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Title: Wind characteristics probability density estimation

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Wind characteristics probability density estimation

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Wind characteristics theoretical probability density function estimation

Wind speed

- Weibull $\mathcal{W}(k, \lambda)$
- log-Normal $\log \mathcal{N}(\mu, \sigma)$
- Gamma $\Gamma(k, \theta)$

Wind direction

- von Mises $\mathcal{VM}(\mu, \kappa)$
- Wrapped Cauchy $\mathcal{WC}(\mu, \rho)$
- Wrapped Normal $\mathcal{WN}(\mu, \rho)$

Wind speed AND wind direction

- Wind speed variation description for wind potential estimation
- Wind direction is also of interest for wind farm design and energy potential estimation

\implies *Bivariate joint distribution*

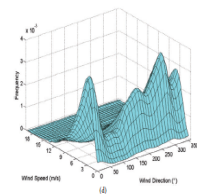
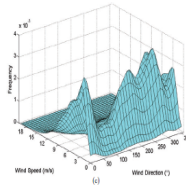
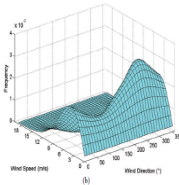
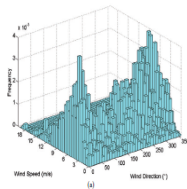
Joint probability density function of wind speed and direction

Several approaches for constructing joint *pdf*

- Anisotropic lognormal approach
- Farlie-Gumbel-Morgenstern families approach
- Angular-Linear approach

$$f(v, \theta) = 2\pi g(\xi) [f_v(v) - f_\theta(\theta)]$$

$$\text{with } \xi = 2\pi [F_v(v) - F_\theta(\theta)]$$



Multimodality

- Empirical distribution is unimodal
 - Parametric *pdf* gives good approximations
- Empirical distribution is multimodal
 - Use of mixture of distributions

\implies *How to specify the right model?*

Definition

Let X and Y be 2 random variables, the Kernel Density Estimator (KDE) is defined as

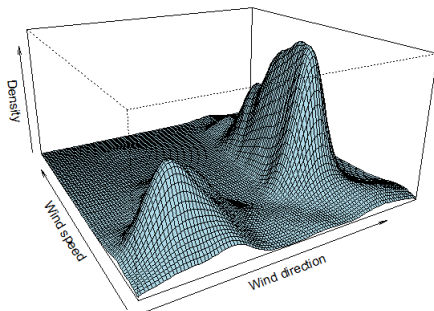
$$\hat{f}(x, y) = \frac{1}{n} \sum_{i=1}^n \frac{1}{h_X} \mathbb{K}_X \left(\frac{x - X_i}{h_X} \right) \frac{1}{h_Y} \mathbb{K}_Y \left(\frac{y - Y_i}{h_Y} \right)$$

- Choice of the kernel \mathbb{K} is not crucial
- Choice of bandwidths h_X and h_Y are of importance

State-of-art

- Zhang (2011) : Multiplicative Gaussian kernels

$$\hat{f}(v, \theta) = \frac{1}{nh_1h_22\pi} \sum_{i=1}^n \exp\left(-\frac{1}{2}\left(\frac{v - V_i}{h_1}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{\theta - \theta_i}{h_2}\right)^2\right)$$



Main issue

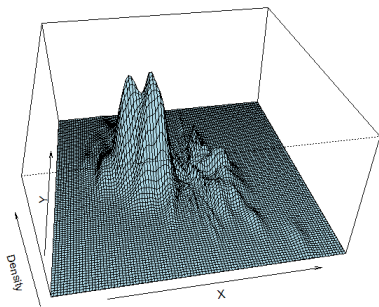
- Wind direction is a circular variable
- Wind direction : 0° and 360° is identical

$$\text{Thus : } \forall v, \hat{f}(v, 0) = \hat{f}(v, 360)$$

Estimation in Cartesian space

- Convert data speed and direction (ρ, θ) into (x, y)
- Fit a KDE with 2 Gaussian kernels

$$\hat{f}(x, y) = \frac{1}{nh_1h_22\pi} \sum_{i=1}^n \exp\left(-\frac{1}{2}\left(\frac{x-X_i}{h_1}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{y-Y_i}{h_2}\right)^2\right)$$

