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Title: The Applied Mathematics for Power Systems (AMPS)

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Abstract

Los Alamos National Laboratory: Applied Mathematics for Power Systems (AMPS)

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Increased deployment of new technologies, e.g., renewable generation and electric vehicles, is rapidly transforming electrical power networks by crossing previously distinct spatiotemporal scales and invalidating many traditional approaches for designing, analyzing, and operating power grids. This trend is expected to accelerate over the coming years, bringing the disruptive challenge of complexity, but also opportunities to deliver unprecedented efficiency and reliability. Our Applied Mathematics for Power Systems (AMPS) Center will discover, enable, and solve emerging mathematics challenges arising in power systems and, more generally, in complex engineered networks. We will develop foundational applied mathematics resulting in rigorous algorithms and simulation toolboxes for modern and future engineered networks.

The AMPS Center deconstruction/reconstruction approach "deconstructs" complex networks into sub-problems within non-separable spatiotemporal scales, a missing step in 20th century modeling of engineered networks. These sub-problems are addressed within the appropriate AMPS foundational pillar—complex systems, control theory, and optimization theory—and merged or "reconstructed" at their boundaries into more general mathematical descriptions of complex engineered networks where important new questions are formulated and attacked. These two steps, iterated multiple times, will bridge the growing chasm between the legacy power grid and its future as a complex engineered network

Project Narrative

Introduction

The Applied Mathematics for Power Systems (AMPS) Center's approach is built around three applied mathematics pillars: complex systems theory, control theory, and optimization theory (Fig. 1). These pillars are inherently interrelated by emerging problems in complex engineered networks and other areas. Different approaches that ignore one or more of these pillars are incomplete because they disregard fundamental couplings among the three pillars that are required to address these problems. These incomplete approaches would result in self-consistent but myopic mathematical formulations. Instead, the AMPS approach integrates these pillars through an iterative, multifaceted center. *A full integration of these three pillars produces the necessary mathematical tools to achieve many far-reaching goals, including, but not limited to, a fully automated, real-time monitoring, analysis, and control system for large-scale electric power grids.*

Complex Systems—The AMPS Center's approach begins with complex systems theory, where the distinct scales of the complete network are specified and the basic static, dynamic, and stochastic phenomena are analyzed. We iteratively partition, or *"deconstruct"* the network and its processes into non-separable spatiotemporal scales and identify separate sub-problems and the crucial couplings between them. Several methods of model reduction are applied at this stage, e.g., smoothing over spatially discrete network flows/injections by conversion of network flow models to Ordinary (ODE) or Partial Differential Equations (PDE) [6, 7], construction of hybrid dynamical system representations [8], and rare event analysis to identify a small number of the most probable, yet damaging, network fluctuation modes out of a continuous space of possibilities [9-12]. Models underlying these methods and phenomena determine scale-specific optimization and control sub-tasks and formulate important practical engineering problems in a mathematically sound form. These well-formulated control problems are passed to the control theory pillar.

Control Theory—The structure of the complex network formulations is influenced by state of the art control concepts. The mathematical building blocks of control theory must also develop to rigorously handle new and difficult problems, e.g., large-scale, distributed control problems with hierarchical constraints that require actions over a wide range of time scales with varying degrees of information available. Problems that are particularly large, discrete, nonlinear, and/or constrained are passed to optimization theory.

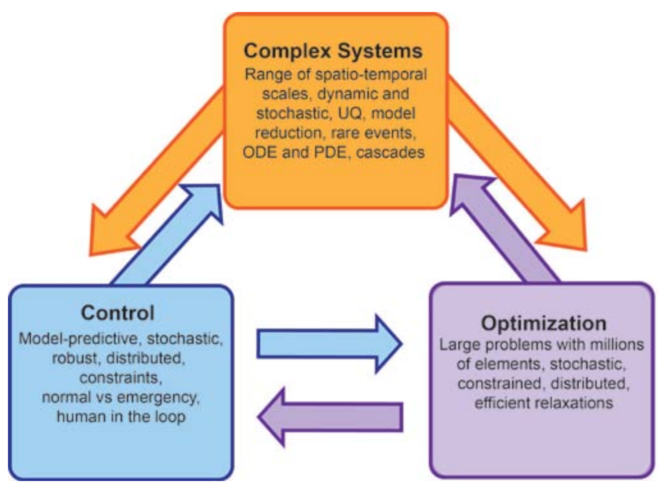

Figure 1: The three applied math pillars of the AMPS Center and the iterative flow of model development and solution.

Optimization Theory—Although related to control theory, optimization theory is better suited to address complicated nonlinear, discrete problems. Their size and structure requires advances in the mathematical foundations of optimization theory and algorithms to improve computational efficiency and accuracy, e.g., novel relaxation methods and approximation techniques [13, 14] that produce effective bounds in practicable CPU time and new methods for multi-level, robust, and stochastic optimization with chance constraints.

Iterative Approach—The three pillars of the AMPS Center are implemented in a hierarchical and iterative manner. During the natural iterative process, applied mathematics experts work across the pillars to jointly develop solvable formulations that are accurate and appropriate descriptions of the network. After the partitioned sub-problem formulations are analyzed and solution methods are developed, another iteration occurs as these models are rejoined, or *"reconstructed,"* using the couplings discovered by the complex systems analysis during the first pass through AMPS. The first-pass models and couplings are revisited and reduced further to integrate them into larger and more general models capable of a fuller and predictive description of the complex engineered network. These iterations are carried out several times as the research progresses towards a comprehensive network description. The AMPS Center's approach of breaking complex engineered networks into irreducible sub-problems and identifying cross-scale couplings provides several important properties. First, as earlier work by the AMPS team demonstrates, the individual sub-problems, although very challenging, are manageable and allow a feasible solution method [14-19]. Second, solutions to sub-problems are combined and reconstructed; and model reduction is applied again to derive a more challenging composite model. Finally, the AMPS Center's "*deconstruction/reconstruction*" approach, done in multiple iterations and with feedback, provides a natural set of goals and milestones against which our success is measured. The AMPS approach is in sharp contrast to existing "software" based approaches that couple complicated problems relying on subject matter expertise, rules-of-thumb, or convenience [20-24]. Instead, AMPS offers a mathematically sound approach to problem-driven model reduction and reconstruction.

The AMPS Center leadership (Fig. 8) is organized around the three applied math pillars. Michael Chertkov, who has 12 years of experience as a research scientist at LANL, will serve as Director. Chertkov's past project leadership includes two directed research projects with a total of $11M in funding over the last 6 years. We use a co-lead structure that pairs a LANL and non-LANL researcher for each pillar. One of the primary tasks of the co-leads is to ensure that the researchers within each pillar are collaborating closely and interacting across pillars and to monitor the progress of this crosscutting research. Each co-lead is responsible for organizing annual meetings of the team, as well as quarterly video teleconferences. Hiskens and Chertkov will lead the complex system pillar. Hiskens leads ARPA-E and OE efforts at the University of Michigan. He also has extensive experience in the power engineering industry and in academia in building complex system models of electric power systems [25-31]. Low and Backhaus lead the control theory pillar. Low is a recognized leader in control theory [32-34] with current experience leading DOE mission-related efforts, as exhibited by his ARPA-E project Scalable Real-time Decentralized Volt/VAR Control. Backhaus leads LANL's DOE-funded collaboration with New Energy and Industrial Technology Development Organization [35] that focuses on design and demonstration of coordinated control of diverse sets of resources for local mitigation of photovoltaic fluctuations. Bienstock and Bent lead the optimization pillar. Bienstock has pioneered development efforts in new optimization techniques for complex power system problems [36, 37] with his ASCR project Reconfiguring Power Systems to Minimize Cascading Failures: Models and Algorithms. Bent is PI or Co-PI on three LANL directed and exploratory research projects related to power grids (including Chertkov's) and leads optimization efforts for the National Infrastructure Simulation and Analysis Center [38].

The AMPS Center team is composed of a core group of LANL researchers and a strong team of participants from academia and other national laboratories. The LANL researchers are centered at LANL's interdisciplinary Center for Nonlinear Studies (CNLS) and are supported by LANL's major institutional investments in Information Science and Technology. The LANL group, and a large fraction of the external collaborative connections, grew naturally out of a LANL-directed research project on Smart Grids led by Chertkov during the past three years. This group of researchers began the development of new mathematical methods for the electrical grid by attacking problems that bridge spatiotemporal scales and making a number of significant contributions in probabilistic network risk measures [9, 39], data-driven network models [40], continuum models of nonlinear networks [39], control of distributed resources [15, 16, 41], and new algorithms for operations-based network expansion [42, 43].

