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Uncertainty Budget Analysis for Dimensional Inspection Processes (U)

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Abstract

This paper is intended to provide guidance and describe how to prepare an uncertainty analysis of a dimensional inspection process through the utilization of an uncertainty budget analysis. The uncertainty analysis is stated in the same methodology as that of the ISO GUM standard for calibration and testing. There is a specific distinction between how Type A and Type B uncertainty analysis is used in a general and specific process. All theory and applications are utilized to represent both a generalized approach to estimating measurement uncertainty and how to report and present these estimations for dimensional measurements in a dimensional inspection process. The analysis of this uncertainty budget shows that a well-controlled dimensional inspection process produces a conservative process uncertainty, which can be attributed to the necessary assumptions in place for best possible results.

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1. Introduction

Measurements are taken in everyday manufacturing processes. It could be as simple as measuring the thickness of a quarter using a caliper to the measuring of a turbine blade on a boat propeller using a coordinate measuring machine (CMM). Measurements are used for a quantitative (numerical) portrayal of a qualitative (description) item. For example, the ambient temperature of dimensional inspection laboratories must be in a range of 20±1ºC for metrology purposes.

All measurements are subject to a measurand, an object being measured. This measurand can be anything from a diameter measured under a standardized temperature to the volume of a vessel with a standard pressure, as long as there are specific conditions to the measurement in question. Measurands are a necessity in all measuring routines and procedures.

With the measurand stated, measurements of the measurand are taken. Measurements, all types, have an associated error. Error is the difference between the "true" values of a measurement to the actual measured values. The true value is never actually known, but a good estimate is assessed by taking repeated measurements of the measurand in question and estimating with the arithmetic mean (average). The average value will give a good indication of where the true value might lie, though it lacks credibility because it does not have a range of possible values. The parameter that quantifies the boundaries of this error of the measurements is called the uncertainty of the measurement. This uncertainty is defined as the parameter associated with the result of a measurement that characterizes the dispersion (spread) of the values that could reasonably be attributed to the measurand. This uncertainty analysis will be used for manufacturing purpose in the dimensional inspection processes.

This paper is intended to explain the detail in understanding, utilizing and reporting an uncertainty in measurements, both generally and for specific processes. Basic theory and mathematics will be included to give the reader the knowledge to understand and use the basic laws of uncertainty for measuring and stating uncertainty. With this analysis an uncertainty budget with tabulated numerical values of uncertainty sources, will be calculated for a specific dimensional inspection process. This paper will not provide derivation of the laws of propagation of uncertainty or the use of higher order terms, but will use assumptions in place for a more practical approach to measurement uncertainty. Higher-ordered terms will be explained, however not derived, in appendix 2. Small examples will be used to emphasize theory, and a specialized uncertainty budget will be provided in the text. All uncertainty principles, formulas and theory can be traced back to the ISO GUM document [1].

2. General Principles

2.1. What is uncertainty?

What exactly is uncertainty? Uncertainty is the measure of the dispersion that may reasonably be associated with the intended measured value. It gives a possible range, centered about the measured value, within which a stated probability of where the "true" value may lie. It is usually a plus or minus limit, though it is not universally always stated in this form.

The measuring instruments and systems usually require a scaling factor or correction for all intended measuring purposes. These corrections are used in order to correct to the necessary standard(s) used in calibrating the measuring instrument. It is essential and applicable to make any necessary corrections before any attempt is made toward assessing the uncertainty of a measurement. When calibrations of the measuring instruments are conducted, correction factors are implemented into the calibration of the instrument, in order to lower the uncertainty value associated with the calibration certificate. Depending on the measurement being conducted, the dimensional inspection group will either correct for large errors (straightness on a long straight edge that is in need of an overlapping algorithm) or assume negligible uncertainty effects do to other sources (temperature compensation on CMMs). The uncertainty stated in this paper is more geared toward testing implementation and not calibration, because all instruments are calibrated to a certified standard through a certified calibration laboratory.

Most uncertainties are assumed to have a 95% confidence level, where a coverage factor k is used to expand a combined standard uncertainty. This coverage factor can range in different values depending on the probability distribution used for the uncertainty. For instance, a coverage factor of two is used for a normal (Gaussian) distribution. This coverage factor of two is also used for the combined standard of an uncertainty budget, because the combined distributions are assumed normal at a 95% confidence level. The 95% is used in all calibration and testing laboratories and is an established practice in Europe, Asia and North America. Again the ISO guide assumes a 95% combined normal distribution on uncertainty budget analysis.

The first step after setting up the measurand, is constructing models of the measurements. A model equation is what is needed in order to figure what factors are affecting the measurand and contributing to the final uncertainty result. This requires a good understanding of metrology principles, the equipment and the environment. This typically is the most difficult and crucial part of calculating uncertainty. The model will be assumed linear, since higher-ordered terms are not considered in this report, however, if higher-order terms are present, do not neglect them and consider them in the model equation. Correlation is when two factors are independent, but can significantly affect one another. Correlation is considered in the specific example approach, considering it is difficult to control all factor influences. These correlations will be detailed later in the paper.

After a model is set, uncertainty must be separated into two types: Type A or Type B. Type A uncertainty is calculated with statistical analysis. A wide group of repeated measurements must be taken in order to use a Type A uncertainty. Type B uncertainty is found by any other means then statistical analysis. This can be done from manufacture specifications, calibration certificates, historical knowledge, etc. Just as a general approach, the specific process approach will use both Type A and Type B methods, based on the influencing quantities and all will factor into the uncertainty budget.

Uncertainty can come from any type of measurement, thus leading to different types of units used. This is acceptable for finding standard uncertainties, but must be converted to the units defined by the measurand. For example, a measurand with units of MPa (Mega Pascal) and one quantity is the sensitivity of a test temperature, so these units must be equated. The temperature measurements and thus uncertainty are in ºC, so a conversion factor or weighing factor will be required to obtain the effects from the temperature uncertainty term. Such terms are known as sensitivity coefficients. Failure to apply these sensitivity coefficients will result in gross errors and non-sensible uncertainty values. Neglecting or incorrectly estimating the sensitivity coefficients are common causes of erroneous uncertainty estimates. In the specified example approach, the conversion factors or sensitivity coefficients will be evaluated where possible or necessary.

Finally the standard uncertainties of all the influencing quantities must be combined to give a combined standard uncertainty, leading to an expanded uncertainty of a measuring system or process. The laws of propagation uncertainty use a Root-Sum-Squared (RSS) method, because of the assumption of a normal distribution of combined standard uncertainties. This will be the provided example's approach, considering most of the factors can be calculated from statistical methods. This may not always be the correct way, considering if higher-ordered terms are involved. Based off assumptions, higher-ordered are negligible to other sources. A coverage factor of two will be used for the expanded uncertainty, since most calibration laboratories adhere to a standard practice of 95% confidence in all calibration certificates.

2.2. Error

Every measurement has an associated error. Therefore, every measurement process also has error. Error is the difference between the "true" value and the measured value. No measurement is ever one hundred percent correct, so error must be associated with this measurement value. Error can be calculated from the following equation:

$$\varepsilon = x_{true} - x_{measured} \quad (1)$$

Where ε , is the error in the measurement, x_{true} is the true value of the measurement, not actually known, and $x_{measured}$ is the actual measured value from the measuring instrument.

The best estimate of the true value or the central value of a measurement is through the arithmetic mean, or average, of the measured values. However, this average may not be the true value. It may have been biased away from the true value by a combination of errors. If these errors could be accurately determined then they could be applied as corrections to the mean value, which would therefore bring an agreement into the true value of the measurement. These errors can be classified into two different categories: random and systematic. These two types of error will be combined to effectively classify the total error in the measurement. In the total error, there will be at least one component of systematic error and one component of random error for the total error. A significant effort will be used to best try to identify and correct, where achievable, errors. Errors will be detailed more in sections 2.3 and 2.4.

