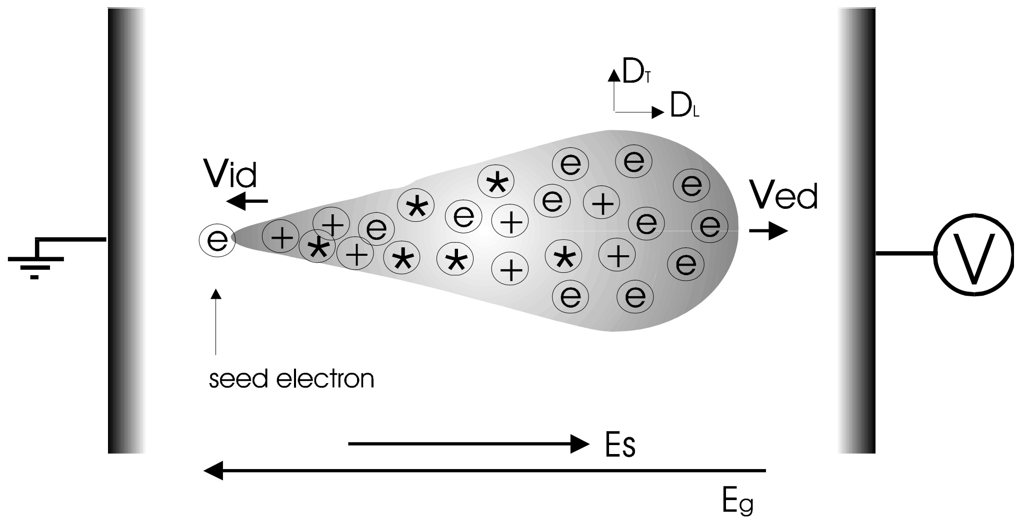
$$\rightarrow V = -RI + \varepsilon$$



I-V characteristics of DC discharge



Drift and diffusion of electron in electric field



$$v_d = \mu E = (\frac{q}{m\nu})E = (\frac{q}{m})(\frac{<v>}{\sigma N})E \quad \propto \quad f(E/p)$$

$$D = \frac{kT}{m\nu} = (\frac{kT}{m})(\frac{<v>}{\sigma N}) \quad \propto \quad f(T/p) \quad \propto \quad f(E/p)$$

Electron collisions in electric field

	electronic reaction	atomic reaction	photon reaction
excitation	$e + A_2 \rightarrow A_2^* + e$		$hv + A_2 \rightarrow A_2^*$
de-excitation	$e + A_2^* \rightarrow A_2 + e$		$A_2^* \rightarrow A_2 + hv$
dissociation	$e + A_2 \rightarrow 2A + e$	$M^* + A_2 \rightarrow 2A + M$ (Penning dissociation)	
ionization	$e + A_2 \rightarrow A_2^+ + 2e$	$M^*+A_2 \rightarrow A_2^+ + M+e$ (Penning ionization)	$hv + A_2 \rightarrow A_2^+ + e$ (photo ionization)
attachment	$e + A_2 \rightarrow A_2^-$		
recombination	$e + A_2^+ \rightarrow A_2$ $2e + A_2^+ \rightarrow A_2 + e$	$A^- + B^+ \rightarrow AB$	$A_2^+ + e \rightarrow A_2 + hv$ (radiative recombination)

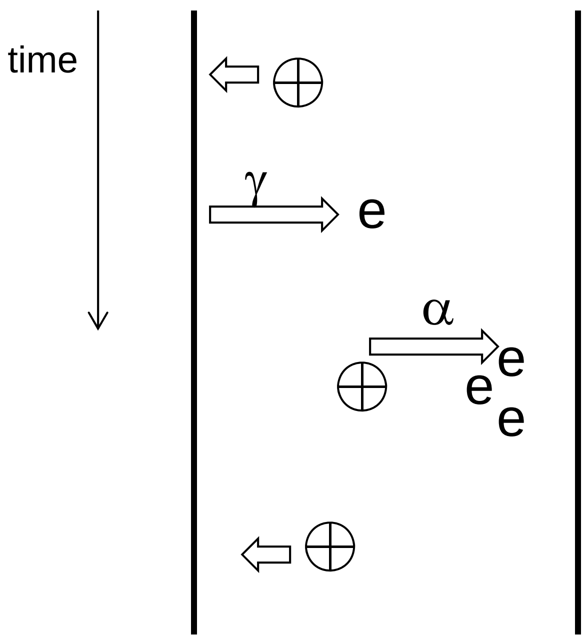
Electron avalanche and Townsend coefficient (α-mode)