One of the LANL team's key strengths is aggressive outreach to complementary expertise necessary to solve emergent mathematics problems. This approach carries over to the AMPS Center, which gathers the additional expertise in complex systems, control theory, and optimization theory needed to develop a comprehensive applied mathematics approach to complex engineered networks. An organizational chart (see the Management Plan) demonstrates the alignment of researchers with the AMPS Center. The Columbia (Bienstock), SNL (Pinar, Chen), Stanford (Boyd), LANL (Bent, Pan), and Cal Tech (Low) groups bring state of the art expertise in modern optimization theory, including contingency analysis [9, 36, 44-47], new message-passing frameworks for distributed optimization and control [48], stochastic optimization [49], and novel convex relaxations of previously intractable optimization problems [13]. UC Berkeley (Poolla, Varaiya), LBNL (Callaway), Cal Tech (Doyle, Low), and Michigan (Hiskens) contribute significant expertise in control theory, including risk-limiting generation control [50-60] analysis of cyber-physical control [61, 62], theory of energy markets [63-65], and statistical modeling and control of electrical loads [33, 34, 66-77]. LANL (Chertkov, Hagberg, Sinitsyn), Columbia (Blanchet), MIT (Turitsyn), and SNL (Najm, Debusschere) bring expertise in complex networks, including rare event analysis [11, 12, 78-83], robust and stochastic network control with chance constraints [84], cascading events on networks [37, 85], and uncertainty quantification [86-89].

Because the expertise required to solve the challenges of the future engineered networks does not exist at a single institution, this proposal organizes the recognized leaders in each of these fields into a tightly integrated team. This team combines expertise in core applied mathematics with experience in advancing applied mathematics to significantly impact complex engineered networks. Such combined experience is difficult to find and takes many years to nucleate. Over the last several years, the AMPS team members already developed a productive, cohesive relationship: they have submitted joint papers, held multiple visits, and exchanged students and postdocs. Collaborations on papers, proposals, and student exchanges are indicated by blue lines in the collaboration diagram in Fig. 9 in the Management Plan. The AMPS team is well positioned to have an immediate impact.

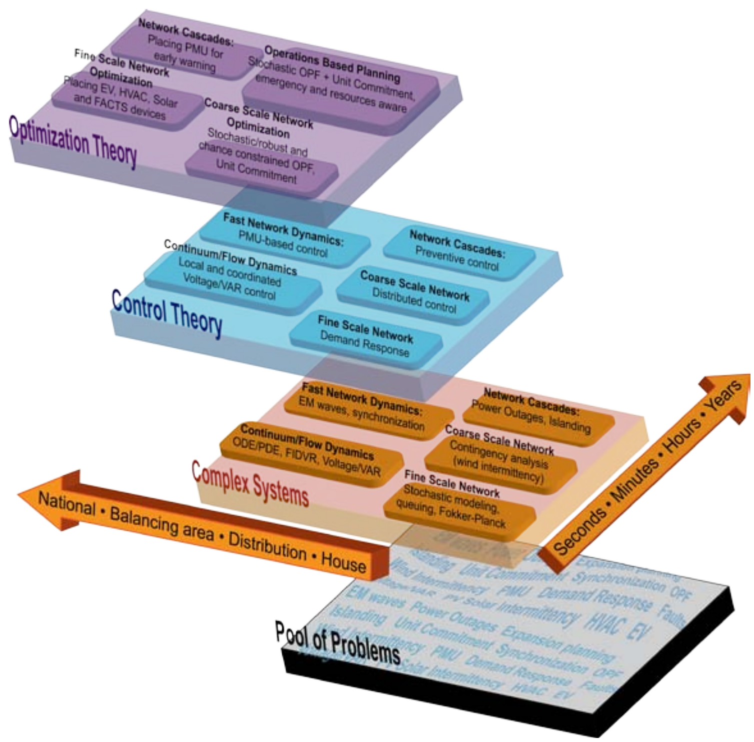

Figure 2. The six representative groups of applied mathematics challenges extracted from the pool of complex engineered network problems and how they crosscut the three pillars of AMPS. The arrows show spatial and temporal dimensions.

The AMPS Center proposed budget includes resources for 50% commitment by the Director, ~35% research commitment by the pillar co-leads, and 25-35% research commitment by the key staff. LANL's interdisciplinary CNLS, which has a long track record of organizing meetings and conferences (10-15 per year), has committed to supporting the conference agenda of AMPS, and will support three post docs and three to five student internships in the related area of complex networks (see Appendix). As the project evolves, scientific problems are solved, and new problems arise. Risk mitigation is performed by reallocating funds for postdocs and students among the key staff to achieve flexibility in scientific emphasis.

Connection to DOE Mission The AMPS Center strongly supports the DOE mission. In particular, this proposal is aligned with the mission of the Office of Electricity Delivery and Energy Reliability (OE), which states its expectation to "lead national efforts to modernize the electric grid; enhance security and reliability of the infrastructure; and facilitate recovery from disruptions to energy supply" [90]. This proposal identifies the core applied mathematics challenges needed to address the open questions of OE that intersect with the scientific goals of DOE's Office of Advanced Scientific Computing Research (ASCR) Applied Mathematics Program [91, 92], including algorithms for solving large-scale, nonlinear optimization problems and uncertainty quantification in complex engineered networks [92]. *The AMPS Center provides a bridge between basic applied mathematics research and the emerging needs of OE.*

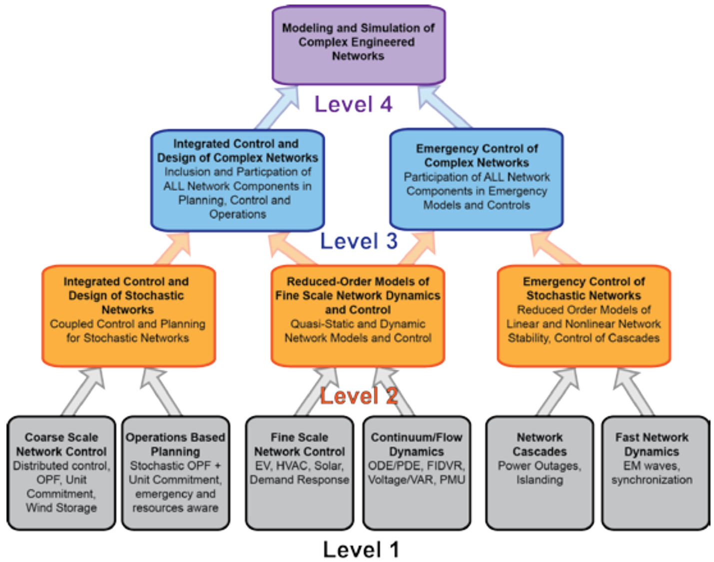

Figure 3. A "reconstruction" roadmap for the applied mathematics challenges identified by the AMPS Center. The roadmap shows the development of a series of applied mathematics toolboxes and their connection to complex engineered networks

Research Plan

Methodological Overview

It is impossible to create a monolithic numerical model of complex engineered networks that spans all spatiotemporal scales and includes all of the relevant phenomena. Instead, the AMPS Center will develop interrelated models that yield a high fidelity representation of complex engineered network behavior and provide the physical intuition, insight, and predictive power crucial for designing, controlling, and assessing risk in these networks. The AMPS Center methodology is built on three applied mathematics pillars—theory of complex systems, control theory, and optimization theory—that are implemented in a "deconstruction-reconstruction" (DC/RC) approach.

In their work over the last several years, AMPS team members used their expertise in complex systems analysis to identify the irreducible classes of spatiotemporal scales associated with the electrical grid and the fundamental couplings between these classes [9, 14-16, 39]. We transformed the unintuitive collection of superficially unrelated electric grid functions (shown on the lowest level in Fig. 2) into rigorous classes of phenomena at the core of applied mathematics. *Each of the representative classes contains several phenomena from the lowest layer, interrelated by fundamental spatiotemporal overlaps in their mathematical description. The classes discussed in this proposal are fast network dynamics, continuum and flow dynamics, network cascades, coarse-scale network analysis, fine-scale network analysis, and operations-based planning.* Additional classes are added when the need arises.

Each of the six classes defines a fundamental applied mathematics problem. By analyzing the mathematical structure within each class (or phenomena within the class), we map the class onto the best-suited applied mathematics pillar: control theory, optimization theory, or additional complex systems analysis. This breakout is shown along the "vertical" axis in Fig. 2. In general, problems involving continuous time and mostly continuous variables map to the control theory pillar, whereas those problems involving discrete time and discrete variables map to the optimization pillar. Nevertheless, the breadth of the phenomena within each class results in crosscuts between the pillars. *This mapping in Fig. 2* *represents the AMPS Center's initial deconstruction of complex engineered networks.* Our vision of complex engineered networks will evolve as we advance the state of the art within each pillar. Our first step in this evolution develops each of the six classes individually to address their underlying applied mathematical challenges and their connection to existing and future problems in complex engineered networks (as identified by the phenomena within each class). If the challenges in any of the classes prove too great, the deconstruction approach is carried out further until classes of tractable problems are reached.

