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Title: Wind characteristics probability density estimation

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# Wind characteristics probability density estimation

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# Wind characteristics theoretical probability density function estimation

## Wind speed

- Weibull  $\mathcal{W}(k, \lambda)$
- log-Normal  $\log\mathcal{N}(\mu, \sigma)$
- Gamma  $\Gamma(k, \theta)$

## Wind direction

- von Mises  $\mathcal{VM}(\mu, \kappa)$
- Wrapped Cauchy  $\mathcal{WC}(\mu, \rho)$
- Wrapped Normal  $\mathcal{WN}(\mu, \rho)$

## Wind speed AND wind direction

- Wind speed variation description for wind potential estimation
- Wind direction is also of interest for wind farm design and energy potential estimation

⇒ *Bivariate joint distribution*

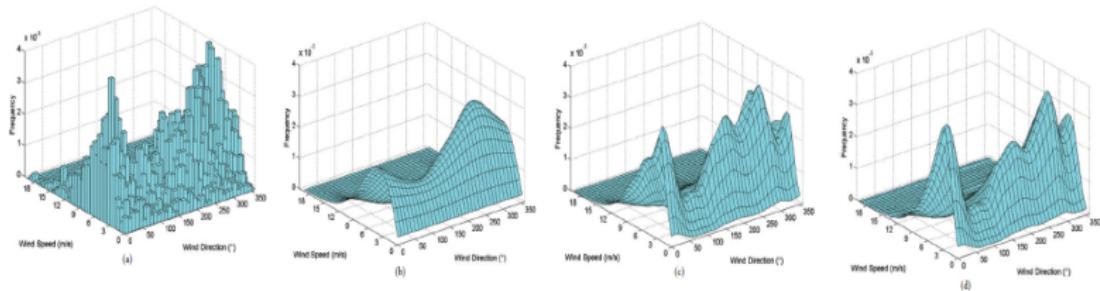
# Joint probability density function of wind speed and direction

Several approaches for constructing joint *pdf*

- Anisotropic lognormal approach
- Farlie-Gumbel-Morgenstern families approach
- Angular-Linear approach

$$f(v, \theta) = 2\pi g(\xi) [f_v(v) - f_\theta(\theta)]$$

$$\text{with } \xi = 2\pi [F_v(v) - F_\theta(\theta)]$$



## Multimodality

- Empirical distribution is unimodal
  - Parametric  $pdf$  gives good approximations
- Empirical distribution is multimodal
  - Use of mixture of distributions

⇒ *How to specify the right model ?*

## Definition

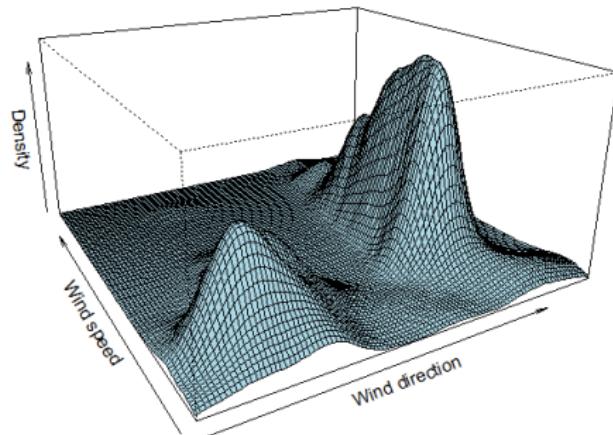
Let  $X$  and  $Y$  be 2 random variables, the Kernel Density Estimator (KDE) is defined as

$$\hat{f}(x, y) = \frac{1}{n} \sum_{i=1}^n \frac{1}{h_X} \mathbb{K}_X \left( \frac{x - X_i}{h_X} \right) \frac{1}{h_Y} \mathbb{K}_Y \left( \frac{y - Y_i}{h_Y} \right)$$

- Choice of the kernel  $\mathbb{K}$  is not crucial
- Choice of bandwidths  $h_X$  and  $h_Y$  are of importance

- Zhang (2011) : Multiplicative Gaussian kernels

$$\hat{f}(v, \theta) = \frac{1}{nh_1h_22\pi} \sum_{i=1}^n \exp \left( -\frac{1}{2} \left( \frac{v - V_i}{h_1} \right)^2 \right) \exp \left( -\frac{1}{2} \left( \frac{\theta - \theta_i}{h_2} \right)^2 \right)$$