- **Electron avalanche = rapid electron density growth by electron collisions**
- **Eqn. of continuity:** $$\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = n n_a < \sigma_i v >$$
- **1-D, steady state case:** $$\frac{dn}{dx} = \left(\frac{n_a < \sigma_i v >}{u} \right) n = \alpha n$$ $$\therefore n(x) = n_o \exp(\alpha x)$$
- **Townsend coefficient (ionization probability per unit length):**

$$\alpha = n_a < \sigma_i v > /u = (Ap)\exp(-Bp / E)$$

Townsend breakdown condition

- **Breakdown = onset of self-sustaining discharge produced by secondary emission from cathode by ion bombardments**



$$\gamma \equiv \frac{\#\,\text{of electrons emitted}}{\#\,\text{of incident ions or photons}}$$

$$\gamma\ \left(\frac{n - n_o}{n_o}\right) \geq 1$$

$$\gamma\ \left(\frac{n}{n_o} - 1\right) \geq 1$$

$$\gamma\ [\exp(\alpha\, d) - 1] \geq 1$$

$$\therefore\ \alpha\, d \geq \ln(1 + \gamma^{-1})$$

Breakdown voltage and Paschen curve (γ-mode)

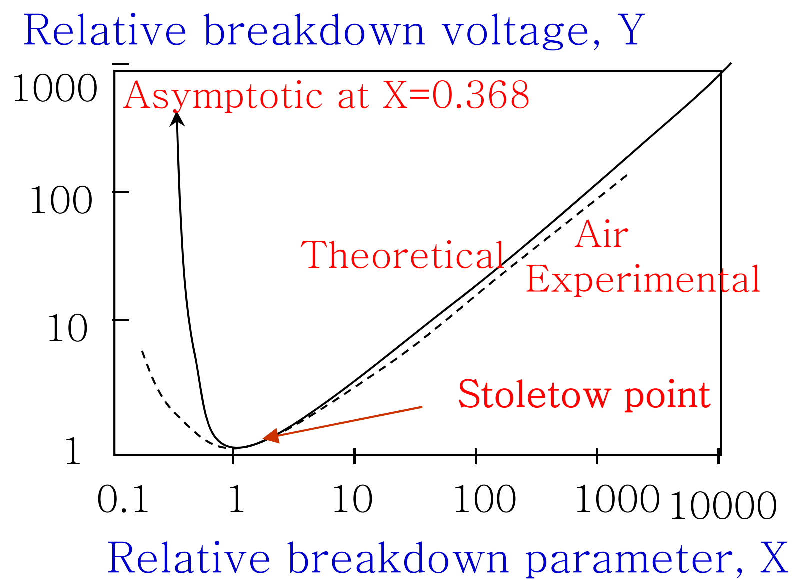
$$\alpha \equiv \frac{Nk_\alpha}{v_d} = Ap\exp(-\frac{Bp}{E})$$

from $\alpha d \geq \ln(1+\gamma^{-1})$

$$Apd\exp(-\frac{Bp}{E}) \geq \ln(1+\gamma^{-1})$$

$$Apd\exp(-\frac{Bpd}{V}) \geq \ln(1+\gamma^{-1})$$

$$\therefore V \geq \frac{Bpd}{\ln\left(\frac{Apd}{\ln(1+\gamma^{-1})}\right)}$$



Gas	Cathode	$V_{B,min}$ (V)	$(pd)_{min}$ (mTorr m)	Reference
Air		360	5.7	Brown (1966)
H_2	Pt	295	12.5	Cobine (1958)
He	Fe	150	25	Cobine (1958)
N_2	Fe	275	7.5	Cobine (1958)
O_2	Fe	450	7.0	Cobine (1958)