2.3. Systematic Error

Systematic error is error which remains constant while the factor that it is influencing remains constant during measuring processes. Systematic errors, like random errors, are complicated to eliminate because the corrections are difficult to assign because their actual magnitude and sign are not precisely known. However, with a well defined measurand and well characterized sampling patterns and measuring instruments, these errors can potentially be reduced. The measurands for the processes are thoroughly thought out and analyzed for best representation of the process and the sampling patterns are well-characterized through historical testing and data.

Some systematic errors can be reduced or possibly eliminated by a careful and well thought out proven measuring method, by an extensive calibration process to determine the actual error sign and magnitude, or in some cases, by changing the influencing quantities over a range and performing repeated measurements as to either characterize or randomize the error. Randomizing the order in which the measurement is made, will help determine if there is a possible way to minimize the systematic error and help minimize the nuisance factors. Corrections should always be applied when possible. This will allow for the best possible measurement estimation and the best representation of the true value.

Using a variety of different measurement systems on the same measurand will help detect the systemic error. This will only be effective if the measuring systems have a comparable resolution. This will not work if say an optical system is used to measure a surface and then compared to a contact surface analyzer. Their resolutions and orientations can be very different from one another. Other useful aides could be inter-laboratory comparisons and proficiency tests. The dimensional inspection group has done both inter-laboratory and inter-complex testing on different measuring techniques and systems. This has proven to be beneficial and helped correct for systematic errors. This is highly recommended when possible.

The dimensional inspection group has defined, as best as possible, measuring methods for reducing as much systematic error as possible. The use of calibrated standards and instruments are implemented into the measuring process to help identify if anything abnormal has happened during the measuring process. These quality checks are used during and after the measuring process to ensure robust quality. These checks are intended to help lower the actual uncertainty of the process.

2.4. Random Error

Random error presumably arises from the unpredictable and spatial variations of influential qualities. It can also be stated as random variation on the measurement. Typical sources for these errors are many and are inherited in the measuring instrument, the artifact or item under testing, the measurement procedure and the test environment. In many cases random error components will be immediately obvious because it occurs at a rate and amplitude sufficient to make taking a particular reading very difficult. If a random error is not detected in repeated observation, then the measuring instrument is not sensitive enough to detect the errors, however, this does not mean it is not good enough for its purpose.

Estimating a random error can be done with a Type A analysis, which will be described in section 3.5. This analysis will require repeated measurements. Performing at least 20 or more repeated measurements is often the standard sample size and produces a more accurate representation of the results. To fully account for all random sources of error the measurement system should be assembled and disassembled between each measurement. The latter statement is not feasible for large scale measuring systems and is intended for smaller scale systems. This will not be done in the dimensional inspection group's case.

When repeated measurements cannot be taken, then predicting the random error is necessary. This is a Type B analysis. The estimate of this component must be based on prior experience on instabilities of the particular measuring instrument or other measuring instruments of the same type. The dimensional inspection group uses calibrated standards, prior working knowledge and experience in characterizing these errors. This can be one of the more challenging aspects if a good understanding of the instrument and short history are not adequate.

2.5. Assumptions

The following assumptions are set to ensure that the best representations of the estimated uncertainties are accounted for. These assumptions are both from a generalized uncertainty budget approach and specific example of the dimensional measurement processes. The assumptions are as follows:

- The measurement process is considered "well-behaved" and under process control to ensure validity to the uncertainty budget

- The effects of higher-ordered term and non-linearity are negligible, compared to other sources
- Uncertainty components must be estimated when not available from a source, such as a calibration certificate and must be estimated, it is sufficient to make a worst case limits and use a rectangular distribution
- A nominal coverage factor may be used. (Using an assumed normal distribution of 95% will be accompanied by a coverage factor of two for expanded uncertainty values.)
- Temperature fluctuates in a constant range of 20±1ºC and instruments use temperature compensation to adjust to these fluctuations. If an instrument does not have temperature compensation, then the Gauge Repeatability and Reproducibility (GR&R) analysis will be used for an estimation of the temperature fluctuations, as well as other environmental effects

- Component's surface finish, surface profiles and flatness all meet drawing specifications

Correlation or interaction between sources will be considered, only during the GR&R portion of the analysis. In turn, it will only compensate for the part-to-operator and part-to-part portion of the data. For testing purposes, other correlated values will be assumed unattainable or negligible to other sources.

These assumptions will be followed for any specified instrument in a dimensional measuring scenario. However, these assumptions can be altered and more can be added if necessary. The dimensional inspection group's laboratories are classified as testing and measuring labs, so the previous assumptions in place are acceptable for a mix of research and development and typical production type work.

2.6. Approach to Errors and Uncertainty

The dimensional inspection group's approach to both errors and uncertainties will follow the traditional methods for creating and presenting an uncertainty budget analysis. All error and uncertainty sources will be considered and either accepted or neglected for the budget as comparable to other sources. Any error that will be documented will be assessed for correction and independence before proceeding with the uncertainty analysis. The uncertainty sources will then be cited and determined if the effects are significant or deemed negligible. Since the laboratory used for measuring is more of a secondary lab on uncertainty analysis, the assessment of uncertainty will be part of a tertiary order from NIST traceable standards to a standards and calibration facility to the dimensional inspection laboratory. Some of the uncertainty may be on the conservative side, considering the budget is border line research and development and production work. All uncertainty sources will be published in a tabulated uncertainty analysis budget, with brief descriptions of sampling patterns, formulas and calibration certificates. An example of a dimensional measuring process of a component will be used as an example and all numbers will be deviations from nominal values.

3. Uncertainty Distributions & Sources

3.1. Gaussian

A Gaussian or normal distribution of many random variables, or random repeated measurements, takes the shape of a symmetric bell-curve. These values tend to focus about the center or average value. This distribution follows the central limit theorem. When a set of readings, or measurements, is taken the best assumption to the type of probability distribution is the normal distribution. The average value acts as a true value, although the true value is not known, and can give a good estimation at a potential value. The average and the spread or variation, represents a possible range for the true value to possibly lie in. In Gaussian distribution, many measurements are needed to help reduce potential random errors and bias. This distribution also helps with Type A uncertainties and combined standard uncertainties. Here is a graph of a normal distribution:

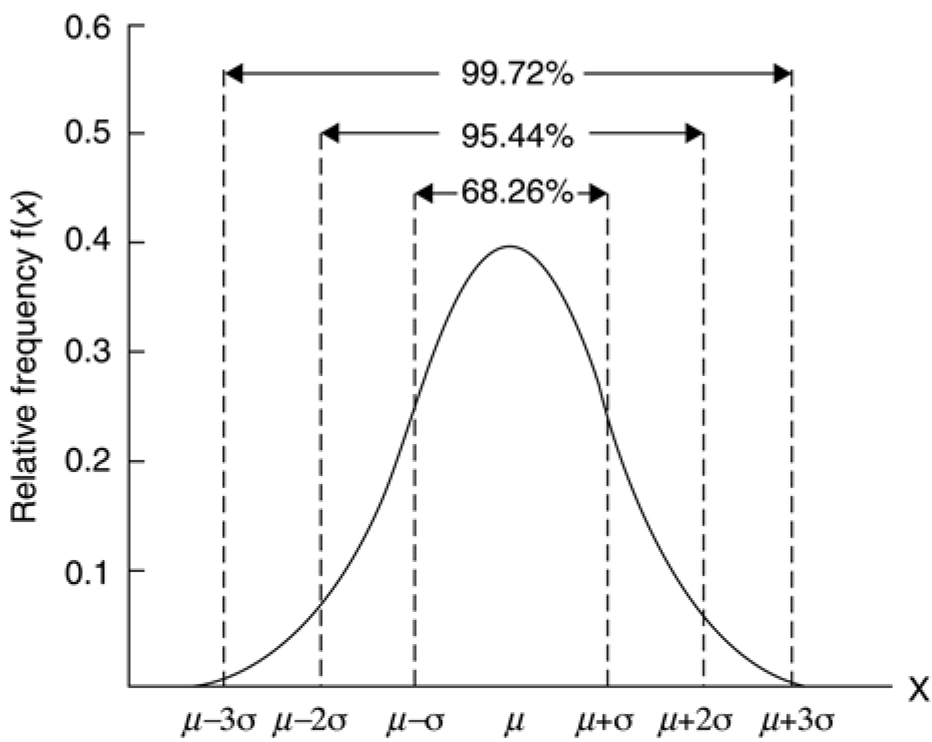

Figure 1: Normal Distribution Curve with Confidence Percentages and Sigma Values

The probability density function used to model normal distribution curve is:

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad (2)$$

Where $f(x)$ is the function of the measurement that describes the measurand, μ is the average or mean of the measured values and σ^2 is the variance or spread of where the average or true value may potentially lie.

