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Author(s): Nguyen, Dinh Cong

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Introduction to Free-Electron Laser Systems

DEPS Short Courses
11 June 2012
Broomfield, CO

Dinh Nguyen
Los Alamos National Laboratory

Course Content

1. Introduction to FEL
2. RF Linac FEL
3. Electron Beam Transport
4. Wigglers
5. Spontaneous Emission, Gain & Efficiency
6. Optical Architectures

Part 1
Introduction to FEL

Light consists of photons

Photons are packets of energy with no mass

Photon energy is proportional to the light frequency, inversely proportional to wavelength

$$\varepsilon = h\nu = \frac{hc}{\lambda}$$

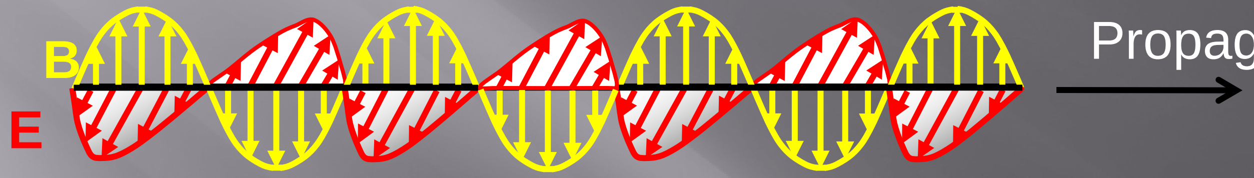
The shorter the wavelength, the more energetic the photons

Light at 1 μm (1,000 nm) wavelength has energy of 1.24 electron volts

$$\varepsilon[eV] = \frac{1.24}{\lambda[\mu m]}$$

Light is also electromagnetic (EM) waves

When there are many photons, light also behaves as a travelling electromagnetic (EM) wave

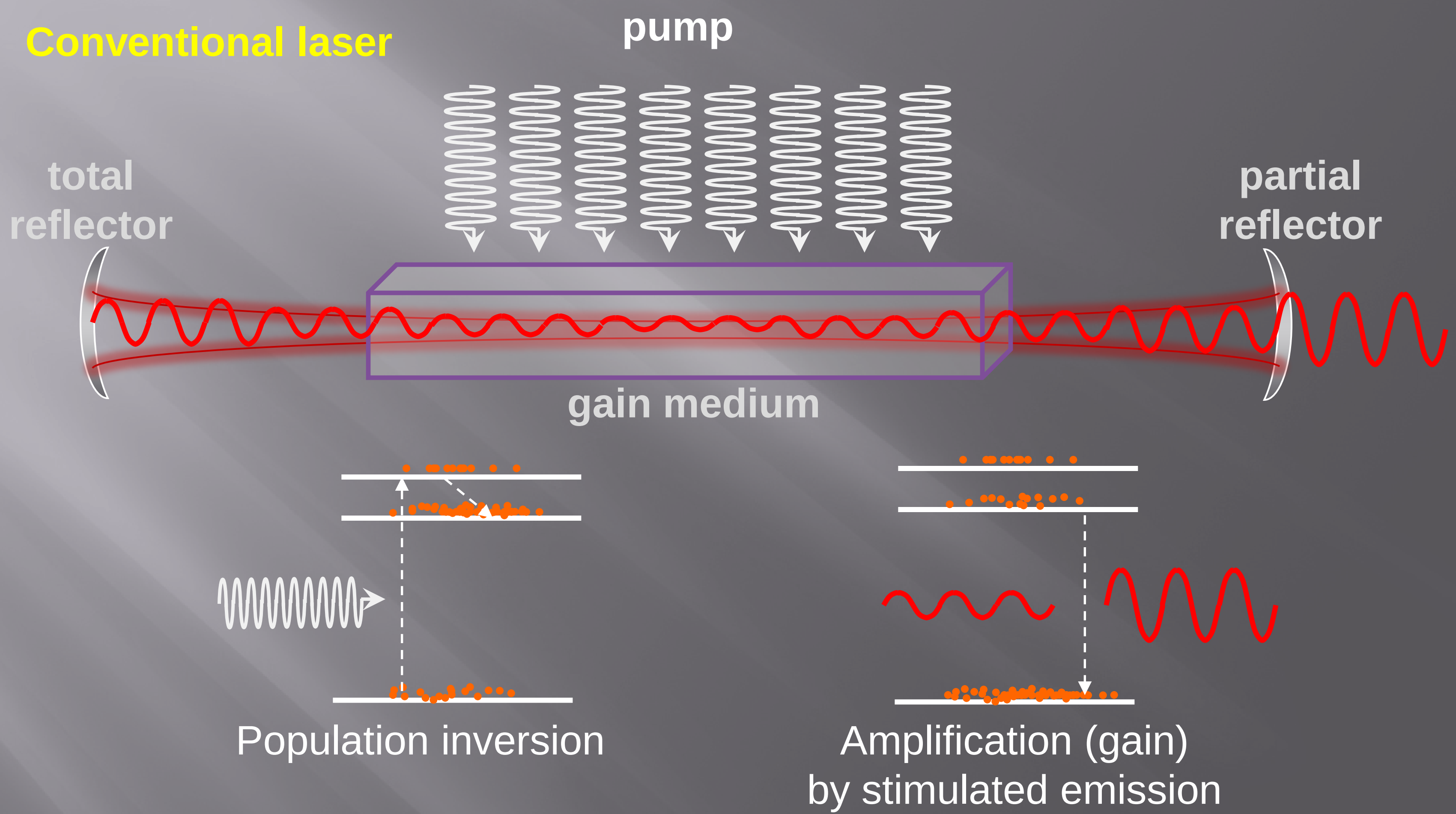


The electric field E and magnetic field B of a plane wave are perpendicular to each other and to the direction of propagation

In our convention, E oscillates along the x direction, B along the y, and z is the direction of light propagation.

$$\mathbf{E}(z,t) = \hat{\mathbf{x}} E_0 \sin(kz - \omega t + \varphi)$$
$$\mathbf{B}(z,t) = \hat{\mathbf{y}} B_0 \sin(kz - \omega t + \varphi)$$

Light Amplification by Stimulated Emission of Radiation (LASER)



Quantum lasers use electrons bound to discrete atomic or molecular energy levels and thus produce fixed wavelengths

FEL gain medium is free electrons traversing a wiggler (undulator)

FEL use free electrons traveling near the speed of light through a series of alternating magnets called a wiggler. Electrons undulating in the wiggler radiate electromagnetic waves at wavelength λ as given by

wiggler period

wavelength

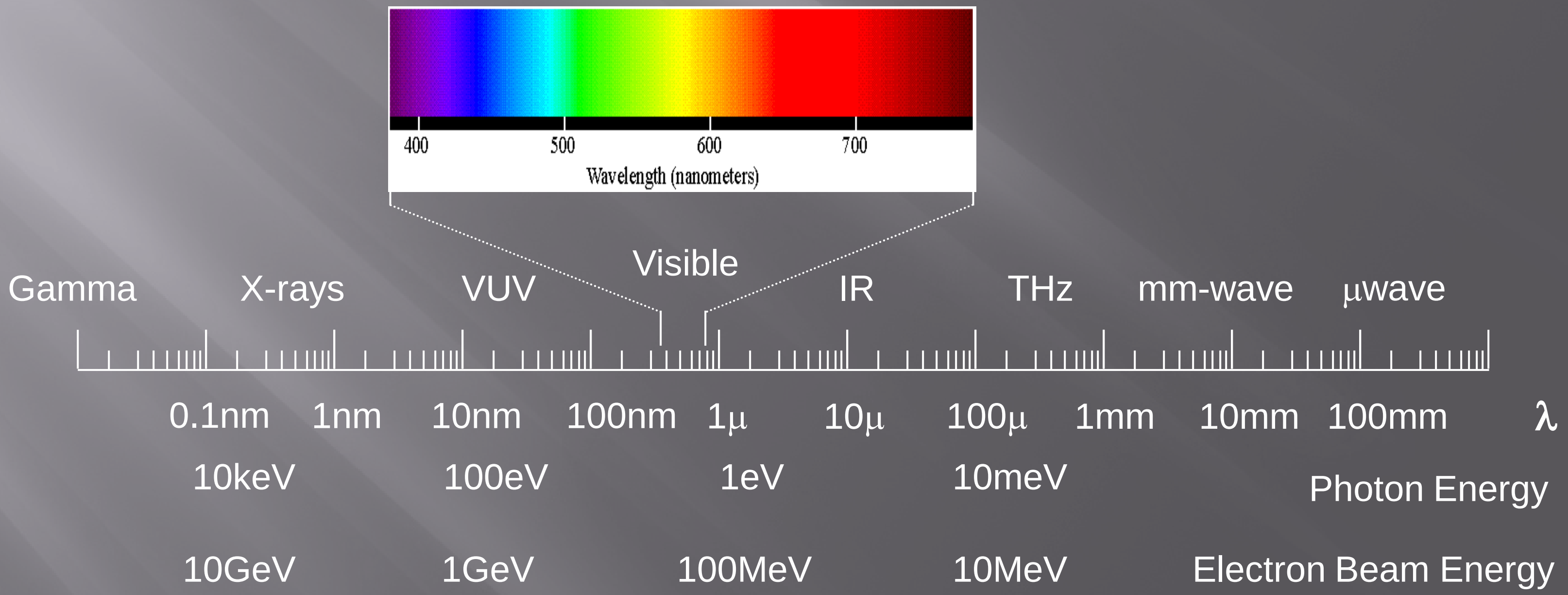
$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1 + a_w^2\right)$$

a parameter that depends on wiggler field and period

a parameter that depends on electron beam energy

Basic features of FEL

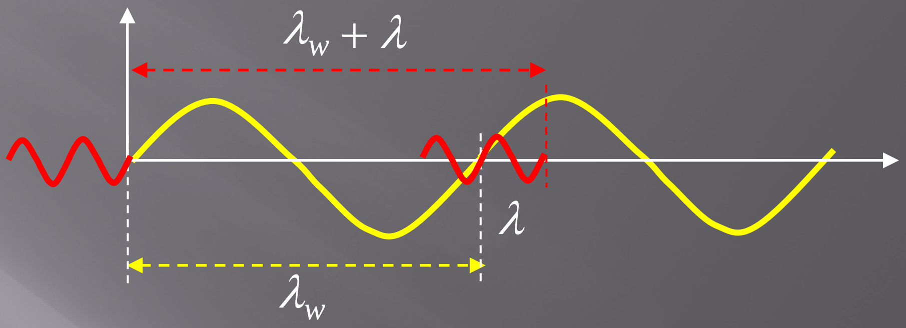
- Wavelength tunable
- Diffraction limited optical beam
- High power (GW peak, 10s of kW average)
- Flexible pulse format



Light slips over the electrons one wavelength every wiggler period

Relativistic factor

$$\gamma = \frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$$



Electron average speed

$$\bar{\upsilon} \approx c\left(1-\frac{1}{2\gamma^2}\right)$$

Electrons travel slightly slower than light so in the time electrons travel one period, light travels one wiggler period plus one wavelength

$$\frac{\lambda_w + \lambda}{c} = \frac{\lambda_w}{\bar{\upsilon}}$$

$$\lambda = \lambda_w\left(\frac{c}{\bar{\upsilon}} - 1\right)$$

$$\lambda = \frac{\lambda_w}{2\gamma^2}$$

Force & Energy Transfer in FEL

Lorentz force on electrons

$$\mathbf{F} = -e\left[\mathbf{E} + \left(\mathbf{v} \times \mathbf{B}\right)\right]$$

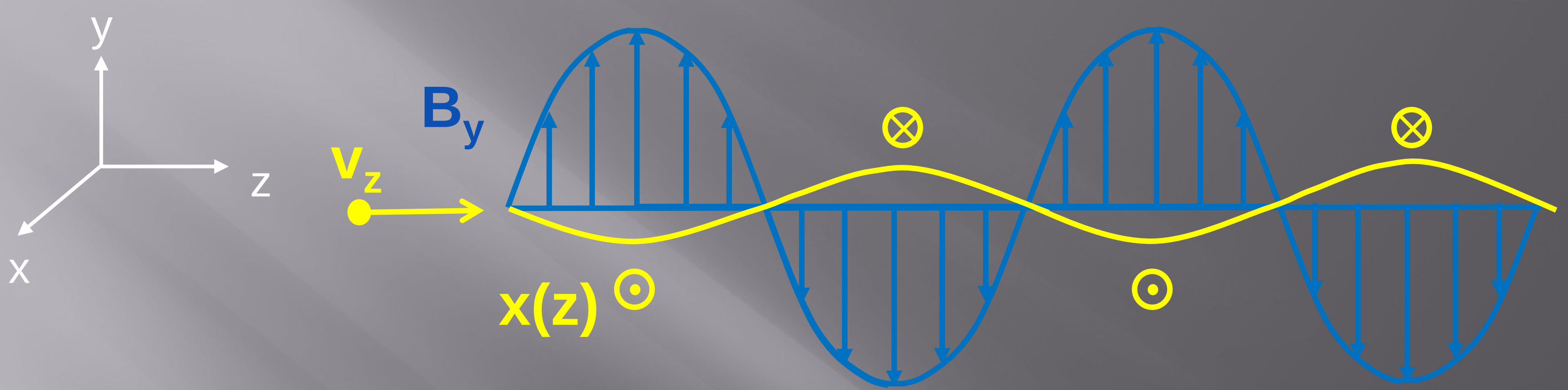
Magnetic field changes the electron beam's momentum but not its energy because the direction of magnetic force is always perpendicular to the electrons' motion.

Energy transfer happens when electrons interact with the light's electric field.

$$\Delta W = \int \mathbf{F} \cdot d\mathbf{s} = -\int e\mathbf{E} \cdot d\mathbf{s}$$

Light only has **transverse electric field**, so for energy transfer between co-propagating electron and light beams to happen, we need a **sinusoidal magnetic field** to give electrons **oscillatory transverse motion** to interact with the light beam's electric field.

Electron Motion in a Wiggler



Sinusoidal magnetic field along the y axis

$$B_y = B_0 \sin\left(k_w z\right)$$

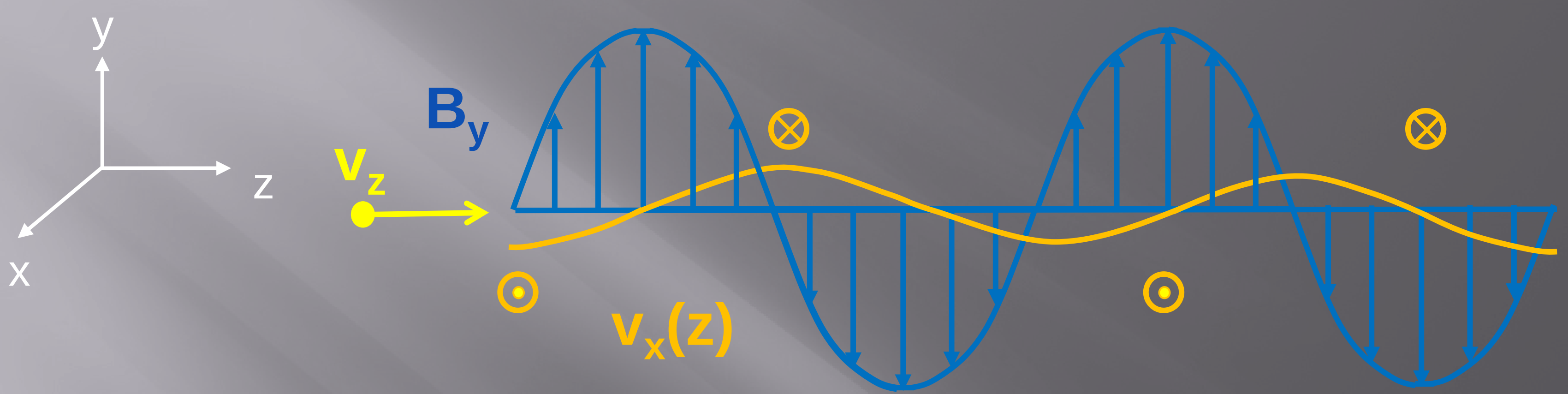
Lorentz force on electrons due to v x B (where v_z is almost c) causes a rate of change of electron momentum in x

$$\gamma m_0 \dot{\upsilon}_x = -e\upsilon_z B_y \approx -ecB_0 \sin\left(k_w z\right)$$

Lorenz force acceleration is in the opposite direction with electrons' motion, similar to the restoring force of a spring.

$$\dot{\upsilon}_x = \frac{-ecB_0}{\gamma m_0} \sin\left(k_w z\right)$$

Transverse Electron Velocity



Rate of change in x velocity

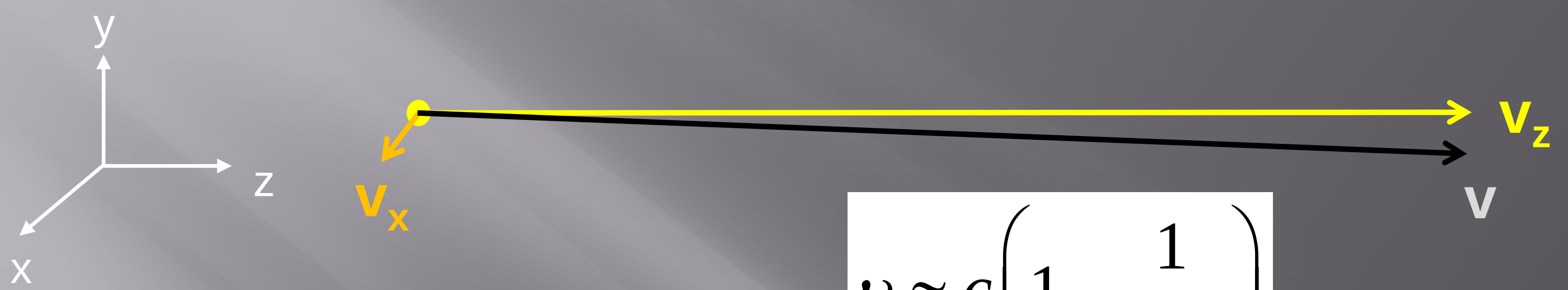
$$\dot{\upsilon}_x = \frac{-ecB_0}{\gamma m_0}\sin\left(k_w ct\right)$$

Integrate to obtain velocity in x

$$\upsilon_x = \frac{eB_0}{\gamma m_0 k_w}\cos\left(k_w z\right)$$

Transverse velocity is 90º out of phase with both motion and acceleration, i.e. transverse velocity is greatest when electrons cross the z axis (B = 0).

Axial Electron Velocity



Using Pythagoras' theorem

$$\upsilon^2 = \upsilon_z{}^2 + \upsilon_x{}^2$$

$$\upsilon \approx c\left(1 - \frac{1}{2\gamma^2}\right)$$

$$\upsilon_x = \frac{c\sqrt{2}a_w}{\gamma}\cos\left(k_w z\right)$$

Calculate the electrons' axial velocity (along z direction)

$$\upsilon_z = c\left[1 - \frac{1}{2\gamma^2}\left(1 + a_w^2 + a_w^2\cos\left(2k_w z\right)\right)\right]$$

Electrons' axial velocity is modulated at twice the wiggling frequency

Figure 8 motion

Average velocity of electrons

$$\bar{\upsilon}_z = c\left(1 - \frac{\left(1 + a_w^2\right)}{2\gamma^2}\right)$$

Electrons' velocity is modulated at twice the wiggler period

$$\upsilon_z = \bar{\upsilon}_z - \frac{c a_w^2}{2\gamma^2}\cos\left(2k_w z\right)$$

Motion in electron's rest frame

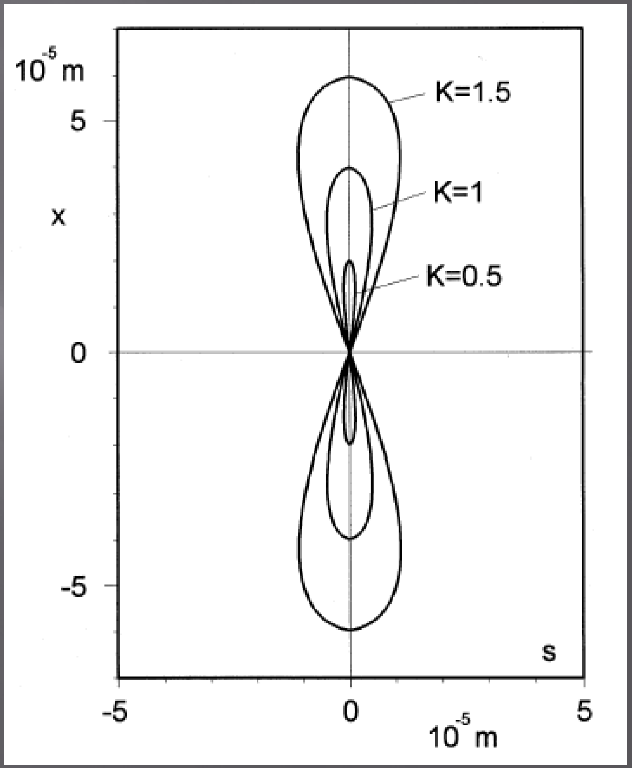


Figure 8 motion gives rise to harmonics in synchrotron radiation

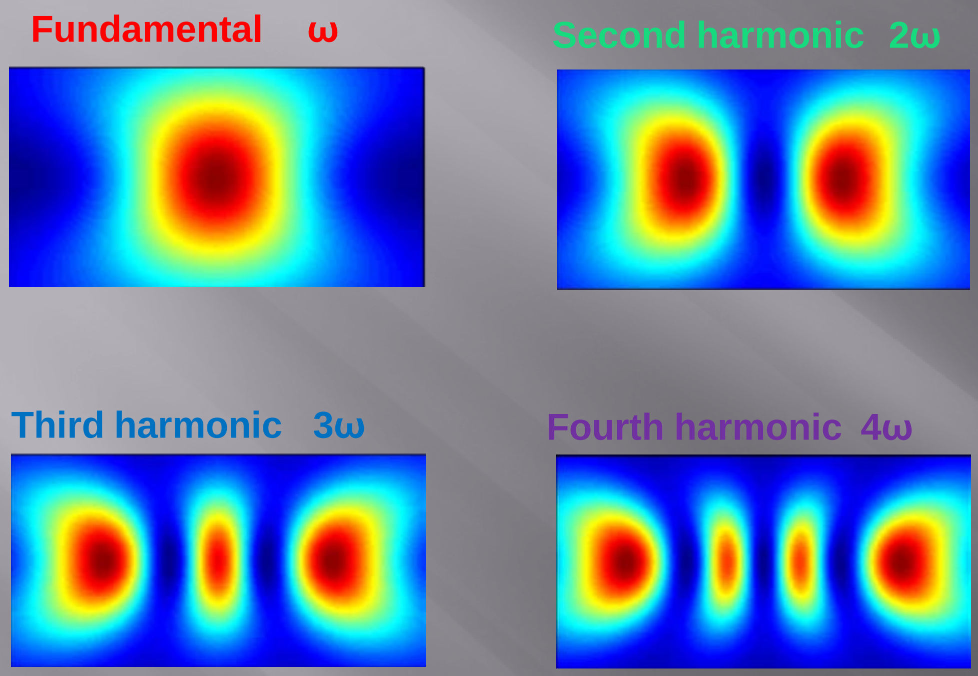
Synchrotron (Undulator) Radiation
3rd Generation Light Source