Outline

- **Early work by Townsend and Paschen**
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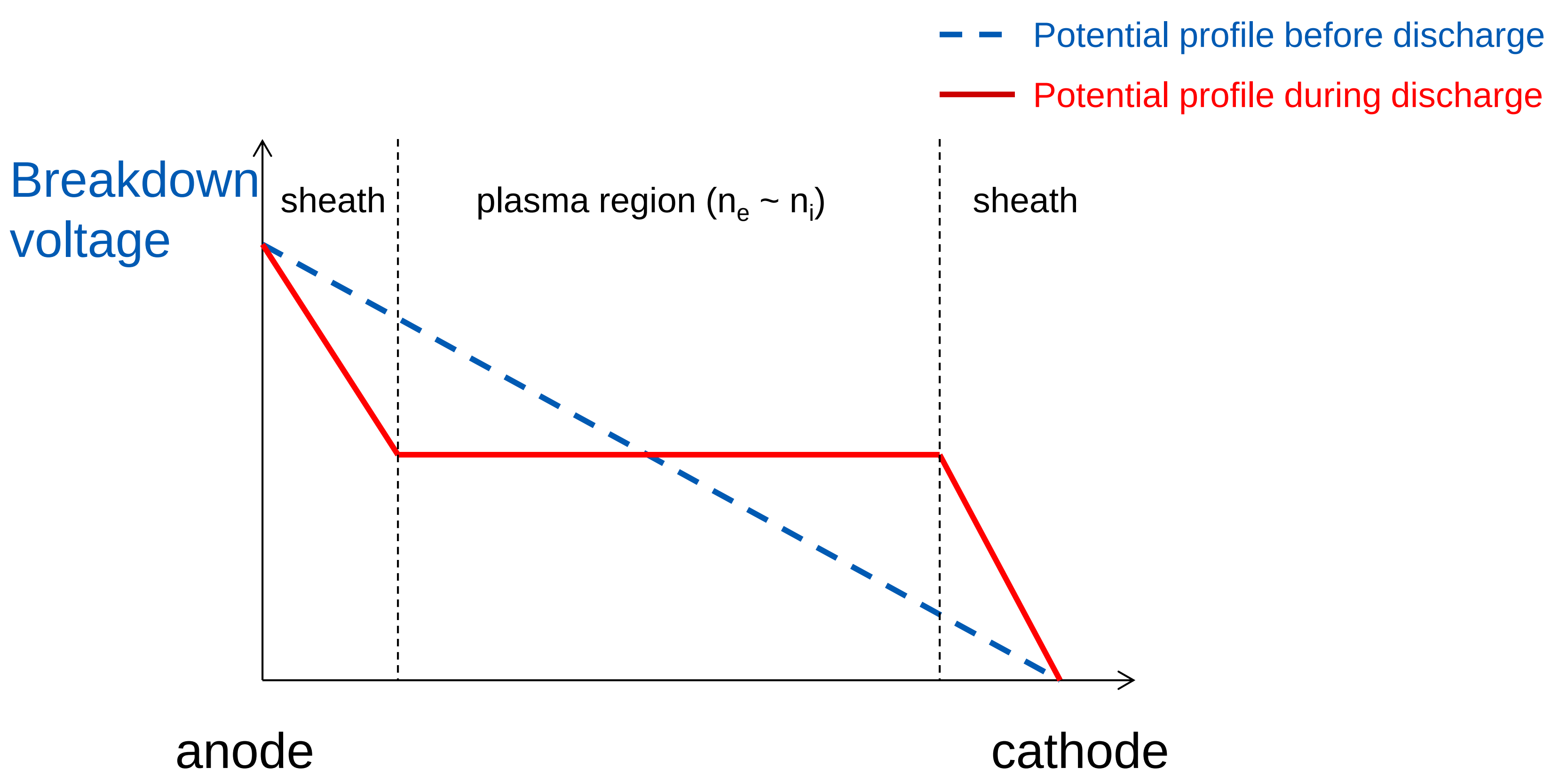
How about potential profile inside DC discharge tube



How to know plasma sheath potential (V_s)?

- **Ion Flux: The ion flux to a solid object is determined by the Bohm velocity (or sound speed) of the ion,**

$$\Gamma_i = u_B n_i = \sqrt{\frac{kT_e}{m_i}} n_i$$

- **Electron Flux: Only the most energetic electrons can overcome the sheath potential,**

  - $\Gamma_e = \frac{1}{4} n_e \langle v_e \rangle \, exp\,(qV_s/kT_e)$

    (the $\frac{1}{4} n_e \langle v_e \rangle$ term is labelled "flux to surface"; the $exp\,(qV_s/kT_e)$ term is labelled "Boltzmann factor")

- **V_s is determined by ion flux = electron flux,** $V_S = -T_e \ln(\frac{m_i}{2m_e}) \sim -5T_e$

