LA-UR-12-22976
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| Intended for: | 53rd Annual INMM Meeting, 2012-07-16/2012-07-19 (Orlando, Florida, United States) |

# THE FEYNMAN-Y STATISTIC IN RELATION TO SHIFT-REGISTER NEUTRON COINCIDENCE COUNTING: PRECISON AND DEAD TIME

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**ABSTRACT**
The Feynman-Y statistic is a type of autocorrelation analysis. It is defined as the excess variance-to-mean ratio, $Y = VMR - 1$, of the number count distribution formed by sampling a pulse train using a series of non-overlapping gates. It is a measure of the degree of correlation present on the pulse train with $Y = 0$ for Poisson data. In the context of neutron coincidence counting we show that the same information can be obtained from the accidentals histogram acquired using the multiplicity shift-register method, which is currently the common autocorrelation technique applied in nuclear safeguards. In the case of multiplicity shift register analysis however, overlapping gates, either triggered by the incoming pulse stream or by a periodic clock, are used. The overlap introduces additional covariance but does not alter the expectation values. In this paper we discuss, for a particular data set, the relative merit of the Feynman and shift-register methods in terms of both precision and dead time correction.

Traditionally the Feynman approach is applied with a relatively long gate width compared to the dieaway time. The main reason for this is so that the gate utilization factor can be taken as unity rather than being treated as a system parameter to be determined at characterization/calibration. But because the random trigger interval gate utilization factor is slow to saturate this procedure requires a gate width many times the effective 1/e dieaway time. In the traditional approach this limits the number of gates that can be fitted into a given assay duration. We empirically show that much shorter gates, similar in width to those used in traditional shift register analysis can be used.

Because the way in which the correlated information present on the pulse train is extracted is different for the moments based method of Feynman and the various shift register based approaches, the dead time losses are manifested differently for these two approaches. The resulting estimates for the dead time corrected first and second order reduced factorial moments should be independent of the method however and this allows the respective dead time formalism to be checked. We discuss how to make dead time corrections in both the shift register and the Feynman approaches.

# The Feynman-Y Statistic in Relation to Shift Register Neutron Coincidence Counting: Precision and Dead Time

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▪INMM-2012



## Advancing Correlated Neutron Analysis

- *This work is part of a larger effort being undertaken at LANL to develop new and/or advanced methods for processing and analyzing correlated neutron data.*
- *Looking into:*
  - *Foundations*
  - *Dead time correction*
  - *Algorithms*
  - *Instrumental affects*
  - *Point Model restrictions*

## Auto-Correlation Analysis: Feynman-Y and SR NCC

▪We imagine the pulse train can be fully described using **reduced factorial multiplets** as the expansion basis set

▪Feynman-Y and SR NCC both use **coincidence gates** to get at the multiplet information and are examples of Time Correlation Analysis (**TCA**)

▪Note: Other statistical games of autocorrelation exist, including Time Interval Analysis (TIA) methods which use various distributions of times between subsequent neutrons to quantify the pulse train



## Reactor Noise Analysis was Born May 1944

▪Los Alamos LOw Power Water Boiler, **LOBO**: **1-ft dia aqueous solution of 14% enriched uranyl sulfate, reflected by Be on a graphite base**.

▪**Feynman** [de Hoffmann et al LA-183(1944)] developed the basic statistical theory of reactor noise analysis and performed the first noise analysis experiment on the **near critical** Los Alamos Water Boiler using a **variance-to-mean technique**

▪**Feynman-Y statistic Y=VMR-1="excess variance"**

## Feynman-Y

▪Y is calculated from the **count distribution**:

$$Y = \frac{\sigma_i^2}{\langle i \rangle} - 1 = \frac{\langle i^2 \rangle - \langle i \rangle^2}{\langle i \rangle} - 1 = \frac{1}{\langle i \rangle} \cdot [\langle i \cdot (i-1) \rangle - \langle i \rangle^2]$$

▪Which in terms of the reduced factorial moments becomes:

$$Y = \frac{1}{m_c(1)} \cdot \left[2m_c(2) - \left(m_c(1)\right)^2\right] = 2\frac{m_c(2)}{m_c(1)} - m_c(1)$$



## Feynman today

▪Data can be collected in **list mode** (which provides a complete experimental record) and analyzed in any way with any gating to achieve optimal results

▪We have also shown that the **MSR Accidental histogram**, with overlapping gates, can also be used to extract correlated information

▪We have shown that **Fast Accidental Sampling (FAS)** at the MSR clock rate is generally better

▪ *Croft et al, "Extraction of Correlated Count Rates Using Various Gate Generation Techniques: Part I Theory", NIMA*
▪*Henzlova et al, Part II Experiental, NIMA*

## Rossi-α

▪During the early days of Los Alamos, prompt neutron behavior of **near critical** configurations was studies using a **pulsed cyclotron** [*Orndoff*]

▪Rossi suggested that such assemblies would be **self-modulated** due to the presence of long chains, and that the prompt neutron mean lifetime could be obtained by measuring the average time distribution of neutrons associated with a **common ancestor** (a preferential starting place).

