LA-UR-12-22852
Approved for public release; distribution is unlimited.

Title: Modeling Compressed Turbulence

Author(s): Israel, Daniel M.

Intended for: 12th International Workshop on the Physics of Compressible Turbulent Mixing, 2012-07-16 (Woburn, ---, United Kingdom)

Modeling Compressed Turbulence

Daniel M. Israel
Los Alamos National Laboratory
LA-UR 12-22852

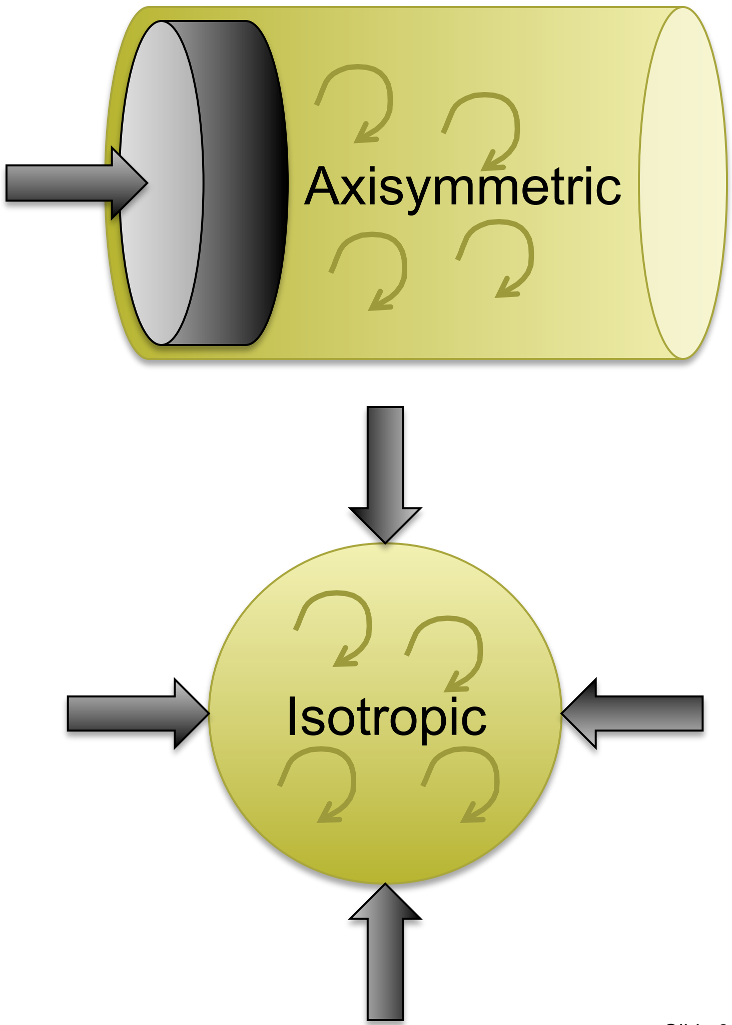
13th International Workshop on the Physics of Compressible Turbulent Mixing
July 17, 2012
Woburn, UK

Motivation

- From ICE to ICF, the effect of mean compression or expansion is important for predicting the state of the turbulence.
- When developing combustion models, we would like to know the mix state of the reacting species.
- This involves density and concentration fluctuations.
- To date, research has focused on the effect of compression on the turbulent kinetic energy.
- The current work provides constraints to help development and calibration for models of species mixing effects in compressed turbulence.

Overview

- **Consider a homogeneous anisotropic turbulent field subject to uniform mean compression.**
- **The flow can be decomposed into a mean flow and turbulent fluctuations.**
  - The mean flow must be treated compressibly.
  - The turbulent fluctuations may be amenable to simpler models.
- **For example, the DNS of Wu, et al. (1985), assumes the turbulent density fluctuations could be neglected.**



Approach

- **Cambon, et al. (1992) demonstrate (based on an observation of Frisch) that a simple rescaling relates homogeneous isotropic compressed turbulence and decaying turbulence.**
- **Their analysis assumes**
  - density fluctuations are not important
  - the fluctuations are homogeneous and isotropic
- **The current work extends this analysis to cases which**
  - include multiple species,
  - include density, concentration, and temperature fluctuations, and
  - are anisotropic.