In a normal distribution, the population size is infinite. This means that an infinite number of measurements are needed to find a possible true value. From the population variance σ the standard deviation also called sigma, can be determined by taking the square root, which denotes the range of the data. However since there cannot be an infinite number of measurements, a sample of the population is used estimate the average, variance and standard deviation. In a sample, the average is $\bar{x}$, the variance is s^2 and the standard deviation is s. With a sample size of about twenty of more measurements, these estimates will give good indications of the actual values.

When measurements of normal distributions are taken, the shape is a bell-shaped curve, and has a standard deviation of approximately one sigma, or 68%, confidence that the values are close to the true value. Normal distributions can be 68%, 95% or 99% confident that the true value is somewhere in the range of values. A 95% level is the usual standard for calibration and testing laboratories in confidence and uncertainty. This is also the typical value for uncertainty in calibration certificates.

Both Type A and B uncertainty analysis can use a normal distribution, either in calibration or testing, depending on what is needed. It can be used in Type A analysis for testing and research and development purposes, where statistical analysis is used. From this analysis, the standard uncertainty from the repeated measurements can be calculated and used for the uncertainty budget analysis. It is also common in Type B uncertainties for calibration and primary labs to state on calibration certificates. This distribution is slightly more difficult in Type B because the potential lack of historical data.

More detail of the probability distribution uncertainties, statistical formulas and methods will be published in appendix 1. These methods follow the same methodology as the ISO GUM standard [1].

3.2. Rectangular

Rectangular or uniform distribution is when the distribution of the measurements, or readings, is evenly spaced between the highest and lowest values of the range. It is often assumed that the errors have an equal probability of having a value anywhere in a specific range and unlikely to have random errors. This distribution leads to the measurements to be closer to the limits rather than the mean value. The limits are often assumed to be equal in magnitude and can actually be considered a half-range. Just like normal distribution, statistical analysis can be use to determine the standard deviation and hence the standard uncertainty. The following is an illustration of a rectangular or uniform distribution:

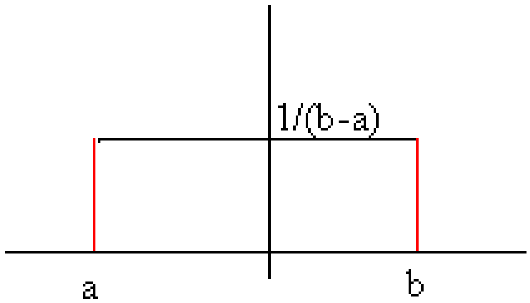

Figure 2: Rectangular or Uniform Distribution

With the rectangular distribution, where a and b values are the limits that are used to define the probability density function. The probability density function is as follows:

$$f(x) = \begin{cases} \frac{1}{b-a}, & for\ a \leq x\ \leq b, \\ 0, & for\ x \leq a\ or\ x\ \geq b \end{cases} \tag{3}$$

Like normal distributions, there is a confidence to the data, it being 58%, 95% and 100%, depending on what is needed for the calibration. Again, the standard uncertainty can be calculated from this expanded uncertainty by dividing by the coverage factor, which is different for the rectangular distribution.

Rectangular distribution is more applicable and mostly used for Type B evaluations. The rectangular distribution may be considered the distribution of minimal knowledge, because only the limits are what is really known and in the absence of further knowledge, assumptions that the value has an equal probability. The dimensional inspections group's use of Type B evaluations occurs when measuring calibrated artifacts. These calibrated artifacts are used for qualifying if the sampling pattern is adding a significant error to the measurement.

3.3. Triangular

Triangular distribution, in essence, can be seen the same way as rectangular distribution. The limits can be estimated in the same way as rectangular distribution; however, it also has a mode, or value that appears most frequently. It is used when there is evidence that the values near the mean are most probable and as the limits are approached, the probability decreases to zero. It requires less knowledge then normal distribution, but more than rectangular distribution. Again statistical analysis can be used to find the standard deviation, therefore giving a standard uncertainty value. Triangular distribution is the least commonly use distribution when doing uncertainty budget analysis.. There will be more focus on the previous two types of distributions.

3.4. Student's *t*

Student's *t* distribution is essentially the same as a normal distribution. It takes multiple readings and the same statistical analysis approach is used for finding the mean and standard deviation. It also has the same bell-shaped curve of the normal distribution, but with a different sigma value. Student's *t* distribution is used when the sample size is small and the population standard deviation is unknown. It therefore will provide a different confidence interval than the normal distribution. The same analysis is used for finding uncertainty in the Student's *t* as in the normal distribution; however there is a coefficient that is multiplied by the standard uncertainty that gives a new confidence interval. The value for t is found by using the degrees of freedom and subtracting one. A table is used to find a value for t and multiplied by the standard uncertainty to get a new confidence interval. When necessary this distribution will be used. When there is a small sample size, or historical data, this approach will be used. This was recently used for determining a new length formula for an older CMM. Again, rectangular and normal distributions will be more of the focus for the laboratories uncertainty budget.

3.5. Type A Uncertainty Evaluations

As stated earlier, Type A uncertainty analysis is evaluated using statistical methods. In most cases, the best available estimate of the true value is to calculate the mean value μ_x of the measured quantity x that varies randomly and for which many independent measurements can be measured under the same conditions each time. This average can be found through the arithmetic mean.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (4)$$

Where n is the number of measurements taken and x_i is the measurement at a specific point. This equation is used for estimating the true value that may potentially lie in the probability distribution used for determining the uncertainty.

From the individual observations x_i, they will differ slightly because of the influence of the random variation effects that are inherited in the testing. The estimated variance is the next necessary calculation for the repeated measurements. This estimated variance will give an estimate of the spread, or range, of the where the true value could lie. The estimated variance is s^2 and is calculated by:

$$s^2(x_i) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad (5)$$

Where the nomenclature is the same as stated previously. This estimate of variance and its positive square root, $s(x_i)$, is termed the experimental or estimated standard deviation and characterizes the variability of the observed x_i, or more specifically, the dispersion about the mean $\bar{x}$.

So to characterize the standard uncertainty from the standard deviation, the standard deviation of the mean will have to be used. This will give an estimate of the standard uncertainty for the measurement in question. It is as follows:

$$s^2(\bar{x}) = \frac{s^2(x_i)}{n} \tag{6}$$

$$s(\bar{x}) = \frac{s(x_i)}{\sqrt{n}} \tag{7}$$

The experimental variance of the mean $s^2(\bar{x})$, equation (6), and the experimental, or estimated, standard deviation of the mean $s(\bar{x})$, are equal to the positive square root of $s^2(\bar{x})$, equation (7), and will quantify how well $(\bar{x})$ estimates the populations mean of the measurement and can be used as a measure of uncertainty. This is sometimes stated as a *Type A variance* and *Type A standard uncertainty.*

This uncertainty analysis can be used in all Type A uncertainty analysis. It can be used as an estimate of a population used for a curve that has been fitted with experimental data by the method of least squares. The estimated variances and resulting standard uncertainties of the fitted parameters characterizing the curve and any other predicted points can usually be calculated by well-known statistical procedures, like the National Institute of Standards and Technology's journal article on reference algorithms using a least squares methods [2]. This method can be used on calibrated artifacts that the dimensional inspection group uses for uncertainty data and modeling actual measures points on specific components.