▪White attributes the suggestion to **1944** [*White*]

▪Such measurements have frequently been called Rossi-α measurements since

*John D Orndoff NSE 2(1957)450-460*

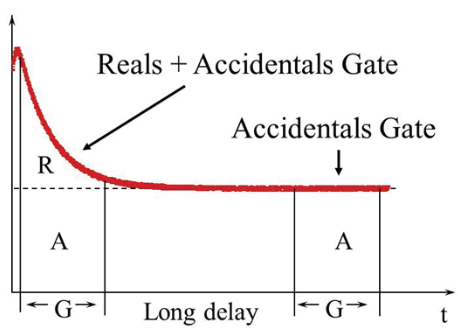
*White NSE 1(1956)53-61*



## The Rossi-α distribution

▪The average probability of detecting either a random neutron or a chain-related neutron in a time interval dt in absence of dead time

$$p(t)dt = Cdt + Ae^{\alpha t}dt$$



## Thus, Two Traditions Developed

- "Feynman": Number distribution histogram form by the periodic (random with respect to the pulse train) inspection of the pulse train using non-overlapping gates

- "Rossi": Event triggered opening of a coincidence gate which is now the foundation of shift-register coincidence counting



## SR NCC in more detail: Point Model Equations

- With the advent of **shift-register logic** NCC become the **work horse** of safeguards for quantifying fission rate [K Böhnel, and the huge benefit of having more than one gate open at a time]

- The general theory in terms of the point-model has been described by **Cifarelli and Hage** [*NIM A251(1986)550-563*]. Rates expressed in terms of properties of the detector and item

$$S = \frac{R_0}{T_M} = F_S \varepsilon \left(\frac{\bar{\nu}_S}{1}\right)(1+\alpha)$$

$$D = f_2 \frac{R_1}{T_M} = F_S \varepsilon^2 \left(\frac{\bar{\nu}_{S2}}{2}\right) f_d M_L^{\,2} \left[1 + (M_L - 1)(1+\alpha)\left[\frac{\bar{\nu}_{S1}}{(\bar{\nu}_I - 1)}\right] \cdot \left[\frac{\bar{\nu}_{I2}}{\bar{\nu}_{S2}}\right]\right]$$

$$\frac{M_L - 1}{\nu_{I1} - 1} = \left[p_f - \frac{p_c/p_f}{\nu_{I1} - 1}\right] . M_T \approx p_f . M_T$$

## Reconciliation of the two Approaches – This Work

- To determine if the two approaches contain similar or unique information about an item in question, we need to determine the relationship between them.
- Creating a relationship between the two techniques will help connect the individuals involved with correlated neutron measurements for safeguards purposes with those performing measurements for criticality studies.
- We therefore revisited Feynman's derivations in the light of modern vernacular



## Different Starting Points to the Point-Model

| Concept | Feynman | NCC |
|---|---|---|
| Item | Homogeneous reactor core | Point-like |
| Idea | Population fluctuation (noise) in steady state | Initiating n's (replace cyclotron) in steady state |
| Reactivity | Near critical, M>100 | Highly sub-critical, M<4 |
| SF | None | Measurement Goal |
| (α,n) | None | Yes |
| dn | Neglected | Include as random along with (α,n) |
| Gating | Random wrt P/T, contiguous | Event triggered, overlapping |
| Detector | Internal probe Tau=M.Tau$_{prompt}$ (Tau$_{prompt}$~20 ns) | Ext, decoupled, $\lambda_{det}$ utterly dominant, super-fission concept |
| Multiplication | Total | Leakage |
| Efficiency | per fission | per neutron |

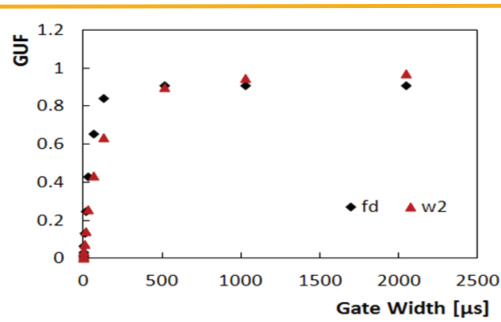
## Key Result

$$\text{Second Factorial Moment} = \frac{D}{f_2} = \frac{\frac{1}{2} \cdot S \cdot Y}{w_2}$$