## Main issue

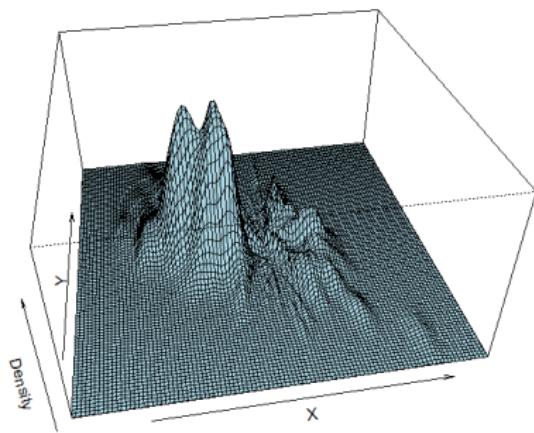
- Wind direction is a circular variable
- Wind direction :  $0^\circ$  and  $360^\circ$  is identical

$$\text{Thus : } \forall v, \hat{f}(v, 0) = \hat{f}(v, 360)$$

# Estimation in Cartesian space

- Convert data speed and direction  $(\rho, \theta)$  into  $(x, y)$
- Fit a KDE with 2 Gaussian kernels

$$\hat{f}(x, y) = \frac{1}{nh_1h_22\pi} \sum_{i=1}^n \exp\left(-\frac{1}{2}\left(\frac{x - X_i}{h_1}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{y - Y_i}{h_2}\right)^2\right)$$

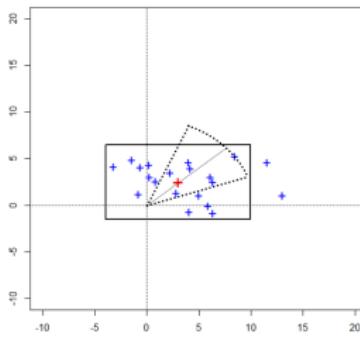
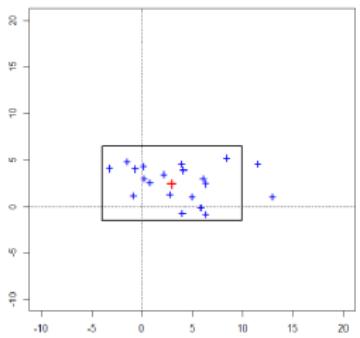
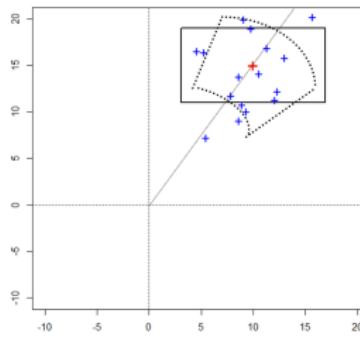
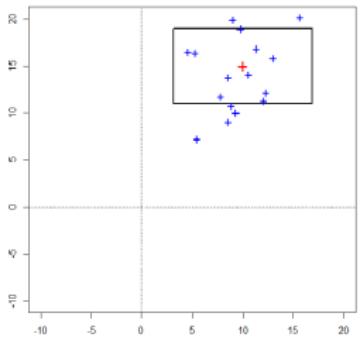


# Estimation in Polar space

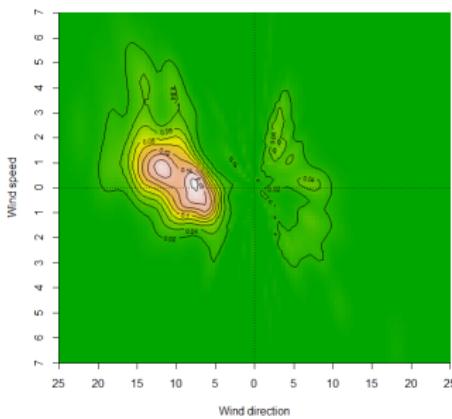
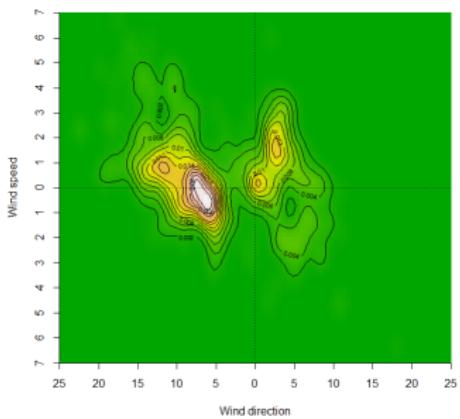
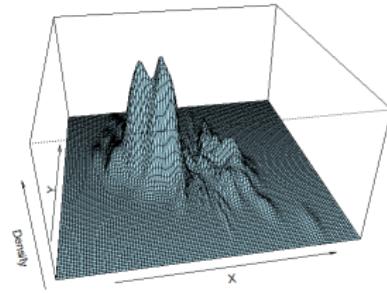
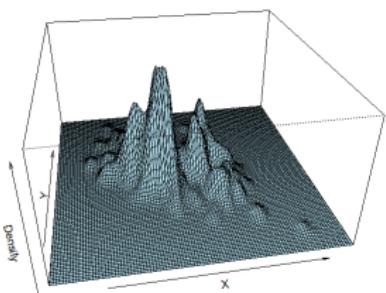
- Work with  $(\rho, \theta)$
- Use of specific kernel to deal with wind direction : circular kernel
  - von Mises  $\hat{f}_k(\theta) = \frac{1}{n} \sum_{i=1}^n \frac{1}{2\pi I_0(k)} \exp(k \cos(\theta - \theta_i))$
  - Wrapped Gaussian  $\hat{f}_h(\theta) = \frac{1}{n} \sum_{i=1}^n \frac{1}{\sqrt{(2\pi)}} \frac{1}{h} \exp\left(-\frac{1}{2} \left(\frac{\theta + 2k\pi - \theta_i}{h}\right)^2\right)$
  - Wrapped Uniforme, etc.
- Fit a KDE with a Gaussian kernel for wind speed and a circular kernel for wind direction