The individual phenomena, or even individual classes, are initially attacked by single researchers or groups of researchers from the same applied mathematics pillar. Then researchers from different pillars will collaborate to reconstruct the individual phenomena and classes to create more general, descriptive, and predictive models of complex engineered networks, justifying the crosscutting research that is only supportable by a multi-disciplinary center such as AMPS. Our reconstruction methodology is illustrated pictorially in Fig. 3. The base of the pyramid is composed of the six applied mathematics problems identified by the deconstruction in Fig. 2. Once these problems are well formulated and computationally tractable, they are reconstructed in a pair-wise fashion on the second level in Fig. 3. The choice of the pairs is dictated by which reconstructions are most useful for the complex network under study and is revisited as priorities change. The combinations at the second level create new applied mathematics toolboxes that are critically important for the evolution of complex engineered networks. Higher levels of reconstruction yield models of complex engineered networks that provide revolutionary capabilities such as real-time simulation and monitoring of the complex engineered networks. Based on our work over the last several years, we expect that the problems on the bottom layer are technically feasible; however, the level of technical risk increases at each layer as we move up the pyramid though the series of pair-wise reconstructions. The higher risk is in the upper levels is compensated by the higher reward of success at these levels.

The Fundamental Example of a Complex Engineered Network

The AMPS Center is focused on key applied mathematics contributions discussed earlier. These contributions are motivated through application to the electrical power grid—arguably the most complex engineered network. The power grid has relied on power engineering best practices and subject-matter expertise to define the appropriate separation of the spatiotemporal scales, but as the grid evolves, many of these separations are quickly losing relevance and utility due to increasing complexity. Starting at the lowest level (shown in Fig. 2), the full range of electrical grid phenomena spans a huge range of spatiotemporal scales. The range of temporal scales is roughly nine orders of magnitude—from tens of milliseconds (i.e., a few AC cycles) to many years. The range of spatial scales is four to five orders of magnitude—from ~10-100 m (i.e., an individual node in an electrical distribution circuit) to the 1000-km national scale of large interconnections. Within this spatiotemporal domain lie phenomena as diverse as 1) grid transients that propagate over 1000-km interconnections in seconds, 2) decade-scale network expansion planning ranging from 1-km distribution circuits to 1000-km interconnections, and 3) consumer demand response acting on the second to hour scale in 1-10 km distribution networks. Using these and other examples from the power grid, we discuss in detail a subset of the problems and applied mathematics challenges arising from a loss of separation of space and time scales and the AMPS Center's approach to addressing the challenges.

Deconstruction—Complex Systems Theory Pillar and Its Challenges

Even when the basic nonlinear evolution equations of a network node and its interaction with other nodes are well known, the aggregate behavior of these networks is often unpredictable. For example, the time dependence of the network injections may be highly uncertain or network parameters and/or nodal injections may not be measured. Further, networks with a large number of nodes can show emergent behavior that is not obvious from the governing equations. *The key mission of the complex system pillar is* *to develop general complex system models at the appropriate level of network detail necessary to isolate and analyze the relevant static, dynamic, and stochastic phenomena.*

The complex system pillar crosscuts and feeds the control and optimization pillars of AMPS. It is utilized in all of the individual problems at the lowest level of the pyramid in Fig. 3. Solving these problems requires three different complex systems approaches. First, when network parameters are known or can be reasonably estimated but the network injections are uncertain, alternative mathematical descriptions are required to describe the probabilistic effects of this uncertainty on the network. Second, when there is a high degree of uncertainty in network parameters or they are not measured, data-driven methods are needed to extract static, dynamical, and stochastic models from the available data. Third, an exact description of the lowest levels in a hierarchical network may not be required, but reduced-order models of these lower levels must appropriately describe the internal network structure, preserve any emergent behavior of the real network, and predict the interaction with the higher-level network.

Complex Systems—Coarse Scale Network: Contingency Analysis through Distance to Failure

Problem: Injections at network nodes are often uncertain, e.g., generation produced by wind farms in power systems, but a probability distribution over the space of the uncontrolled injections of the network may be known or reconstructed from measurements by Uncertainty Quantification (also Machine Learning) techniques [93-95]. It is then natural to ask—what are the most probable states in the space of uncontrolled injections that cause failure? The dimensionality of this space is very large; methods are needed to efficiently search and quantify the distance to and likelihood of rare, yet most probable failure modes.

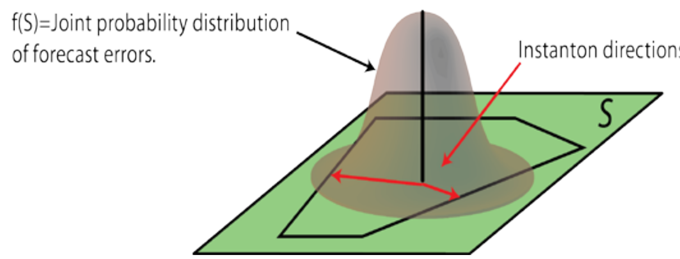

Figure 4. An instanton measures the distance and direction from the most probable configuration of the uncertain resources to the boundary of the feasibility domain in the space S of fluctuating network injections. The instanton also quantifies the probability of encountering failure along this direction.

Challenge: Searching through high-dimensional spaces of probability distributions for the most probable network failure mode is often intractable, but the properties of the boundary between the feasible and infeasible regions in the probability space depend on the mathematical structure of the network flow and the network control equations. The applied mathematical challenge is how to exploit the structure of these equations to simplify the search and devise computationally tractable algorithms.

Approach: For linear power flow models of electrical networks, the feasibility domain is a polytope in a multi-dimensional space of parameters characterizing renewable resources. The structure of this polytope depends on the spatiotemporal properties of other components in the network. For example, when the controllable generation is adjusted via secondary control [96], the output of these generators are rescaled to maintain net power flow balance. The resulting polytope is tractable in the sense that it has as many facets as the number of linear inequalities defining the feasible region, i.e., twice the number of edges in the system. In previous work [9, 39, 97], we exploited this structure to develop an "instanton" search algorithm based on approaches from mathematical physics that search for a rare but important instance of interest in huge probability spaces [98] (Fig. 4). As we showed in [10], the tractability property is the key to resolving the resulting inference (maximum a posteriori) problem exactly and efficiently.

AMPS generalizes this methodology to a number of other challenging problems in the complex network domain. Our first step extends the instanton approach to more realistic nonlinear power flow models and incorporates a broader range of feasibility conditions, including voltage collapse [6, 7] and loss of synchrony [99]. Other generalizations include extending the methodology to a spatially continuous setting, formalized in terms of spatial ODEs [6, 7], where the network is treated as a one- or twodimensional inhomogeneous medium. Our more challenging extensions will include dynamical failures of the network at different temporal and spatial scales. Connecting to current electrical-grid operating practices, the instanton methodology fundamentally extends the commonly accepted N-1 contingency (robustness) analysis [96]. Like N-1, it warns of potential future operational trouble, but it does so in a much broader and more ambitious setting by accounting for fluctuations in resources and dynamics. We will use the results of our approach to formulate objectives for the failure-avoiding schemes of the control theory pillar and risk-sensitive formulations of the optimization theory pillar.

Complex Systems—Fast Network Dynamics: Data-driven Models

Problem: Large, complex engineered networks that span thousands of kilometers may support transient wave propagation initiated by a disturbance at one of thousands of nodes, e.g., fast-propagating electro-mechanical dynamical transients launched by the loss of a large electrical generator. Direct time simulation of these waves may not be feasible because the network parameters cannot be updated at a rate sufficient for near real-time simulation. Methods are needed to estimate dynamical response with large parameter uncertainty.

Challenge: System identification via purposeful probing of the network can provide estimates of the dynamical responses, but most complex engineered networks are critical infrastructure and such probing interferes with network reliability. The applied mathematical challenge is to develop real-time, model-independent descriptions of the network dynamics using ambient network noise collected during normal operations, e.g., using Phasor Measurement Unit data from the electrical grid [100].

Approach: System identification methods [101] are typically used to extract dynamical model parameters via purposeful probing or by leveraging noise in dynamical systems. If all the parameters are identified, the resulting model is used to predict dynamics or for system control. However, large electrical grids can have over 100,000 dynamical parameters. Even if all the parameters could be identified, the speed of the disturbance propagation and the stochastic nature of future electrical grid are unlikely to allow enough time for computations to predict the impact of all possible disturbances. In previous work [40], we demonstrated how online analysis of electrical grid noise data from modern grid sensors can be used to estimate the dynamical directed sensitivity between pairs of network nodes. These pair-wise responses, or pair-wise Green functions [40], are used to forecast network-wide impacts of an initially localized disturbance. These data-driven methods are fast because they bypass the model construction and simulation stages of dynamical prediction.

The AMPS Center will extend these techniques by developing new mathematical methods aimed at solving systems of inhomogeneous linear ODEs to deconstruct the Green functions and resolve the problem of learning the hidden network parameters from available data. We will utilize these new representations of network dynamics in an automatic or semi-automatic form for new models of control. We will apply similar techniques to extract the dynamical models of aggregate, reduced-order models of distribution networks (see below). The lack of such models is a major uncertainty in assessing the dynamical stability of electrical grids. We will leverage our expertise in analysis of uncertain ODE models [86-89], model reduction [102, 103], and compressive sensing [104, 105] to quantify the uncertainty, thereby enabling the control theory and optimization theory pillar activities that require these reduced model representations.

