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Title: Wind characteristics probability density estimation

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# Wind characteristics probability density estimation

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1ères Rencontres R

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# Wind characteristics theoretical probability density function estimation

### Wind speed

- Weibull $\mathcal{W}(k, \lambda)$
- log-Normal $\log \mathcal{N}(\mu, \sigma)$
- Gamma $\Gamma(k, \theta)$

### Wind direction

- von Mises $\mathcal{VM}(\mu, \kappa)$
- Wrapped Cauchy $\mathcal{WC}(\mu, \rho)$
- Wrapped Normal $\mathcal{WN}(\mu, \rho)$

### Wind speed AND wind direction

- Wind speed variation description for wind potential estimation
- Wind direction is also of interest for wind farm design and energy potential estimation

$\Longrightarrow$ *Bivariate joint distribution*

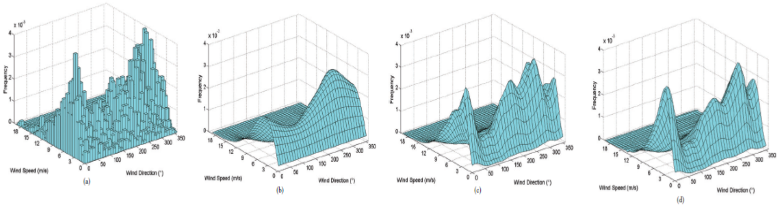
# Joint probability density function of wind speed and direction

## Several approaches for constructing joint *pdf*

- Anisotropic lognormal approach
- Farlie-Gumbel-Morgenstern families approach
- Angular-Linear approach

$$f(v,\theta) = 2\pi\; g(\xi)\,[f_v(v) - f_\theta(\theta)]$$

$$\text{with } \xi = 2\pi\,[F_v(v) - F_\theta(\theta)]$$



## Multimodality

- Empirical distribution is unimodal
  - Parametric *pdf* gives good approximations
- Empirical distribution is multimodal
  - Use of mixture of distributions

$\Longrightarrow$ *How to specify the right model ?*

# Joint probability density function estimation with KDE

**Definition**

Let $X$ and $Y$ be 2 random variables, the Kernel Density Estimator (KDE) is defined as

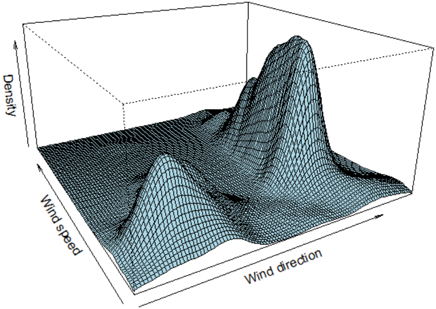
$$\hat{f}(x,y) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{h_X} \ \mathbb{K}_X\left(\frac{x - X_i}{h_X}\right) \ \frac{1}{h_Y} \ \mathbb{K}_Y\left(\frac{y - Y_i}{h_Y}\right)$$

- Choice of the kernel $\mathbb{K}$ is not crucial
- Choice of bandwidths $h_X$ and $h_Y$ are of importance

## State-of-art

- Zhang (2011) : Multiplicative Gaussian kernels

$$\hat{f}(v,\theta) = \frac{1}{nh_1 h_2 2\pi} \sum_{i=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{v - V_i}{h_1}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{\theta - \theta_i}{h_2}\right)^2\right)$$



## Main issue

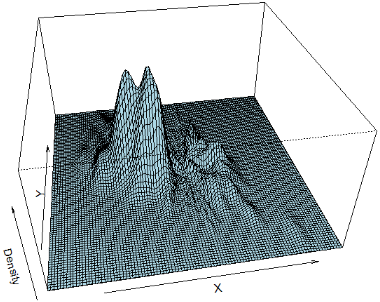
- Wind direction is a circular variable
- Wind direction : 0 $^\circ$ and 360 $^\circ$ is identical

$$\text{Thus : } \forall\ v, \quad \hat{f}(v,0) = \hat{f}(v,360)$$

# Estimation in Cartesian space

- Convert data speed and direction $(\rho, \theta)$ into $(x, y)$
- Fit a KDE with 2 Gaussian kernels

$$\hat{f}(x,y) = \frac{1}{nh_1 h_2 2\pi} \sum_{i=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{x - X_i}{h_1}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{y - Y_i}{h_2}\right)^2\right)$$