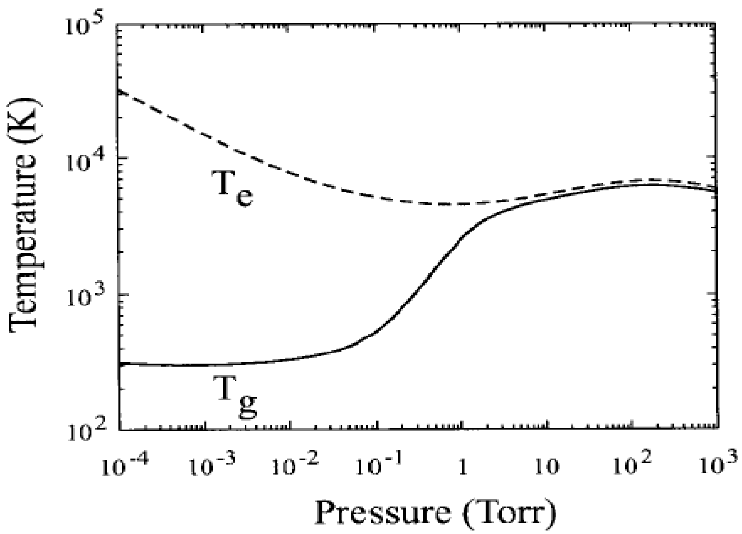
After breakdown, plasma temperature is determined

- kinetic energy exchange = energy gain by field

$$\frac{3}{2}k(T_e - T_h)\frac{2m_e}{m_h} = eEv_d t_e$$

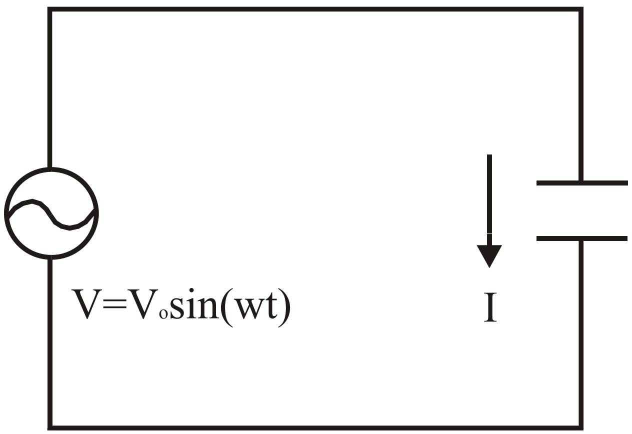
$$= eE(\frac{e}{m\nu}E)(\frac{\lambda_e}{v_e})$$

$$\Rightarrow \frac{T_e - T_h}{T_e} = \frac{\pi m_h}{24 m_e}\frac{(\lambda_e eE)^2}{(kT_e)^2} \propto f(E/p)$$



In reality, RF discharge replaces DC discharge to minimize electrode contamination and other issues

Capacitively-coupled rf discharge (before breakdown)

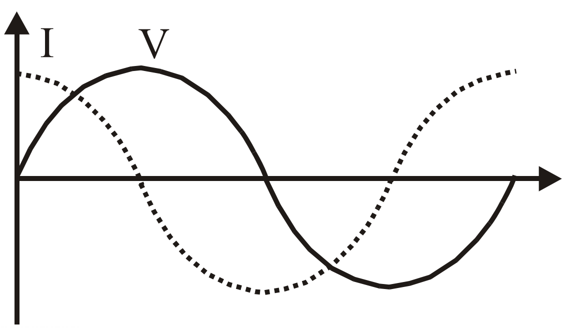


$$I = \frac{dQ}{dt} = \frac{d}{dt}(C_g V)$$

$$= C_g w V_0 \cos(wt)$$

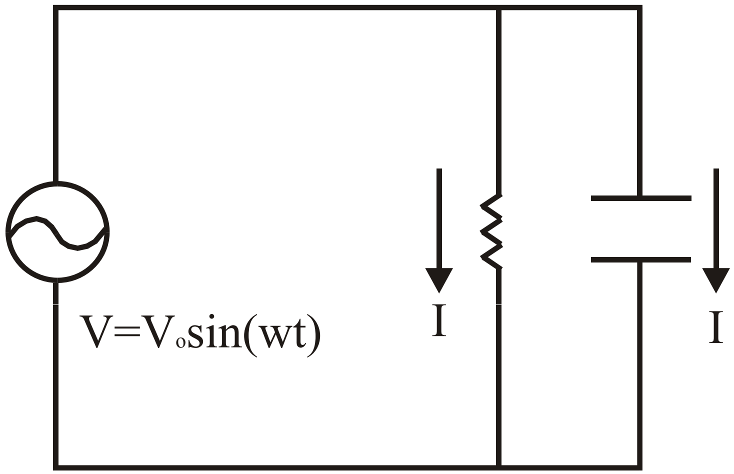
$$= C_g w V_0 \sin(wt + \frac{\pi}{2})$$