Advanced Photon Source

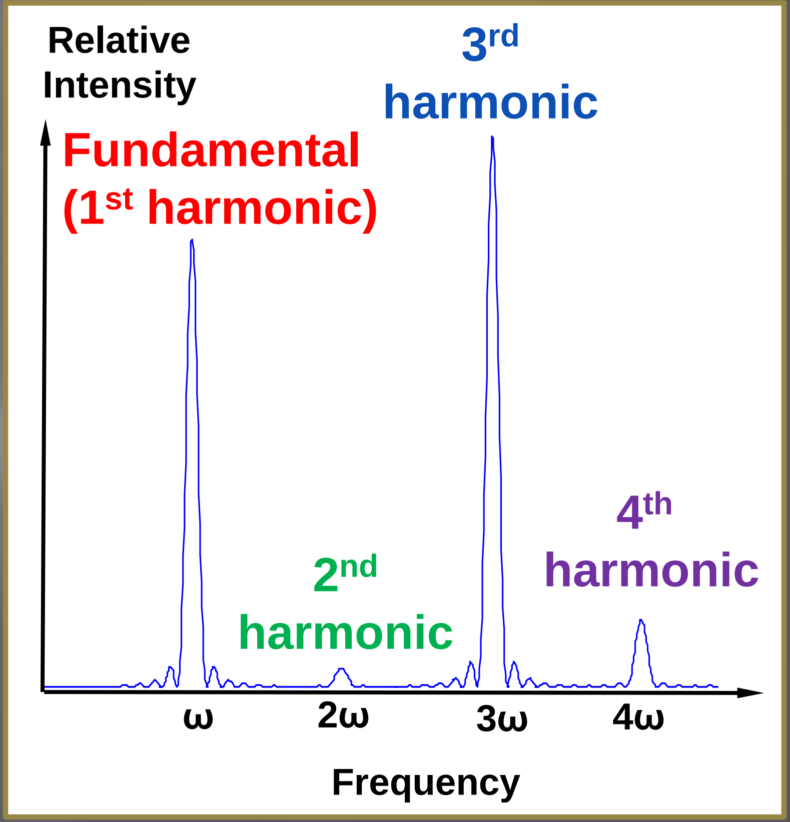
Undulator (aka

Insertion Devices)

Undulator Radiation Harmonics



Color codes depict radiation intensity not wavelength

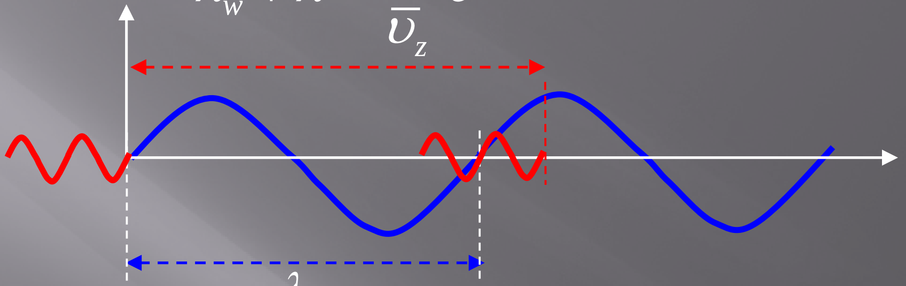


Odd harmonics are stronger and emitted on-axis (e- beam direction)

Even harmonics are weaker and emitted off-axis

Resonance Wavelength

$$\lambda_w + \lambda = \frac{\lambda_w}{\bar{\upsilon}_z} c$$



Wiggler period

rms wiggler parameter

Wavelength

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1 + a_w^2\right)$$

$$a_w = 0.66 B_0\left(T\right)\lambda_w\left(cm\right)$$

period

magnetic field

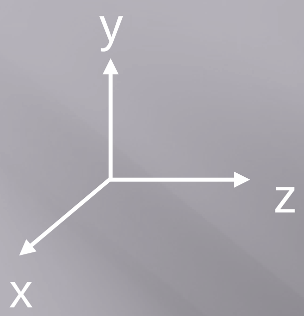
Electron kinetic energy

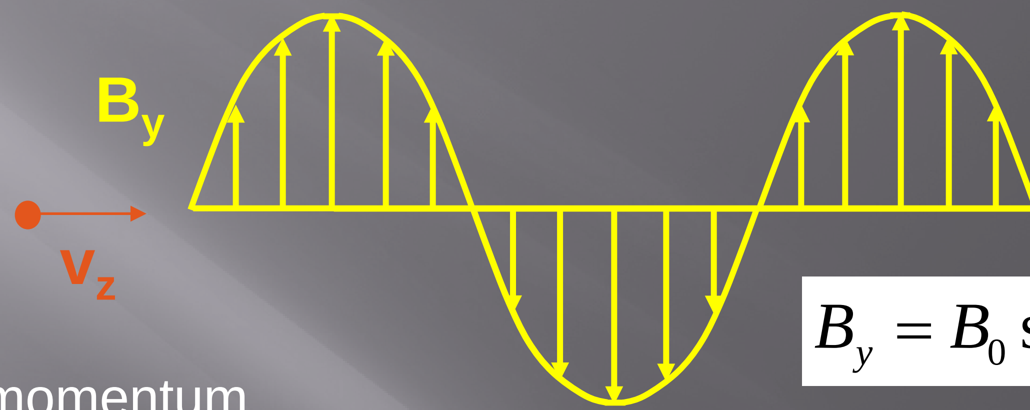
Total energy

$$\gamma = \frac{E_k + m_0 c^2}{m_0 c^2}$$

Electron rest energy
(0.511MeV)

Electron Beam Velocity and Motion inside a Wiggler





$$B_y = B_0 \sin\left(k_w z\right)$$

Rate of change in x momentum

$$\frac{d}{dt}\left(\gamma m_0 \upsilon_x\right) = -e\upsilon_z B_0 \sin\left(k_w z\right) = -eB_0 \frac{dz}{dt} \sin\left(k_w z\right) = \frac{eB_0}{k_w}\frac{d}{dt}\cos\left(k_w z\right)$$

$$\upsilon_x = \frac{eB_0}{\gamma m_0 k_w}\cos\left(k_w z\right) = \frac{cK}{\gamma}\cos\left(k_w z\right)$$

Dimensionless undulator parameter

$$K = \frac{eB_0}{m_0 c k_w}$$

Motion in x direction

$$x = \frac{K}{\gamma k_w}\sin\left(k_w z\right)$$

Energy Exchange between Electrons and Optical Field

$$\frac{dW}{dt} = j_\perp \cdot E_s$$

Transverse electron current

$$j_\perp = -ec\upsilon_x = \frac{-ecK}{\gamma}\cos\left(k_w z\right)$$

Plane-wave transverse electric field

$$E_s(z,t) = E_{s,0}\sin(kz-\omega t)$$

$$\frac{d\left(\gamma m_0 c^2\right)}{dt} = \frac{-ecKE_{s,0}}{\gamma}\sin(kz-\omega t)\cos\left(k_w z\right)$$

$$\frac{d\left(\gamma m_0 c^2\right)}{cdt} = \frac{-eKE_{s,0}}{2\gamma}\sin\left(\left(k+k_w\right)z-\omega t\right)$$

Divide both sides by m_0c^2

$$\frac{d\gamma}{dz} = -\frac{eKE_{s,0}}{2\gamma m_0 c^2}\sin\left(\left(k+k_w\right)z-\omega t\right)$$

Evolution of Energy Change with respect to Resonance Energy

Electron energy loss is small relative to its initial energy, i.e. $\Delta\gamma \ll \gamma$. Define change in energy as relative difference between the electron energy compared to the resonance energy

$$\frac{\Delta\gamma}{\gamma_R} = \frac{\gamma - \gamma_R}{\gamma_R}$$

$$\frac{d}{dz}\left(\frac{\Delta\gamma}{\gamma_R}\right) = \frac{d}{dz}\left(\frac{\gamma - \gamma_R}{\gamma_R}\right) = \frac{1}{\gamma_R}\left(\frac{d\gamma}{dz}\right)$$

Dimensionless optical field

$$a_s = \frac{eE_{s,0}}{km_0c^2}$$

Rewrite energy evolution in terms of relative change in energy to the resonance energy $\Delta\gamma/\gamma_R$ and a_s

$$\frac{d}{dz}\left(\frac{\Delta\gamma}{\gamma_R}\right) = -\frac{ka_sK}{\gamma_R^2}\sin\left[\left(k + k_w\right)z - \omega t\right]$$

Evolution of Electron Phase

Electron phase

$$\psi = \left(k + k_w\right) z - \omega t$$

Electron phase velocity

$$\frac{d\psi}{dt} = \left(k + k_w\right)\bar{\upsilon}_z - \omega$$

Wiggler wave-number

$$k_w = \frac{2\pi}{\lambda_w} = k\frac{\left(1 + a_w^2\right)}{2\gamma^2}$$

$$\frac{d\psi}{dt} \approx k_w c - \frac{kc}{2\gamma_R^2}\left(1 + a_w^2\right)$$

$$\frac{d\psi}{dt} = \frac{kc}{2}\left(1 + a_w^2\right)\left(\frac{1}{\gamma_R^2} - \frac{1}{\gamma^2}\right)$$

Rate of change in phase versus distance is proportional to electron energy relative to the resonance energy

$$\frac{d\psi}{dz} = \frac{k\left(1 + a_w^2\right)}{2\gamma_R^2}\left(\frac{2\Delta\gamma}{\gamma_R}\right) = 2k_w\left(\frac{\Delta\gamma}{\gamma_R}\right)$$

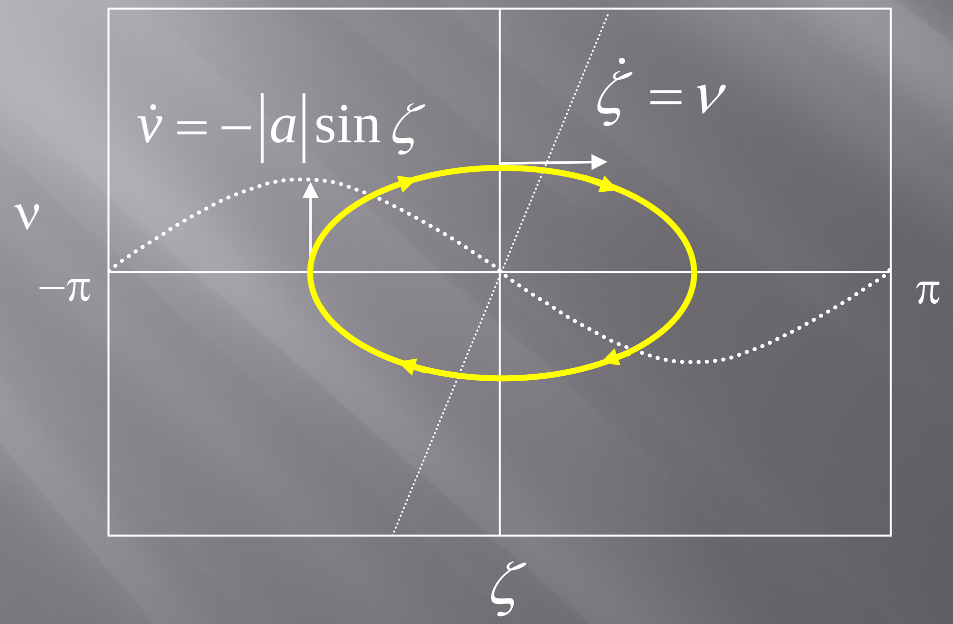
Trajectory in Phase Space

Evolution of energy difference relative to resonance energy depends on sine(phase) →

$$\frac{d}{dz}\left(\frac{\Delta\gamma}{\gamma_R}\right) = -\Omega^2 \sin\psi$$

Evolution of phase depends on energy difference relative to resonance energy →

$$\frac{d\psi}{dz} = 2k_w\left(\frac{\Delta\gamma}{\gamma_R}\right)$$



Convert to two new units

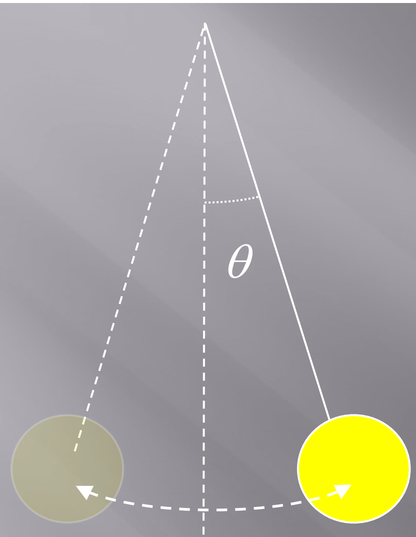
$$\zeta = \frac{\psi}{2k_w} \qquad \nu = \frac{\Delta\gamma}{\gamma_R}$$

Coupled 1st order differential equations

$$\dot{\zeta} = \nu$$
$$\dot{\nu} = -|a|\sin\zeta$$

Particles follow **elliptical trajectories** corresponding to different energies.

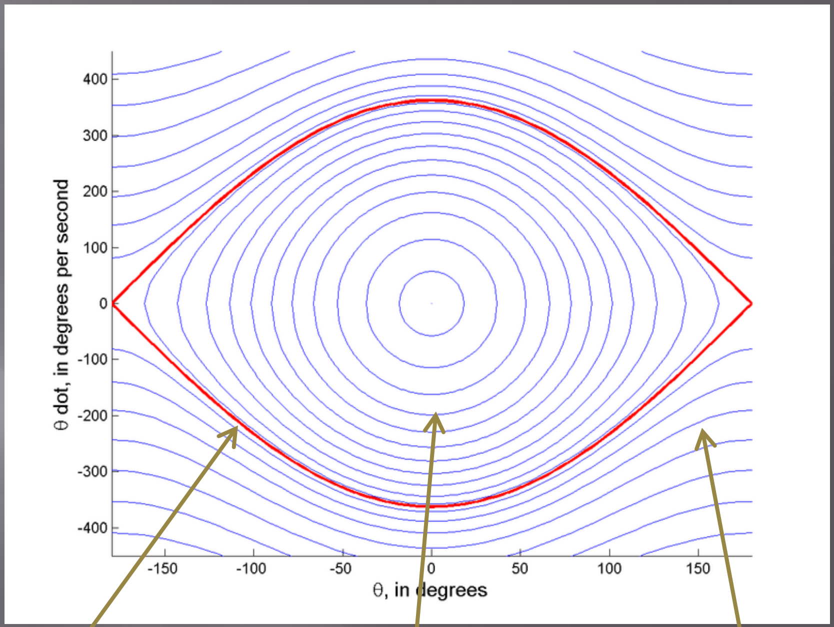
Pendulum Equation



Hamiltonian = Total energy

$$H = \frac{v^2}{2} - \frac{g}{l}\cos\theta$$

The separatrix defines the boundary between closed (oscillatory motion) and open (rotational) orbits



Separatrix

$$H = \frac{g}{l}$$

Closed orbits

$$-\frac{g}{l} < H < \frac{g}{l}$$

Open orbits

$$\frac{g}{l} < H$$

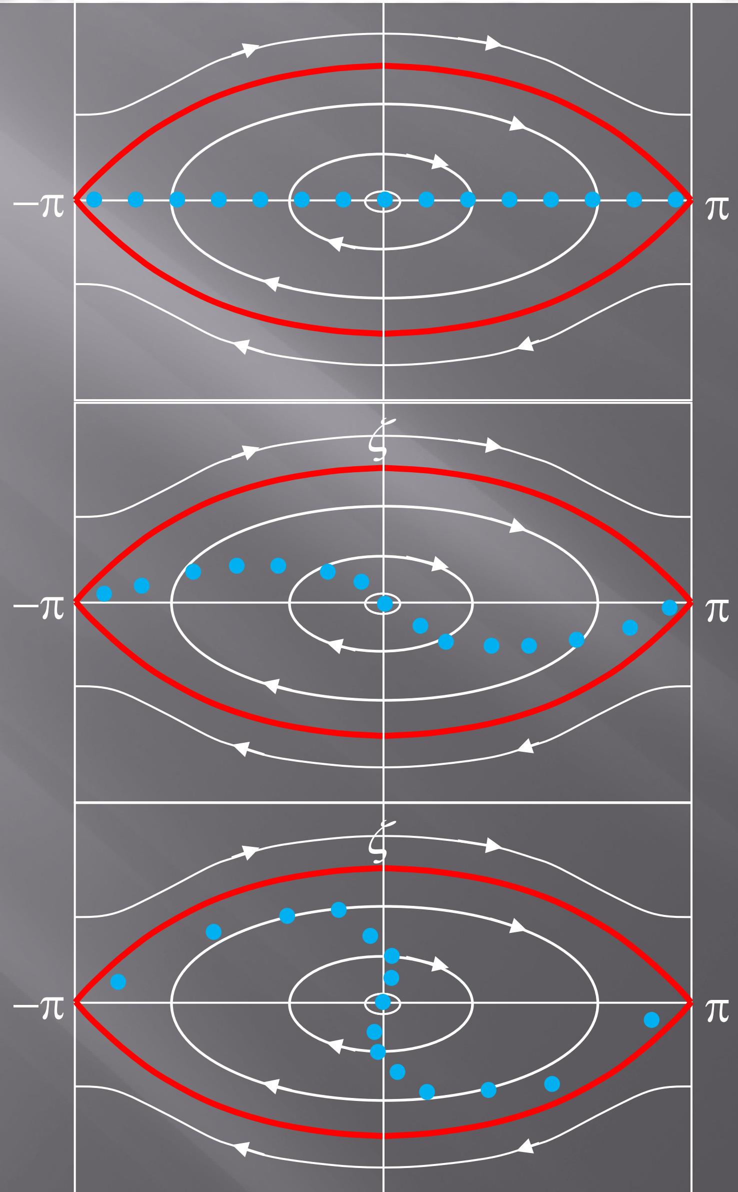
Courtesy of Wikipedia

Phase-space Motion of Electrons in Ponderomotive Wave

At z = 0
(wiggler entrance)

At z = 10 L_g
(middle of wiggler)

At z = 16 L_g
(wiggler exit)

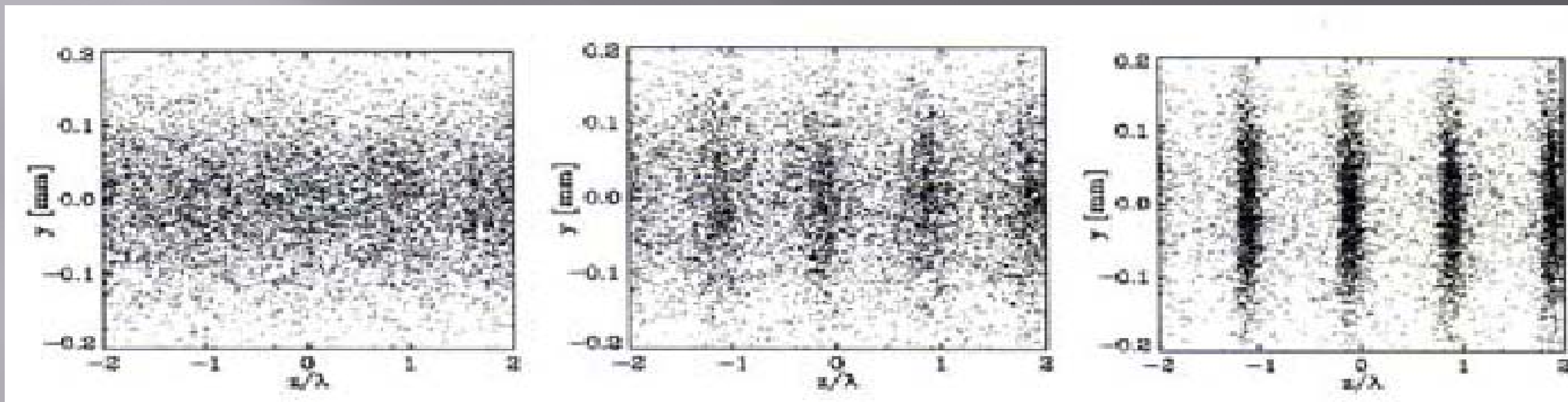


Electrons are unbunched

Electrons have energy modulation

Electrons are bunched in time (density modulation)

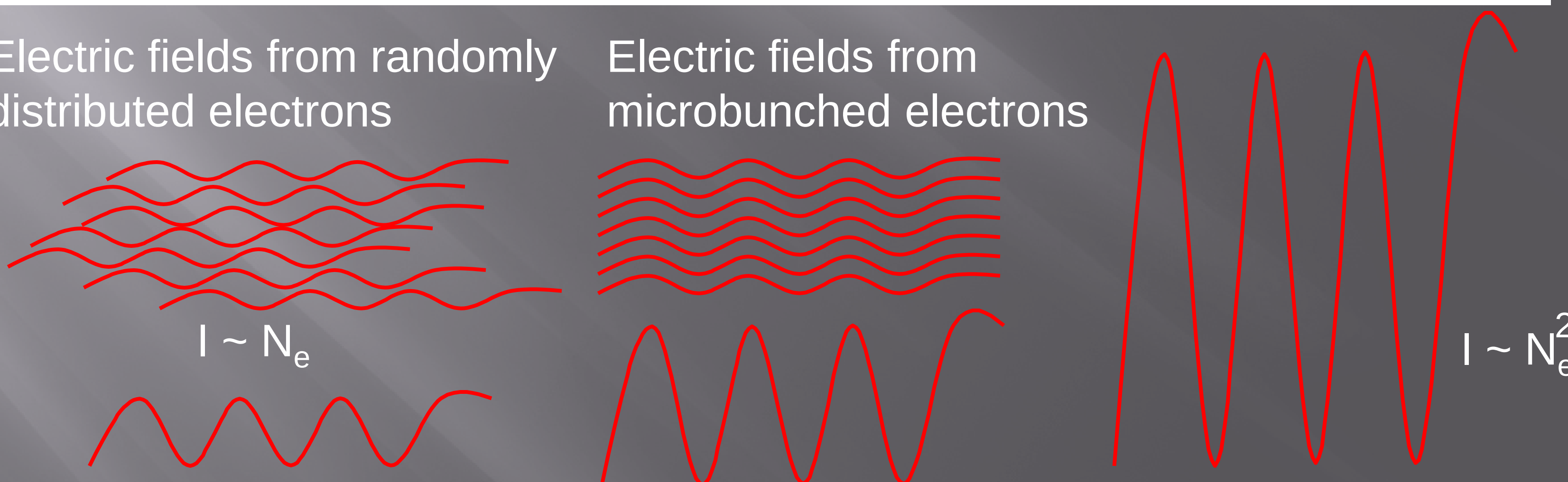
Electrons are microbunched with periodicity of one wavelength