- All standard NCC results then follow
  - e.g. Known-ε and known-α solution for multiplication corrected $m_{eff}$
- We have a quantitative link between the two traditions
- Extension to Triples and higher counting also possible



## Gate Factor Behavior




$$f_d = e^{-\lambda T_p}\left(1 - e^{-\lambda T_g}\right)$$

$$w_2 = 1 - \frac{\left(1 - e^{-\lambda T_g}\right)}{\lambda T_g}$$

- Ex. Dieaway 50µs. SR predelay 4.5µs
- Feynman-Y much slower to saturate (200 vs 5.3 dieaway times to reach 99.5%) therefore traditionally wide gates are used **BUT** in many safeguards applications we can treat the gate factor as a constant and achieve good precision with short gates one to two times the dieaway.

## DT Extraction

- Feynman-Y is directly proportional to D
- D = 0 for a random source when there is no system DT
- But when DT is present we find that extracting Feynman-Y from the MSR A-histogram provides a simple and precise estimate of the DT parameter using an AmLi source:

$$d = T_g \phi = T_g \left(1 - \sqrt{1 - \left[\frac{\bar{c} - \sigma_c^2}{\bar{c}^2}\right]}\right) = T_g \left(1 - \sqrt{1 + \frac{Y}{\bar{c}}}\right)$$



## NCC DT Corrections

- **Extending the work of Mattes and Hass, ANE 12(12)1985)693 we find:**

$$S_m = S_c . \left(1 - \frac{D_c/f_d}{S_c} . \frac{d}{\tau}\right) . exp\left[-S_c . d . \left(1 - \frac{1}{2} . \frac{D_c/f_d}{S_c} . \frac{d}{\tau}\right) + 0(\left[\frac{d}{\tau}\right]^4)\right]$$

$$D_m = D_C . \left(1 - 2.S_c . d . \left[1 - \frac{D_c/f_d}{S_c} . \frac{d}{\tau} . \left(1 + \frac{1}{2} . \frac{\theta_2}{\theta_1}\right)\right]\right) . exp\left[-2.S_C . d . \left(1 - \frac{D_c/f_d}{S_c} . \frac{d}{\tau}\right) + 0(\left[\frac{d}{\tau}\right]^4)\right]$$

## This justifies the Empirical 2-parameter approach

$$CF_S \approx e^{\frac{1}{4}\delta \cdot S_m} = e^{\frac{1}{4}\cdot(A+B\cdot S_m)\cdot S_m}$$

and to first order $A = 4d, and\ B = \frac{A^2}{4}$

with $CF_D \approx (CF_S)^4$



## For Feynman-Y we use a Padé approximation here

See Croft et al INMM-2011: Just an expansion of our Mattes & Hass result in terms solely of the observed singles rate

$x = d.S_m$

$$CF_Y = \frac{R_{2c}/R_{2m}}{R_{1c}/R_{1m}} \approx \frac{1 + x + \frac{3}{2}.x^2 + \frac{8}{3}.x^3}{1 - 2x(1 + x + \frac{3}{2}.x^2 + \frac{8}{3}.x^3)}$$

▪There is nothing special about this choice. We could have used the M&H expressions directly or the empirical forms

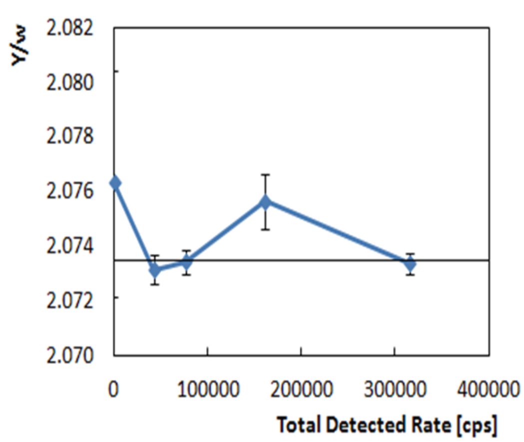
## Cf-252 Experiments with the LANL ENMC

- 720x10sec runs in a 65% counter with a 22μs dieaway

| | Approximate source strength [n/s] | SR NCC | Random triggered gates |
|---|---|---|---|
| | | $T_g$ [us] | $T_g$ [us] |
| Cf_1 | $6.8x10^2$ | 64 | 128 |
| Cf_2 | $7.0x10^4$ | 8 | 16 |
| Cf_3 | $1.3x10^5$ | 8 | 16 |
| Cf_4 | $2.7x10^5$ | 8 | 16 |
| Cf_5 | $5.2x10^5$ | 8 | 16 |