Complex Systems—Continuum/Flow Dynamics: ODE/PDE Models for Emergent Behavior

Problem: Analysis of the electrical grid generally focuses on detailed models of generation and transmission, with connections to the distribution network commonly represented by deterministic, aggregate load models. Such representations are notoriously inaccurate [106] and lead to large uncertainties in dynamical models of transmission networks. In addition, these models are completely inadequate as distributed generation grows and loads become actively controlled. More sophisticated models are needed to combine higher fidelity models with the reality that the composition of distribution-level devices is continually changing, and thus can only ever be known approximately.

Challenge: The high density and large number of devices in distribution networks preclude the detailed component-level modeling used in transmission network models. A fundamentally different approach to distribution-level models is required to capture the interactions within the distribution network and the single-point interaction with the transmission network while avoiding parametric details that are impossible to ascertain. Figure 5 gives an example of how complex the single-point interaction can become.

Approach: In previous work [6], we derived abstract models of distribution networks using ODEs that resolved many individual components (e.g., loads) in an aggregated, spatially continuous fashion. We have also derived PDE versions of these models that capture both the continuous spatial and temporal behavior of these systems. Rigorous and consistent derivation of ODE/PDE models from the spatially discrete models is a challenging task. It involves homogenization in space to account for disorder in characteristics of lines, loads, and generation with a minimal number of key parameters while not losing the mathematical description of important constraints on device operation [15]. In addition, stochastic representations are needed to account for temporal uncertainty. The initial ODE/PDE models we developed in [6] are superior to discrete-element models for capturing dynamic interactions and emergent behavior. Examples include Fault-Induced Delayed Voltage Recovery (FIDVR) that manifests as two distinct, overlapping, stable solutions to an ODE model of a distribution network [107] and limit cycle behavior attributed to time-dependent interactions between the distribution grid and the power electronic controls in network with even relatively small amounts of solar PV [57].

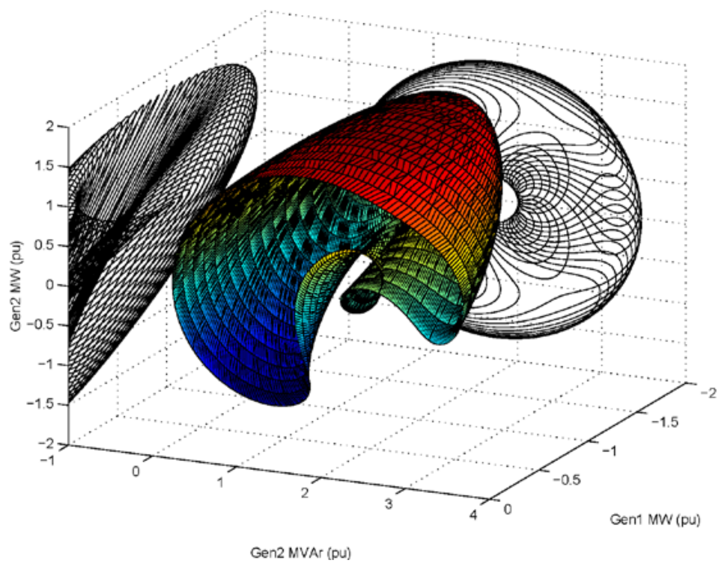

Figure 5. Power injection region for a simple three--bus network demonstrating the high degree of complexity, even for such a simple system. The projection to the left shows an elliptical shape, which is the space of all possible combinations of real power injections for two of the system buses. The projection to the rear shows a donut shape; it is the space of all possible combinations of real and reactive power injections at one of the buses.[3]

The AMPS Center will extend our previous work by creating comprehensive ODE/PDE models that incorporate the different devices and controls that are making inroads into electrical distribution networks, e.g., PV systems, electric vehicle charging, and frequency responsive loads. The interaction of new components and sophisticated controls will generate a richer set of emergent behaviors in future distribution grids; these collective behaviors will impact the higher-level transmission grid. This was illustrated in [7], where emergence of multiple competing and dynamically stable low voltage solutions in a feeder with distributed generation was discovered. We will use spectral analysis techniques to isolate and analyze these behaviors, creating formulations that can be used by the control theory pillar to manage them. Our ODE/PDEs approach will be computationally tractable and easy to adjust, making it the only reasonable approach to model such emergent phenomena.

Solution Methods I—Control Theory and Its Challenges

The future controls of complex engineered networks are fundamentally different from today in several ways. First, to gain the flexibility needed to integrate stochastic network injections, future controls will reach deep into fine network scales and control many millions of small injections, instead of the few thousand, high-level large injections controlled today. Second, future complex engineered networks will be subject to far greater local and global fluctuations, and fast emergency control actions intended to protect locally may lead to an unintended cascade of additional control events that disable the network globally. *Significant advances in control theory that enable scalable, coordinated control of millions of distributed assets and incorporate global situational awareness into local protective control decisions are the key missions of the AMPS control theory pillar*. Problems of interest include developing new methods that provide performance guarantees in aggregated or autonomous distributed control frameworks where control is required over different time scales with different levels of information, e.g., slow control modes where centrally communicated information is available to fast emergency control with local information and some inferred global information. Developing adaptive controls is also important because failures, corrective actions, or network expansion can cause network structural changes. To address these problems, we propose novel control schemes that control large numbers of devices with a small number of signals, PDE formulations of control that provide robust guarantees on performance, and incorporation of rapid simulation of global network behavior into local protective control decisions.

The control theory pillar is strongly connected to the complex systems and optimization pillars. The complex systems pillar partitions complex network models into irreducible spatiotemporal couplings that yield reduced-order models. Control theory is the mechanism by which controls are designed and analyzed to extract the desired behavior from these models. Advances in control theory enable the analysis of more difficult models, thereby driving the complex systems pillar to address more difficult problems. Models that are too large or involve difficult nonlinearities are formulated in control theory, but are solved in the optimization pillar.

Control Theory—Network Cascades: Modeling and Controlling Undesired Propagations

Problem: Emergency control of complex networks is traditionally performed at the local level to provide fast response to initially local problems, e.g., a local network overload. These automated control actions are designed to protect the network in the immediate area, but lack of global information and situational awareness can lead to poor global outcomes, such as the initiation of a chain of cascading local actions that disable the network globally [108, 109] (Fig. 6). There are a small number of specially designed protective schemes at critical locations in the electrical grid that incorporate global information via "hardwired connections." Nevertheless, the increasing level of fluctuations from stochastic generation will cause a proliferation of an unsustainable number of such schemes. Instead, global information must be incorporated into local decisions in a principled manner; simulation of possible cascades is a promising route.

Figure 6. Simulation of the 2011 San Diego cascading black out. The green lines are operating normally. The blue lines were the lines that tripped. Red, orange, yellow, white are lines that were progressively more overloaded. Details of the approach are found in reference [2]

Challenge: From an applied mathematics standpoint, cascading behavior on complex engineered networks epitomizes a multi-scale stochastic process, both in time and space, giving rise to two complementary technical challenges: the accurate simulation of a cascade based on initial conditions and the rapid computation of mitigating actions. Current techniques [110-112] can simulate complex networked systems with tens of thousands of edges and nodes in practicable computational time, albeit at the cost of simplification in the modeling of some dynamics important during cascading processes and not fully accounting for wide stochastic variance. New methods and models are required that faithfully represent these dynamics while enabling fast computation.

Approach: Cascade simulations generally include random behavior of the initial timing of the event. Moreover, as shown in [85], different sequences of the cascading events result in undesirable final network states (such as outages) of greatly varying size. For cascades in the electrical grid, these results suggest that controlled tripping (topology modifications) of lines at the early stages of the cascade development is extremely advantageous, but algorithms are needed to identify the optimal control actions. Our ongoing research [37] suggests that real-time algorithms based on massive and parallel simulations of the cascade using noisy and partial observations of a cascade's progress yield optimal controls to arrest the cascade. The algorithms for computing optimal control are akin to learning (or "boosting") methods, sifting in real time through a large amount of synthetic data to deduce the control. Developing these computational learning algorithms is a natural extension of our ongoing work [37]. To improve the simulation component, we envision a parallel approach designed to capture the complexities of cascades in a methodologically valid manner, while yielding effective algorithms to mitigate cascades. First, we will develop reduced-order stochastic models of the short-term dynamics that exert great influence at pivotal points in a cascade. Second, we will develop new approximate statistical methods based on deep understanding of the underlying interactions and scale separations and importance sampling of entire sequences of steps to reduce the inherent combinatorial complexity of cascades.

Control Theory—Fine-Scale Network Control: New Principles and Challenges of Distributed Control of Millions of Grid-Interactive Devices

Problem: Integrating significant levels of stochastic generation (e.g., wind and photovoltaic) into the future electrical grid requires reaching deep into fine distribution network scales to tap into the latent control flexibility of the millions of small injections (loads, generation, and storage) in these networks. Exploiting this latent resource, however, presents problems [32]. Foremost is the extreme computational burden of coordinating the control of the millions of injections required to create a significant impact on the grid as a whole. Controlling the loads individually is not feasible so new methods are required. In addition, when these injections are controlled, the electric power flows in these fine-scale (distribution) networks are modified from their nominal design values, resulting in networks that are significantly more fragile. Therefore, the algorithms that implement the control over the injections must be locally aware to avoid violations of flow and nodal constraints in these distribution networks.