$$= jwC_g V$$



$$Z_c = \frac{V}{I} = \frac{V}{jwC_g V} = -j\frac{1}{wC_g}$$

After breakdown, impedance of rf discharge is changed by plasma resistance



$$Z_c = -j\frac{1}{wC_p}$$

$$Z_R = R_p$$

$$\therefore Z_p = \frac{1}{\frac{1}{Z_c}+\frac{1}{Z_R}} = \frac{1}{jwC_p + \frac{1}{R_p}}$$

$$= \frac{R_p}{1+w^2C_p{}^2R_p{}^2}(1-jwC_pR_p)$$

Dynamics of rf discharge operation is that plasma properties are varied as a function of electron density

Plasma capacitance

$$C_p = \varepsilon_p \frac{A_g}{d_g} = \varepsilon_o (1 - \sigma_p) \frac{A_g}{d_g}$$

$$\sigma_p = \frac{w_p{}^2}{w^2 + \nu_m{}^2} \quad , wp = \sqrt{\frac{ne^2}{m\varepsilon_o}} \quad , \nu_m = N\sigma(v)v$$

Plasma resistance

$$R_p = \frac{d_g}{\sigma_p A_g}$$

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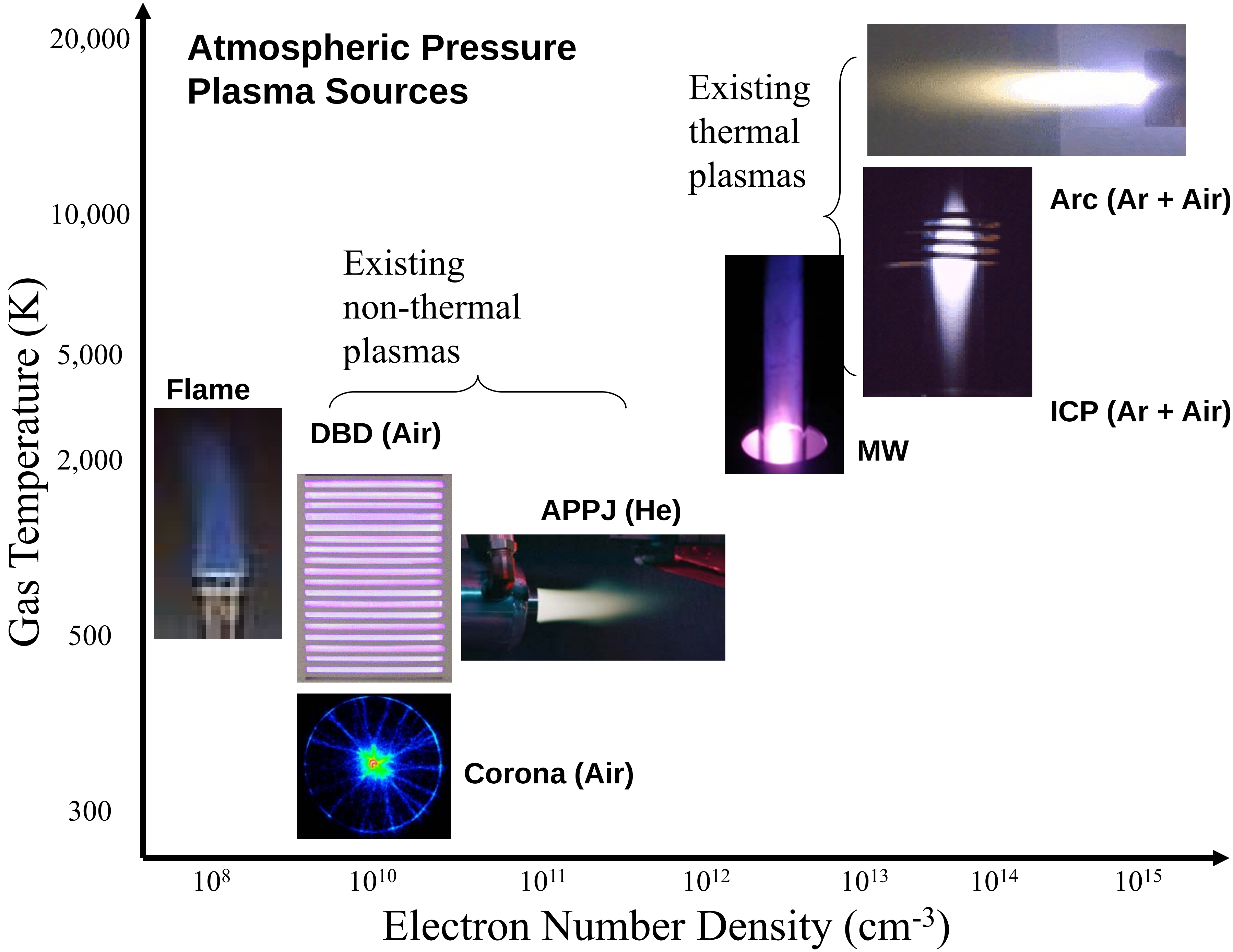
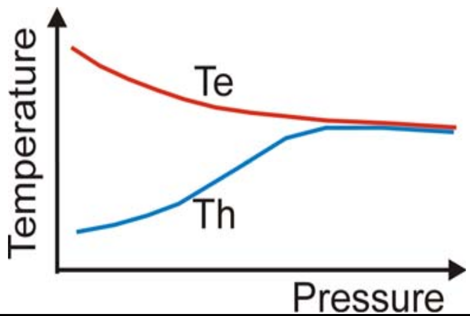