At entrance to the undulator — Exponential gain regime — Saturation(maximum bunching)

Electric fields from randomly distributed electrons

Electric fields from microbunched electrons



Intensity is proportional to E^2

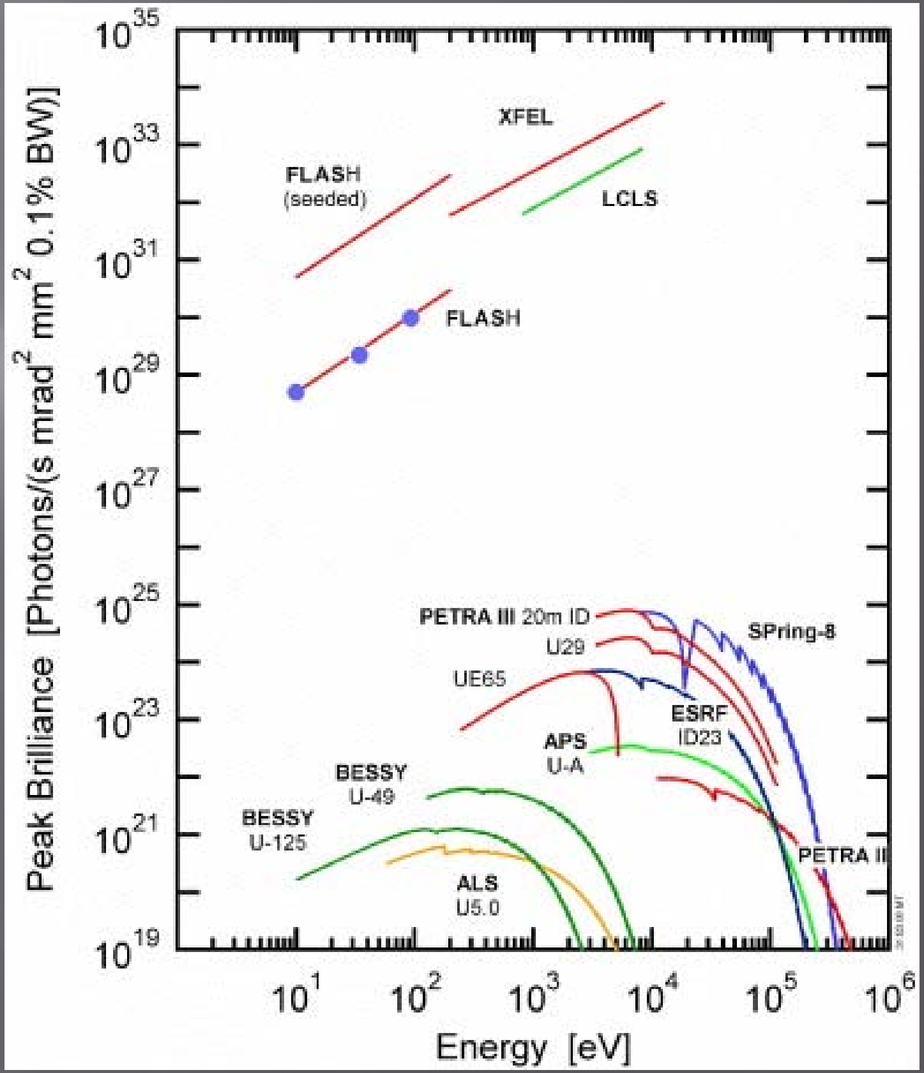
Light is amplified by N_e

FEL is a 4th Gen Light Source

Brilliance

$$\mathfrak{B} = \frac{N_p}{\pi^2 \varepsilon_x \varepsilon_y \Delta t \frac{\Delta\omega}{\omega}}$$

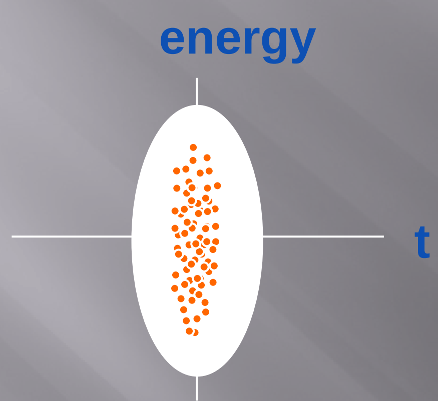
FEL peak brilliance is orders of magnitude above synchrotron radiation because FEL light is **magnified** by the **# of electrons** in each bunch and the FEL phase space area is small.



Brilliance

$$\text{Brilliance} = \frac{\text{Number of photons}}{\text{time} \quad \text{bandwidth} \times \text{beam area} \times \text{solid angle}}$$

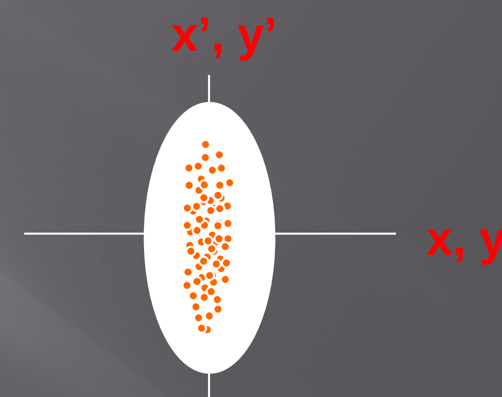
longitudinal phase space



$$\sigma_z \left(\frac{\sigma_\gamma}{\gamma} \right) \leq \left(\frac{\Delta\lambda}{\lambda} \right) c\sigma_t$$

Longitudinal emittance is expressed in time % bandwidth

transverse phase space



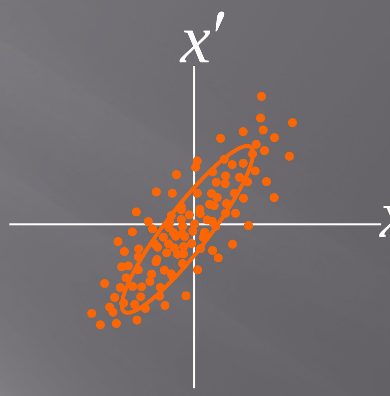
$$\varepsilon_{x,y} \leq \frac{\lambda}{4\pi}$$

Transverse emittance is expressed in microns

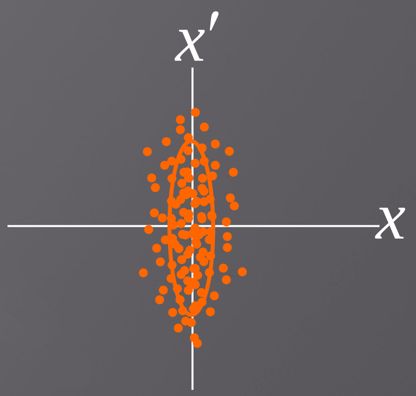
Transverse Emittance

Root-mean-square x emittance

$$\varepsilon_{rms,x} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$



<x-x'> ≠ 0



<x-x'> = 0

Emittance is defined using ensemble averages, denoted by < >, of x^2 and x'^2 and *x'-x* correlation. The correlation vanishes at the waist (upright ellipse) and rms beam emittance becomes $\sigma_x \sigma_{x'}$ where $\sigma_x = \sqrt{\langle x^2 \rangle}$ is the rms radius in *x* and $\sigma_{x'} = \sqrt{\langle x'^2 \rangle}$ is the rms spread in *x'*.

At the beam waist

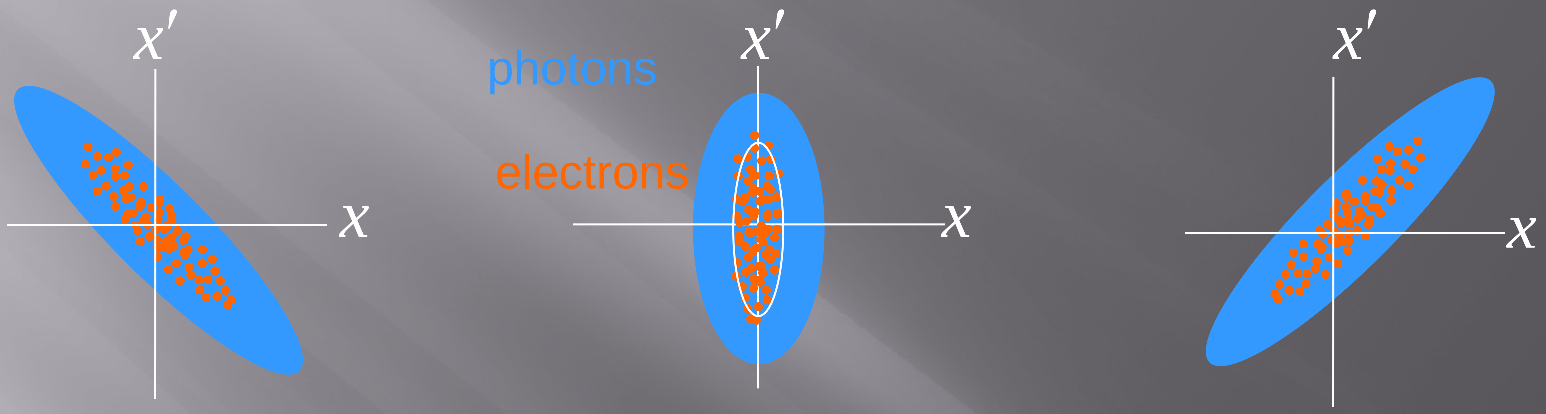
$$\varepsilon_{rms,x} = \sigma_x \sigma_{x'}$$

Normalized emittance

$$\varepsilon_n = \beta\gamma\varepsilon$$

Required Transverse Emittance

Electrons' phase-space area must be less than photons' phase space area for efficient energy exchange between electrons and photons



Required un-normalized emittance

$$\varepsilon_u \le \frac{\lambda}{4\pi}$$

Required normalized emittance

$$\varepsilon_n \le \frac{\beta\gamma\lambda}{4\pi}$$

Required Longitudinal Emittance

Electron beam's energy spread must be smaller than the electrons' velocity spread over the interaction length.

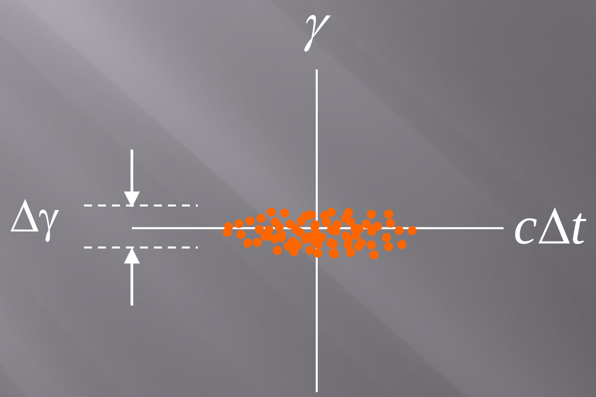
For oscillator FEL, interaction length ~ wiggler length

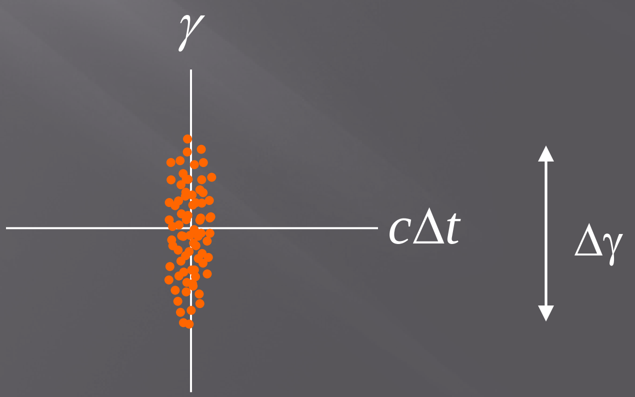
$$\frac{\Delta\gamma}{\gamma} \le \frac{1}{2N_w}$$

For SASE and amplifier FEL, interaction length ~ gain length

$$\frac{\Delta\gamma}{\gamma} \le \rho$$

Uncompressed electron beams have small energy spread and low peak current. Compressed beams have high current and large energy spread.





Summary of Part 1

FEL produce wavelength-tunable, picosecond and sub-ps pulses of coherent radiation over a broad spectrum from **mm-wave to x-rays**.

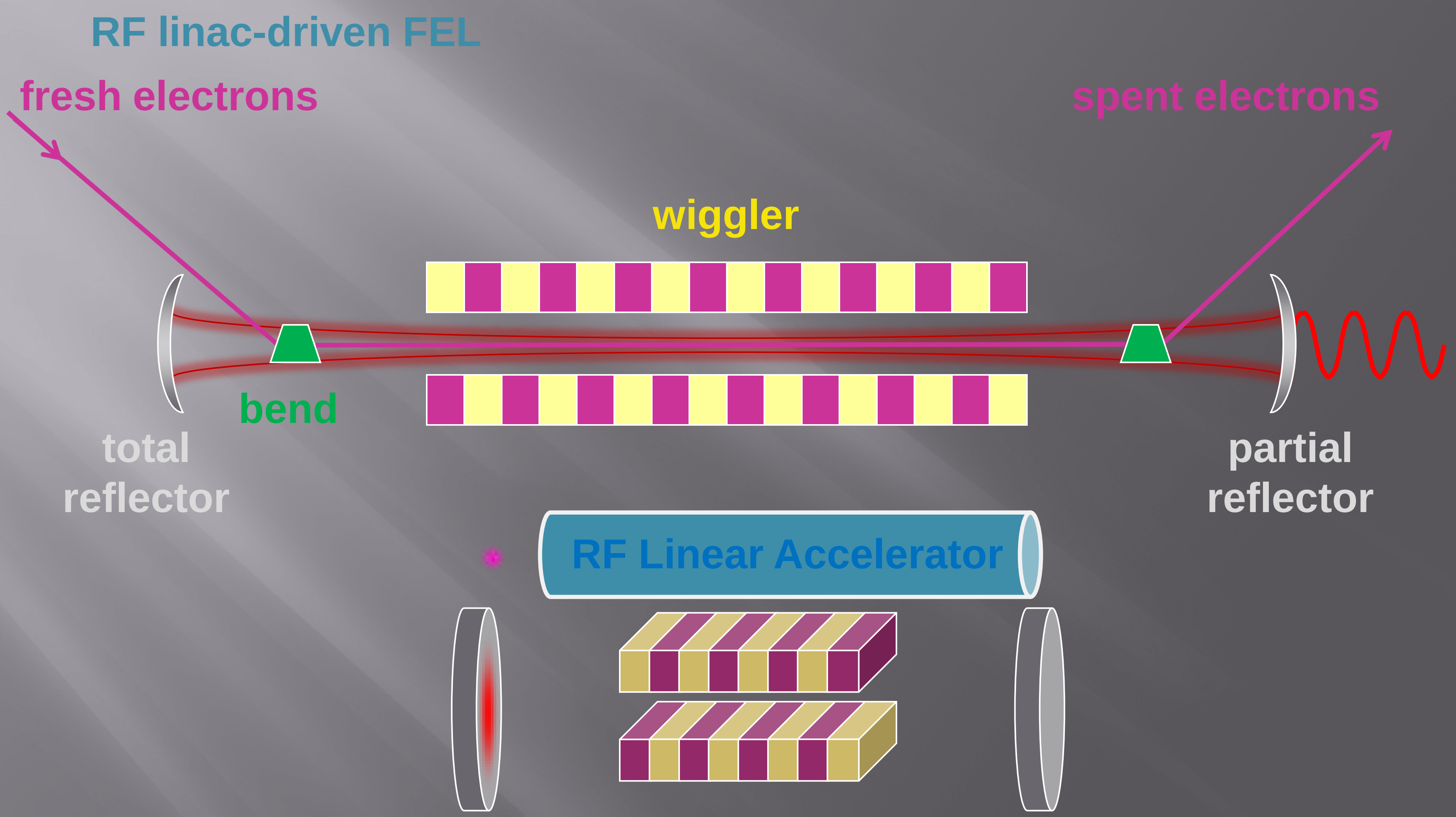
The FEL gain medium is **relativistic electron beams** traveling in a vacuum through the alternating magnetic field of a wiggler.

The interaction between the electrons' transverse motion with the optical electric field causes some electrons to gain energy and other to lose energy. The **energy modulation** causes electrons bunch up along the longitudinal axis with a period equal to the radiation wavelength.
The longitudinally bunched electron beam emit coherent radiation with intensity much higher than the input optical signal, thus providing gain.

FEL wavelength dictates the required electron beam's **transverse emittance**; the FEL interaction length dictates the required electron beam's **longitudinal emittance** .

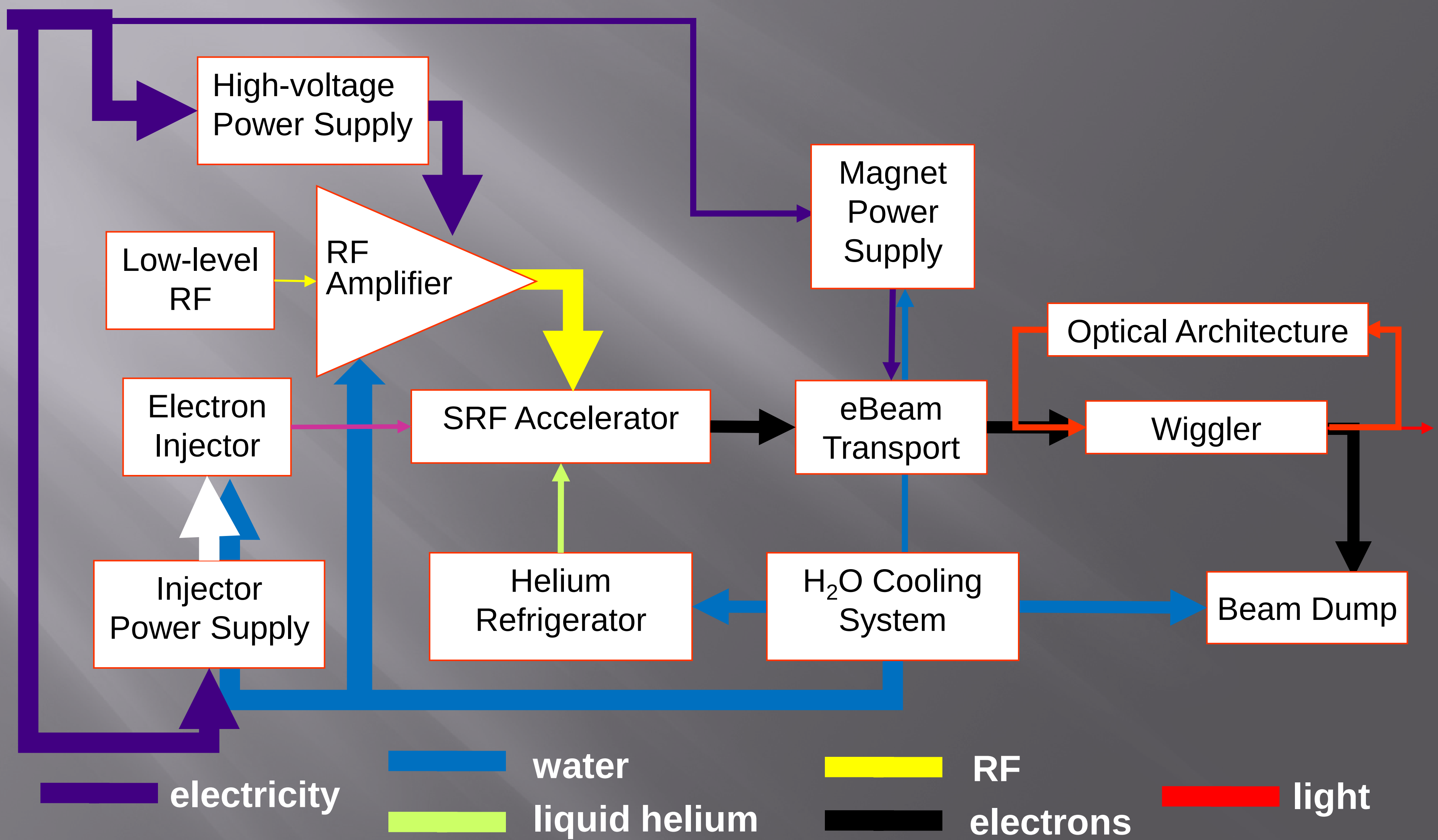
Part 2
RF Linac FEL

How an RF-linac FEL works

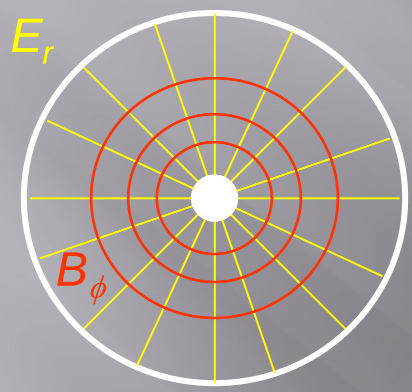


Courtesy of John Lewellen

RF-linac Driven FEL Sub-systems

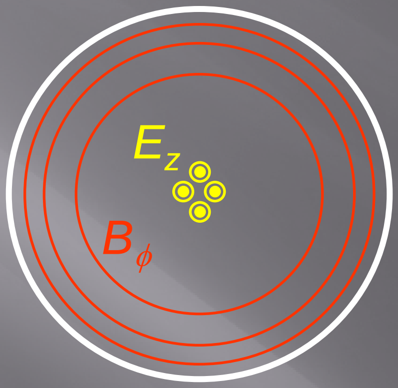


RF Waveguide Modes



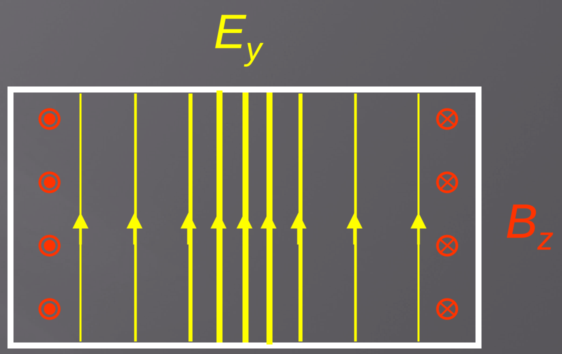
TEM
Transverse electric, magnetic

TEM modes in coaxial transmission lines; used for power transmission



TM
Transverse magnetic

TM modes in cylindrical waveguides; used for **acceleration**



TE
Transverse electric

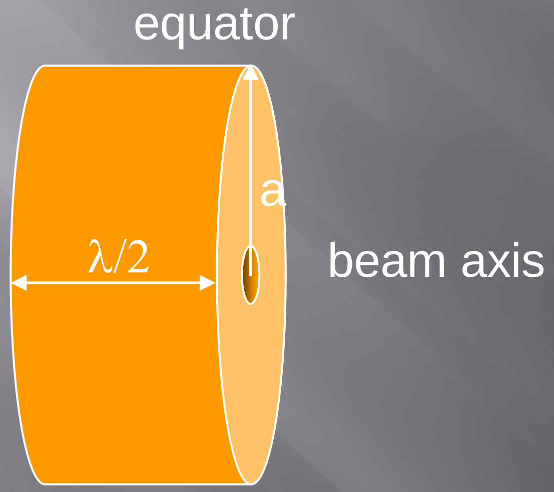
TE modes in rectangular waveguides; used for high-power transmission

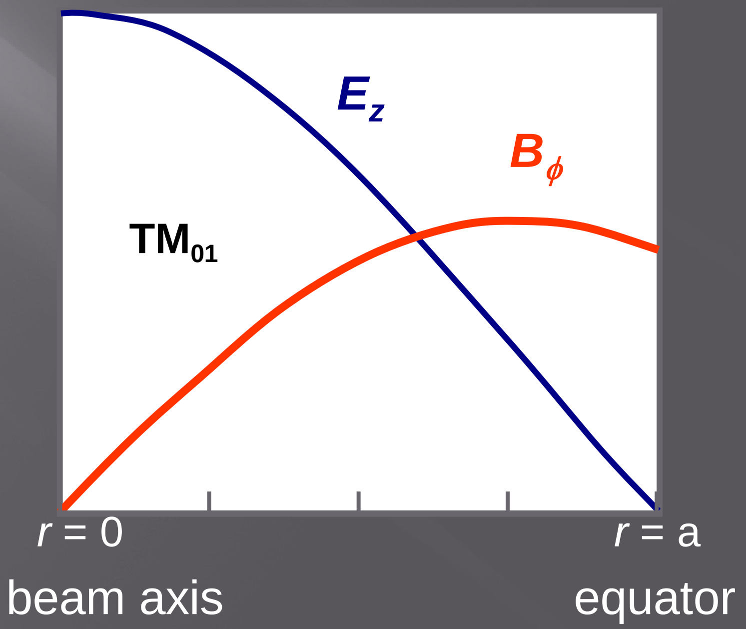
How an RF accelerator works

Electric field accelerates electrons and increases their energy.