Challenge: A crucial challenge is overcoming the computational burden of controlling a large number of injections. The true challenge is not just developing algorithms that are computationally feasible for large problems, but developing algorithms whose performance improves as the number of devices increases. Communication limitations suggest a need for algorithms that are fully distributed or centrally implemented with a small number of control signal/information channels. In addition, mathematical methods are required to provide guarantees of algorithm performance under widely variable disturbance and network conditions. Analysis of such guarantees must be extended to include the violation of nodal and flow constraints.

Approach: In previous work, we developed a range of different algorithms for control of electric vehicle (EV) charging. These range from stochastic direct load control using a single number disseminated via broadcast communication [16, 33, 66, 69, 71, 72, 113], broadcast of a single dynamic pricing signal combined with hysteretic local controllers [57, 74, 114-116], and game-theoretic methods involving bi-directional, individually addressed communication [26, 113, 115]. The first two methods demonstrate excellent scaling properties where the variance in the controlled EV charging load decreases relative to mean load as the number of EV under control increases. All of these methods, however, are capacity based; they do not consider the effects of the controlled power flows on the internal constraints in the distribution network. Within the AMPS Center, we will extend these methods to include models of network flow constraints.

The integration of distributed photovoltaic (PV) generation into low-level electrical distribution networks is a good example of how low-level network internal constraints are violated when the network is forced into configurations for which it was not designed. The spatiotemporal correlations of solar irradiance fluctuations can create sudden and large reversals of electrical power flow causing nodal voltage to drift outside the bounds of normal operations, and resulting in damage to consumer devices. One straightforward control solution limits PV generation by acting directly on the fluctuating injections, but this very suboptimal solution severely constrains the ability of the low-level networks to supply generation to the higher-level network. Alternative formulations by AMPS team members [15, 17, 117] leverage control of reactive power generated by the PV inverters to mitigate this effect. Both control algorithms rely on distributed control because it is robust to communications failure and the temporal scales of the irradiance fluctuations are fast enough to preclude centralized control. The performance of these distributed control algorithms [15] was validated using Monte Carlo sampling of irradiance conditions, network load, and network configuration. Within the AMPS Center, we will develop more principled probabilistic measures of control performance using stochastic and uncertainty quantification methods. These new probabilistic measures will enable design of novel distributed and robust control schemes that provide coordinated and guaranteed operations for the grid consisting of millions of integrated devices.

Control Theory—Fine-Scale Network Control: Limited Control Flexibility and Coupling across Time Scales

Problem: The operation of some electrical loads (e.g., thermal processes) is deferrable without significant impact on the end-use function. Nevertheless, there are constraints—based on the physical characteristics of the loads and user preferences—on how much load shifting is possible. These constraints, which are directly analogous to those incurred by energy storage [118], create new couplings over time where control decisions made at one moment affect the flexibility of the control at future times. These future effects must be included in the immediate decision-making. Further complications arise because these loads are typically discrete, i.e., on/off loads that are not continuously adjustable. New control formulations are needed that address the unique nature of these loads.

Challenge: The couplings created by limited deferability of loads over time create a need for models of time evolution, but the discrete, on/off nature of these loads presents a mathematical challenge because the time-dynamics involve discontinuous state transitions that are not easily addressed via continuous-time control formulations. In addition, individually modeling and controlling millions of these discrete loads is not computationally feasible.

Approach: In previous work [41, 114], we explored first-principles methods for modeling and controlling large homogeneous collections of these unique loads. We developed [41] a first-principles approach, based on two Fokker-Planck PDEs, that describes the continuous-time evolution of probability distributions of temperature for the on and off states. Coupled boundary conditions on these PDEs at the extremes of the temperature range create a flow of probability between the on and off states, representing the discontinuous state transitions of individual loads. Analysis of these PDEs allows extraction of response functions that guide design of controls for the aggregate load and the probability distributions. We also explored feed-forward methods [41] that leverage the unique dynamics of these loads to preplan controlled responses. The AMPS Center will extend this work to descriptions of heterogeneous collections of loads. We propose a completely new perspective on the Chapman-Kolmogorov equations through a rigorous treatment of randomness. We will develop new mathematical tools to represent system energy and thermal states with dynamic nonparametric probability distributions, where state transitions evolve according to Markov chains. This discrete state model formulation is represented in standard linear time invariant-state space formulation and facilitates the large-scale state estimation methods for which this formulation is especially well suited.

Solution Methods II—Optimization Theory and Its Challenges

Complex engineered networks can be very large, spanning a wide range of spatiotemporal scales. Optimization is required for coordinating large numbers of network resources to manage uncertainty in demands, availability, and capacity. The resulting optimization problems are often nonlinear, non-convex, mixed integer, and stochastic, making them computationally intractable (NP-Hard), and placing them far beyond the reach of commercial software. The key mission of AMPS in the optimization pillar is to *develop significant advances in optimization theory to address currently intractable problems.* Problems of interest include multi-level models, where the lower-level optimization problems are themselves NP-Hard. The lower levels may also include non-standard constraints that encapsulate results from complex network analysis, e.g., new dynamical stability criteria [99], models of nodal responses to exogenous information [39], and functional evaluation through simulations [1, 14]. To solve these problems, we propose novel relaxation methods and approximation techniques that produce effective bounds in practicable CPU time and new methods for multi-level, robust, and stochastic optimization with constraints accounting for risk—all of these together with careful leveraging of massive distributed computing resources.

The optimization pillar is strongly connected with the control theory and complex systems pillars. The complex systems pillar provides information about temporal and spatial couplings required to build the optimization models. Optimization is then a mechanism by which complicated sub-models of control theory can be solved. Advances in optimization inform complex systems about the problem structures and details that are tractable, which may change the spatial and temporal couplings that are modeled.

Optimization Theory—Fine-Scale Networks: Decentralized (Message Passing) Optimization

Problem: Increasingly, optimization is faced with models that are so large that the problem cannot be constructed in one place because the communications bandwidths are insufficient. This is especially true for optimization of networks of physical devices, where network-state information can be quite large. Decentralized optimization techniques are required that utilize more limited communication between individual devices, with components exchanging messages about their state and planned actions (decision variable assignments) and negotiating to minimize network-wide cost.

Challenge: The challenge in this area is to develop distributed decentralized algorithms for complex networks that not only converge to a feasible solution, but also provide guarantees on optimality or approximations to optimality. Distributed optimization with optimality guarantees is required for coordinating the large numbers of devices that manage uncertainty in demand, availability, and capacity in future complex engineered networks, including the future electrical grids, where vast numbers of devices are employed to control power flows, consumption, and generation locally. This distributed optimization represents a fundamental break from current practices that optimize a comparatively smaller number of aggregated devices in a centralized fashion.

Approach: We have recently demonstrated the basic concept of completely decentralized optimization [48] by developing a method for coordinating time-dependent flows on a network. We have shown that guaranteed optimality can be achieved in decentralized optimization while only passing simple messages and maintaining the privacy of the devices. Much like internet protocols, decentralized peer-to-peer exchange of information is a very resilient, secure, and reliable system that automatically coordinates individual devices, and optimizes an overall network objective. Such a structure for distributed optimization reacts to changes in real-time (as a feedback system does) and also provides a feed-forward ability to anticipate known or predicted upcoming events. Our approach can solve a problem involving around 30 million optimization variables, e.g., a future electrical grid with 100,000 devices exchanging energy for one day in 15-minute intervals over 250,000 capacity-limited, lossy lines–in minutes on a basic desktop 8-core machine. This fundamental advance demonstrates the potential of decentralized optimization in the domain of complex engineered networks. Within the AMPS Center, we will develop the fundamentals of distributed optimization science including the generalization of a message-passing methodology to account for loops in power flows and integer constraints and controls (non-convex NP-Hard problems). We will produce new convex relaxations of complex nonlinear optimization problems that may be implemented on a distributed platform [119].

Optimization Theory—Operations-based Planning: Multi-Level Optimization

Problem: Traditionally, the design of complex networked systems has been optimized using simplified models of network operations. These simplified models reduced the computational complexity and yielded results that were considered *good enough*, e.g., the outer optimization of grid expansion planning has historically approximated the inner optimization of the bi-level problem with linear power flow models [1, 4, 14, 42, 120-135]. Situations are arising, however, where using overly reduced models has led to network designs with adverse operational characteristics, e.g., loop flows [128] and negative energy prices [136] in power systems, pointing to the need for fundamentally new network expansion planning algorithms that integrate high fidelity models of operations. These examples provide motivation for new results in one of the emerging challenges in optimization—bi-level or multi-level optimization of nonlinear and discrete models, where one or more lower-level optimization problems are embedded in another optimization problem [1, 4, 14, 42, 120-135, 137, 138]

Challenge: Purely linear multi-level optimization problems are NP-Hard but there are well known solution concepts for such problems [138]. In contrast, addressing multi-level problems important for complex engineered networks requires considerable advances. These problems have nonlinear or discrete aspects [139], such as optimal electrical grid design coupled to models of power flows, operations, and security.