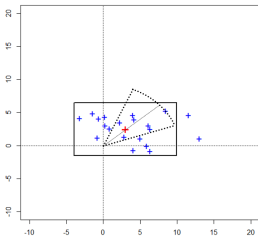
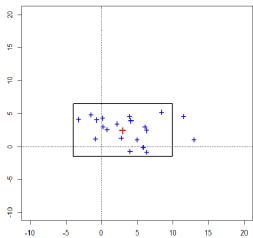
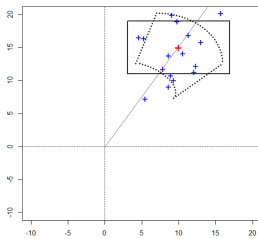
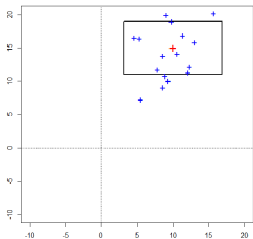


Estimation in Polar space

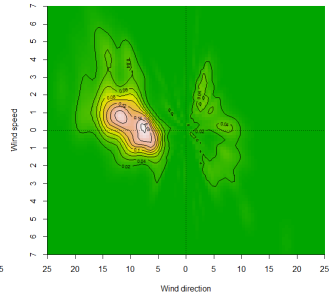
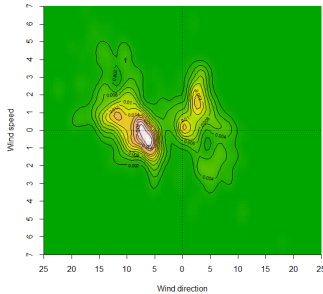
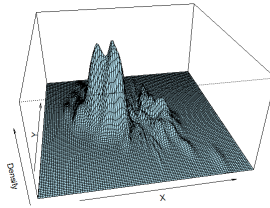
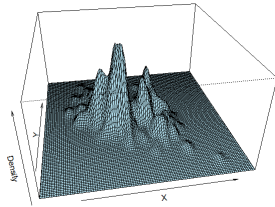
- Work with (ρ, θ)
- Use of specific kernel to deal with wind direction : circular kernel
 - von Mises $\hat{f}_k(\theta) = \frac{1}{n} \sum_{i=1}^n \frac{1}{2\pi I_0(k)} \exp(k \cos(\theta - \theta_i))$
 - Wrapped Gaussian $\hat{f}_h(\theta) = \frac{1}{n} \sum_{i=1}^n \frac{1}{\sqrt{(2\pi)h}} \frac{1}{h} \exp\left(-\frac{1}{2} \left(\frac{\theta + 2k\pi - \theta_i}{h}\right)^2\right)$
 - Wrapped Uniforme, etc.
- Fit a KDE with a Gaussian kernel for wind speed and a circular kernel for wind direction

$$\hat{f}_h(v, \theta) = \frac{1}{nh_v h_\theta 2\pi} \sum_{i=1}^n \exp\left(\frac{v - V_i}{h_v}\right) \exp\left(-\frac{1}{2} \left(\frac{\theta + 2k\pi - \theta_i}{h_\theta}\right)^2\right)$$

Cartesian vs Polar



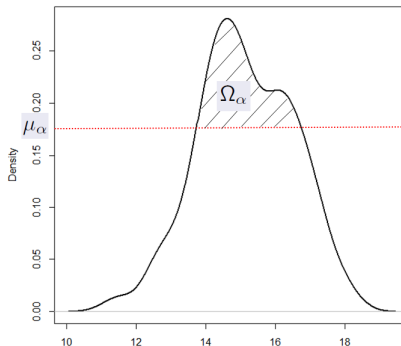
Cartesian vs Polar (cont'd)



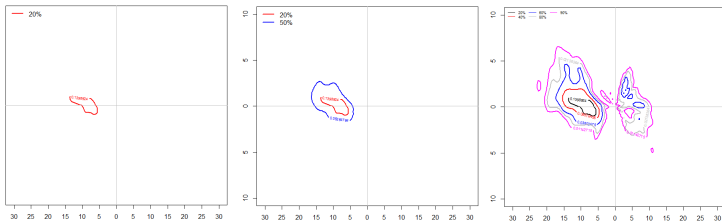
Contour plot

Plotting chosen contour lines

- Let's define α so that $\int_{\Omega_\alpha} \hat{f}(\rho, \theta) d\rho d\theta = \alpha$.
- Let μ_α is the contour line depending on α
- With $\Omega_\alpha = \{(\rho, \theta) : \hat{f}(\rho, \theta) > \mu_\alpha\}$



Contour plot (cont'd)



Conclusions and Perspectives

Conclusions

- KDE better than fitting parametric distributions
- Use of circular kernel to estimate wind direction *pdf*
- Algorithm to estimate bivariate joint distribution
- Representation in polar coordinates
- Contour lines according to quantiles

Perspectives

- Package R in progress
- Apply KDE with MCP method for long term correlation
- Extend the results to energy yield estimation

Thank you for your attention !