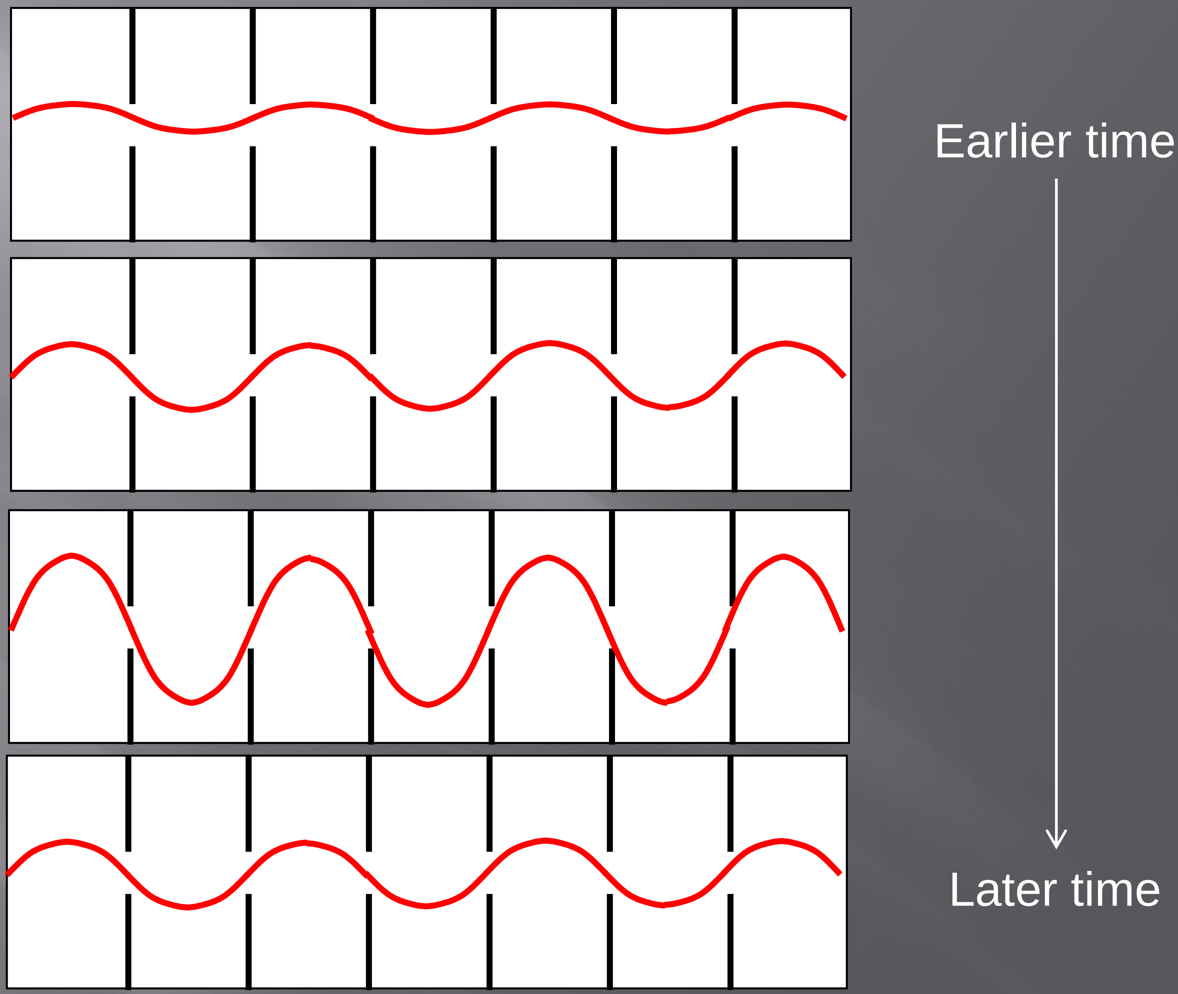
$$\Delta W = \int \mathbf{F} \cdot d\mathbf{s} = -\int e\mathbf{E} \cdot d\mathbf{s}$$

RF cavities store electromagnetic energy to produce high electric fields. Electric fields are maximum on axis and magnetic fields are maximum near the equator.



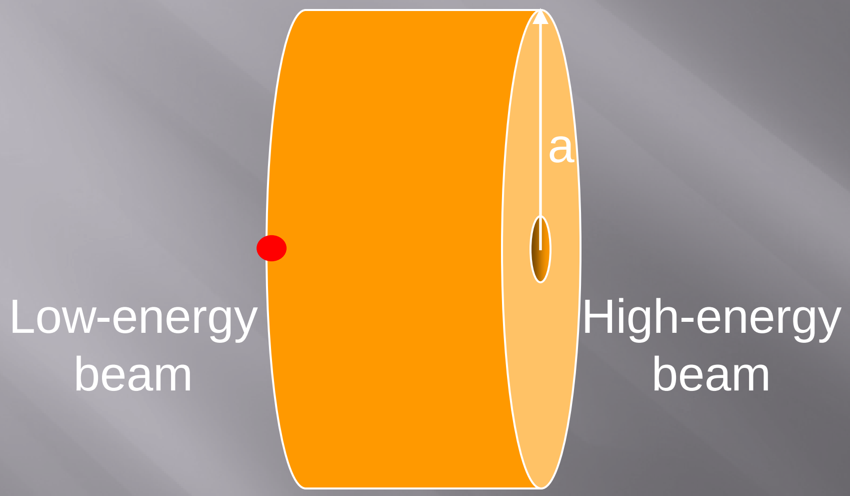


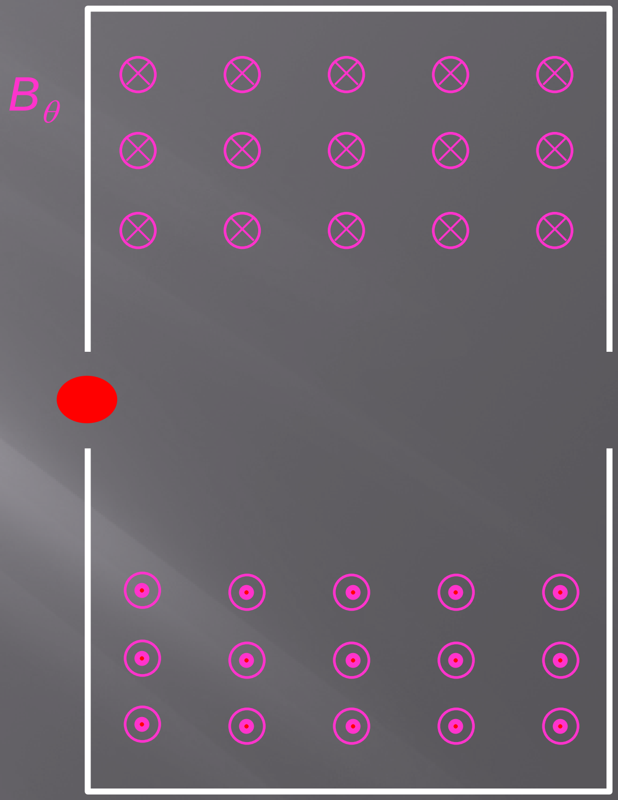
Standing-wave π-mode Cavity



Standing waves exist in multi-cell cavity if each cell is one-half wavelength long.

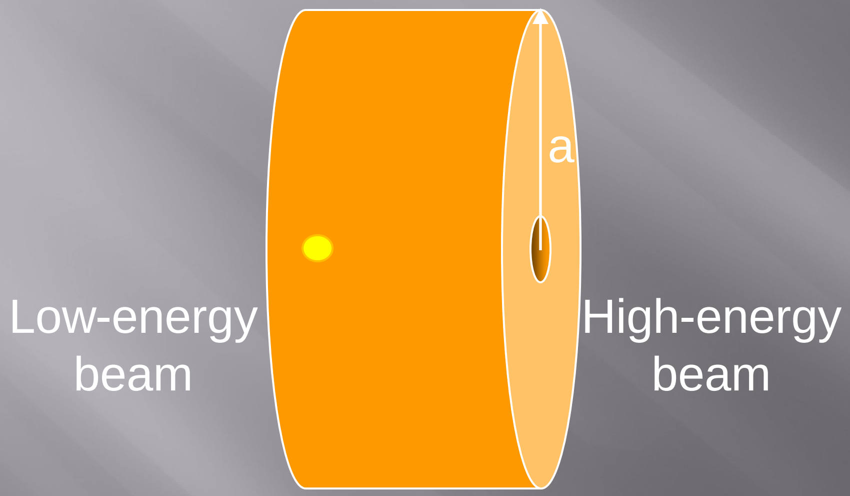
Single Pillbox Cavity

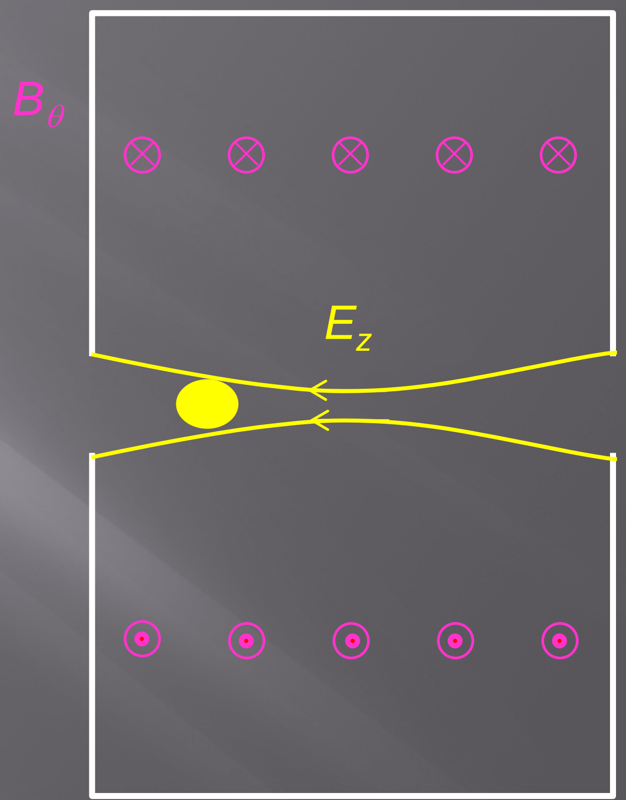




The color of the electron bunch denotes its energy (red = low; blue = high)

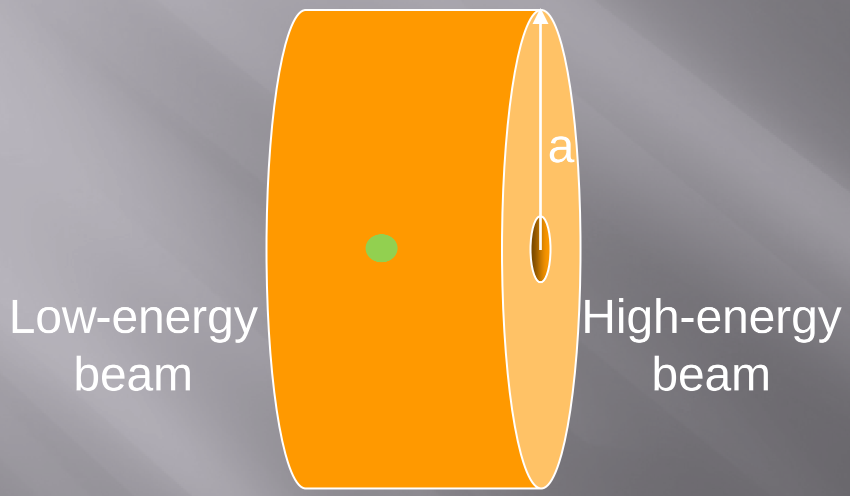
Single Pillbox Cavity

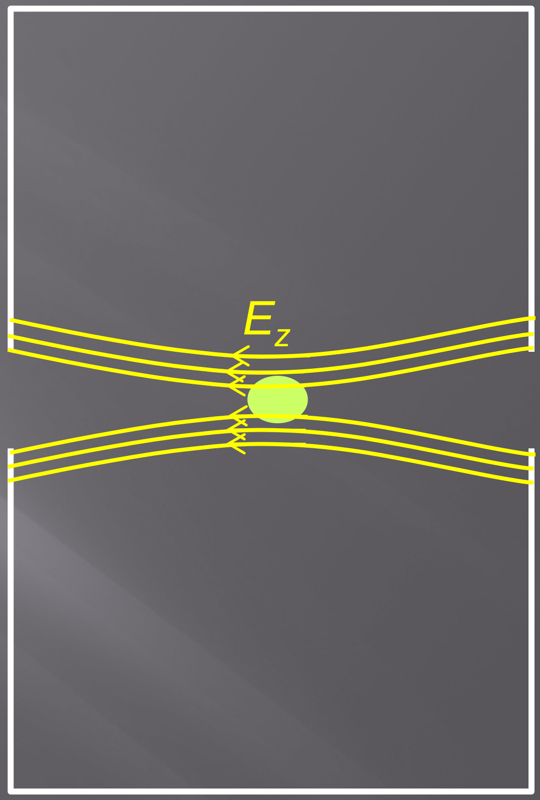




The color of the electron bunch denotes its energy (red = low; blue = high)

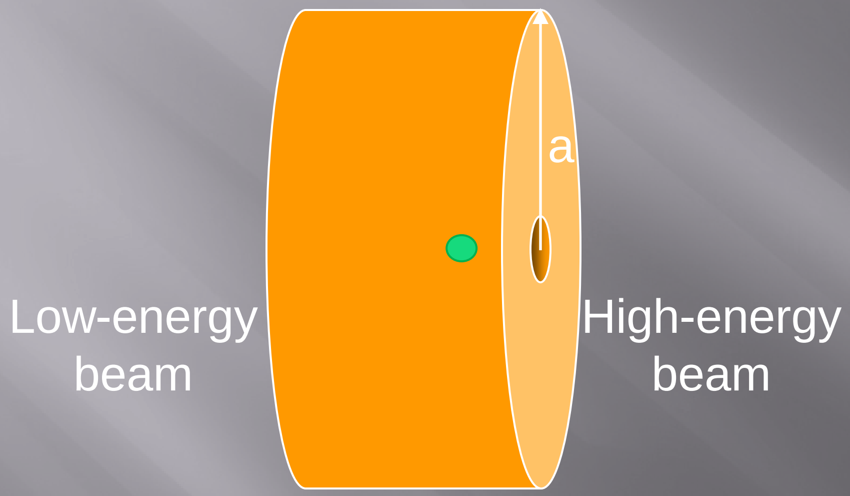
Single Pillbox Cavity

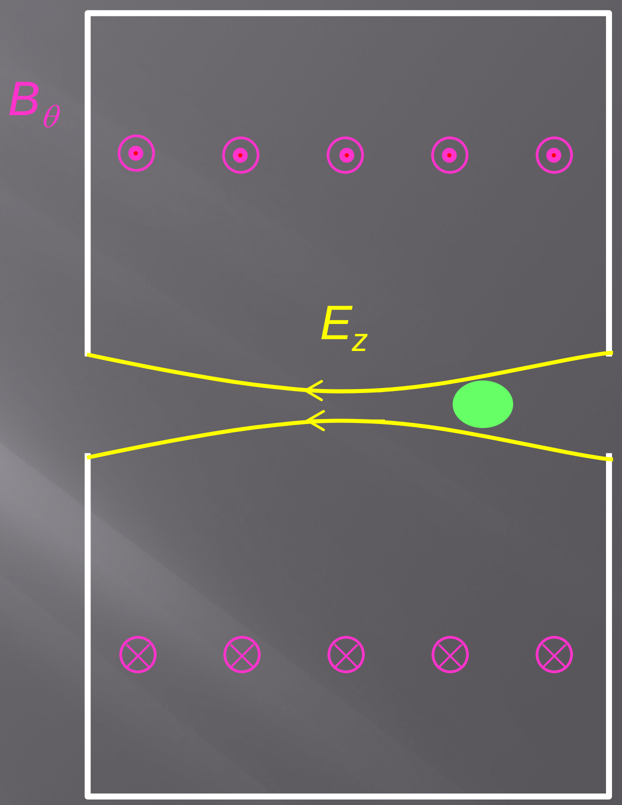




The color of the electron bunch denotes its energy (red = low; blue = high)

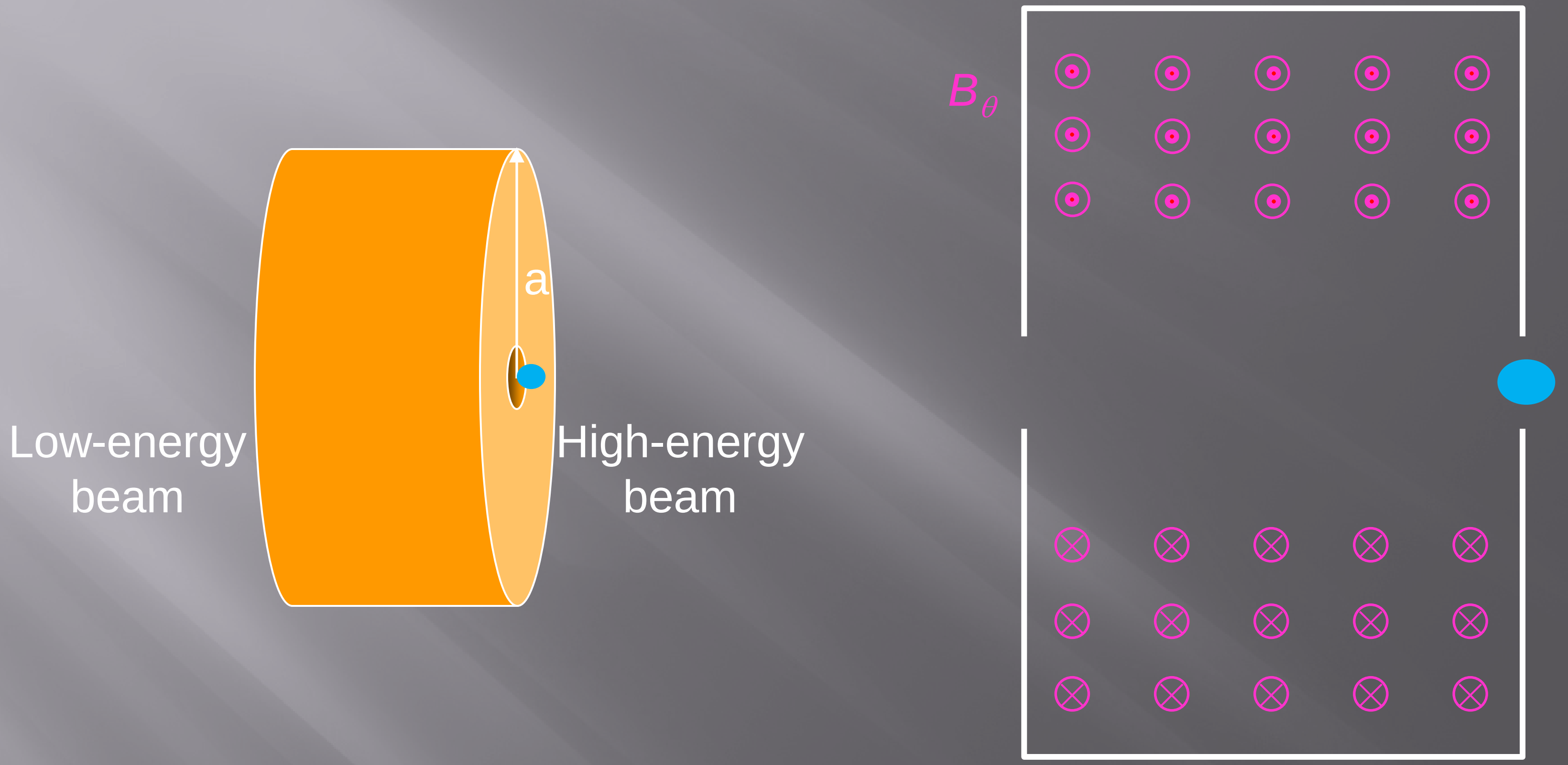
Single Pillbox Cavity





The color of the electron bunch denotes its energy (red = low; blue = high)