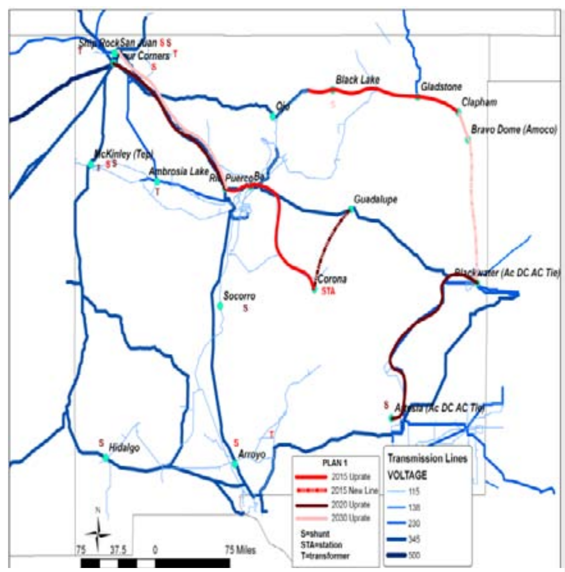

Figure 7. The work of [1] was applied to aid the state of New Mexico in its transmission planning studies [4, 5]

Approach: We adopt three strategies for solving multi-level problems: 1) improving the quality of the lower bounds, 2) creating decomposition algorithms to improve the quality of the upper bounds, and 3) generalizing the cutting plane algorithms. The lower bound strategy builds on our team's recent efforts in developing convex relaxations of complex nonlinear optimization problems, such as AC optimal power flows [13] and iterated improvements of linear approximations to nonlinear problems [140]. The decomposition strategy considers methodologies that split complex problems across their natural temporal, spatial, and level boundaries, while at the same time providing mechanisms to pass information between the different decompositions to improve global solution quality (upper bounds). In previous work, we pioneered efforts to improve the bi-level couplings through decomposition for planning utilizing nonlinear (AC) power flow models [1, 4, 14, 17-19, 42, 117, 128-130, 141] (concurrently with a handful of other researchers in the field [131-133]). This work has demonstrated how our basic applied mathematics has transitioned to impact DOE-related missions, as shown in Fig. 7. The cutting plane approach is based on our success in screening power system vulnerabilities via worst-case interdiction analysis [36, 44-47] and bringing those models into problems like unit commitment and transmission planning [134, 135, 142]. In order to bring N-k security operations into planning, we propose to extend linear bi-level and multilevel optimization methods in security-constrained power system optimization to consider discrete components (in the lower-levels). Efficient flow-cover algorithms will be used to find small sets of contingency elements that fail a power system. This approach relies on solving a bi-level integer program using inequalities derived from the generalized flow-cover inequalities [143, 144] and disjunctive cutting planes [145, 146].

Optimization Theory—Operations Based Planning: Stochastic/Robust Optimization with Probabilistic Risk Constraints

Problem: The control of complex engineering networks takes place on several levels and time scales. The lowest level is often preprogrammed as continuous-time automatic feedback control that responds to fast changes in network conditions without any external intervention. Often, the next highest level is an optimization problem that deploys network assets for optimal steady-state performance. This step effectively designs or plans the resources available for automatic control in future time periods. When fluctuations in the network conditions between optimizations are relatively minor, utilization of mean conditions is sufficient. However, complex engineered networks, especially the electrical grid, are expected to survive large fluctuations in network injections and network flows in the future. Deploying networks assets by “optimizing for the mean” or through traditional scenario-based stochastic programs with recourse is no longer sufficient. Instead, methods of stochastic and/or robust optimization that incorporate models of automatic control system response are needed to balance optimal network performance with resilience to rare but damaging fluctuations.

Challenge: Robust and stochastic optimizations are two methods of accounting for uncertainty. Formulating and incorporating new constraints expressing the uncertainty in a combined fashion presents significant mathematical and computational challenges because of the mathematical structure of the models of network flows and automated control responses. The main challenge is the probabilistic nature of the so-called “chance constraints” and their intricate dependence on the optimization parameters. An example is a chance constraint that expresses that the probability of overloading a line being less than a pre-defined small number, which follows from solving the power flow equations. The challenge is to model these constraints so that the problem is computationally tractable.

Approach: In previous work [84], we used real-time control and optimal dispatch of traditional generators in electrical networks to pioneer the development of stochastic and robust optimization formulations with chance constraints. Instead of optimizing the output levels of renewable generators with mean forecasts, we developed a stochastic optimal power flow model that minimizes the expected generation cost under chance-constraints and ensure that the likelihood of transmission line overloads is below a small but non-zero probability. In this approach, we leveraged properties of the wind forecast errors and a simplified model of power flow (DC approximation) to derive a computationally tractable conic optimization problem. Within AMPS, we will continue this research direction. First, we seek to utilize more realistic models of electric power flows, even though these models introduce additional nonlinear complications in expressing implicit dependence on optimization parameters for robust and stochastic optimization. Second, we extend this approach to additional control schemes at other spatiotemporal scales, including the operation of electrical distribution networks, e.g., control of real and reactive power dispatch from storage in these networks. In both situations, we will develop novel, computationally tractable, conic derivations.

Challenges of Reconstruction

Figure 3 shows how our deconstruction/reconstruction DC/RC approach reduces complex engineered networks into classes of computationally feasible sub-problems based on analysis of the spatiotemporal scales and the primary applied mathematics pillars. This complexity reduction identifies the crucial couplings between these sub-problems and across spatiotemporal scales. Figure 3 also shows how our multifaceted, integrated approach describes a path for discovering relevant new models of increasing complexity that are addressed with the broad expertise available across the center. The structure in Fig. 3 is informed by our DC/RC approach as we integrate and develop control and optimization algorithms by reconstructing the Level 1 problems that are most closely coupled across spatiotemporal scales. Members of the AMPS team are already developing the six primary focus areas at *Level 1* in the pyramid. These include coarse-scale network control [84], operations-based planning [42, 43], fine-scale network control [15, 16, 26, 57, 113-115], continuum/flow dynamics [6, 107], network cascades [37, 85], and fast network dynamics [99, 147]. Developing these individual areas is crucial for attacking design, control, and risk assessment for complex engineered networks. However, it is the integration of these areas into a logical framework that creates a set of applied mathematics toolboxes that are more powerful than the constituent pieces. Next, we describe how AMPS will pursue this integration.

Level 2—Integrated Control and Design of Complex Stochastic Networks

Problem: The need for operations-based planning motivates the discovery of new applied mathematics for solving nonlinear, discrete, multi-level optimization problems—a challenge that is amplified when control is embedded as a decision variable in a design problem. It is easy to envision that a distributed control strategy would find a particular network design to be optimal, whereas a centralized control strategy would find an entirely different design is required. Thus, it is important to co-design the control and network configuration to discover the optimal combined control and network design. Critically, this requires a novel integration of control algorithms and optimization algorithms.

Challenge: In independent optimization problems, it is reasonable to expect that a single optimization strategy yields good results (mixed-integer optimization with Bender's decomposition, column generation, etc.). In contrast, combined control and optimization problems are multi-level problems that may have very different and challenging structures requiring advances in the applied mathematics to find high quality solutions, e.g., advances in the relatively new field of *hybrid optimization* [148]. The challenge is to discover structural properties of classes of problems that lend themselves to being solved efficiently by different optimization strategies, e.g., constraint programming for feasibility problems, mixed-integer program for discrete optimization with tight linear relaxations, and semi-definite programming for problems with tight convex relaxations. Novel combinations of these strategies are required to solve classes of problems that exhibit two or more such structures.

Approach: The AMPS Center will advance to Level 2 in this area by developing a new hybrid control-optimization paradigm for problems that are currently intractable using existing approaches, e.g., co-design of network control and topology. Our initial approach involves modeling control strategies (policies) as 0-1 variables and embedding them directly into multi-level optimization problems. A second approach relaxes this set of variables to allow the selection of multiple control strategies across spatiotemporal scales or operating modes, where the modes are selected through column generation. A third approach combines constraint programming with convex optimization [119] to produce a powerful new paradigm for solving optimization with difficult feasibility constraints such as chance constraints. There are compelling reasons to believe that combining approaches will prove to be effective, including recent work demonstrating that previously unsolvable scheduling (pure optimization) problems [149] can be solved by combining mixed integer programming with constraint programming and our own work in combining constraint programming with local search to solve network restoration scheduling [150-155].

Level 2—Reduced-Order Models of Fine-Scale Network Dynamics and Control

Problem: The ODE/PDE models from continuum/flow dynamics are powerful techniques for describing fine-scale networks where the large numbers of nodes are homogenized to create a continuous representation. However, significant mathematical difficulties arise when these models are combined with control schemes to create composite models for designing and analyzing fine-scale network controls, e.g. infinite dimensionality of the control, model parameter uncertainty, and dynamics which may have been ignored at the homogenization step.

Challenge: New applied mathematics techniques are required to discover distributed, decentralized, and low-order control schemes and to analyze these schemes for control performance and stability guarantees.

Challenges in this process arise from model uncertainty from homogenization of PDEs and the need to include the myriad of new distributed devices of the fine-scale networks.