Outline

- **Give an high-level overview of the rescaling procedure.**
  - Mathematical details are being prepared for publication.
- **Present the scaling results.**
- **Briefly describe the physical case described by the rescaling.**
- **Apply the rescaling to some example turbulence model equations to illustrate the method for model calibration and development.**
- **Show two applications of the theory for model development and calibrations.**
  - Constraints for BHR model constants
  - Modeling the “rapid” pressure-strain

Governing equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i$$

$$\frac{\partial \rho c}{\partial t} + \frac{\partial \rho u_j c}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\rho D \frac{\partial c}{\partial x_j}\right]$$

$$\frac{\partial \rho c_v T}{\partial t} + \frac{\partial \rho c_v u_j T}{\partial x_j} = -p\frac{\partial u_i}{\partial x_i} + \sigma_{ij}\frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left[k \frac{\partial T}{\partial x_j}\right]$$

Decomposition

- **Introduce an ensemble average,**

$$f = \langle f \rangle + f'$$

- **and a Favre (density-weighted) average**

$$\langle \rho f \rangle = \langle \rho \rangle \tilde{f}$$

$$f = \tilde{f} + f''$$

- **and decompose the Navier-Stokes equations using**

$$\rho = \langle \rho \rangle + \rho' \quad u_i = \tilde{u}_i + u_i'' \quad p = \langle p \rangle + p'$$

$$c = \tilde{c} + c'' \quad T = \tilde{T} + T''$$

Averaged equations

- **Assuming spatially homogeneous turbulence, the mean equation are:**

$$\frac{1}{\langle\rho\rangle}\frac{\partial\langle\rho\rangle}{\partial t}+S_{ii}=0$$

$$\left(\dot{S}_{ij}+S_{ik}S_{kj}\right)x_j=-\frac{1}{\langle\rho\rangle}\frac{\partial\langle p\rangle}{\partial x_i}+g_i$$

$$\dot{C}_i+C_jS_{ji}=0$$

$$\left(\dot{A}_i+A_jS_{ji}\right)x_i=\frac{\langle\sigma_{ij}S_{ij}\rangle-\langle pS_{ii}\rangle}{\langle\rho\rangle c_v}-\dot{B}$$

$$\langle\rho\rangle\equiv\text{constant}$$

$$S_{ij}(t)=\frac{\partial\tilde{u}_i}{\partial x_j}$$

$$C_i(t)=\frac{\partial\tilde{c}}{\partial x_i}$$

$$\tilde{T}=A_i(t)x_i+B(t)$$

- **Blaisdell, et al. (1991) give examples of meanflows that satisfy these constraints.**
- **The state equation will impose an additional constraint.**
- **Note that B is an unclosed quantity that would need to be solved for.**

Re-scaling

- **Introduce a coordinate transform to remove the mean compression:**

$$x_i^* = J^{-1/3} x_i \qquad \frac{dt^*}{dt} = J^{-2/3}$$

where

$$J^{-1} = \exp\left(-\int_0^t S_{ii}(t')\,dt'\right) = \frac{\langle \rho \rangle}{\langle \rho(t=0) \rangle}$$

and the following re-scaling:

$$\rho(\mathbf{x},t) = J^{-1}(t)\,\rho^*(\mathbf{x}^*,t^*) \quad p(\mathbf{x},t) = J^{-5/3}(t)\,p^*(\mathbf{x}^*,t^*) \quad T(\mathbf{x},t) = J^{-2/3}(t)\,T^*(\mathbf{x}^*,t^*)$$

$$u_i(\mathbf{x},t) = S_{ij}(t)\,x_j + J^{-1/3}(t)\,u_i^*(\mathbf{x}^*,t^*) \quad c(\mathbf{x},t) = c^*(\mathbf{x}^*,t^*)$$

Invariance

- **Applying the coordinate transform and re-scaling, the resulting equations are identical to those for homogeneous turbulence with no mean compression, with the following scalings**

$$S_{ij}^{*} = J^{2/3}\left(S_{ij} - \frac{1}{3} S_{kk}\delta_{ij} \right)$$

$$g_i^{*} = J g_i$$

$$\nu^{*} = \nu \quad \alpha^{*} = \alpha \quad D^{*} = D$$

- **Note the time-dependent body force.**
- **Note that kinematic viscosity is not rescaled, but the viscosity will change in time due to mean temperature change caused by the compression.**