The dimensional inspection group's main use of Type A uncertainty is when a component is measured extensively, meaning it has produced tens of thousands of data points. This method makes calculation easier because of the use of spreadsheets are possible and reliable, up to a certain precision in the data. If the random variation in the measurements of the quantity that is being measured has a correlation, in time the mean and the estimated standard deviation of the mean may be inappropriate estimators of the desired statistics. In such a case, the observations will be analyzed by specifically designed software and statistical methods to treat the correlation of the randomly varying measurements. For example, Minitab® software will be used for the Gauge Repeatability and Reproducibility (GR&R) calculations, because it is specifically designed for the correlation, or interaction, between the parts and operators. The use of Type A uncertainty evaluation is meant to help with complex measurements such as short-term versus long-term random variations in reference standards and use more simple statistical methods to analyze these measurements. It is also helpful with simpler measurement scenarios such as measuring set of gauge pins for thermal expansion uncertainty at different temperatures. The use of spreadsheets and statistical packages makes it easier for uncertainties to be calculated and documented. The dimensional inspection group's uncertainty budget will be presented later where Type A uncertainty is used in three of the five uncertainty sources. More detail on the measurand and measuring process will be documented and specified on why that certain type was chosen for the sources.

3.6. Type B Uncertainty Evaluations

Unlike Type A uncertainty analysis, Type B uncertainty analysis is determined by any other means than statistical analysis. This means that measurements were not obtained in repeated observations and were more specifically evaluated from other scientific means based on the available information of the variability. Thus they are not found by standard variances and standard deviations. These uncertainties can be found from:

- Previous measurement data
- Experience with a general knowledge of the behavior and properties of relevant materials and instruments
- Manufacture's specifications
- Data provided in calibration and other certificates
- Uncertainties assigned to reference data taken from a handbook

For convenience, the estimated variance will be denoted by $u^2(x_i)$ and the standard uncertainty will be denoted by $u(x_i)$, respectively. These can also be called *Type B variance* and *Type B standard uncertainty*.

When the needed information is not available for a Type B evaluation of the standard uncertainty, a call for more insight based on experience, knowledge of the measurement and skill in taking the measurement, hence a judgment call is issued. This can be recognized as a Type B analysis of standard uncertainty. Type B evaluations can be just as reliable as a Type A uncertainty analysis, with the potential of being a better representation of the uncertainty when it is very well characterized. This can be especially true in measurement situations where a Type A evaluation is based on a comparatively small number of statistically independent measurements. In actuality, Type B evaluations are more reliable when an analytical model is constructed and used for measurements. This is because there will be a model to predict the measurement with all its influences (providing the model is correct), less error from round-off, etc. Again though, this is also one of the more difficult portions of uncertainty analysis. Model equations will be discussed in section 4.1.

If the estimate $u(x_i)$ of the uncertainty is taken from a manufacturer's specification, a calibration certificate, either standards lab or manufacturer, an engineering handbook or any other source and its quoted uncertainty is stated to be a particular multiple of a standard deviation, the standard uncertainty $u(x_i)$ is simply the quoted value divided by the multiplier, and the estimated variance $u^2(x_i)$ is the square of the quotient. In laymen terms, the expanded uncertainty is divided by the coverage factor for the specific distribution, and therefore the value is brought back to a one sigma, or standard deviation value, that can be used for the combined standard uncertainty in the uncertainty budget. Thus squaring that value will also produce the variance of the measurement.

From previously stated, the quoted uncertainty of x_i may not necessarily be from a multiple of standard deviation, thus potentially not from a normal distribution. Instead, it may be state that the quoted uncertainty is defined by an interval from 90, 95 or 99 percent level of confidence or a different distribution altogether. The quoted uncertainty may need to be divided by the appropriate coverage factor for the stated distribution. Unless otherwise stated in the document or source, the assumption that a normal distribution was used to calculate the quoted uncertainty and the standard uncertainty of x_i is thus found by dividing the quoted values by the coverage factor for the distribution type.

With Type B uncertainty analysis, it is important not to count the same uncertainty components more than once. If a component for a certain effect arises and is obtained in a Type B method, then the component should be treated as independent in the calculation of the combined standard uncertainty and only to the extent that the effects do not contribute to the repeated measurements of the measurand. This means that if a component is not totally independent of a Type A analysis, then the same uncertainty can be calculated again and has already been included in the statistical analysis, thus giving a flawed affect to that source. Correlation is a special acceptance, which will be explained in section 4.4. So with Type B uncertainty analysis, all methods besides statistical means can be used to state the standard uncertainty and Type B uncertainty is meant only to be indicative and further evaluation should be done out of necessity.

4. Estimating Uncertainty Sources

4.1. Model

Before any calculations are attempted and recorded, it is necessary to consider the measurement system being used and the environment that it is in. A model of the measurement should be developed, either through explicit or implicit means, depending on the complexity of the measurement. A simple sketch or equation may also be the only model that is needed to explain the measurement. The model is intended to provide a simple means of describing the relationship between the input parameters and the influencing quantities to the measurand. This is the step that metrologists find most difficult. Generally, it will also be necessary that the sensitivity coefficients, be calculated (explained in section 4.5). These sensitivity coefficients are necessary for conversion from one unit of measure to another to satisfy the measurand. They also help out with scaling the sources to balance out the total uncertainty. Without a model some significant uncertainties may be overlooked or it may be difficult to determine the sensitivity coefficients.

In most cases, a measurand is not measured directly but found from a number of different quantities found through a relationship. This is when multiple measurements are not taken and an analytic model must be found. The analytic model can be found through the following function:

$$Y = f(X_1, X_2, \ldots, X_N) \quad (8)$$

Where X_1, X_2,…, X_N are quantities, N is the number of quantities and f is a functional relationship between the quantities. The same notation will be used for physical quantities (the measurand) and for the random variables that represent the possible outcomes of the observations of the quantities. When it is stated that X_i has a particular probability distribution, the symbols will be used in the latter sense; it is assumed that the physical quantity itself can be characterized by essentially a unique value.

It is very important that the model be correct and as accurate as possible for the measurands. If the input quantities X_1, X_2,…, X_N upon which the output quantity Y depends on, may themselves be viewed as possible measurands and may themselves depend on other quantities, including corrections and correction factors for systematic effects, thus leading to a complicated functional relationship that may never be written down explicitly. Furthermore, f may have to be determined experimentally or exist only as an algorithm that must be evaluated numerically. If a model of the measurement cannot be determined analytically, then numerical solutions through Monte Carlo and various experimental measurements must be taken and analyzed for determining the uncertainties. This method is done on certain processes where a direct model cannot be found analytically, but can be measured both experimentally and numerically solved using specific algorithms. An example for determining estimated densities of an odd-shaped geometrical artifact using thousands of slices of data points. Another example is taking a production lot of mock parts and doing a GR&R study to find out how much influence, or variation, a certain factor can add to a measuring process.

For accuracy purposes, if the data indicates that f does not model the measurement to the degree of accuracy needed, more quantities may need to be imputed. These input quantities may reflect incomplete knowledge of an unknown source that is affecting the measurand. Models should be used all the time when permissible, however only if the model has a strong characterization to it.

In summary, the quantities that affect the measurand model are factors whose values and uncertainties are directly determined in a current measurement. These values and uncertainties may be obtained from a single observation, multiple observations or judgment based on experience and may involve corrections to the measuring instrument readings and corrections for influence quantities. More quantity values and uncertainties are also brought from external sources, such as those attributed by calibration measurement standards, certified materials and reference data from handbooks. Again all these input quantities can be found from both Type A and B sources.

4.2. Indirect Measurements

Uncertainty components can be subject to more than one possible quantity of influence in the uncertainty calculation. It is more common the case when a quantity of interest is only accessible indirectly, or having a result that must be inferred from results of another measurement. Therefore, the uncertainty of an indirect source must be obtained from the uncertainties of those influential quantities. An example could be when determining an air density correction for measuring ionization in a dosimeter. The air pressure and temperature measurements are the influencing quantities in the correction factor. Thus both of these quantities must be incorporated into a model or equation that represents the affects they have on the indirect measurement.