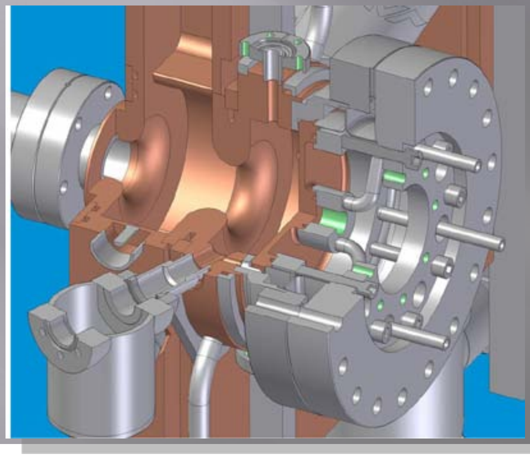
Single Pillbox Cavity



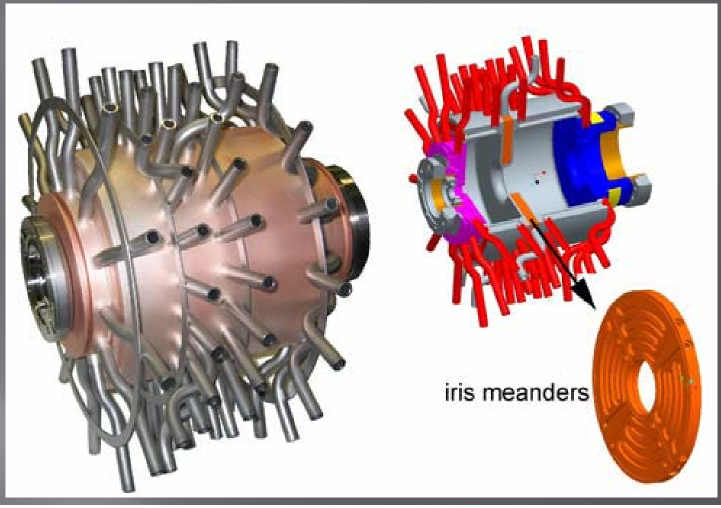
The color of the electron bunch denotes its energy (red = low; blue = high)

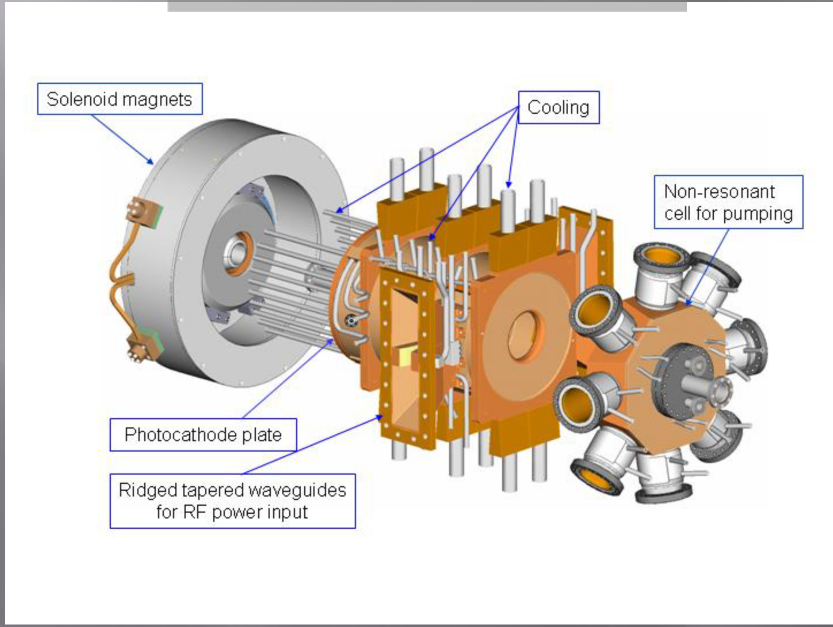
Normal-Conducting RF Guns

LCLS 3GHz NCRF Gun

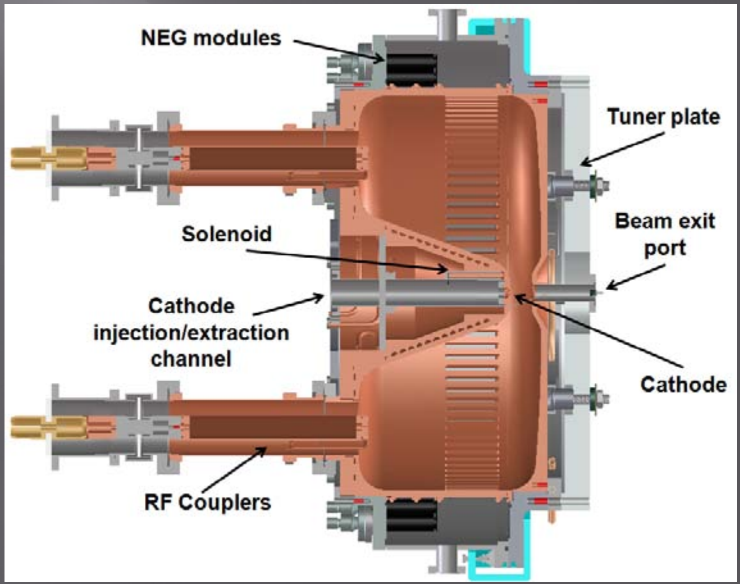


PITZ 1.3GHz NCRF Gun



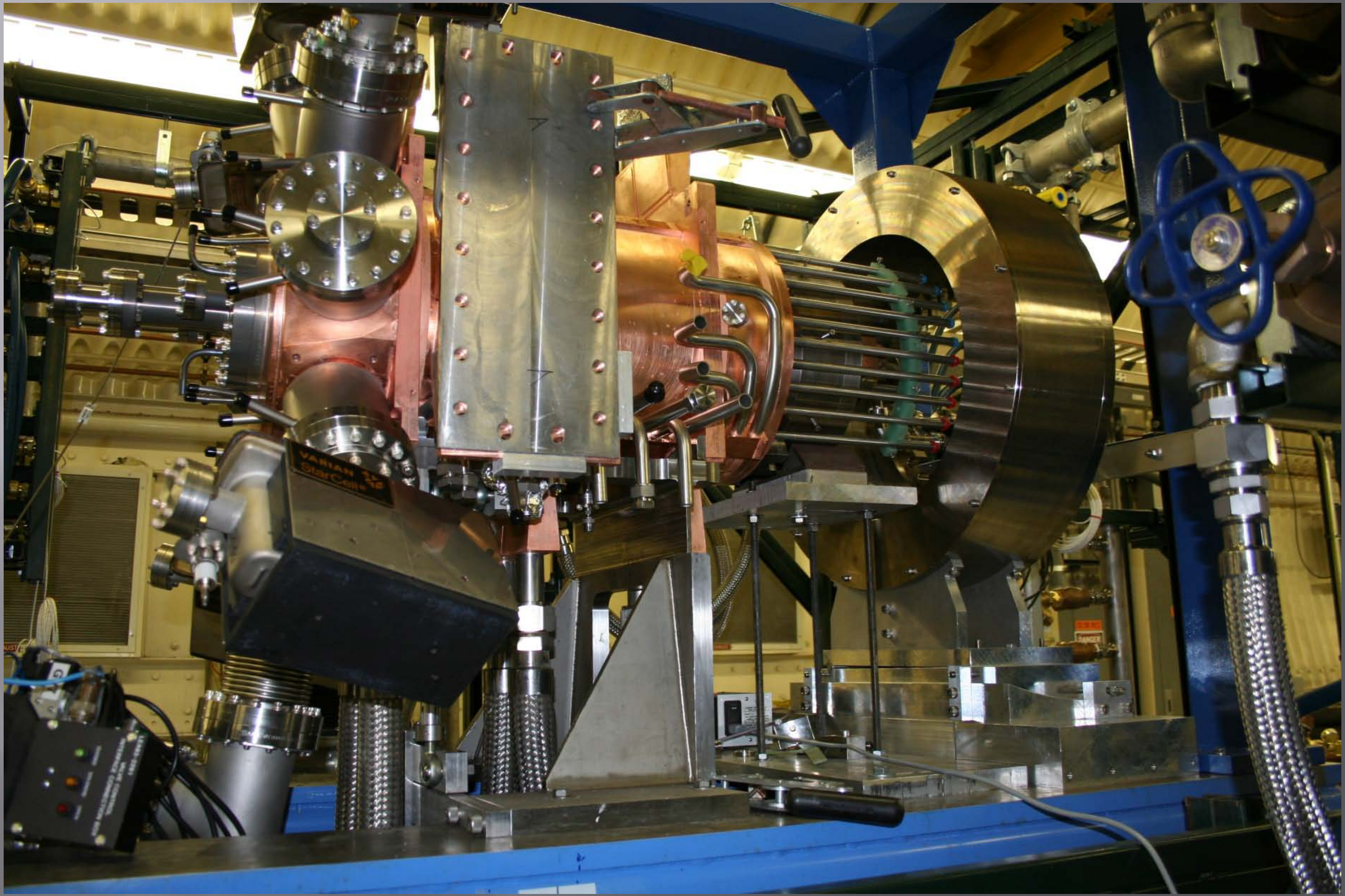


LANL 700MHz NCRF Gun



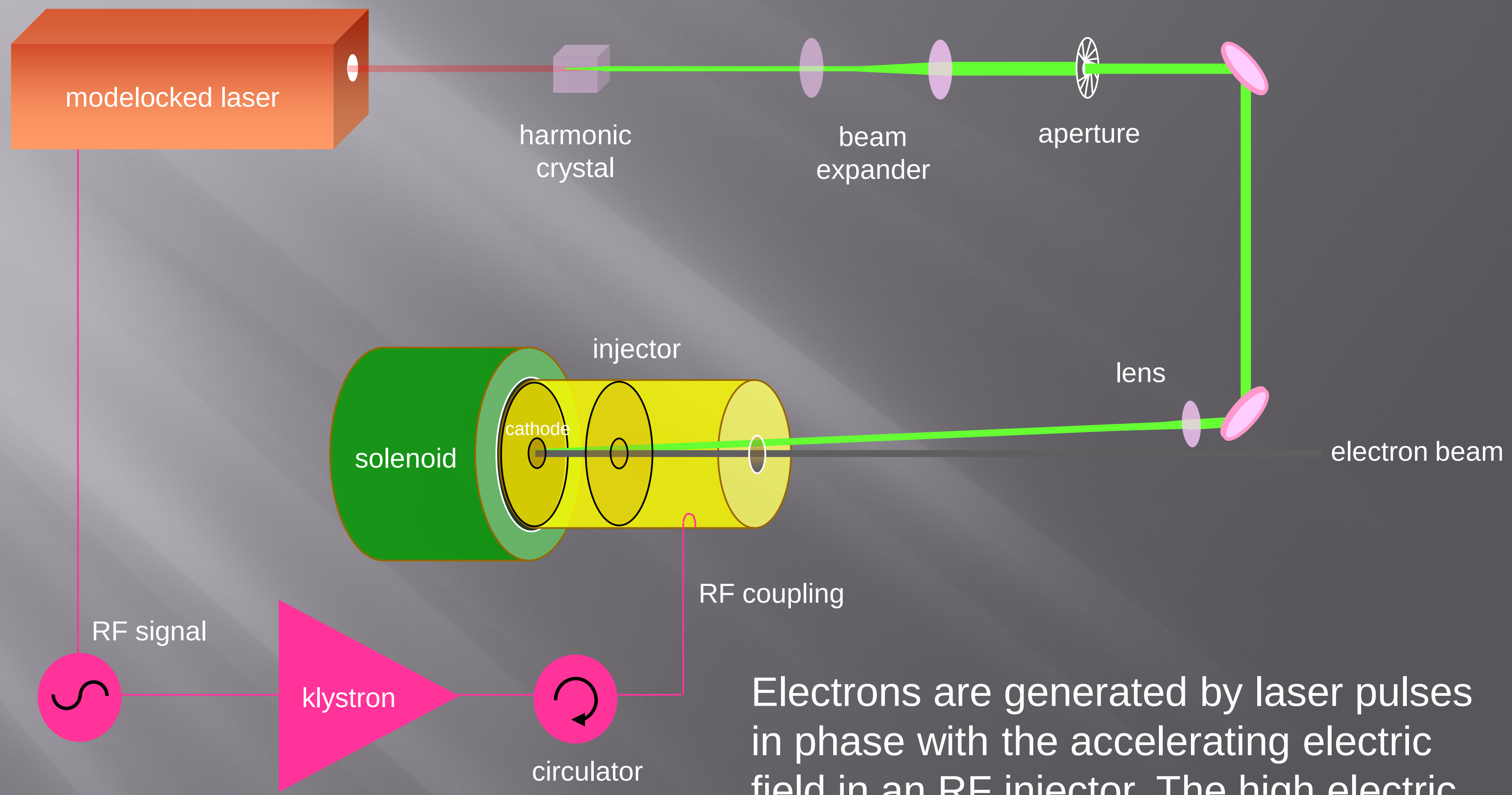
LBNL 187MHz NCRF Gun

Electron Injectors



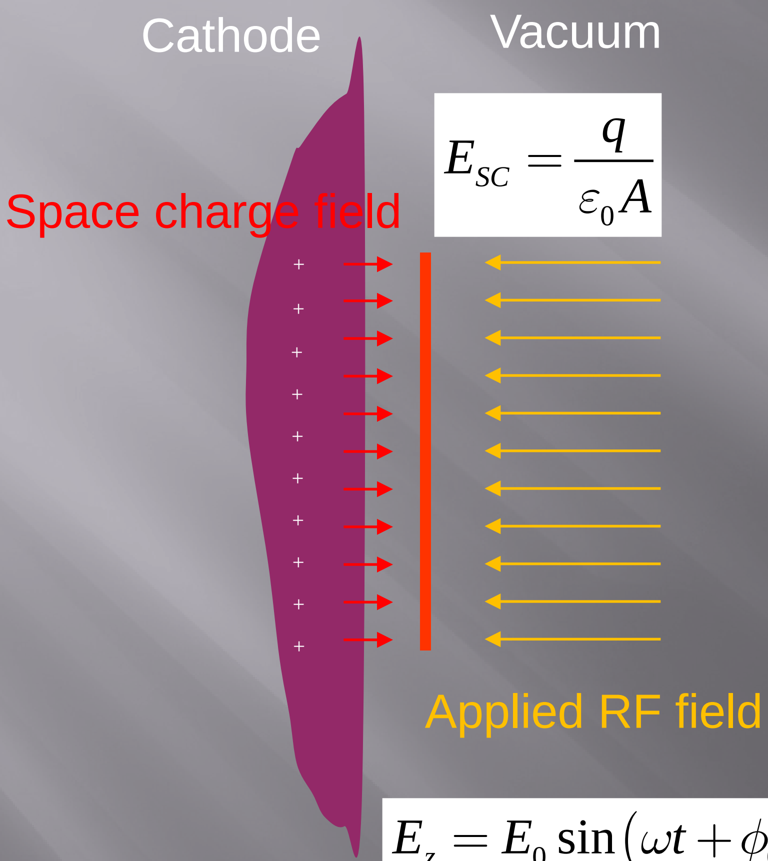
LANL 700MHz NCRF Gun

RF Gun Schematic



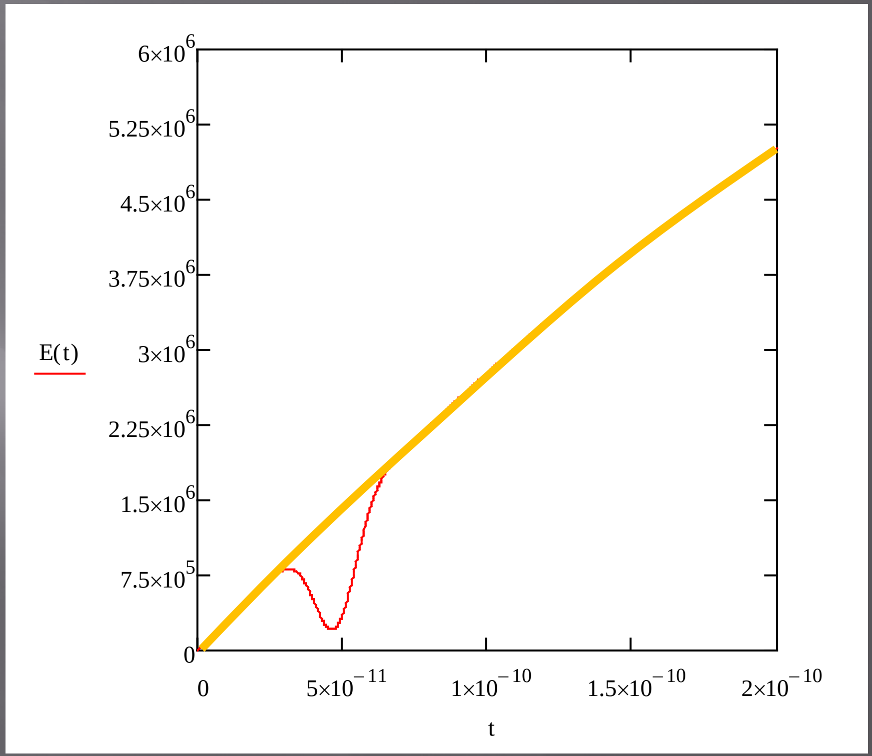
Electrons are generated by laser pulses in phase with the accelerating electric field in an RF injector. The high electric fields produce electrons with relativistic energy at the injector exit.

Space Charge Limited Emission



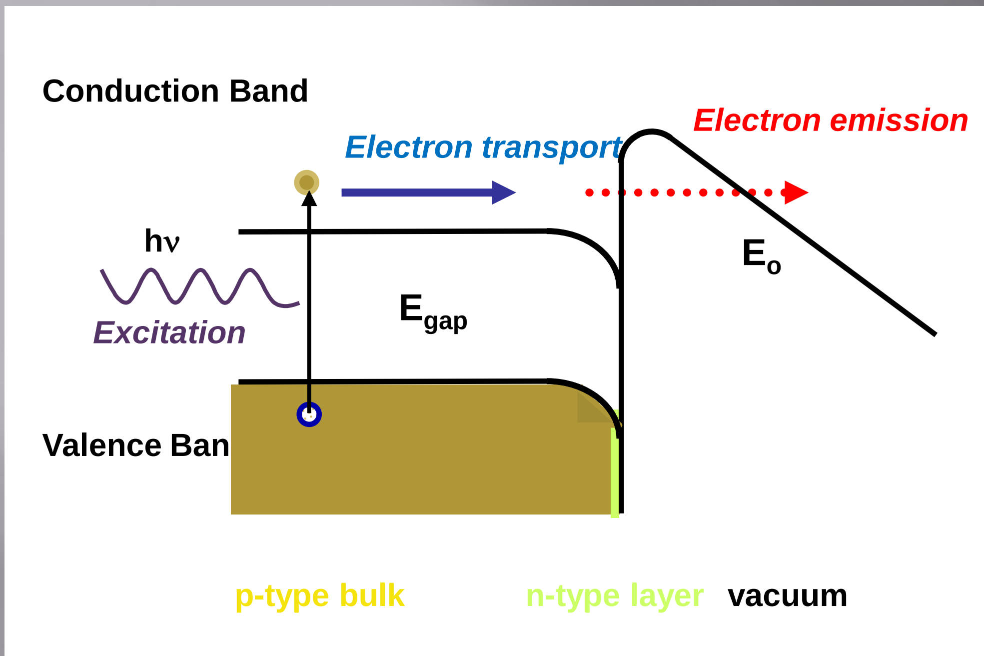
$$E_{SC} = \frac{q}{\varepsilon_0 A}$$

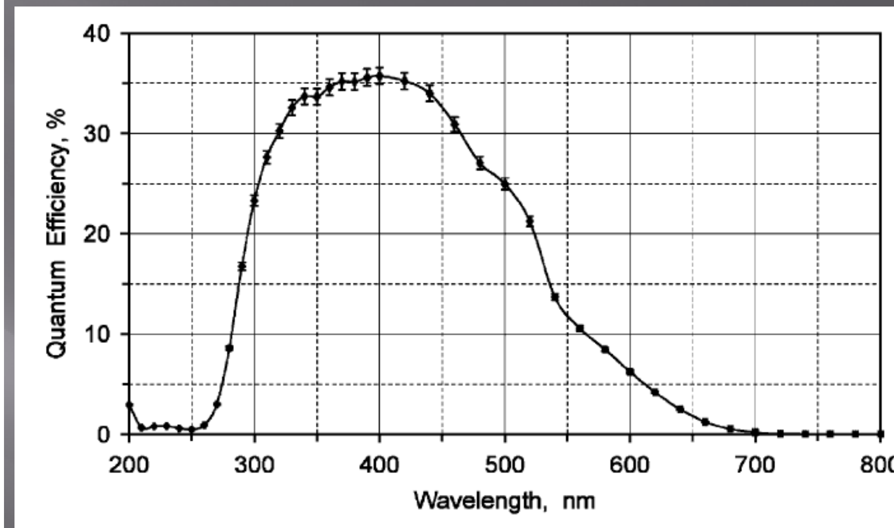
$$E_z = E_0 \sin(\omega t + \phi_0)$$



Applied RF field must exceed space charge field to avoid shutting off emission of photoelectrons during the laser pulse.

Electron Photoemission from Semiconductor Photocathodes

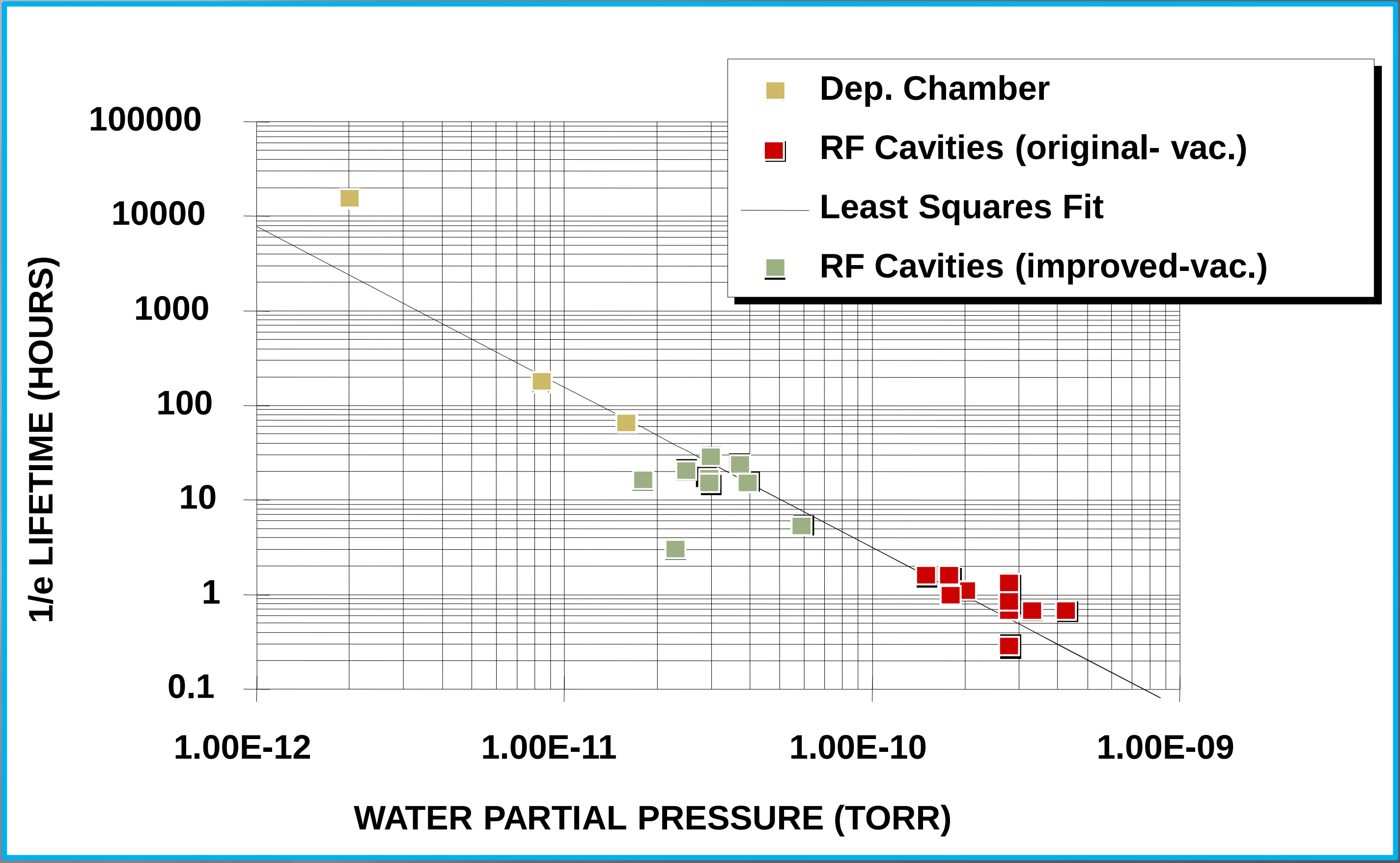




Quantum efficiency of CsK_2Sb

Photoelectron emission from a semiconductor photocathode is a three-step process, involving first **excitation** of electrons from valence band to conduction band, followed by **electron transport** from the bulk to the cathode-vacuum boundary, and finally **electron emission** via tunneling through a potential barrier.