Realizability

- **Of course, pressure, temperature, and concentration gradients cannot extend infinitely,**
  - but this is no more non-physical than the assumption of an “infinite box.”
- **The pressure gradient must be balanced by the concentration and/or temperature gradient,**
  - or the scale-height must be large.
- **Either the Atwood number must be small, or, temperature fluctuations must be negligible.**
- **In what follows, the internal energy equation will not be used.**

Constraints for RANS

- **Homogeneous buoyancy driven turbulence subject to an isotropic compression corresponds to freely evolving buoyancy driven turbulence with a time depended body force.**
- **Since this re-scaling is an exact analytic result, any RANS model should preserve it.**
- **Following Cambon, et al., we can neglect the viscosity variation, since in general, RANS models do not account for time variations in viscosity.**
  - However, this indicates that the term $(1/\nu)\, d\nu/dt$, which is typically neglected in the derivation of the dissipation equation, must be included for compressed turbulence, as noted by many researchers (e.g. Coleman & Mansour)

Setting the BHR coefficients

- **The two-equation version of the BHR model (Besnard, et al., 1992) for homogeneous buoyancy driven turbulence undergoing mean compression reduces to the following set of ODEs:**

$$\frac{\partial k}{\partial t} = a_3 g - R_{ij} S_{ij} - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} = -C_{\varepsilon 1} R_{ij} S_{ij} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + C_{\varepsilon 3} \varepsilon S_{kk} - C_{\varepsilon 4} \frac{\varepsilon}{k} a_3 g$$

$$\frac{\partial a_3}{\partial t} = bg - C_{a1} \frac{\varepsilon}{k} a_3 + \left(C_{a2} - 1\right) a_3 S_{33}$$

$$\frac{\partial b}{\partial t} = -C_{b2} \frac{\varepsilon}{k} b$$

Re-scaling the turbulence quantities

- **Based on the exact re-scaling for the turbulent fluctuations, we can write the following re-scalings for the turbulence model quantities:**

$$k(t) = \frac{1}{2}\left\langle u_i''(t) u_i''(t) \right\rangle = J^{-2/3} k^*\left(t^*\right)$$

$$\varepsilon\left(t\right) = \upsilon \left\langle \frac{\partial u_i''(t)}{\partial x_j} \frac{\partial u_j''(t)}{\partial x_i} \right\rangle = J^{-4/3} \varepsilon^*\left(t^*\right)$$

$$a_3\left(t\right) = -\left\langle u_3''(t) \right\rangle = J^{-1/3} a_3^*\left(t^*\right)$$

$$b\left(t\right) \approx \frac{\left\langle \rho'\left(t\right)^2 \right\rangle}{\left\langle \rho \right\rangle^2} = b^*\left(t^*\right)$$

- **Inserting these into the BHR equations, and making use of $S_{ij}=(d/3)\delta_{ij}$ and $R_{ii}=2k$ we find the following:**

Re-scaled BHR equations (k and ε)

- **Turbulent kinetic energy**

$$\frac{\partial k^*}{\partial t^*} = a_3^* g - \varepsilon^*$$

- **Dissipation Rate**

$$\frac{\partial \varepsilon^*}{\partial t^*} = -C_{\varepsilon 2}\frac{\varepsilon^{*2}}{k^*} - C_{\varepsilon 4}\frac{\varepsilon^*}{k^*} a_3^* g^* + J^{2/3}\left(\frac{4}{3} - \frac{2}{3}C_{\varepsilon 1} - C_{\varepsilon 3}\right)$$

- **So, to preserve the scaling we must require**

$$C_{\varepsilon 3} = \frac{2}{3}\left(2 - C_{\varepsilon 1}\right)$$

- **This is consistent with the result found by Cambon, et al., for the simpler case, and assuming $C_{\varepsilon 1}=1$, following Reynolds.**

Re-scaled BHR equations (a and b)

- **Density-velocity correlation:**

$$\frac{\partial a_3^*}{\partial t^*} = b^* g^* - C_{a1} \frac{\varepsilon^*}{k^*} a_3^* + J^{2/3} \frac{1}{3} \left(C_{a2} + 3 \right) a_3^* d$$