For the uncertainty budget, when possible, indirect measurements will be specified with an analytic approach. This approach will be more useful for environmental effects. Again the GR&R will help with the most influential aspects of the measurand, but can possibly be lacking in the determination of indirect results. This process will be considered carefully, but only if a complete understanding the influencing quantities are apparent.

4.3. Corrections

It is a common mistake that small corrections are not applied to raw measured data or instruments but are still applied to calculating the uncertainty analysis. The assumption is that the correction of measurements could be replaced by a rectangular distribution where the semi-range of the distribution is equal to the magnitude of the correction. This is not a highly recommended process and should not be implemented into practice.

First, corrections do not have a rectangular distribution. They have a discrete magnitude and sign. If corrections are negligible then neglect them, if not negligible, then they should be applied. The uncertainty of the corrections must be considered for inclusion in the assessment of the measurement uncertainty. It can be shown that, in some cases, the uncertainty will not be appreciably increased by including small corrections, however, the measured value may be in error by an amount approaching the uncertainty due to this sloppy procedure alone.

For example, consider a measurement where they have neglected four small corrections estimated to be +0.02%. Assume that the combined standard uncertainty based on all other terms is 0.3%. If the Root-Sum-Squared (RSS) approach is taken, then the corrections give a new combined uncertainty of 0.3009%, not significantly different from the initial value. However, the combination of the four corrections is 0.08%. If the expanded uncertainty is 0.6%, then there has been an unaccounted bias around 13% in the measurand, something that was unknowingly and unintentionally introduced. Thus all corrections must be considered, even though they may seem negligible, it must be determined that they will not grossly affect the combined and expanded uncertainty.

For all measurement types, the appropriate corrections will be done to ensure the best possible outcome in the data and results. These corrections will be done for the certain type of geometries that can be encountered such as radius corrections to spherical surfaces.

4.4. Correlations

Correlations can be defined as two or more input quantities that are not independent of one another, and can have an effect on the measurand. So if two or more systematic errors are correlated, then the RSS combination for uncertainty is not appropriate. It is then recommended that the correlated uncertainty values be added algebraically and this value combined with the other components in the normal RSS method. This can be considered a simple approach and may overestimate the uncertainty, so caution should be used. It is justified on the basis that, in many cases where correlations are apparent, that the correlated component uncertainty are completely correlated. In more complex cases, the statistically rigorous calculations should be used.

The models used for correlated values can be found form the ISO GUM reference. Here the equations are summed for potentially multiple correlated values. The uncorrelated uncertainty formula can be seen as:

$$u_c^2(y) = \sum_1^n \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) \qquad (9)$$

Where $u_c(y)$ is the combined standard uncertainty, $\frac{\partial f}{\partial x_i}$ is the sensitivity coefficients and $u(x_i)$ is the standard uncertainty. This formula stays true for non-correlated quantities, which are completely independent of one another. For the correlated terms, the ISO GUM states that the formula is the same, however with an extra term for the correlated values.

$$u_c^2(y) = \sum_{i=1}^n \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) + 2\sum_{i=1}^{n-1} \sum_{j=i+1} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) \qquad (10)$$

Where the first term is the same as equation (9) and the second term has $\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}$ for the correlated values. These correlated values can be seen as covariance, where a variance and those of a standard deviation can be calculated for the correlated values. This term can be positive or negative and may reduce the uncertainty. These equations are the ISO GUM method for calculating uncertainty for components, both uncorrelated and correlated. Equation (10) will be looked at again later in the section, in a simplified manner. The software package that will be used for the GR&R uses a similar methodology for computing the operator-to-part and part-to-part interaction or correlation, with a slight variation in the equation. More will be explained in section 4.8.

Next is how to determine if an apparently independent parameter is correlated with another independent parameter. A simple test for correlation is to graph the two parameters against measurement numbers. If they can be seen to change together, say both increase for the second measurement, both stay the same for the same third measurement and reduce for the fourth, then there is clearly correlation between the two parameters. This can be seen as similar to a Design of Experiments (DoE) interaction scenario; however, the interaction graphs show intersection between two factors. Utilizing this simple technique can help detect a correlation and help understand the appropriate model to use for the uncertainty analysis.

Covariances are another important aspect of correlation. The definition of a covariance is a type of variance that depends on each of the correlated values and their correlation coefficients. Covariances are calculated through standard formulas or determined experimentally. The estimated covariance is $u(q_i r_i)$, where q_i and r_i are the estimates (or measured values) of the input parameters. The ISO GUM can be referenced for more detail.

If q_i and r_i are independent pairs of simultaneous measurements, the estimates of the covariance, $u(q_i r_i)$, is:

$$u(q_i, r_i) = \frac{1}{(n-1)} \sum_1^n (q_i - \bar{q})\,(r_i - \bar{r}) \qquad (11)$$

Where $u(q_i, r_i)$ the estimated covariance, n is is the number of measurements, q_i and r_i are the same as previously stated. If correlation between the means is what is wanted of the two independent pairs of simultaneous readings then the equation changes to:

$$u(\bar{q}, \bar{r}) = \frac{1}{n(n-1)} \sum_1^n (q_i - \bar{q})\,(r_i - \bar{r}) \qquad (12)$$

Where $\bar{q}\ and\ \bar{r}$ are the mean values of the two simultaneous quantities. These equations hold true for any two simultaneous quantities and also hold true for higher-ordered terms.

If the correlation is a complete correlation or if the correlation is going to have little effect, then correlation coefficients need to be determined. The correlation coefficient is stated as r and will give a correlation of +1 or -1 for complete correlation or close quantities to zero if there is not a significant correlation. The degree of correlation is given by the estimated correlation coefficient $r(x_i,x_j)$ and the formula is:

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)} \quad (13)$$

Where $u(x_i)$ and $u(x_j)$ are the standard uncertainties of x_i and x_j. Now examining Equation (10), the second term is the product of the standard uncertainties and correlation coefficients of the quantities. Hence, when there are no correlations, the correlation coefficient goes to zero, so the second term in Equation (10) vanishes. If the correlation coefficient is +1 then the uncertainty is at its maximum and vice versa when it is at -1. This can be determined from the model or experience, depending on the scenario.

If complete correlation is determined and stated, then Equation (10) can be simplified to an easier model. It translates to combining the two components by simply adding the standard uncertainty to get to a combined standard uncertainty. So Equation (10) can be reduced to the following form:

$$\left[\sum_{i=1}^{N} \frac{\partial f}{\partial x_i} u(x_i)\right]^2 \; or \quad (14)$$

$$u_c(y) = \sum_{i=1}^{N} \frac{\partial f}{\partial x_i} u(x_i) = c_1 u_1 + c_2 x_2 + c_3 x_3 + \cdots + c_N x_N \quad (15)$$

Where c_N is the sensitivity coefficient and will be discussed in section 4.5. These equations are what the statistical software package uses for determining the GR&R portion of the uncertainty budget. These equations are also easier to implement in the analytical uncertainly portion when the models are of a simpler form.

4.5. Sensitivity Coefficients

As mentioned earlier, sensitivity coefficients are used as scaling or converting factors of measurement units to those of the measurand. This is a necessary precondition to combining the components of uncertainty, which are of different units like, meters and kilograms. This is also one of the other aspects of uncertainty analysis that can cause considerable difficulty.

Sensitivity coefficients serve as scaling or weighing factors. They are used to figure out how sensitive the selective component is to the measurand, through an analytic and numerical means. It can be obvious when a sensitivity coefficient is needed. For example, the temperature coefficient of linear expansion converts uncertainties in measured temperature with units of degrees into uncertainties with units of length.