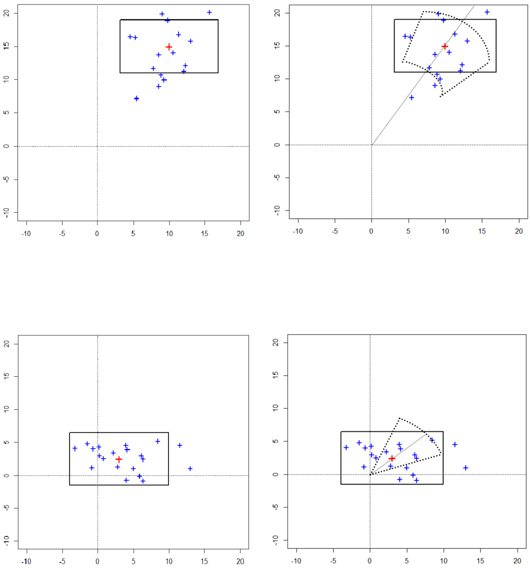


# Estimation in Polar space

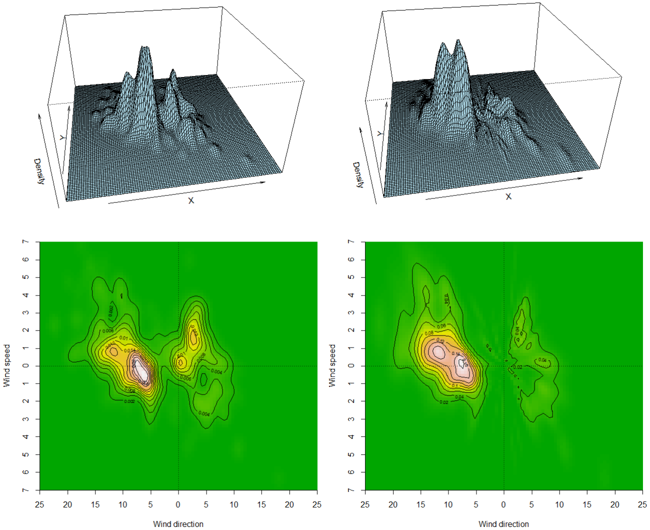
- Work with $(\rho, \theta)$
- Use of specific kernel to deal with wind direction : circular kernel
  - von Mises $\hat{f}_k(\theta) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2\pi \ I_0(k)} \exp\left(k \ cos(\theta - \theta_i)\right)$
  - Wrapped Gaussian $\hat{f}_h(\theta) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\sqrt{(2\pi)}} \frac{1}{h} \exp\left(-\frac{1}{2}\left(\frac{\theta+2k\pi-\theta_i}{h}\right)^2\right)$
  - Wrapped Uniforme, etc.
- Fit a KDE with a Gaussian kernel for wind speed and a circular kernel for wind direction

$$\hat{f}_h(v,\theta) = \frac{1}{nh_v h_\theta 2\pi} \sum_{i=1}^{n} \exp\left(\frac{v - V_i}{h_v}\right) \exp\left(-\frac{1}{2}\left(\frac{\theta + 2k\pi - \theta_i}{h_\theta}\right)^2\right)$$

# Cartesian vs Polar



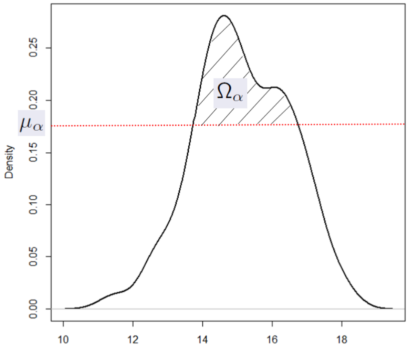
# Cartesian vs Polar (cont'd)



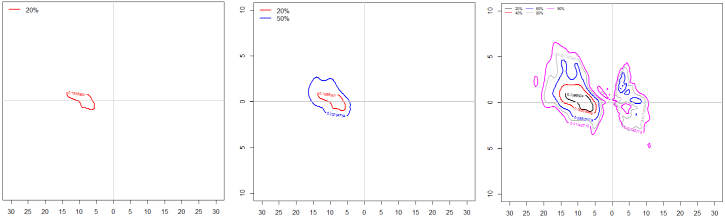
# Contour plot

## Plotting chosen contour lines

- Let's define $\alpha$ so that $\int_{\Omega_\alpha} \hat{f}(\rho,\theta)d\rho d\theta) = \alpha$.
- Let $\mu_\alpha$ is the contour line depending on $\alpha$
- With $\Omega_\alpha = \left\{(\rho,\theta) : \hat{f}(\rho,\theta) > \mu_\alpha\right\}$



# Contour plot (cont'd)



# Conclusions and Perspectives

## Conclusions

- KDE better than fitting parametric distributions
- Use of circular kernel to estimate wind direction *pdf*
- Algorithm to estimate bivariate joint distribution
- Representation in polar coordinates
- Contour lines according to quantiles

## Perspectives

- Package R in progress
- Apply KDE with MCP method for long term correlation
- Extend the results to energy yield estimation

Thank you for your attention !