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Title: Constrained Graph Optimization: Interdiction and Preservation Problems

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Constrained Graph Optimization: Interdiction and Preservation Problems

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Motivation

- Want to detect and block drug traffickers/illegal weapon shipments as much as possible with a limited budget

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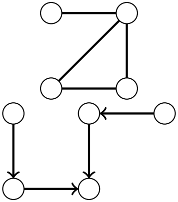
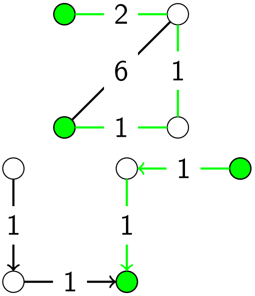
Graphs

- A network of vertices connected by edges
- **Directed graph**: edges have a start and end vertex
- **Undirected graph**: edges connect two (not necessarilly different) vertices
- Can model
  - Road networks
  - Power grids
  - Social networks
  - and much more



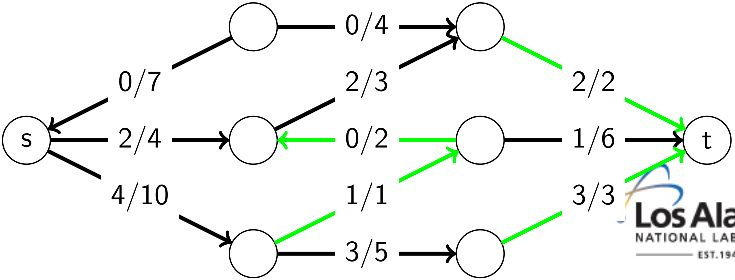
Shortest path problem

- Directed/undirected graph with weights on the edges
- Find directed/undirected path with lowest total weight between two vertices
- Applications: navigation, planning, etc.



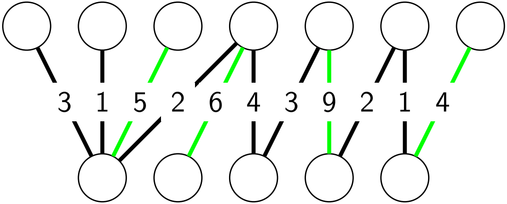
Maximum flow problem

- Directed graph with capacities on each edge
- Pump as much flow as possible from one vertex to another so that
  - the flow through each edge does not exceed the capacity
  - the flow into a vertex equals the flow out of a vertex
- Applications: flow of water through pipes, road networks, etc.



Maximum matching

- Weighted undirected graph
- Find a set of edges with maximum weight so that no two edges share a common endpoint
- Applications: scheduling jobs on a machine, assigning people to tasks efficiently

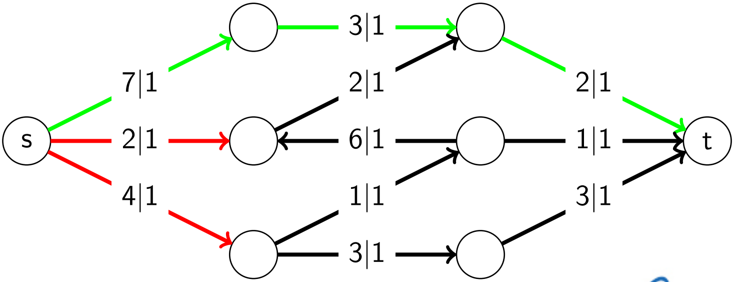


Interdiction problems

- Weighted (directed) graph with a weight and a cost on each edge
- Consider an optimization problem (i.e. shortest path, maximum flow) and fix a budget $B \geq 0$
- Find the set of edges with total cost at most B such that when the edges are removed from G
  - the length of the shortest path between two particular points is maximized
  - the maximum flow from one point to another is minimized
  - the maximum matching in the graph is minimized
  - the optimizer is impeded as much as possible
- Applications: monitoring drug trafficking, nuclear smuggling