Evaluating the sensitivity coefficients can be done through algebraic partial differentiation of the equation that models the measurement or, in more complex cases, by numerical calculations which approximate the differentiation process. The general formulas for the sensitivity coefficients can be found in the ISO GUM. The formula is as follows:

$$c_i = \frac{\partial y}{\partial x_i} \tag{16}$$

Where c_i is the sensitivity coefficient for the component x_i, y is the measurand as a function of x_i and $\frac{\partial y}{\partial x_i}$ is the partial derivative of y with respect to x_i. The partial derivative gives the slope of the curve that results when the function y, the measurand, is plotted for the appropriate range of x_i values. The slope or the derivative is the sensitivity of the measurand to a particular component of the function.

It will be convenient in some cases to calculate the measurand in terms of proportional parts (part per million) deviation from a nominal value. This also means that the percentage from nominal can be used for determining the sensitivity of the measurand components and may sometimes be best suited for a variety of different units that can be difficult to convert to one single unit. However, this may only be applied for determining the major effects of a singled out factor and may not provide a best representation of the overall uncertainty of the measurand. Again, this method works only if all the influences are calculated in the parts per million terms.

Next to forming the model, determining the sensitivity coefficients is the second most difficult section of uncertainty analysis. It is often best to use the SI units for all inputs as errors of three orders of magnitude are easily made when calculating sensitivity coefficients. All units for the dimensional inspection group's approach will rely on SI units as to ease the transition between measurement data and using the data from other experimental processes. These coefficients will be used when an analytic approach is used for input quantities and when rigorous numerical calculations are done.

4.6. Degrees of Freedom

Degrees of freedom are mainly used for determining the correct selection of a coverage factor from a Student's t distribution, but they also give a good indication of how well a components uncertainty can be relied upon. The higher the number of degrees of freedom is associated with a higher sampling size thus presenting a value with a lower variance or lower spread of the range of data. A lower number of degrees of freedom, the lower the sample size, presents a larger variance and dispersion, which leads to a poorer level of confidence in the data.

Every component has an associated degree of freedom, v, and the degree of freedom is calculated by subtracting one from the number of readings or $v = n-1$ where n is the number of measurments. This assessment is straightforward for Type A uncertainty, but more complicated with Type B uncertainty. When the uncertainty has limits, cutoff for the numerical value, for different distributions in a Type B uncertainty, then there can be an infinite number of degrees of freedom, because there is usually a complete confidence in the limits chosen for worst cases. This can simplify the calculations for degrees of freedom for the uncertainty components. If the limits chosen themselves have an uncertainty association, then there will be less degree of freedom assigned. The ISO GUM gives a formula for all types of distributions.

$$v = \frac{1}{2}\left[\frac{\Delta u(x_i)}{u(x_i)}\right]^{-2} \quad (17)$$

Where $\frac{\Delta u(x_i)}{u(x_i)}$ is the relative uncertainty in the uncertainty. This number is less than one and can be thought of as a percentage of the actual uncertainty. A smaller number is a better defined magnitude of the uncertainty, thus giving a more accurate representation of the value.

Once the uncertainty components have been combined, all that remains is to find the number of degrees of freedom in the combined uncertainty. The degrees of freedom for each component must be combined to find the effective degrees of freedom to be associated with the combined uncertainty. This is calculated using the Welch-Satterthwaite equation, which is:

$$v_{eff} = \frac{u_c^4(y)}{\sum_1^n \frac{u_i^4(y)}{v_i}} \quad (18)$$

Where v_{eff} is the effective number of degrees of freedom for u_c, the combined uncertainty, v_i is the number of degrees of freedom for u_i, the *ith* uncertainty term, a $u_i(y)$ is the product $c_i u(x_i)$, with the sign c_i, being neglected and the other terms being the usual meaning.

The degrees of freedoms and effective degrees of freedom will be used where necessary in the uncertainty budget. If a small number of runs are used in an uncertainty budget, then the degrees of freedom will be used for a Student's *t* distribution. They will also be used for getting a feel for an uncertainty source that may not be reliable and used as a quality check for determination. Using degrees of freedom is good idea for uncertainty analysis, though often is underutilized.

4.7. Confidence Intervals and Confidence Levels

Most individuals get confidence interval and confidence level confused and interchanged, because one supports the other. In statistics, the confidence interval is an interval estimate of a population parameter and is used to indicate the reliability of that estimate. It is an observed interval, in principle is different from sample to sample that frequently includes parameters of interest, if the measurement is repeated. In statistics, a confidence level is a measure of the reliability of a result. More specifically, the term "confidence level" is meant that if confidence intervals are constructed across different data analysis of repeated, and possibly different, experiments, the proportion of such intervals that contain the true value of the parameter will approximately match the confidence level: this is guaranteed by reasoning underlying the construction of confidence intervals. However, a confidence interval does not predict that the true value of the parameter has a probability of being in the confidence interval given that the data actually obtained.

All confidence intervals will be brought forth by calibration certificates and manufacture specifications. All confidence levels in the uncertainty budget will be 95%, which can be seen as a standard method for calibration and primary laboratories, thus being enough for the testing purposes.

4.8. Combined Uncertainty

Once the standard uncertainties and accompanying sensitivity coefficients for each of the components for the measurand have been evaluated, they all need to be combined to get the combined standard uncertainty. This is done using Equation (9), which is from the ISO GUM. This equation holds true for all correlated and non-correlated uncertainty components. Again, the higher-ordered terms in Equation (9) have been neglected for this paper. So a simplified version of Equation (9) can be used for the combined standard uncertainty.

$$u_c(y) = \sqrt{\sum_1^n [c_i u(x_i)]^2} \qquad (19)$$

Where $u_c(y)$ is the combined standard uncertainty of the measurand, c_i is the sensitivity coefficient for the *ith* term, $u(x_i)$ is the standard uncertainty for the *ith* input of the estimate and Σ is the summation of all the terms, of which there are *n* number of terms.

This means that the uncertainty components are converted to all the same units as the measurand using the sensitivity coefficients then these products are square rooted. The combined standard uncertainty is the square root of the sum. This is the same as previously stated RSS. For the combined uncertainty, all components are converted to the SI unit of millimeters (mm). This is true for all physical dimensional measurement uncertainties in all the dimensional inspection processes.

4.9. Expanded Uncertainty

In order to have an adequate probability where the value of the measurand lies within the range given by the uncertainty, the combined uncertainty is multiplied by a coverage factor. This coverage factor may be selected or it may be calculated to reflect the stated confidence level. For example, a coverage factor of two gives an expanded uncertainty (*U)* of 95% confidence level, for a normal distribution.

This is acceptable and perhaps the best approach for testing situations where only a few worst case estimates are used for determining the test uncertainty. For high level calibration the more rigorous method may be preferred. This is what primary laboratories strive for.

When the uncertainty has been estimated from data with poor reliability, a large coverage factor may be required to maintain a 95% confidence interval. The CMM data that most of the uncertainty that will be calculated, is reliable in the sense that the manufacturing guidance known as the 9900000, calls for a 4:1 Test Accuracy Ratio (TAR), which states that a measuring instrument must be able to measure within 25% of a desired specification, like an engineering drawing. The 9900000 guidance also states a Test Uncertainly Ratio (TUR), which is more of a collective effort of multiple measuring instruments, can be used for uncertainty analysis. This can be seen as an alternate version to an uncertainty budget. The expanded uncertainty is calculated from:

$$U_{95} = ku_c(y) \tag{20}$$

Where U_{95} is the expanded uncertainty at a 95% confidence limit, k is the coverage factor, and $u_c(y)$ is the combined standard uncertainty. Again, the sub-script on the expanded uncertainty can also change to the level of confidence that one desires, however, it is standard practice the 95% is used throughout industry and government.

By assuming that the combined uncertainty has essentially a normal distribution, which will also be the case for the uncertainty budget, one can also use the Student's t factor as a coverage factor k. This is justified by invoking the Central Limit Theorem, which in essence states that if many distributions are combined, irrespective of their own shape, the combined distribution will approximate a normal distribution. Hence the combined uncertainty will tend toward a normal distribution as more and more components are included. Again, if the sample size is small due to economic reasons, measuring instruments reasons, or even time constraints, then the Student's t distribution will be used in place of the normal distribution. Assuming that a normal distribution is in effect will be the case for the uncertainty budget. The components of the uncertainty budget will be labeled as Types A or B, along with the types of distribution that were used to find out the uncertainty value.