Approach: The analysis of the PDE models provides significant insight into the dynamics of fine-scale networks. It is expected that the continuum dynamics have a set of fast modes that are stable and decay rapidly and a smaller set of slow modes that are potentially unstable and need to be controlled. We will segregate the fast and slow modes using spectral analysis to verify the damping of the fast modes and isolate the critical slow modes. Examples of slow modes include oscillatory and wave-like disturbances such as voltage oscillations caused by power flows interacting with the collective behavior of power electronics [57] and power flow-voltage waves caused by coupling to induction motor dynamics [107]. Other slow modes are hysteretic in nature, e.g., collective behaviors associated with FIDVR [147]. By isolating the slow modes, we enable coarse-grained formulations in a frequency-wave vector space suitable for the design of low-order controls. Our approach incorporates model uncertainty and stochastic information by utilizing concepts from stochastic and robust control [156] in our new formulations

Level 2—Emergency Monitoring and Control of Networks

Problem: Fast dynamics can generate short-term, locally large deviations from the predictions of static network models. The deviations trigger local protective actions that are often not detected by network cascade simulations based on static network models. These interactions elucidate the need for combining two very challenging subjects in applied mathematics–fast network dynamics and network cascades [157, 158]. Efficient combination of these models will contribute a comprehensive computational, adjustable, and much needed toolbox for emergency control that currently does not exist [159].

Challenge: The disparity in time scales between models of fast dynamics and the quasi-static network models used in cascade simulations creates a significant challenge. The computational expense of the dynamical simulations and stochastic nature of the cascades makes the direct integration of the dynamical simulations and cascade models computationally intractable. However, fast dynamics must be included to yield an accurate and predictive model of cascading failures in complex engineered networks. New applied mathematics methods are required to merge these two disparate models.

Approach: We will address this challenge by bypassing the direct time simulations and instead integrate fast dynamics with cascade simulations using the data-driven methods of the Complex Systems pillar. By monitoring ambient electrical grid frequency and voltage noise, we will create a large-scale, multi-scenario database of pair-wise electromechanical-wave Green’s function responses that will be used to estimate the fast dynamics between all pairs of network nodes. Our approach enables the rapid estimation of the fast dynamics at each node in the network following discrete cascading events such as the sudden removal of a transmission line or the emergency disconnect of a large generator. Estimates of the dynamic excursion at each node allow the cascade simulation to efficiently scan for dynamically triggered failures that propagate the cascade farther.

Levels 3 and 4—Higher Levels of Integration

The Level 2 models discussed in the prior section are beyond the reach of current network analysis tools. Resolving these models requires coherent efforts from all the participants of AMPS. Nevertheless, the applied mathematics challenges of complex engineered networks are evolving and new challenges will arise. Levels 3 and 4 in Fig. 3 exist to ensure flexibility to meet such challenges and reduce risk. The solutions at Level 1 and Level 2 provide the fundamental tools to address emergent questions. The combinations at higher levels also provide a mechanism for transitioning the applied mathematics to tools that support DOE mission. For example, when the science of new instabilities matures and the science of EM wave diagnostics becomes computationally efficient, a full-scale stability analysis is feasible. Such a combination allows the extraction of qualitative understanding of how collective distribution effects, such as FIDVR voltage collapse and collective transmission systems failures (e.g., of the loss of synchrony type), affect and interact with each other. *Ultimately, the research outlined in this proposal provides foundational applied mathematics to bring the grand challenge of real-time simulation and monitoring of complex engineered networks, including the power grid, to fruition.*

Management Plan

This large, multi-institutional project requires a strong management structure to maintain project cohesion and to build and maintain momentum. Here we outline our plans for managing the overall effort.

Management Structure & Budget

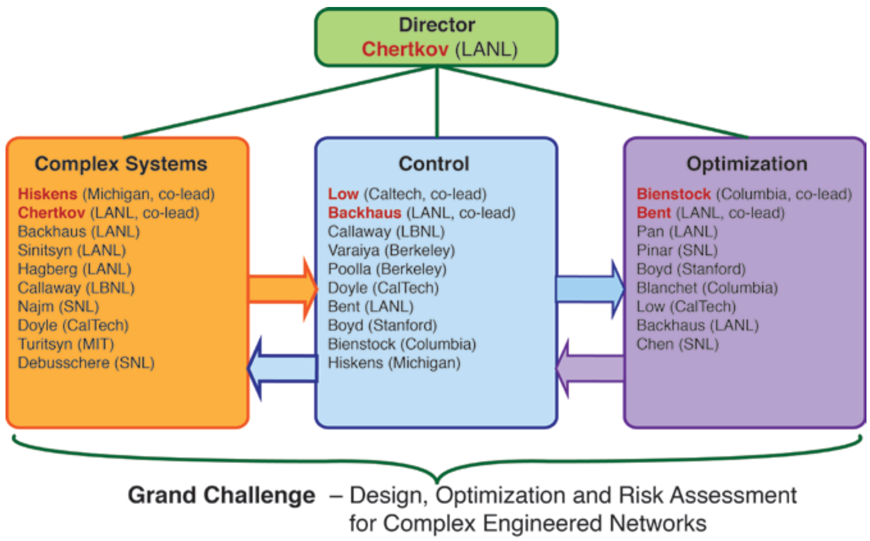

Figure 8. Breakdown of the team and leadership between the three pillars of AMPS.

Management of the AMPS project will be shared among the team, including the Director, the Science Team co-Leads, Laboratory and University Leads, SciDAC Institute Liaisons, and Senior Personnel (Fig. 8 and Table 1). The Director, Michael Chertkov is responsible, along with Science Team and Laboratory leads, for overall project coordination, including organization of meetings and conference calls, tracking progress, and reporting to DOE Program Managers. Chertkov has demonstrated both strong scientific and organizational leadership. He has set the scientific direction of two LDRD projects (totaling \$11 million over 6 years), including directing research by many students, postdocs, other LANL staff, and collaborating researchers. During these projects, he went well beyond stated requirements by organizing unique project reviews that brought together a diverse set of researchers into mini-conferences. These conferences helped to establish additional collaborative connections and stimulate discussions that resulted in concurrent DITRA/DOD and NSF funding. The most recent project on smart grids results in more than 30 publications in peer-reviewed journals and conferences (over three years), and more than 10 invited talks, including a super-session presentation at IEEE/PES and a plenary talk at SIAM/DS. The AMPS Center team is stronger because of connections made in this manner. Chertkov has organized 15 high visibility conferences, the most recent of which was a technical conference on mathematical advances for power grids, held May 21-25, 2012. This conference was the culmination of the LDRD project on Smart Grids [160]. In consultation with the AMPS Center's pillar leads, Chertkov will take on a similar role for this work, setting the broad scientific direction in the three pillars. He will take responsibility for overall project coordination, including managing the budget, reporting requirements, and tracking the scientific progress against project milestones via quarterly project meetings. Further, Chertkov will build a robust applied math and scientific community around complex engineered networks by continuing his outreach. He will disseminate the results of the AMPS Center and maintain and create new collaborative relationships by organizing seminars and conferences. These seminars and conferences will provide an opportunity for applied mathematicians and other scientists to interact with the AMPS Center

Table 1. AMPS's *Center annual budget breakdown by institution and applied mathematics pillar.*

Institution	Lead	Director ($K)	Complex Systems ($K)	Control Theory ($K)	Optimization Theory ($K)	Total ($K)
LANL	Chertkov	250	525	125	450	1,350
SNL	Pinar		200		250	450
LBNL	Callaway		100	100		200
Cal Tech	Low		100	200	100	400
Columbia	Bienstock			100	200	300
Berkeley	Poolla			275		275
Stanford	Boyd				200	200
Michigan	Hiskens		100	100		200
MIT	Turitsyn		125			125

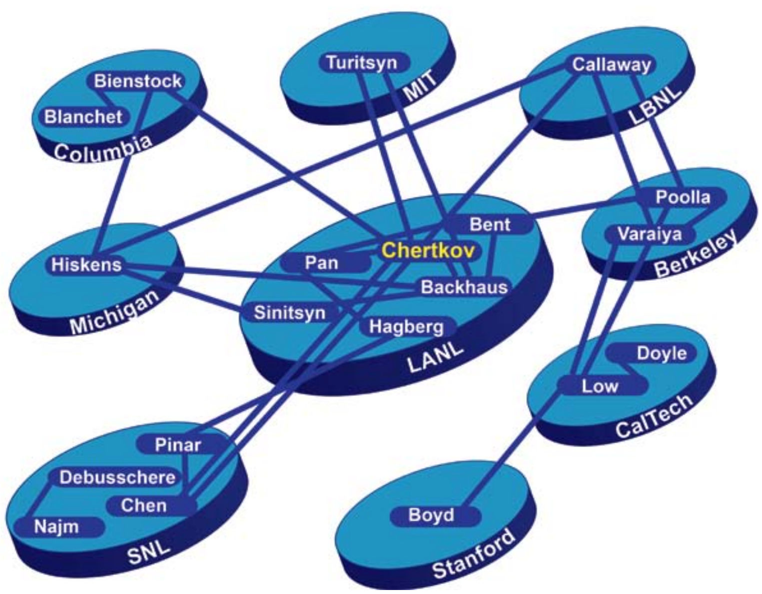

Figure 9. Pre-existing collaborations between AMPS team members. A line represents collaboration in the form of a paper, a student, or a postdoctoral researcher

The Science Team leads are responsible for tracking progress on relevant tasks for each applied mathematics pillar. They also form a committee that will help with decisions on scientific direction and will communicate with each other on shared tasks. In the complex system pillar, Chertkov and Hiskens form the leadership; for the control theory pillar, Low and Backhaus form the leadership, and Bent and Bienstock lead the optimization pillar. Laboratory and University leads are responsible for tracking budget and deliverables for all work performed by their institutions and will ensure timely reporting. The Science Team leads are also responsible for organizing annual workshops and meetings to reach out to the greater scientific community. Senior personnel will not only perform work, they will also advise students and postdocs on the project. The team's focus on student and postdoc education ensures that the impact of the AMPS Center extends beyond its duration, as the center will train the scientists needed to contribute in this area over the next 20-30 years. Table 1 demonstrates how the annual funding is allocated between applied math pillars and institutions. Annual funding for individual science topics is listed with the Science Team leads, and annual funding for each institution is listed with the Lab and University leads. Annual funding for Senior Personnel is specified in the budget documents.