Shortest path interdiction example

$B = 2$

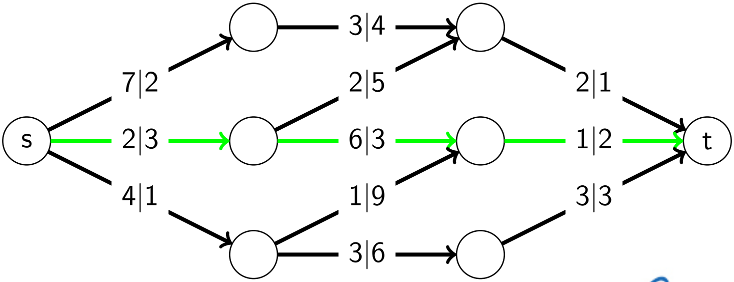


Preservation (multiple-objective) problems

- Weighted (directed) graph with a weight and a cost on each edge
- Consider an optimization problem (i.e. shortest path, maximum flow) and fix a budget $B \geq 0$
- Find the set of edges with total cost at most B such that
  - the length of the shortest path between two points using only those edges is minimized
  - the maximum flow from one point to another that is nonzero only on those edges is maximized
  - the maximum matching only containing those edges is maximized
  - the optimizer is helped as much as possible
- Applications: maintaining road networks (network flow interdiction)

Shortest path preservation example

$B = 8$

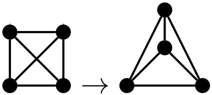
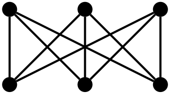


Difficulty of optimization problems

Shortest path	P
Maximum flow	P
Maximum matching	P
Shortest path interdiction	not 2-approximable
Maximum flow interdiction	Strongly NP-complete
Matching interdiction	Strongly NP-complete
Shortest path preservation	Pseudo-P
Maximum flow preservation	**Strongly NP-complete**
Matching preservation	PTAS

- P: polynomial time solvable (fast to solve)
- NP-complete: not polynomial time solvable unless $P = NP$ (hard)
- Strongly NP-complete: not pseudo-polynomial time solvable unless $P = NP$ (very hard)
- *PTAS*: polynomial-time approximation scheme (approximable)

Planar graphs

- A graph that can be represented in the plane with no edge crossings
- Why we care:
  - Road networks, power grids, and other important networks often are almost planar
  - NP-complete problems can often be approximated well on planar graphs
- Example 
- Nonexample 

Difficulty of hard optimization problems on planar graphs

Shortest path interdiction	not 2-approximable	Unknown
Maximum flow interdiction	Strongly *NP*-complete	Pseudo-*P*
Matching interdiction	Strongly *NP*-complete	**Pseudo-*PTAS***
Shortest path preservation	Pseudo-*P*	N\ A
Maximum flow preservation	**Strongly *NP*-complete**	Unknown
Matching preservation	PTAS	N\ A

- *P*: polynomial time solvable (fast to solve)
- *NP*-complete: not polynomial time solvable unless $P = NP$ (hard)
- Strongly *NP*-complete: not pseudo-polynomial time solvable unless $P = NP$ (very hard)
- *PTAS*: polynomial-time approximation scheme (approximable)

References

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- Leonid Khachiyan, Endre Boros, Konrad Borys, Khaled Elbassioni, Vladimir Gurvich, Gabor Rudolf, and Jihui Zhao. On shortest paths interdiction problems: Total and node-wise limited interdiction. *Theory of Computing Systems*, 43(2): 204-233, 2008.
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Summary of results

- A pseudo-polynomial time approximation scheme for
  - Matching interdiction on planar graphs
- Proof that maximum flow preservation is strongly NP-complete on general graphs

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Abstract

The maximum flow, shortest path, and maximum matching problems are a set of basic graph problems that are critical in theoretical computer science and applications. Constrained graph optimization, a variation of these basic graph problems involving modification of the underlying graph, is equally important but sometimes significantly harder. In particular, one can explore these optimization problems with additional cost constraints. In the preservation case, the optimizer has a budget to preserve vertices or edges of a graph, preventing them from being deleted. The optimizer wants to find the best set of preserved edges/vertices in which the cost constraints are satisfied and the basic graph problems are optimized.

Abstract (cont.)

For example, in shortest path preservation, the optimizer wants to find a set of edges/vertices within which the shortest path between two predetermined points is smallest. In interdiction problems, one deletes vertices or edges from the graph with a particular cost in order to impede the basic graph problems as much as possible (for example, delete edges/vertices to maximize the shortest path between two predetermined vertices). Applications of preservation problems include optimal road maintenance, power grid maintenance, and job scheduling, while interdiction problems are related to drug trafficking prevention, network stability assessment, and counterterrorism.

Abstract (cont.)

Computational hardness results are presented, along with heuristic methods for approximating solutions to the matching interdiction problem. Also, efficient algorithms are presented for special cases of graphs, including on planar graphs. The graphs in many of the listed applications are planar, so these algorithms have important practical implications.