4.10. Gauge Repeatability & Reproducibility

This portion of the uncertainty analysis budget is not covered in the ISO GUM or any other guide to measurement uncertainty. This component is used in the dimensional inspections groups' uncertainty budget for many different factors and reasons. It is a commonly used method for understanding and correcting processes and deals with a great range of variables in dimensional inspection process. A brief description of the Gauge Repeatability and Reproducibility study will be explained, with a much greater explanation in the reference "*Gauge Repeatability and Reproducibility Study on a Hemi-Component with a Brown & Sharpe® Coordinate Measuring Machine (U)*" [5].

Gauge Repeatability and Reproducibility, or GR&R, is a measure of the total variability of a gauge or measuring instrument to obtain the same measurement reading every time the measurement process is undertaken for the same characteristic or parameter. In other words, the GR&R indicates the consistency and stability of the measuring instrument and operator. The ability of a measuring instrument to provide consistent measurement data is important in the control of any process. Operator consistency is also important because a good process should be able to be done by any qualified person. Repeatability is the variation in the measuring instrument and can be traced back to the precision. Reproducibility is variation due to the operator and can be traced to the accuracy. The GR&R study will determine and quantify where most of the process variability exists.

There are two statistical methods for analyzing GR&Rs. One method is the $\bar{x}$ (average) and R (range) charts and the second is Analysis of Variance (ANOVA). $\bar{x}$ and R is a set of control charts for variable data (data that is both quantitative and continuous in measurement, such as a measured dimension or time). The $\bar{x}$ chart monitors the process location over time, based on the average of a series of observations, called a subgroup. The R chart monitors the variation between observations in the subgroup over time.

The ANOVA method is a statistical method using the statistical approach of Analysis of Variance. Analysis of Variance is a collection of statistical models, and their associated procedures, in which the observed variance is partitioned into components due to different sources of variation. ANOVA uses either fixed-effect or random-effect modeling systems to assess the statistical system. ANOVA is a chosen method for measurement systems, because of better accuracy in the results.

ANOVA GR&R considers several factors that affect the measuring system: operators, testing methods, part setup, performance specifications and the measuring instrument itself. ANOVA GR&R methodology is more accurate because it not only captures the repeatability and reproducibility, but it also breaks down the reproducibility portion into part to part interaction and operator to part interaction, which can be seen as correlation in the uncertainty components. This can be explained by one operator having more variation between measuring components of smaller size compared to measuring components of larger size. The GR&R will also be used for the assumptions being made at the beginning of the paper. The environment, correlations/interactions, sampling pattern, operator influences on setup, etc., will all be accounted for in the GR&R calculations.

Figure 3 demonstrates the methodology of an ANOVA GR&R

Figure 3: ANOVA GR&R Tree.

With a GR&R, the analysis method is different for both $\bar{x}$ and R and ANOVA. The $\bar{x}$ and R method uses a root-summed-square analysis, while the ANOVA uses a sum-of-squares or standard deviations analysis to calculate the Precision-to-Tolerance (P/T) ratio. The P/T ratio, also known as the gauge capability ratio, is the measure of the precision of the measurement to the given performance specifications. Both methods quantify total process variability, which will be the basis for an uncertainty component in the uncertainty budget calculations. The total variation in the process will be calculated through a Type A uncertainty using a normal distribution and a 95% confidence level, thus using variations and standard deviations as the standard uncertainty from multiple repeated measurements.

5. Compiling an Uncertainty Budget

5.1. General Approach

The following is a generalized approach to evaluating and expressing the uncertainty of the result of a measurement which can be traced back to the ISO GUM method and any lower-echelon laboratories.

1. Express mathematically the relationship between the measurand Y and the input quantities X_i on which Y depends: $Y = f(X_1, X_2\ldots, X_N)$. The function f should contain every quantity, including all corrections and correction factors, which can contribute a significant component to the result of the measurement.
2. Determine x_i, the estimated value of input quantity X_i, either on the basis of the statistical analysis of series of observations or by other means.
3. Evaluate the *standard uncertainty* $u(x_i)$ of each input estimate x_i. for an input estimate obtained from the statistical analysis of a series of observations, the standard uncertainty is evaluated as a *Type A evaluation of standard uncertainty*. For input estimated by other means, the standard uncertainty $u(x_i)$, is evaluated by *Type B evaluations of standard uncertainty*.
4. Evaluate the covariances associated with any input estimates that are correlated.
5. Calculate the results of the measurements, that is, the estimate y of the measurand Y, from the functional relationship f using the input quantities X_i the estimates x_i obtained from step 2.
6. Determine the *combined standard uncertainty* $u_c(y)$ of the measurement result of the measurement result y from the standard uncertainties and covariance associated with the input estimates. If the measurements determine simultaneously more than one output quantity, calculate their covariance.
7. If it is necessary to give an *expanded uncertainty* U, whose purpose is to provide an interval $y\text{-}U$ to $y+U$ that may be expected to encompass a large fraction of the

distribution of values that could reasonably be attributed to the measurand *Y*, multiply the combined standard uncertainty $u_c(y)$ by a *coverage factor k,* typically in the range of 2 to 3, to obtain $U=k\ u_c(y)$. Select *k* on the basis of the level of confidence required of the interval.

8. Report the result of the measurement *y* together with its combined standard uncertainty $u_c(y)$ or expanded uncertainty *U*.

More detail can be found in more formal guidelines for measurement uncertainty.

5.2. Dimensional Inspection Group Approach

The dimensional inspection group's approach will, in essence, be approached in the same method as the general approach in section 5.1. All the necessary steps to determining the standard uncertainty will be done to the determined input quantities. Not all of input quantities can be derived from a model. The geometry of the components is much more complex than measuring the length of a rod. Of course the rod can expand or contract, depending on the material, so an analytical model can be derived and the CTE can be used to calculate the sensitivity coefficients for the actual length of the rod. The components will be measured using both Type A and Type B evaluations, depending on which will be needed. The main sources of uncertainty that will be looked at will be dependent on the dimensional inspection process being used. The main differences will be the type of instrument that will be used and the sampling patterns needed for the analysis section. Since the laboratory is a testing lab, calibrated artifacts, not necessarily standards will be used as quality and performance checks. Again this uncertainty will also be considered.

For general dimensional inspection processes, the following uncertainties will be stated. A recent research and development project performed, called Gemini, used multiple parts and sampling patterns to get the appropriate data needed for the analysis. The sources considered for uncertainty budget analysis are:

- The machine (Precision Measuring Machine/PMM-C) calibration certificate with a Type B evaluation
- Surface Measurement of Inner Surface Standard Deviation with a Type A evaluation
- Surface Measurement of Outer Surface Standard Deviation with a Type A evaluation
- SS Sphere Standard Form with a Type B evaluation
- GR&R with a Type A evaluation

A table with the measurement deviations, distributions, divisors, standard uncertainties and combined uncertainties can be seen in section 6. These sources were specifically chosen for this type of process because the nature of the environment, part forms, inspectors/operators, data density, etc. These sources can also change when necessary.

6. Stating Uncertainty in Testing

The following table describes and states the uncertainty sources and actual deviations from the uncertainty budget for a Gemini component wall thickness inspection:

Component Wall Thickness Uncertainty Budget: Gemini Measurement Process (Non-Nuke)				
Source of Uncertainty: Type A&B	**value ± (mm)**	**Probability Distribution**	**Divisor**	**Standard Uncertainty (ST) (mm)**
PMM-C Calibration Certificate (S&CL File:026734)* B	0.00231	Normal	2	0.00116
Profile Measurement of Inner Surface Standard Deviation** A	0.00047	Normal	1	0.00047
Profile Measurement of Outer Surface Standard Deviation**A	0.00033	Normal	1	0.00033
SS Sphere Standard Form (S&CL File: 041013)*** B	0.00057	Rectangular	√3	0.00033
Gauge R&R**** A	0.00213	Normal	2	0.00213
Combined Standard Uncertainty		Assumed Normal		±0.00250
Expanded Uncertainty (K=2)		Assumed Normal		**±0.00500@ 95% Confidence**

Table 1. Uncertainty Budget for Non-Nuke Dimensional Inspection Process

Assumptions:

1. Temperature fluctuates in a constant range of 20±1ºC and machines use temperature compensation, when available, to adjust for these fluctuations.
2. All component's surface finish, surface Surfaces and flatness all meet drawing specifications.
3. Wall standard uncertainty calculated at pole, midpoint and equator, though equator can differ because of the different geometrical influence.