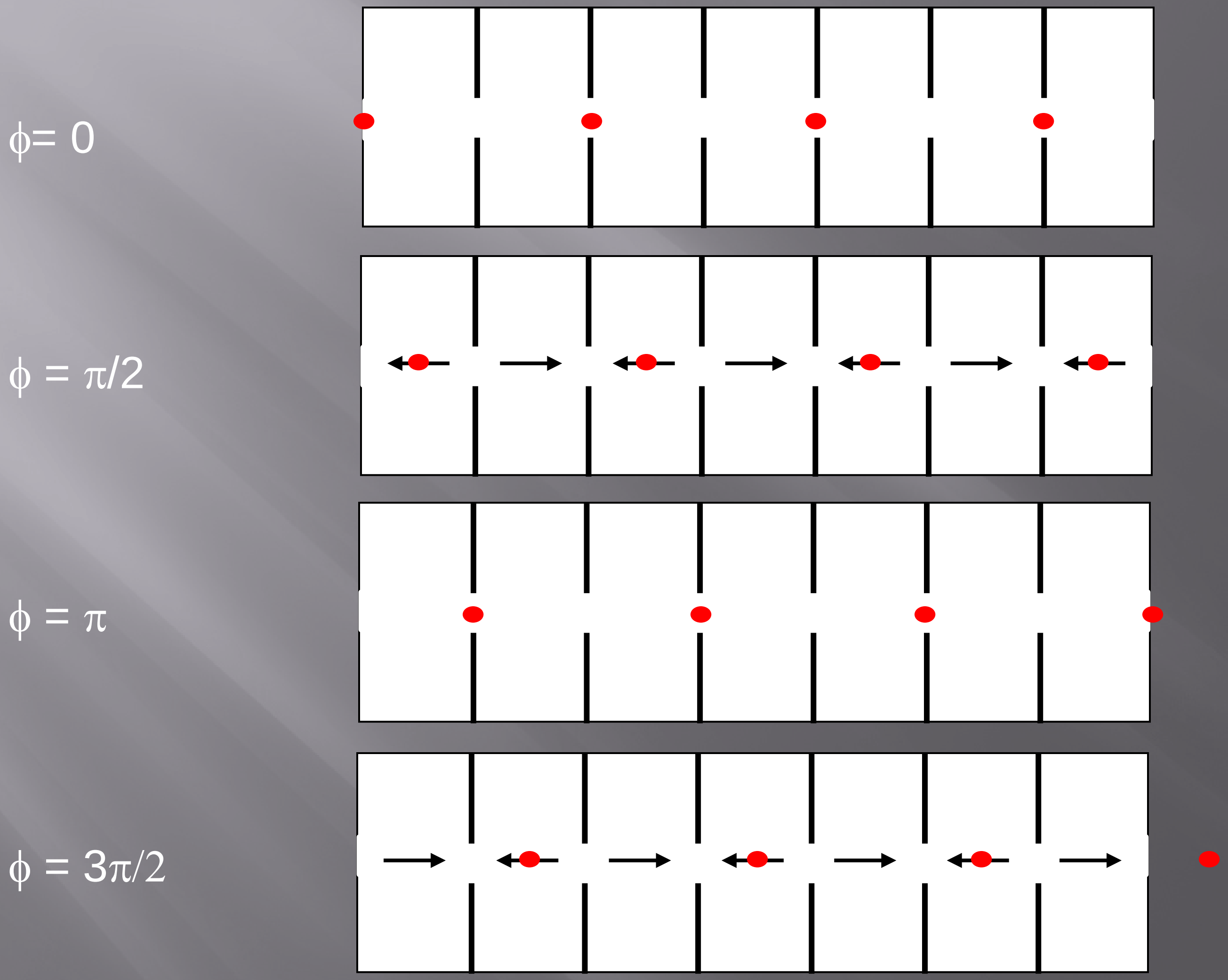
Photocathode Lifetime



Cesiated cathode lifetime depends strongly on partial pressures of water and other oxygen-containing species in the RF cavity.

Courtesy of Dave Dowell

Multi-cell Acceleration

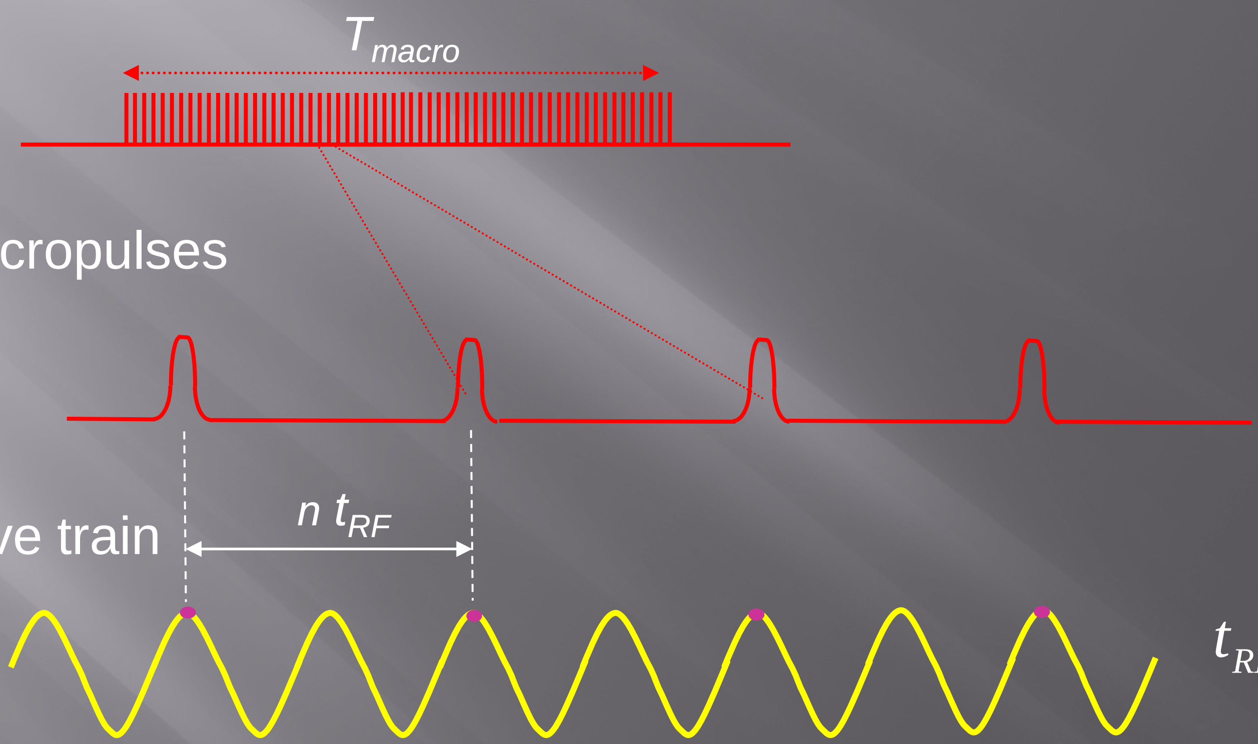


RF-linac FEL Pulse Format

FEL macropulses

FEL micropulses

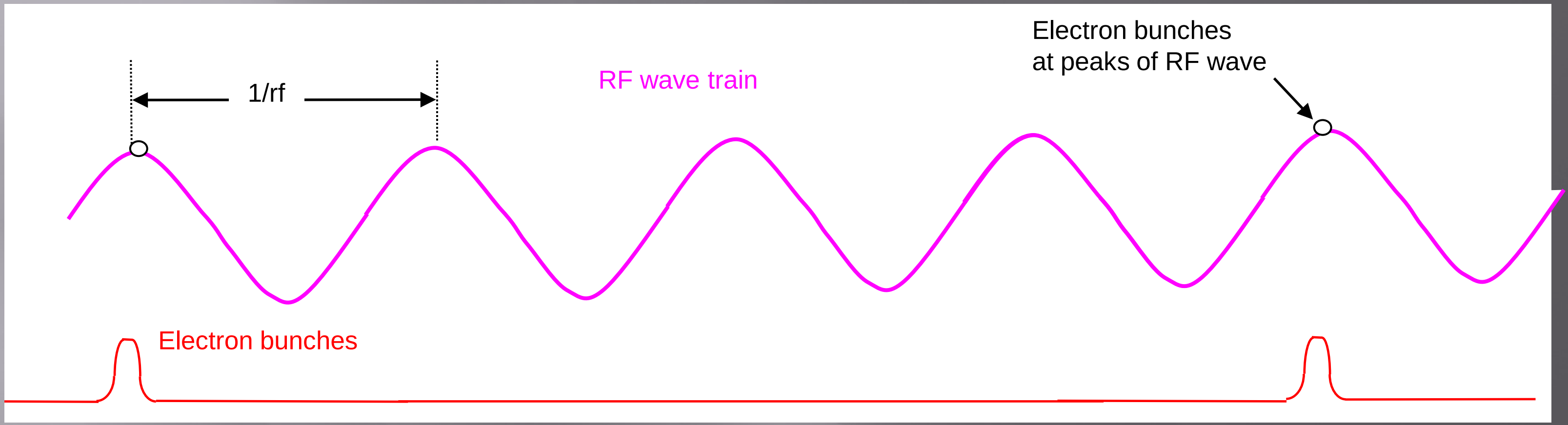
RF wave train



$$t_{RF} = \frac{1}{f_{RF}}$$

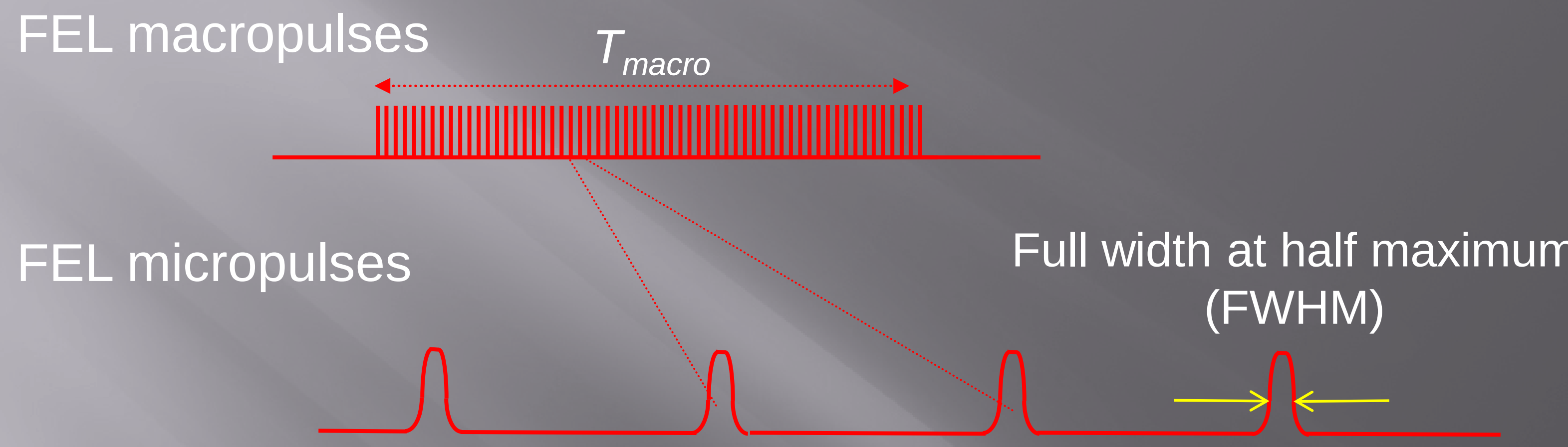
FEL micropulses are typically a few picoseconds long and separated in time by a few nanoseconds. Macropulses range from μs to seconds.

Typical Parameters of RF Linac



Typical RF frequency	400 MHz
Electron charge	1 nC
Bunch repetition rate	100 MHz
Electron average current	0.1 A
Electron beam energy	100 MeV
Electron beam's average power	10 MW
FEL extraction efficiency	1%
FEL average power	100 kW

Peak, Macropulse and Average Power



Peak power is approximately the micropulse energy divided by FWHM

Example: Micropulse energy = 1 mJ

FWHM = 1 ps P_{peak} = 1 GW

Macropulse power is macropulse energy divided by T_{macro}

Example: Macropulse energy = 1 J

T_{macro} = 10 μs P_{macro} = 100 kW

For continuous-wave FEL, average power is P_{macro}

Summary of Part 2

The main function of the accelerators is to accelerate electrons to **high energy** so FEL lasing can occur at the correct wavelength.

The main function of the electron guns is to produce a **high-quality electron beam** with suitable properties for FEL gain.

Most FEL use **RF linac** and **RF electron guns** that produce ps pulses of high-current, low-emittance electrons.

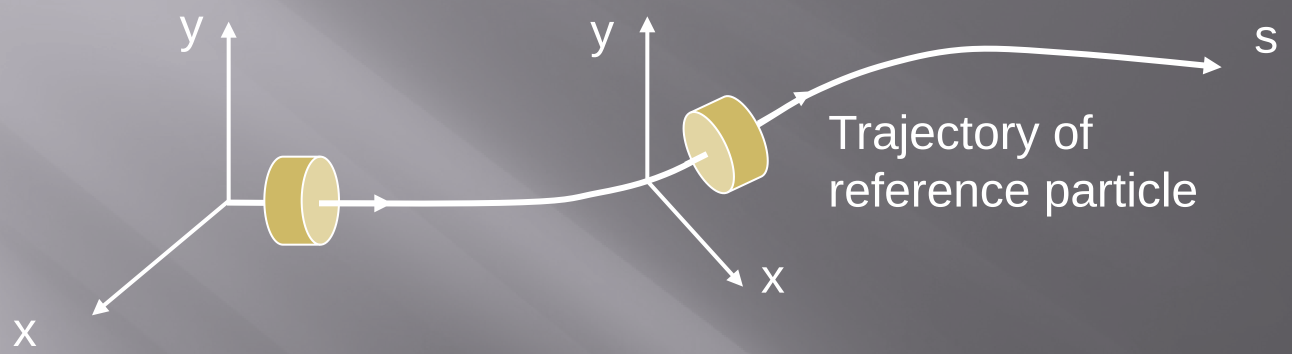
Typical radio-frequency ranges from a few MHz to a few GHz

The temporal pulse format of an RF-linac FEL consists of **micropulses** and **macropulses**.

The RF-linac FEL's **peak power** is 3-4 orders of magnitude higher than its **average power**.

Part 3
Electron Beam Transport

Curvilinear Coordinate



Electrons travel in the s direction. Use (x, y, s) coordinate system to follow the reference electron, an ideal particle at the beam center with a curvilinear trajectory. The reference particle trajectory takes into account only pure dipole fields along the beam line. The x and y of the reference trajectory are thus affected only by the placement and strength of the dipole magnets.

For other electrons, define x' and y' as the slopes of x and y with respect to s

$$x' = \frac{dx}{ds} \qquad y' = \frac{dy}{ds}$$

Lorentz Force

$$\mathbf{F} = -e\left[\mathbf{E} + \left(\mathbf{v} \times \mathbf{B}\right)\right]$$

In MKS units, e = 1.6 x 10^{-19} coulomb, electric field is in volts/m and magnetic field is in tesla.

Electric force acts on electrons along their direction of motion and thus changes the electrons' kinetic energy.

$$\Delta T = \int \mathbf{F} \cdot d\mathbf{s} = -\int e\mathbf{E} \cdot d\mathbf{s}$$

Magnetic force is perpendicular to direction of motion and does not change the electrons' kinetic energy. Magnetic field can be used to change momentum, i.e. bend electron beams.

$$\Delta\vec{p} = \int \mathbf{F}\,dt = -e\int \left(\vec{\upsilon} \times \mathbf{B}\right) dt$$

Dipoles bend electron beams

Bend angle and radius

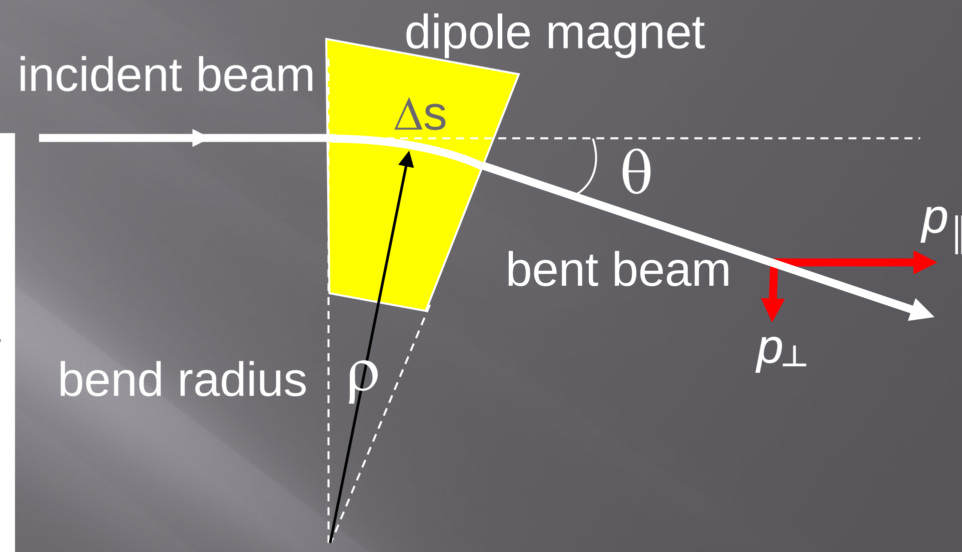
$$\tan\theta = \frac{p_\perp}{p_\parallel}$$

$$p_\perp = F\Delta t = -e\upsilon B\Delta t = -eB\Delta s$$

$$p_\parallel = \beta\gamma m_0 c = \frac{E_b}{c}$$

$$\tan\theta = \frac{\Delta s}{\rho} = \frac{-ecB\Delta s}{E_b}$$

$$\frac{1}{\rho} = \left|\frac{ecB}{E_b}\right|$$

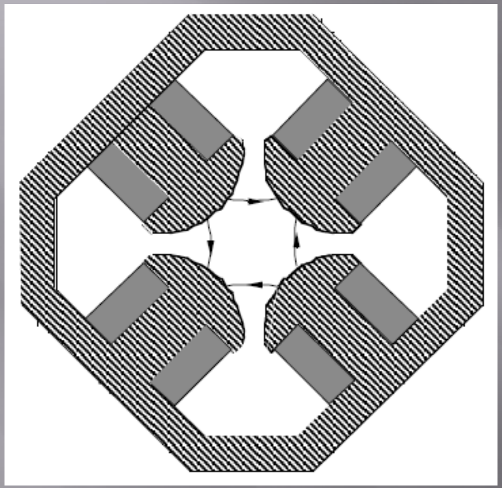


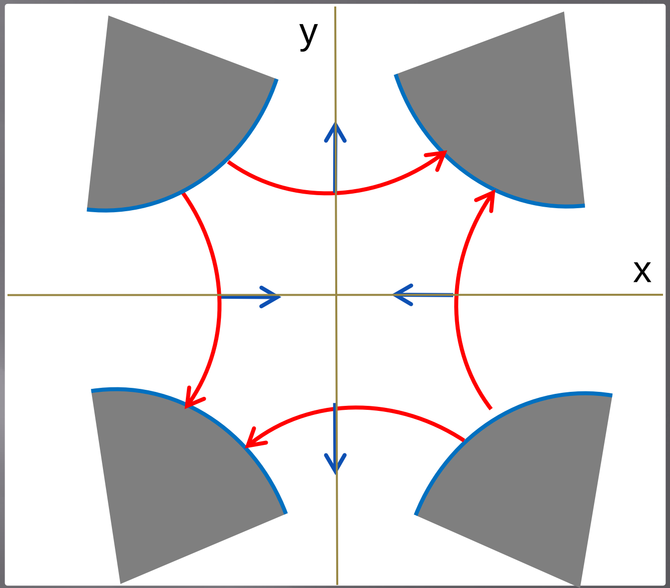
$$\frac{1}{\rho}\left(m^{-1}\right) = 299.8\frac{B(T)}{E_b\left(MeV\right)}$$

Magnetic rigidity

$$B\rho\left(T-m\right) = \frac{1}{299.8}E_b\left(MeV\right)$$

Quadrupole Magnets





Quadrupoles are used to focus and contain electron beams

Quad magnetic fields are hyperbolic

Magnetic fields get stronger as one goes away from center

$$\vec{B}_{quad} = \begin{pmatrix} -yG_0 \\ xG_0 \\ 0 \end{pmatrix}$$

Quadrupoles focus and defocus electron beams

Lorentz force

$$\frac{d\vec{p}}{dt} = -e\left(\vec{\upsilon} \times \vec{\mathbf{B}}\right)$$

Transverse momentum

$$p_x = \gamma m_0 \upsilon_x = \gamma m_0 \upsilon_z x'$$

Constant energy in B-field $\rightarrow$ relativistic γ is constant

Use chain rule: $d/dt = v_z\, d/dz$

$$\gamma m_0 \upsilon_z \frac{dx'}{dt} = \gamma m_0 \upsilon_z^2 x'' = -e\upsilon_z B_y = -e\upsilon_z G_0 x$$