Project Coordination

Success of AMPS depends critically on the strength of the partnerships formed between participating complex systems, control, and optimization scientists. This partnership is natural, as many of the team members have worked together previously and are working together on other projects (Fig. 9). We present here additional mechanisms for maintaining project cohesion in a distributed multi-institution project. Project managers and science team leads will coordinate regular conference calls for sharing their latest results and planning of upcoming collaborations and visits. When new expertise is required, relevant researchers outside of the team will be invited to share their work at one of our seminar series (at LANL, Caltech, and Berkeley). These seminars will occur on a monthly basis. On a quarterly basis, DOE program managers will be invited to join the calls as an informal form of project reporting. Physical meetings will take place periodically, taking advantage of existing conferences or meetings when possible. Dedicated project meetings for all project participants will occur at least annually in the form of workshops and more frequent if significant fractions of people are present at a large meeting (e.g., SIAM/DS, CDC, IEEE PES, INFORMS). Other meetings will be scheduled as needed to address specific issues.

Our primary means of collaboration is through the team's students and postdocs. Students funded under AMPS will spend summers at CNLS, interacting with one another and LANL scientists. Many of the post-doctoral positions will be joint appointments, which will ensure inter-institutional collaboration. Indeed, this team's existence is a product of our recent success using this structure to promote collaboration and group cohesiveness.

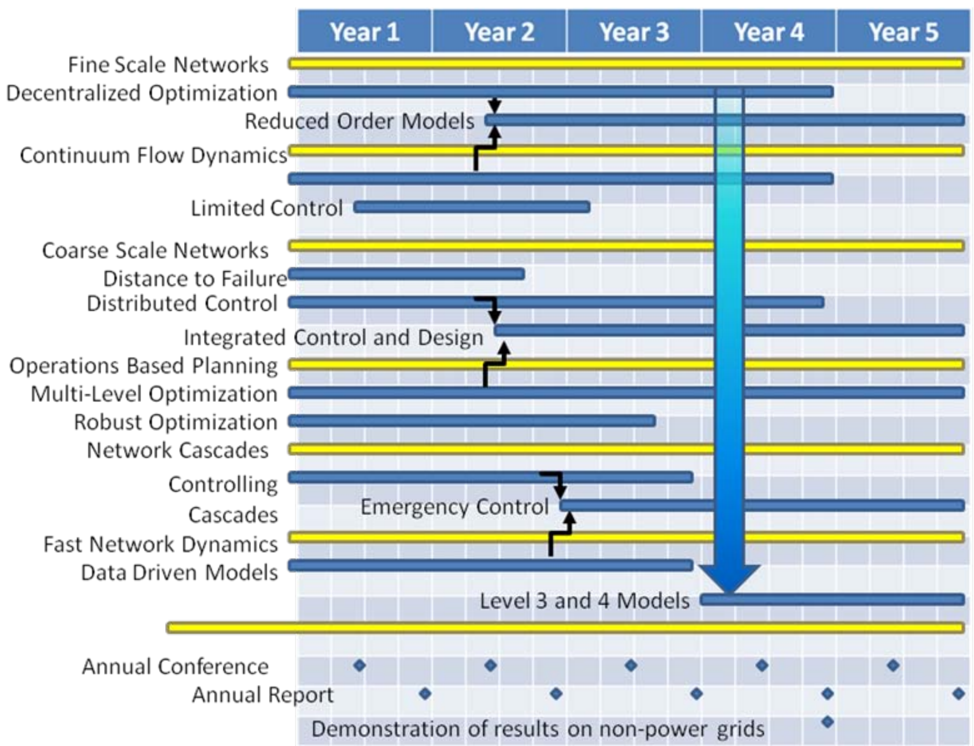

Figure 10. AMPS Center Project timeline.

We highlight two examples of this collaboration. First, Sean Harnett a graduate student at Columbia under direction of Bienstock, spent summer 2011 at LANL working on incorporating the concept of robustness in optimization of power flows. This developed into collaboration of Chertkov with Bienstock and Harnett on stochastic and robust OPF with probabilistic (chance) constraints. Harnett will spend summer 2012 at LANL, extending the subject that now forms the basis for his Ph.D. thesis. Second, in September 2011, Bent (LANL) hired Annarita Giani, formerly a postdoc at Berkeley under the direction of Poolla, as a postdoctoral researcher. In just six months, this collaboration has led to a conference paper on power grid security [61] and two (nearly completed) journal submissions.

Metric for Success (Deliverables)

There are several key measurements for project success. First, for any science project, peer-reviewed publications are a critically important metric; publications in applied math, network science, statistics, control theory, operations research, and optimization theory are expected from this project. As one of the projects goals is creating and building a set of analysis and mathematical tools, as well as novel control and optimization algorithms for complex engineered networks, we set aggressive publication targets of five per year for each mathematics areas. Second, we will issue annual reports at the end of each fiscal year. The reports will be delivered and presented to ASCR and DOE program offices, such as OE, to ensure the results of this project have impact on DOE mission. Third, at the conclusion of the fourth year of the project, the annual report will include results of impact outside of the power grid application domain, demonstrating that the new applied mathematics of AMPS has far-reaching impacts. Fourth, as the power grid is evolving rapidly and there are numerous opportunities for "game changing" technologies to be introduced, we emphasize flexibility in our metrics, as noted in the previous discussion of Levels 3 and 4. A section of the annual reports will survey emerging mathematical challenges, discuss how the existing program plan will address those challenges, and suggest modifications to the research agenda of AMPS to handle those challenges. Our fifth metric of success is a demonstration of collaboration outside the confines of the AMPS Center. We have the unique opportunity to exchange ideas with the SciDAC institutes and other ASCR projects at LANL, SNL, and LBNL, as well as Columbia. These include Bienstock's Reconfiguring Power Systems to Minimize Cascading Failures: Models and Algorithms project (ASCR); Germann's Exascale Co-Design Center for Materials in Extreme Environments (ExMatEx) and Algorithms (ASCR); Najm's QUEST—Quantification of Uncertainty in Extreme Scale Computations; Hagberg's Dynamics through randomness: New mathematical approaches for complex networks (ASCR); Pinar's Scalable methods for representing, characterizing and generating large graphs (ASCR), and Debuscherre (PECASE Early Career). Success here is measured through joint papers and reports. Finally, in the last year of the project we will provide a transition plan for the program offices of DOE (such as OE) to adopt and use the science developed under the project.

Project Timeline

The primary tasks of the AMPS Center are described in Figs. 2 and 3. We will work on all three major project pillars for the duration of the project, following the time line sketched in Fig. 10, but also we will adjust as needed after receiving reviews, organizing workshops, and receiving ASCR/DOE guidance. Work will begin on different tasks, shown in Fig. 2, at different stages of the project, moving from the Complex System pillar to the Control Theory pillar and Optimization Theory pillar (in this order), and then returning back and extending formulations when the next level of understanding and sophistication is achieved. As our project continues (beyond year 2), we will work on progressively more complex/multi-faceted problems, advancing up the levels shown in Fig. 3. Our progression up the pyramid of Fig 3 is adaptable to an environment of changing resources.

Conclusion

This proposal describes our vision for developing novel applied mathematics for impact in complex engineered networks, such as power grids. This proposal has identified three pillars of applied mathematics: complex systems, control theory, and optimization, which require basic mathematical advances to meet the future needs of engineered networks. The advances include crosscutting technologies in the fields of fast network dynamics, continuum and flow dynamics, network cascades, coarse-scale network control, fine-scale network control, and operations-based planning. The key contributions of this AMPS Center are the development of general applied mathematical solution concepts in each of these areas, and mathematically sound methods for decomposing and reconstructing problems that cross-temporal and spatial boundaries. AMPS will *provide the foundational applied mathematics to bring the grand challenge of real-time simulation and monitoring of the complex engineered networks, including the electric power grid, to fruition*

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