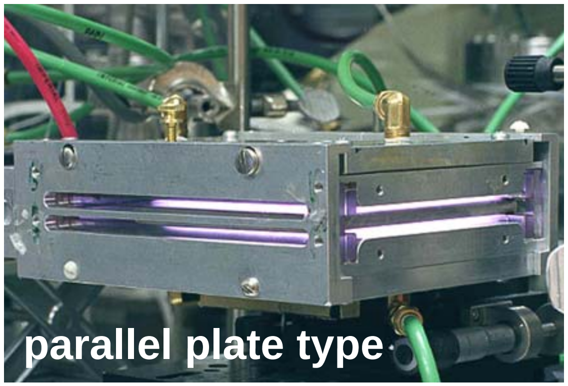
Atmospheric Pressure Plasma Sources
Existing thermal plasmas
Existing non-thermal plasmas
Arc (Ar + Air)
ICP (Ar + Air)
MW
Flame
DBD (Air)
APPJ (He)
Corona (Air)
Gas Temperature (K)
20,000
10,000
5,000
2,000
500
300
10^8
10^10
10^11
10^12
10^13
10^14
10^15
Electron Number Density (cm^-3)

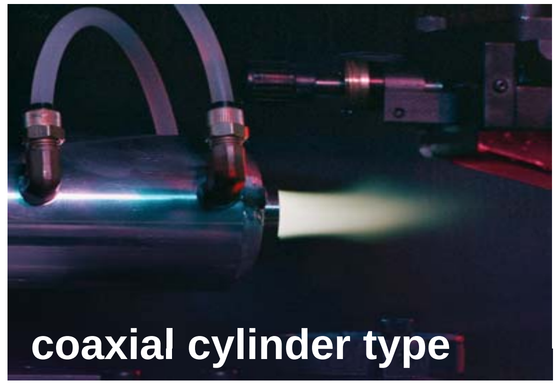
Existing Atmospheric Pressure Plasmas

	Thermal Plasma	Non-Thermal Plasma	
		Transient	Stationary
Electron Temperature (eV)	~ 1	~ 10	~ 2
Gas Temperature (K)	5000 ~ 15000	~ 300	~ 300
Electron Density (cm^{-3})	10^{16} ~ 10^{19}	10^{10} ~ 10^{12}	10^{11} ~ 10^{12}
Sources	DC Arc Torch RF-ICP **Microwave**	PCD **DBD**	**APPJ** MHCD OAUGDP