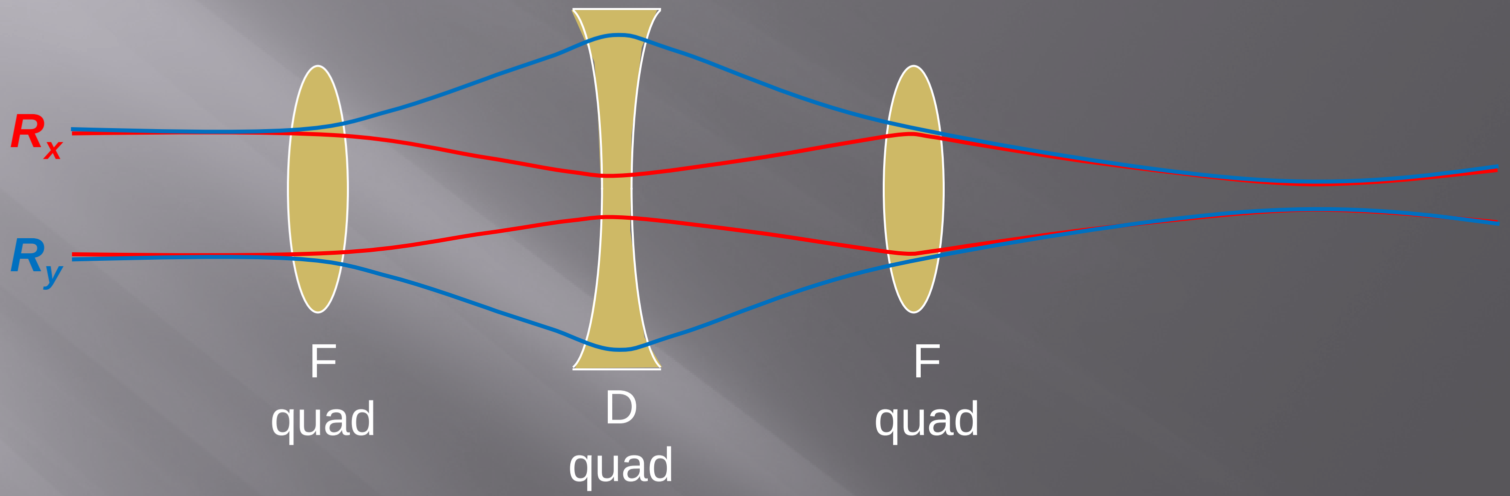
Rearrange to get Hill's equation for x

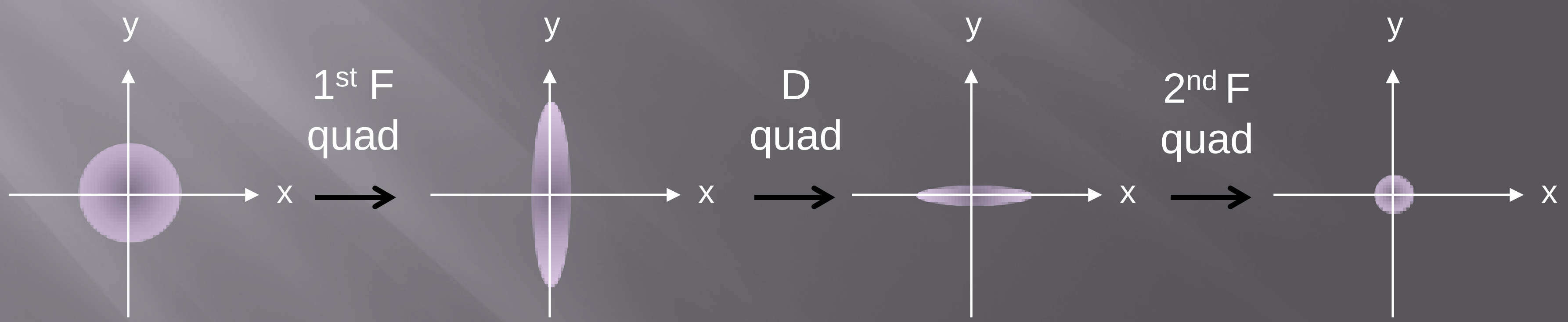
$$x'' + \left(\frac{eG_0}{p_z}\right) x = 0$$

and similarly for y

$$y'' - \left(\frac{eG_0}{p_z}\right) y = 0$$

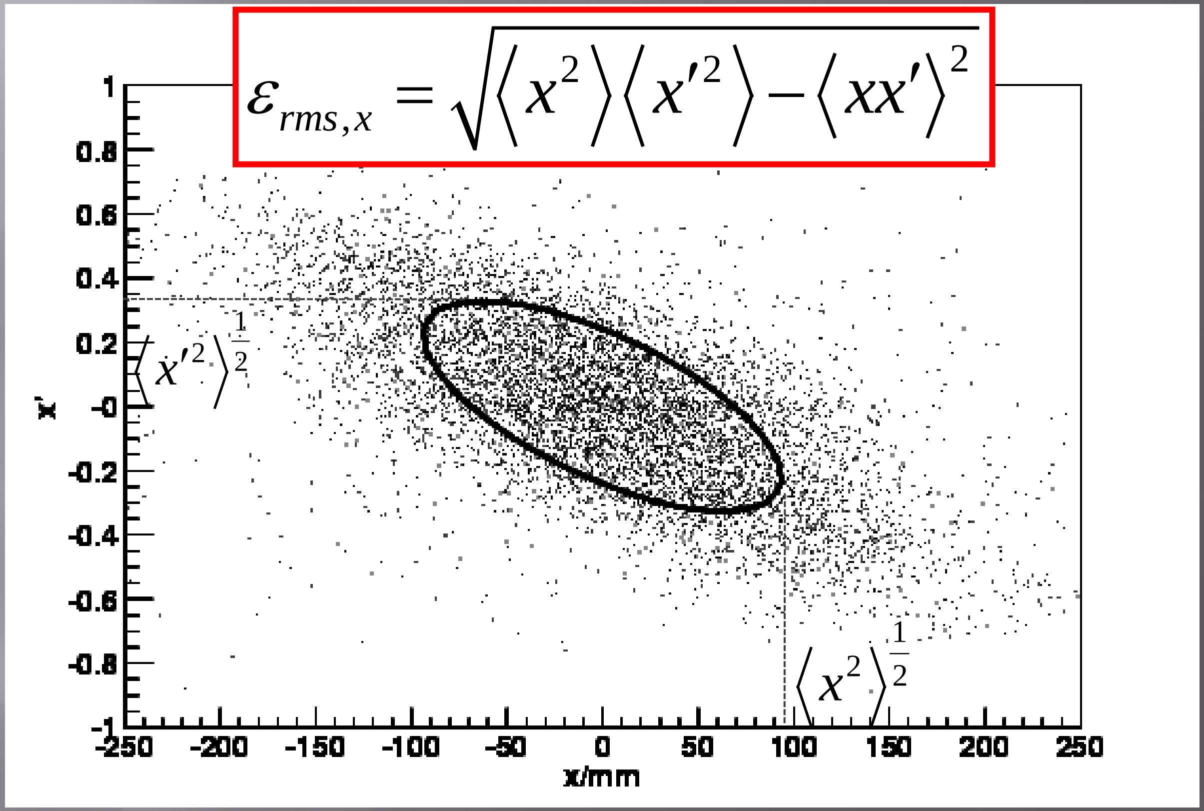
Triplets focus electron beams in both x and y directions





Evolution of electron beam envelope through a quadrupole triplet

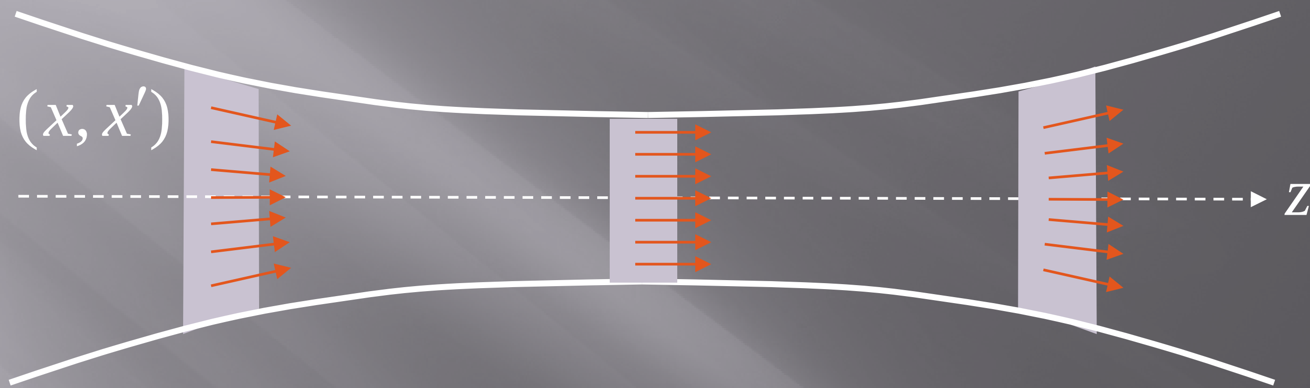
Phase Space Ellipse & Emittance

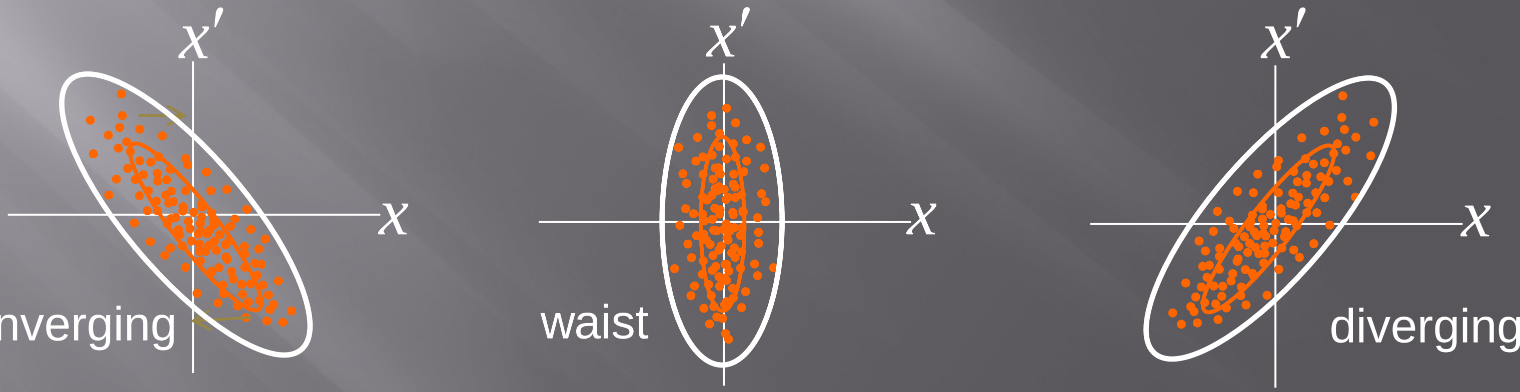


Beams are treated as a statistical distribution of particles in *x'-x* (also in *y'-y* and γ-*ct*) phase space (particles on x'-x plot). We can draw an ellipse around the particles such that 50% of the particles are found within the ellipse. The area of this ellipse is π times the beam emittance.

Beam Envelope

Consider x and x’ phase space in a drift after a quadrupole lens





Electrons with x’ > 0 drift to the right; those with x’ < 0 drift left

Electron Beam Radius

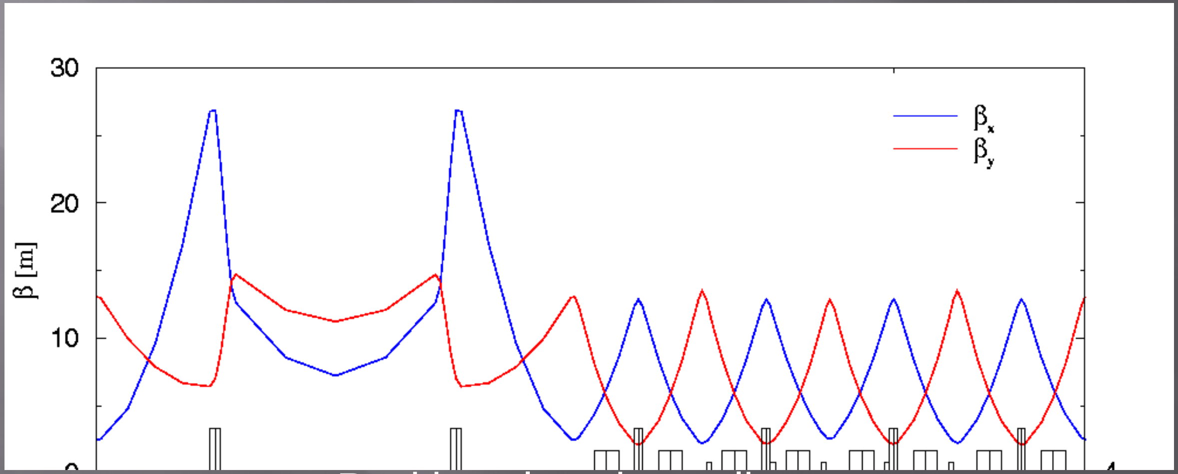
rms electron beam's radius
at the waist

$$\sigma_{x0} = \sqrt{\beta_x \varepsilon_x}$$

β function
Property of the lattice

Emittance (un-normalized)
Property of the beam

The beta function is used to describe the beam's size (square root of beta) and to identify the maximum amplitude of beam's envelope; beta also tells us where the beam is most sensitive to perturbations.



Position along beam line

Beam Transfer Matrix

Dispersion

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ c\tau \\ \delta \end{bmatrix}_{out} = \begin{pmatrix} M_{11} & M_{12}\ldots\ldots & M_{16} \\ M_{21} & M_{22}\ldots\ldots & M_{26} \\ M_{31} & M_{32}\ldots\ldots & M_{36} \\ M_{41} & M_{42}\ldots\ldots & M_{46} \\ M_{51} & M_{52}\ldots\ldots & M_{56} \\ M_{61} & M_{62}\ldots\ldots & M_{66} \end{pmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \\ c\tau \\ \delta \end{bmatrix}_{in}$$

$$\tau = t - t_0$$

$$\delta = \frac{E - E_0}{E_0}$$

Matrix element that converts energy spread into time within the bunch – compresses bunches with energy chirp

Transfer matrix is a 6x6 matrix that performs linear transformation of a beam vector at the input location to another beam vector at the output location.

Thin-lens 2x2 Transfer Matrices for Quadrupole & Drift

Transfer matrix for focusing quad

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{in}$$

Transfer matrix for a drift with length L

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{out} = \begin{pmatrix} 1 & L \\ 1 & 1 \end{pmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{in}$$

Transfer matrix for defocusing quad

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ \dfrac{1}{f} & 1 \end{pmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{in}$$

Transfer Matrices of a Quadrupole Triplet

Combining matrices in the order: elements closest to the input beam on the right hand side (next to input vector) and work toward the left.

Transfer matrix in x plane

$$A_x = \begin{pmatrix} 1 & 1 \\ -\frac{1}{f_5} & 1 \end{pmatrix} \begin{pmatrix} 1 & l_4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1_3 \\ \frac{1}{f_3} & 1 \end{pmatrix} \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

Transfer matrix in y plane

$$A_x = \begin{pmatrix} 1 & 1 \\ \frac{1}{f_5} & 1 \end{pmatrix} \begin{pmatrix} 1 & l_4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1_3 \\ -\frac{1}{f_3} & 1 \end{pmatrix} \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{f_1} & 1 \end{pmatrix}$$

5	4	3	2	1
F quad	drift	D quad	drift	F quad

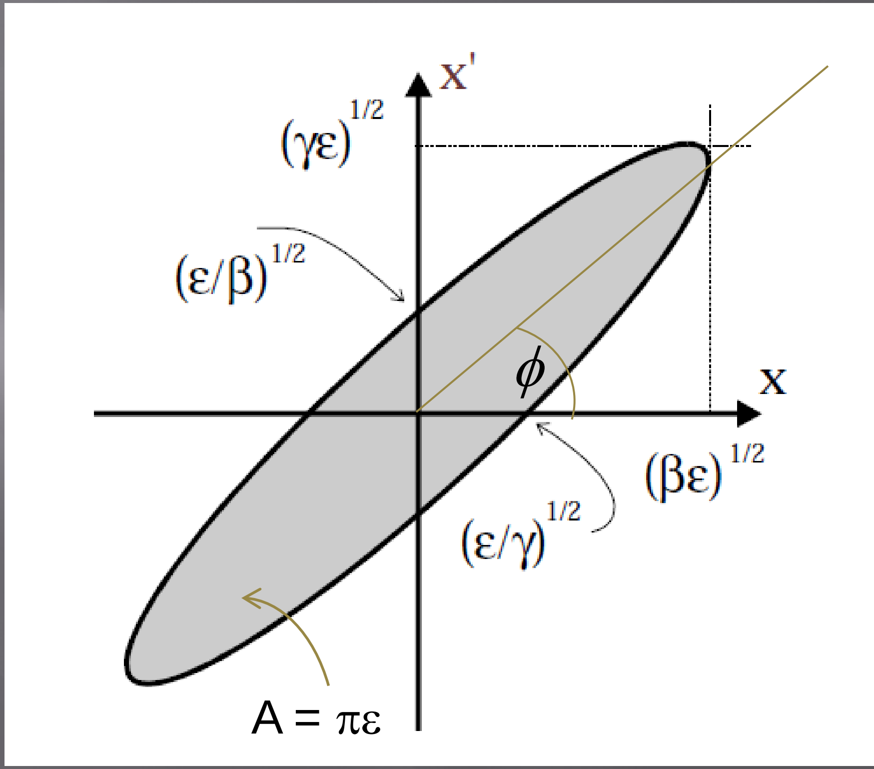
Twiss Parameters

β ~ (beam envelope)^2

γ ~ (beam divergence)^2

ϕ ~ angle in phase space

$$\tan(\phi) = \frac{-\alpha}{\beta}$$



α = 0 upright ellipse (waist)

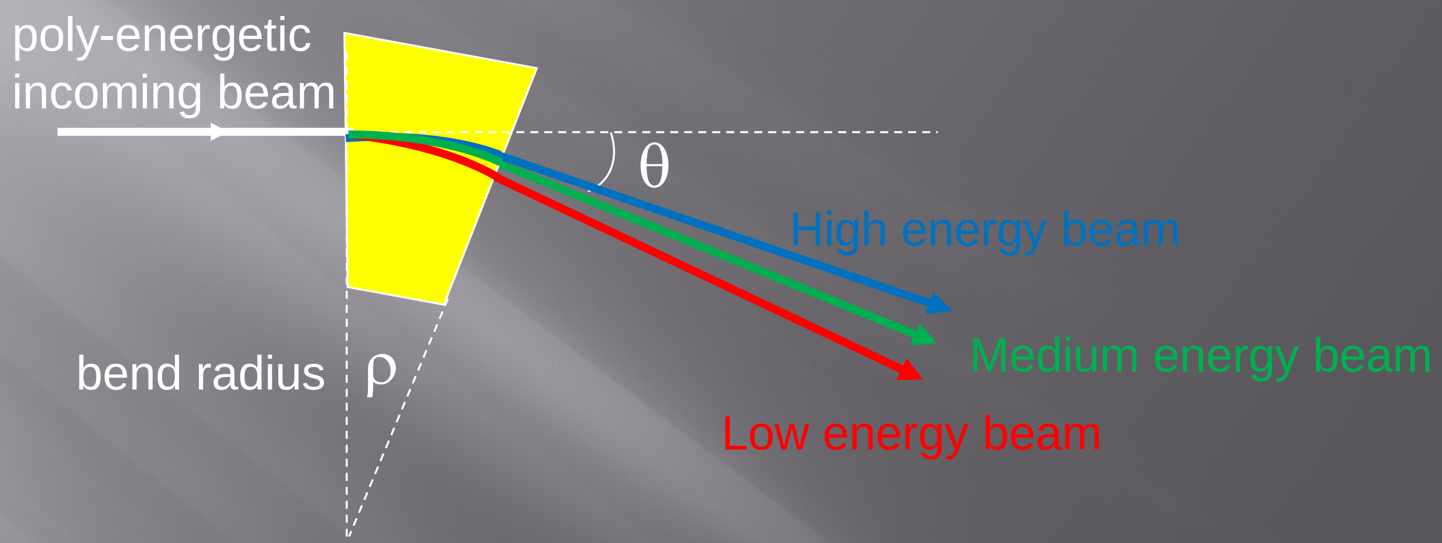
α < 0 diverging beam

α > 0 converging beam

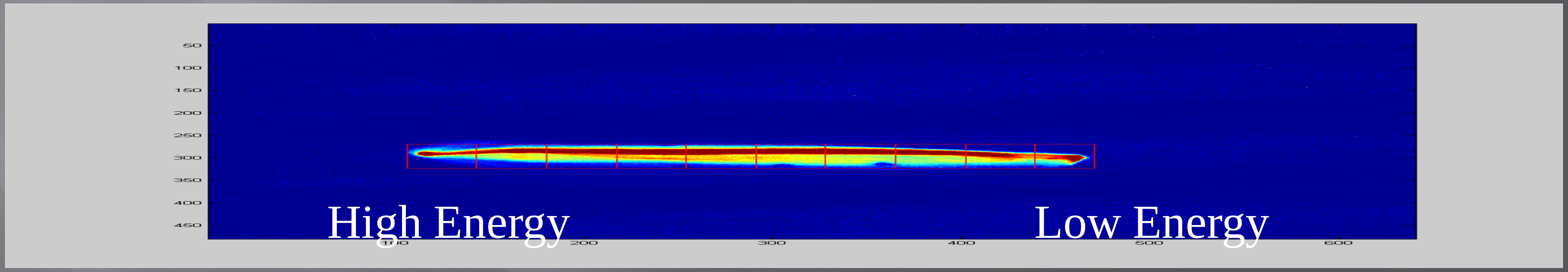
Courant-Snyder invariant

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon$$

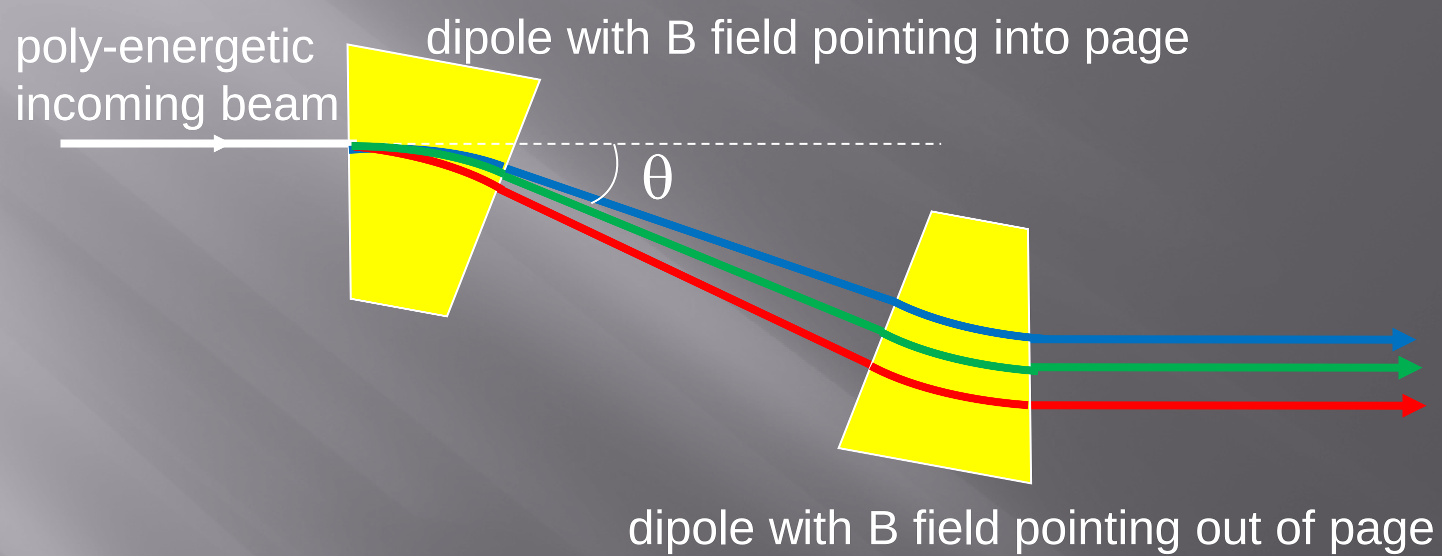
Dipole's dispersion spreads electron beam in the bend plane



Dipoles are used as **energy spectrometers** to measure beam's energy (from known bend radius and magnetic field) and energy spread.