## Results: $Y/w_2$ is flat after DTC (small genuine variations do exist between some sources)



## Results Cont.

| Source | SR NCC | σ | Random triggered gates | σ | Ratio of σ-values |
|---|---|---|---|---|---|
| Cf_1 | 2.0693 | 0.0051 | 2.0763 | 0.0041 | 1.2 |
| Cf_2 | 2.0666 | 0.0029 | 2.07316 | 0.00054 | 5.4 |
| Cf_3 | 2.0679 | 0.0029 | 2.07346 | 0.00044 | 6.7 |
| Cf_4 | 2.0721 | 0.0031 | 2.0757 | 0.0010 | 3.1 |
| Cf_5 | 2.0734 | 0.0029 | 2.07340 | 0.00038 | 7.8 |

- NCC (8µs) and Feynman-Y (16µs) agree on DTC doublet
- But note too the dramatic uncertainty difference! Both use FAS at 1MHz and the SAME pulse train.



## Conclusions

- We have shown explicitly the direct link between the Feynman-Y statistic and SR NCC.
- Known ε known α point model inversion applies to both.
- The reduced factorial moments which form a basis set to describe the composition of the pulse train can be extracted from the (R+A), A or mixed (R+A) & A histograms.
- Fast Accidental Sampling is always preferred
- The same gate width works reasonably well for both types of analysis
- Therefore present Multiplicity Shift Register 's and future multi-MSR devices can be used to perform both analyses.
- List Mode provide a complete record and one can explore the temporal behavior in an item specific way … but must be analyzed off-line and the files can become large.
- The same established dead time corrections can be applied to BOTH Feynman-Y and SR NCC.

## Future Work

- Exploit this common language to pull knowledge developed for sub-critical assembly studies and reactor noise analysis into the safeguards domain
- We are free to use what ever expressions give best precision and greatest accuracy … practical comparisons are needed especially to explore higher rates and deeply multiplying items
- We shall report another time in detail aspects of gate optimization
- Treatment of covariance's in the data and uncertainty reporting
- Test the simple way proposed to estimate the dead time parameter when a Poisson source is available
- Real systems exhibit non-ideal behaviors such as baseline shifts following events that may manifest differently and need studying



## Acknowledgements

- This work was sponsored by the U.S. Department of Energy (DOE), National Nuclear Security Administration (NNSA), Office of Nonproliferation Research and Development (NA-22).

## Thank you

What Questions Do You Have?




## Supporting Material

## Correlated Neutron Analysis – A Workhorse

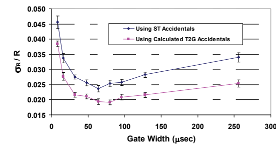
- **Temporally correlated neutron emission**
  - Spontaneous and induced fission
  - Unique signature for the detection of special nuclear material
- **Passive neutron multiplicity counting**
  - Based on multiplicity shift register (MSR) logic and electronics
    - Signal triggered gate inspection
    - Randomly triggered gate inspection
  - Records the number of occurrences of neutron multiplicity to generate MSR histograms
  - <u>Derive</u> correlated event rates (Doubles, Triples)
  - Singles rate is just the MSR trigger rate



## The statistical game

- **Multiplicity Shift Register Time Correlation Analysis**
- **We collect two histograms – one emphasizes short term correlations the other chance behavior**
- **Trigger + 1 in the gate = a pair**
- **Trigger +2 in the gate = a triple**
- **Then its just mathematics**
- **Shift-Register theory was significantly advanced by the work of K Böhnel, Karlsruhe, who was a visiting scientist at Los Alamos in 1971.** e.g. [*Trans ANS 15(2)(1972)671-672.*]. *Huge Benefit of having more than one gate open at a time.*

## Fast Accidental Sampling (FAS)




▪Traditional: Every logic pulse opens an A-gate after a long delay of 4096 µs

▪But we could any time >> the dieaway time, or even prior time. It is just one sample per event

▪FAS recognizes this and simply says why not massively over sample the pulse train at the capability of the MSR electronics to get the best average estimate of the A-histogram possible. This improves the **precision**. Like using calculated Accidentals

▪FAS: Sample at a **fast clock rate (typ. $\nu >> S_m$)**

▪*Croft, Blanc, Menaa, WM'10*

▪*Provided a systematic study of when FAS is beneficial*