DBD: Dielectric Barrier Discharge, APPJ: Atmospheric Pressure Plasma Jet

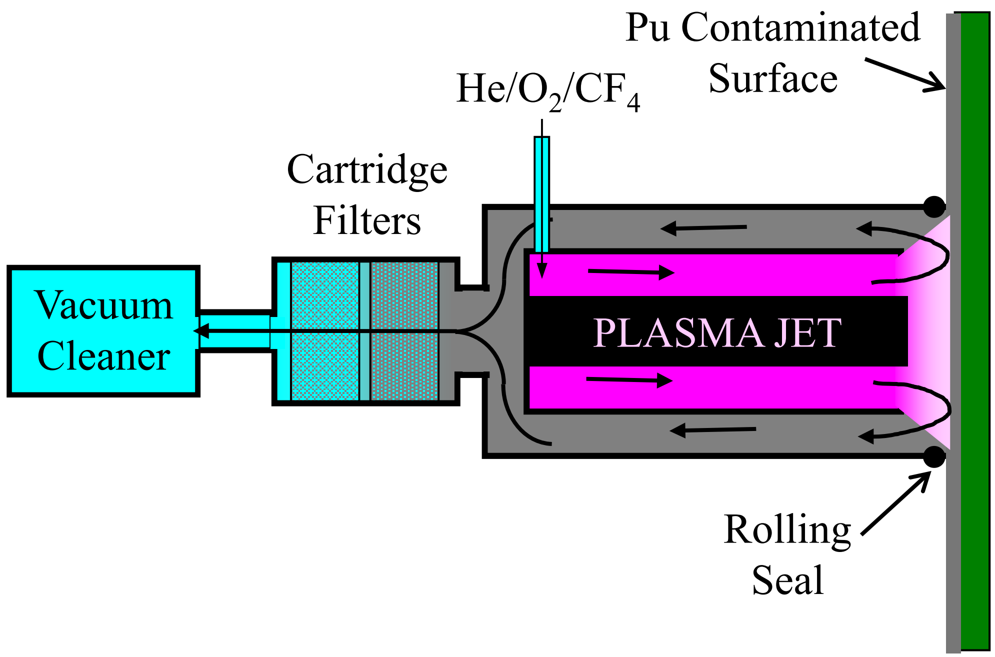
Atmospheric Pressure Plasma Jet (APPJ)





- **Capacitively coupled rf discharge** operated at atmospheric pressure
  - free of arcing (streamers) without dielectric barrier
  - rf α-mode operation at 13.56 MHz
  - helium based (~ 96 %) + additive gas (~ 4 % O_2, CF_4)
  - narrow gap (1~3 mm) operation
- **Volumetric non-equilibrium** plasma
  - oscillating steady state discharge with $N_e \sim 10^{11}$ cm^{-3}
  - T_e (1 ~ 2 eV) >> T_{gas} (50 ~ 300 ˚C)
  - abundant radicals/metastables in plasma and effluent

APPJ is designed for Decontamination of Actinides (Pu and U)



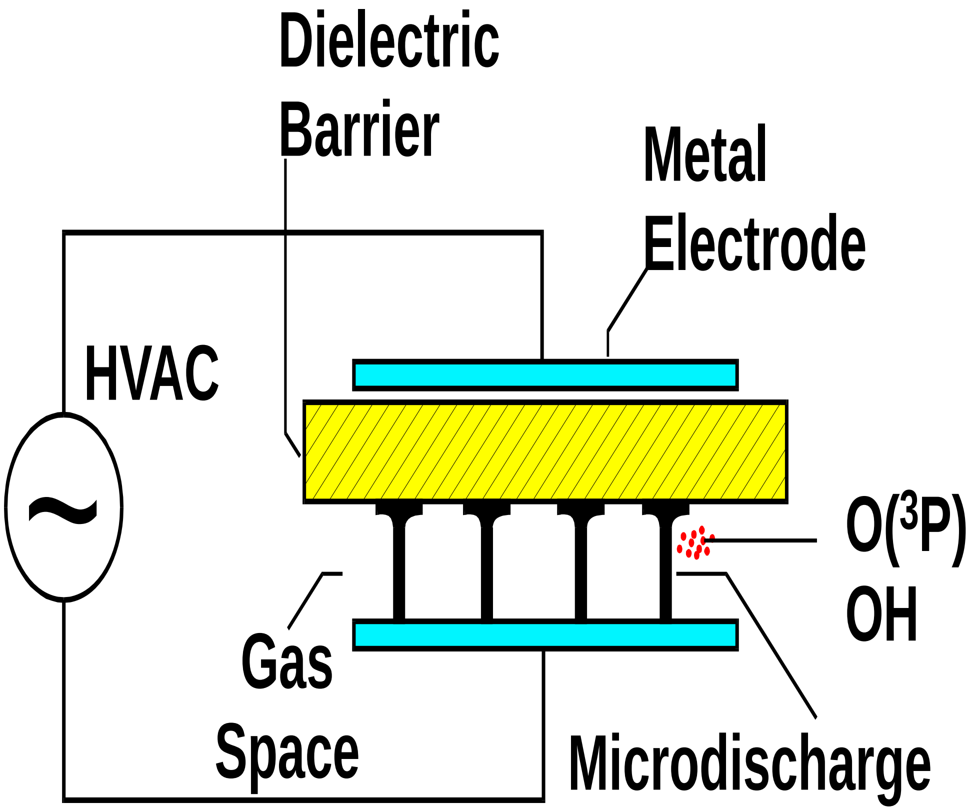
(1) In plasma, energetic electrons dissociate CF_4 forming F atoms