* Data used from Standards and Calibration (S&L) Certificate. The volumetric scanning uncertainty will be calculated with the equation

$1.2 + \frac{L}{400}$, where L is the measured length in mm, but stated in micrometers. A diagram can be seen below.

** Standard deviations are calculated from n number of measurements, taken at 1.5º azimuthally (0-360º) and 1º (0-mº) in the polar direction; n = data density, m = degree of polar direction.

*** Stainless steel sphere that has been qualified as a standard by S&CL, thus the uncertainty of the sphere will come from measured values (five) and then used for the standard uncertainty.

**** Gauge R&R will be serve as part of the statistical analysis of the process and will incorporate machine and environmental uncertainty as well as operator influences. It also takes into account covariance's (interactions) between the operator and the actual part.

Notes:

1. Type A uncertainty is achieved through statistical analysis. Type B is non-statistical uncertainty, i.e. calibration certificate, manufacture specification, etc.
2. All data is assumed continuous, though if not continuous, the equations are stated for discrete data, so continuous data can still be used.
3. Utilizing the assumptions, the uncertainties from the assumptions are assumed to be negligible to the process.
4. All uncertainty calculations are estimations.
5. Combined Uncertainty calculated by the "summation in quadrature" of the standard uncertainties (ST): $\sqrt{ST_1^2 + ST_2^2 + ST_3^2 \dots ST_n^2}$
6. All measurements are deviations from drawings.
7. Divisor value is used to determine the standard uncertainty, if greater than one sigma value.

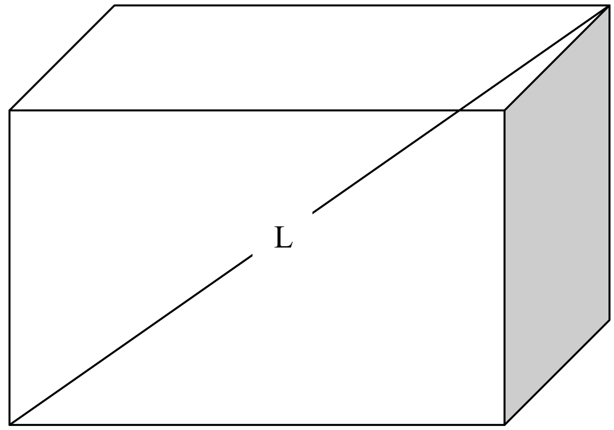

Figure 4. Estimated Volume of Measurements for Measuring Instrument

Again, this methodology is specifically designed for this type of process which is specific for the dimensional inspection group. This is not the absolute method for determining uncertainty in a measurement process. The ISO GUM is the absolute method and should be referenced for a more detailed general view of uncertainty analysis. This uncertainty budget can be used to help determine where the most error in a process can reasonably be, help determine if the tooling path of a machine tool needs to be adjusted for a re-machine or for helping with simulations of complex numerical algorithms for design work.

References

[1] – "*Evaluation of Measurement data- Guide to the Expression of Uncertainty in Measurement"*, First Edition, ISO, 2008

[2] – R.R.Cook, "*Assessment of Uncertainties of Measurements"*, National Association of Testing Authorities, Second Edition, 2002

[3] – S. Bell, "*A beginner's Guide to Uncertainty of Measurement",* National Primary Laboratory, Issue 2, Number 11, 2001

[4] - *G*. Recktenwald, "*Uncertainty Estimation and Calculation",* Portland State University, 2006

[5] – L. Valdez, "*Gauge Repeatability and Reproducibility Study on a Hemi-Component with a Brown & Sharpe® Coordinate Measuring Machine (U)",* LA-UR-11-02559 Los Alamos National Laboratory, 2011

Appendix 1 – Statistics and Uncertainty Formulas

The following statistics and uncertainty formulas are used for all uncertainty input quantity estimations and will be broken up into Type A and Type B evaluations. Each will come with a brief statement or definition for explanation purposes.

Type A Evaluation Statistics and Uncertainty Formulas:

If the number of measurements is *n*, and x_i is the *ith* measurement (and *x* is the mean) then:

Mean

$$\bar{x} = \sum_{1}^{n} \frac{x_i}{n}$$

Variance

$$s^2 = \sum_{1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

Standard Deviation

$$s = \sqrt{\sum_{1}^{n} \frac{(x_i - \bar{x})^2}{n-1}}$$

Estimated Standard Uncertainty of Type A Evaluations

$$u(y) = \frac{s}{\sqrt{n}}$$

Degrees of Freedom for Standard Uncertainty

$$v = n - 1$$

Type B Evaluation Statistics and Uncertainty Formulas:

Degrees of freedom, from relative uncertainty $\frac{\Delta u(x_i)}{u(x_i)}$

$$v = \frac{1}{2}\left[\frac{\Delta u(x_i)}{u(x_i)}\right]^{-2}$$

If:

$$R = \frac{\Delta u(x_i)}{u(x_i)} x100\%$$

Then:

$$v = \frac{1}{2}\left[\frac{100}{R}\right]^{2}$$

Rectangular Distribution

If the semi-range is a, then the standard uncertainty, u, is given by:

$$u = \frac{a}{\sqrt{3}}$$

The degrees of freedom (v) for a rectangular distribution are infinite if the semi-range represents absolute limits.

Sensitivity Coefficients, c_i

If y is a function of x, then

$$c_i = \frac{\partial y}{\partial x_i}$$

Combined Standard Uncertainty, $u_c(y)$

$$u_c(y) = \sqrt{\sum_{1}^{n} [c_i u(x_i)]^2}$$

Effective Degrees of Freedom, v_{eff}

$$v_{eff} = \frac{u_c^4(y)}{\sum_1^n \frac{u_i^4(y)}{v_i}}$$

Coverage Factor, k

$$k = Students's\ t - factor$$

Expanded Uncertainty, U

$$U_{95} = ku_c(y)$$

These formulas and statistics are what were used for the analysis of the uncertainty budget. And Excel spread sheet was used for calculating the standard uncertainties of the Surfaces. Minitab® statistical software package was used for determining the GR&R analysis.

Appendix 2 – Higher Ordered Terms

As stated earlier, higher-ordered terms were not considered on the specific inspection process uncertainty budget, however, when they are present, they must be incorporated into the budget as to not understate or be too conservative with the uncertainty of the process. The ISO GUM states that the combined standard uncertainty is the positive square root of the combined variance. This is given from Equation (9) in the report which is:

$$u_c^2(y) = \sum_{1}^{n} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i)$$

This is derived from a Taylor series and neglects the higher ordered terms. If the next highest ordered term is considered then the uncertainty is increased significantly if the second ordered effect cannot be neglected. The additional term is:

$$\sum_{i=1}^{n-1} \sum_{j=1} \left[\frac{1}{2}\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)^2 + \frac{\partial f}{\partial x_i} \frac{\partial^3}{\partial x_i \partial x_j^2}\right] u^2(x_i) u^2(x_j)$$

Those with well-developed mathematical skills will be able to follow the method given in the ISO GUM. There are examples of higher-ordered terms in the ISO GUM for reference.

Higher-ordered terms are always present. However, in linear measurements model they can usually be neglected. When the measurements model is non-linear, the probability of their terms being significant increases. Fortunately, even with non-linear models, if the uncertainties are small and are calculated at specific point values, then the higher-ordered terms may still be negligible.