$$\hat{f}_h(v, \theta) = \frac{1}{nh_v h_\theta 2\pi} \sum_{i=1}^n \exp\left(\frac{v - V_i}{h_v}\right) \exp\left(-\frac{1}{2} \left(\frac{\theta + 2k\pi - \theta_i}{h_\theta}\right)^2\right)$$

# Cartesian vs Polar



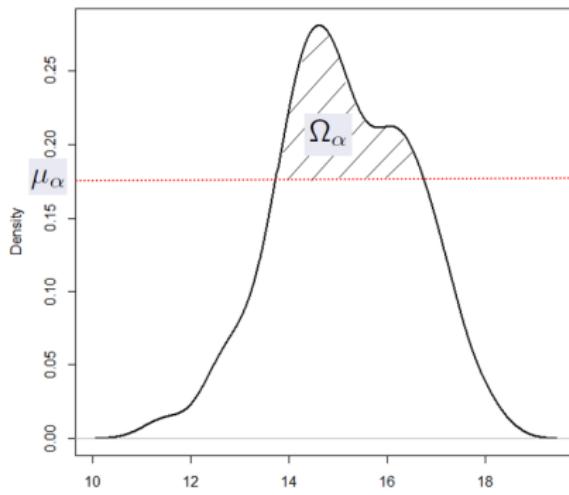
# Cartesian vs Polar (cont'd)



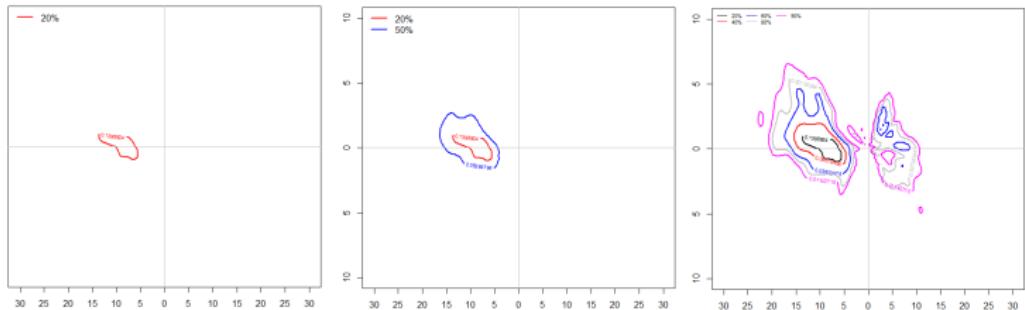
# Contour plot

## Plotting chosen contour lines

- Let's define  $\alpha$  so that  $\int_{\Omega_\alpha} \hat{f}(\rho, \theta) d\rho d\theta = \alpha$ .
- Let  $\mu_\alpha$  is the contour line depending on  $\alpha$
- With  $\Omega_\alpha = \{(\rho, \theta) : \hat{f}(\rho, \theta) > \mu_\alpha\}$



# Contour plot (cont'd)



## Conclusions

- KDE better than fitting parametric distributions
- Use of circular kernel to estimate wind direction *pdf*
- Algorithm to estimate bivariate joint distribution
- Representation in polar coordinates
- Contour lines according to quantiles

## Perspectives

- Package R in progress
- Apply KDE with MCP method for long term correlation
- Extend the results to energy yield estimation

Thank you for your attention !