(2) Atomic Fluorine may etch contamination: $Pu(s) + 6F(g) \Rightarrow PuF_6(g)$

(3) Volatile byproducts (e.g., PuF_6) can be captured in adsorbent filters

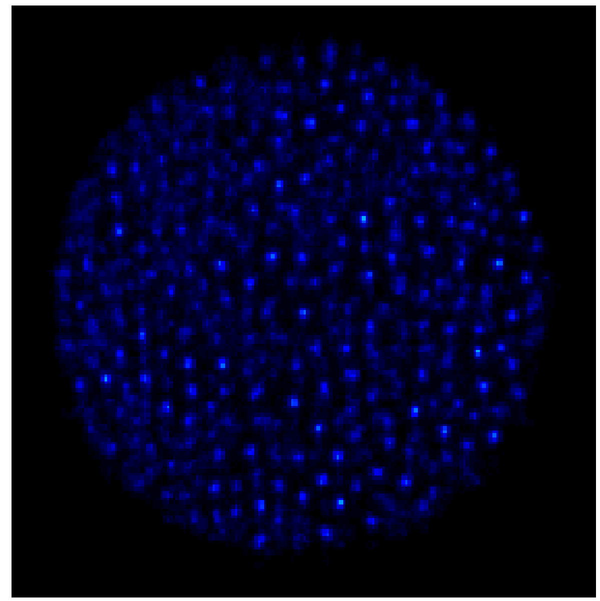
(4) We can re-cycle Pu and reduce Contaminated Volume

Dielectric Barrier Discharge (DBD)



Schematic Drawing by Louis Rosocha

Top view



ICCD image of microdischarge channels taken by Yongho Kim in 2002

Can Plasma improve Combustion Efficiency?

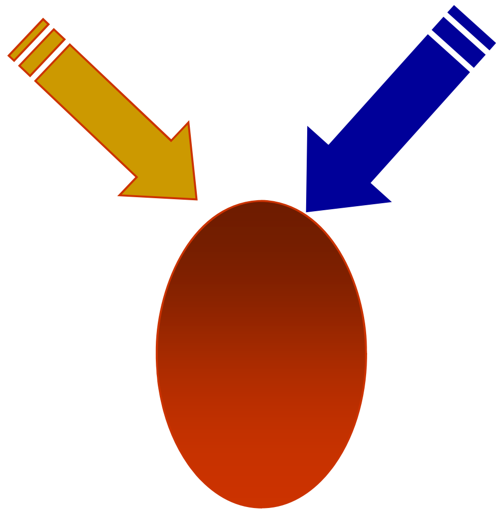
Fuel Discharge

$e + CH_4 \rightarrow CH_3 + H + e$

$CH_3 + CH_4 \rightarrow C_2H_6 + H$
$H + CH_4 \rightarrow CH_3 + H_2$

$H + H \rightarrow H_2$
$CH_3 + H \rightarrow CH_4$
$CH_3 + CH_3 \rightarrow C_2H_6$

Fuel modification
H_2 and H production



Combustion enhancement

Oxidant Discharge

$e + O_2 \rightarrow O + O + e$
$e + O_2 \rightarrow O_2^* + e$
$e + H_2O \rightarrow H + OH$

$O + O_2 \rightarrow O_3$
$H + O_2 \rightarrow O + OH$

O, O_2^*, O_3, OH production

- **plasma ignition inside combustion chamber**
- **plasma injection into combustion chamber**
- **plasma treatment of fuel**