- **To preserve the scaling:**

$$C_{a2} = -3$$

- **Density self-correlation:**

$$\frac{\partial b^*}{\partial t^*} = -C_{b2} \frac{\varepsilon^*}{k^*} b^*$$

- **Does not impose further constraints**

Modeling the "rapid" pressure-strain

- **Applying the transform to the exact Reynolds-stress equation gives:**

$$\langle\rho\rangle\frac{\partial R_{ij}}{\partial t}=\langle\rho\rangle\left(R_{ik}S_{jk}+R_{jk}S_{ik}\right)+\langle\rho\rangle\left(a_i g_j+a_j g_i\right)+\overline{p'\frac{\partial u_i'}{\partial x_j}}-\overline{\sigma_{kj}'\frac{\partial u_i'}{\partial x_k}}$$

$$J^{-7/3}\rho_0\frac{\partial R_{ij}^*}{\partial t}-\frac{2}{3}J^{-5/3}\rho_0 R_{ij}^* S_{ll}=J^{-5/3}\rho_0\left(R_{ik}^*S_{jk}+R_{jk}^*S_{ik}\right)+J^{-7/3}\rho_0\left(a_i^*g_j^*+a_j^*g_i^*\right)$$
$$+J^{-7/3}\overline{p^*\frac{\partial u_i^*}{\partial x_j^*}}-J^{-7/3}\overline{\sigma_{kj}^*\frac{\partial u_i^*}{\partial x_k^*}}$$

$$J^{-7/3}\rho_0\frac{\partial R_{ij}^*}{\partial t}=J^{-7/3}\rho_0\left(R_{ik}^*S_{jk}^*+R_{jk}^*S_{ik}^*\right)+J^{-7/3}\rho_0\left(a_i^*g_j^*+a_j^*g_i^*\right)$$
$$+J^{-7/3}\overline{p^*\frac{\partial u_i^*}{\partial x_j^*}}-J^{-7/3}\overline{\sigma_{kj}^*\frac{\partial u_i^*}{\partial x_k^*}}$$

- **As expected, the exact equation is invariant.**
- **All terms scale with $J^{-7/3}$.**

Tensor form of the "rapid" pressure-strain

- **The pressure-strain is decomposed into a "slow" and "rapid" part.**
- **The standard incompressible model for the pressure strain assumes a linear tensor form:** $\left(\Pi_R^r\right)_{ij} = A_{lj}^{ki}\left(R_{ij}\right)S_{kl}$
  - Assume A scale as $J^{-2/3}$ ($\sim R_{ij}$).
  - Applying the invariance property, this form is not invariant, because it will have a new term containing the mean compression rate.
- **Instead, we should use the form:**

$$\begin{aligned}\left(\Pi_R^r\right)_{ij} &= A_{lj}^{ki}\left(S_{kl} - \frac{1}{3}S_{nn}\delta_{kl}\right)\\ &= J^{-2/3}A_{lj}^{*ki}\left(S_{kl} - \frac{1}{3}S_{nn}\delta_{kl}\right)\\ &= J^{-4/3}A_{lj}^{*ki}S_{kl}^{*}\\ &= J^{-4/3}A_{lj}^{*ki}\left(S_{kl}^{*} - \frac{1}{3}S_{nn}^{*}\delta_{kl}\right)\end{aligned}$$

Conclusions

- **The Cambon, et al., re-scaling has been extended to buoyancy driven turbulence, including the fluctuating density, concentration, and temperature equations.**
- **The new scalings give us helpful constraints for developing and validating RANS turbulence models.**
  - For BHR,

$$C_{\varepsilon 3} = \frac{2}{3}\left(2 - C_{\varepsilon 1}\right)$$

$$C_{a2} = -3$$

  - For "rapid" pressure-strain

$$\left(\Pi_R^r\right)_{ij} = A_{lj}^{ki}\left(S_{kl} - \frac{1}{3} S_{nn}\delta_{kl}\right)$$

- **Future work**
  - Further investigation into constraints on tensor forms
  - Examining time dependent viscosity effects
  - Application to other models (e.g. Grégoire, et al., 2005)

Acknowledgements

- Rob Gore for his support and discussions.
- Ray Ristorcelli for his encouragement and helpful feedback.