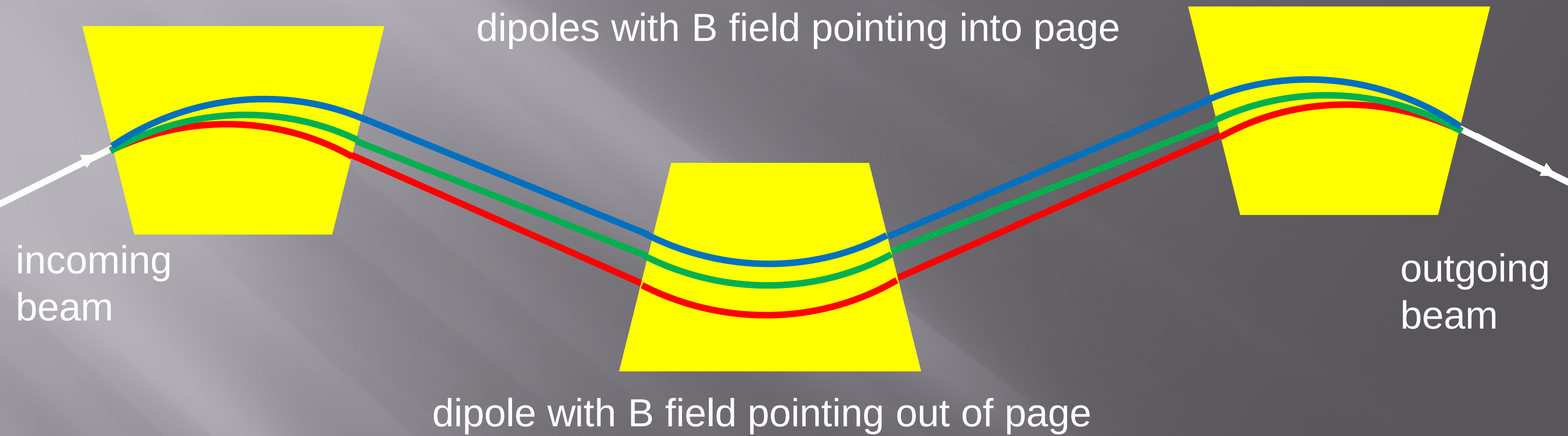
Two dipoles translate & disperse electron beam into a new line



Achromatic: different energies come back together in space
Isochronous: different energies come back together in time

The above two-dipole bend is neither achromatic nor isochronous; low-energy beam (red) takes longer path than high-energy (blue).

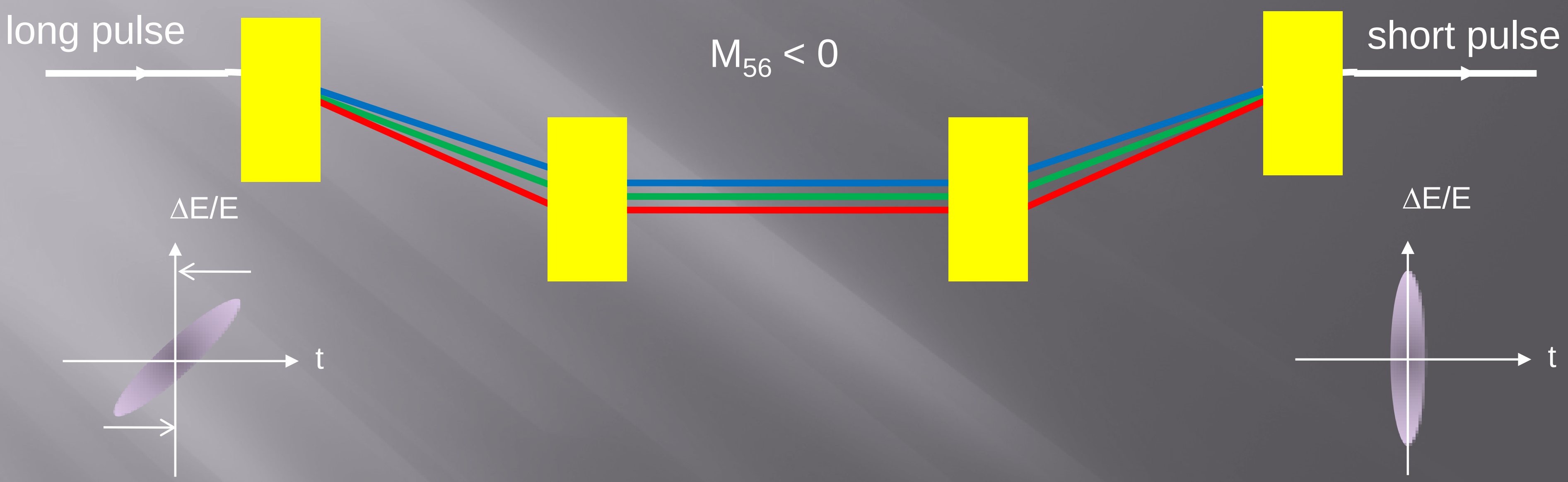
Achromatic Bunch Compressor



This two-dipole bend is achromatic but not isochronous.
Three dipole bend can be used to compress electron bunch via M_{56}

$$c\Delta\tau = M_{56}\frac{\Delta E}{E}$$

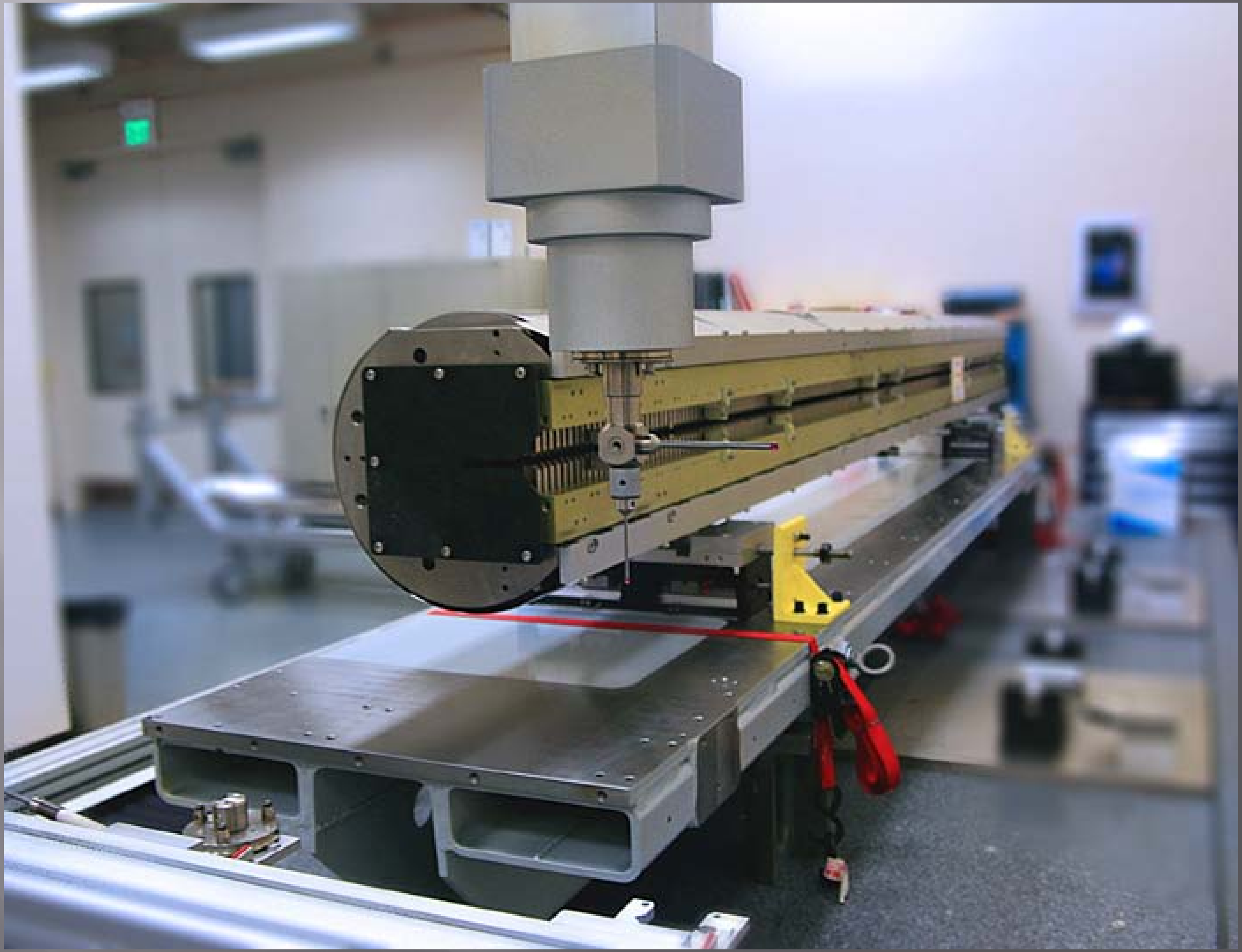
Chicane Compressor



- Chirp electron bunch by accelerating it off-crest so the leading edge (left) has lower energy than the trailing edge (right)
- Put the energy-chirped electron bunch through a chicane
- Chicane compresses long bunch to shorter bunch (higher peak current)

Part 4
Wigglers

Linac Coherent Light Source Wiggler (Undulator)



Magnetic Field B and Magnetizing Force H

Magnetic field B (aka magnetic induction) is in unit of tesla (T). Magnetizing force H is in unit of A/m. In vacuum, B and H are related by the permeability of free space, μ_0

$$\mathbf{B} = \mu_0 \mathbf{H}$$

$\mu_0 = 4\pi \times 10^{-7}$ T-m A^{-1}

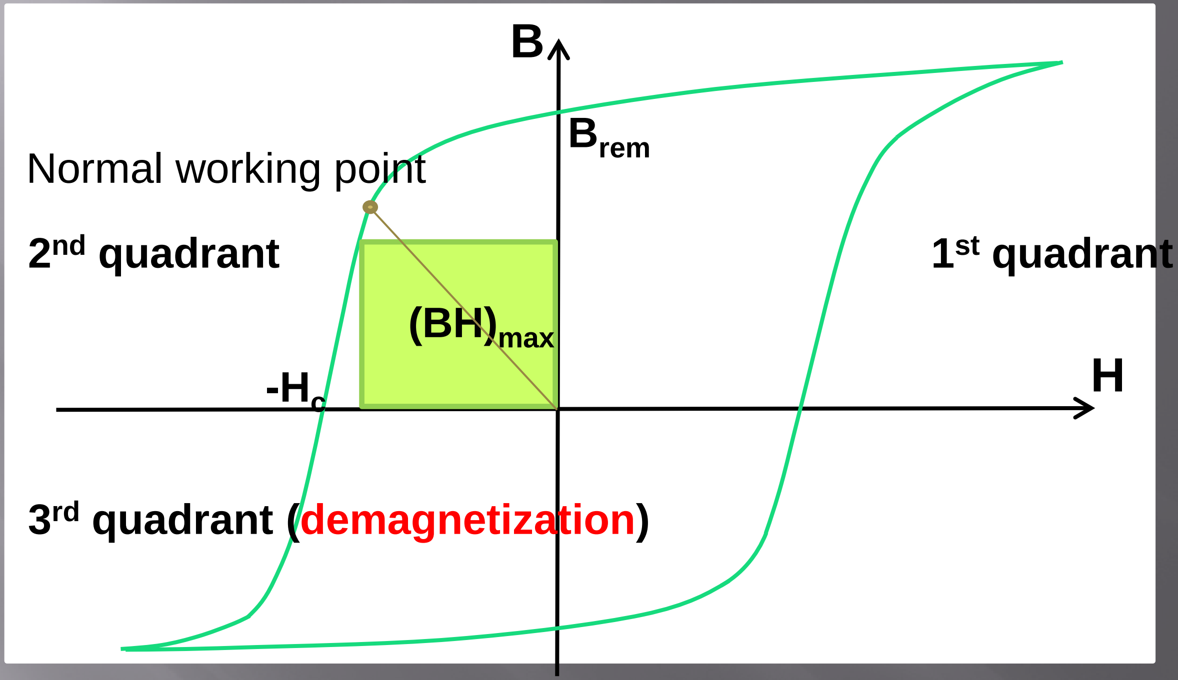
Magnetic field is increased in the presence of a ferromagnetic material by its relative magnetic permeability, μ_r

$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$

Relative permeability of common magnet materials ~ 1.05

Relative permeability of vanadium permendur ~ 7,000

B-H Curve of Magnet Materials



Remanence - coercivity trade-off

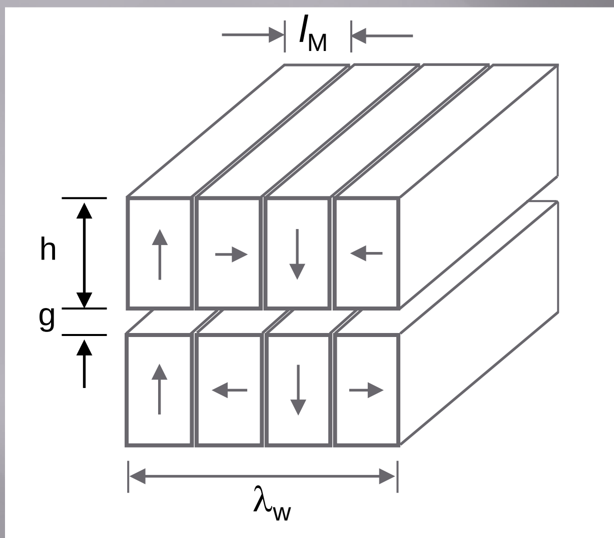
SmCo have high H_c and lower B_{rem}

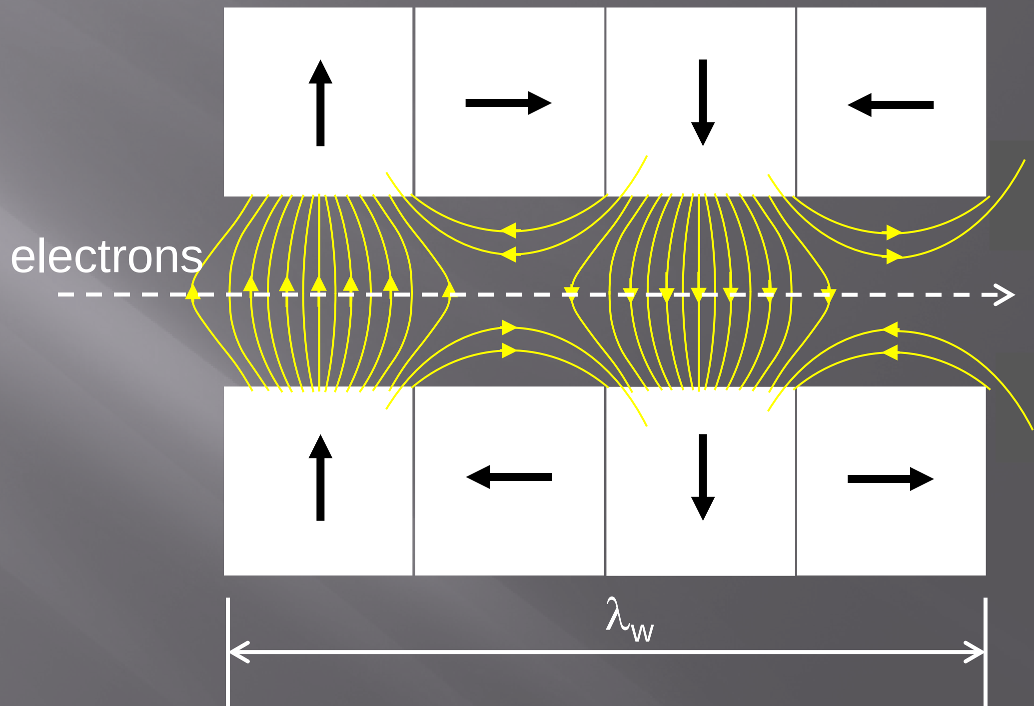
NdFeB have high B_{rem} but low H_c

PM Material	$(BH)_{max}$ (kJ/m^3)	Remanence (mT)	H_c Coercivity (kA/m)	Radiation Hardness
$SmCo_5$	170	800-1000	2400	High
Sm_2Co_{17}	220	1000-1100	2000	Medium
NdFeB	300	1100-1400	1400	Low - Medium

Permanent Magnet Wiggler Halbach Design

Halbach Design





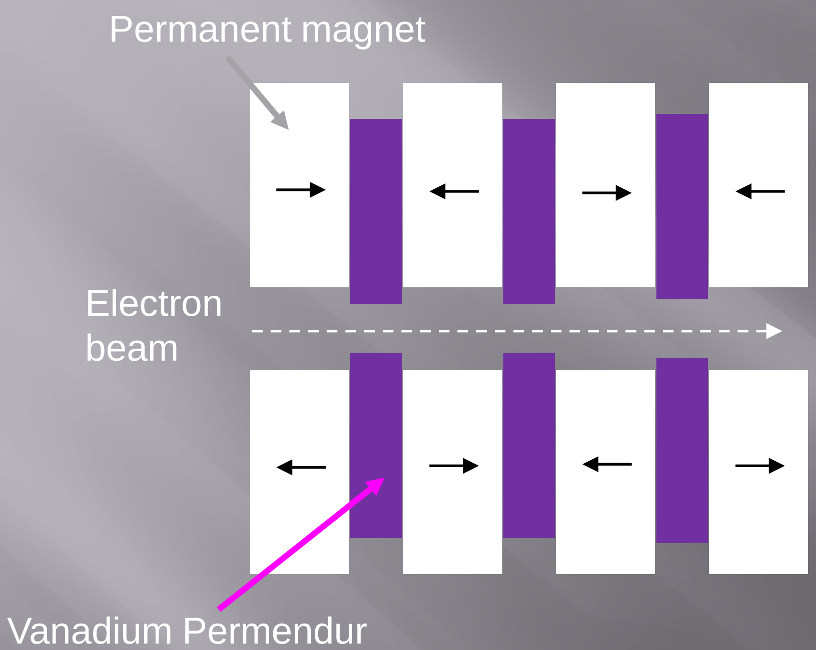
Peak magnetic field on axis

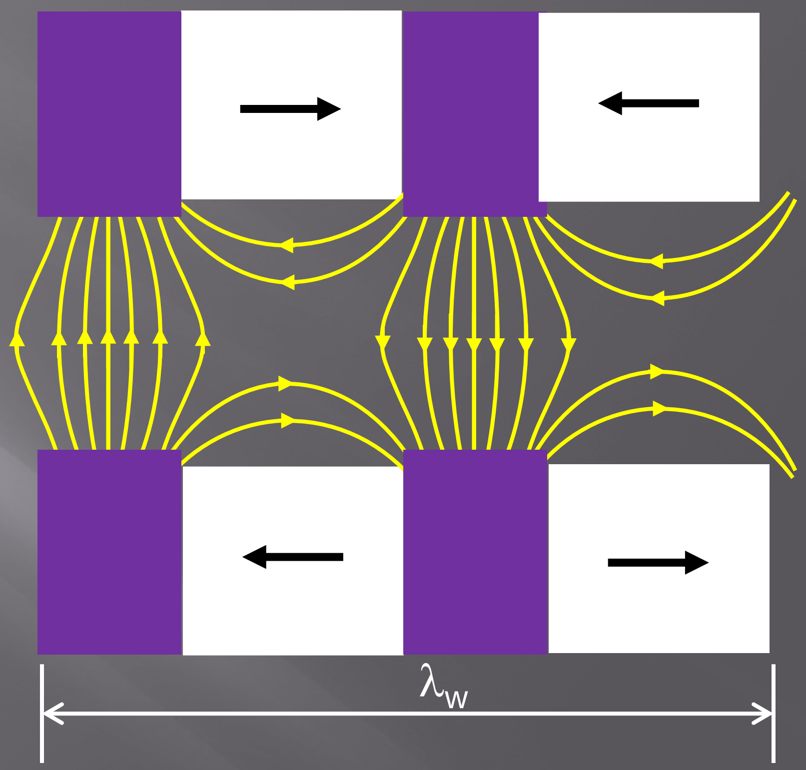
$$B_0 = 1.78 B_r e^{-\frac{\pi \cdot g}{\lambda_w}}$$

Note that magnetic field increases as we go away from electron beam axis

Magnetic field is near zero outside the wiggler due to cancellation of fields.

Hybrid Wiggler Design





Hybrid wiggler design replaces up/down magnets with vanadium permendur

$$B_0 = (3.69T)\, e^{-\frac{g}{\lambda_w}\left(5.07 - 1.5\left(\frac{g}{\lambda_w}\right)\right)}$$

Planar wigglers focus e- beams in the y (vertical) plane

Planar wiggler with infinite x dimension

$$B_x = 0$$
$$B_y = \hat{y} B_0 \cosh(k_w y)\cos(k_w z)$$
$$B_z = -\hat{z} B_0 \sinh(k_w y)\sin(k_w z)$$

Equation of motion in the y direction

$$\frac{d\left(\gamma m_0 \upsilon_y\right)}{dt} = e\left(\overline{\upsilon_x B_z} - \overline{\upsilon_z B_x}\right)$$

Velocity in x

$$\upsilon_x = -\left(\frac{a_w c}{\gamma}\right)\sin\left(k_w z\right)$$

B_x is zero

Expand B_z about y=0

$$B_z = B_0 k_w y \sin\left(k_w z\right)$$

$$\ddot{y} = -\left(\frac{e B_0 k_w c}{\sqrt{2} m_0 \gamma}\right)\left(\frac{a_w}{\gamma}\right) y$$

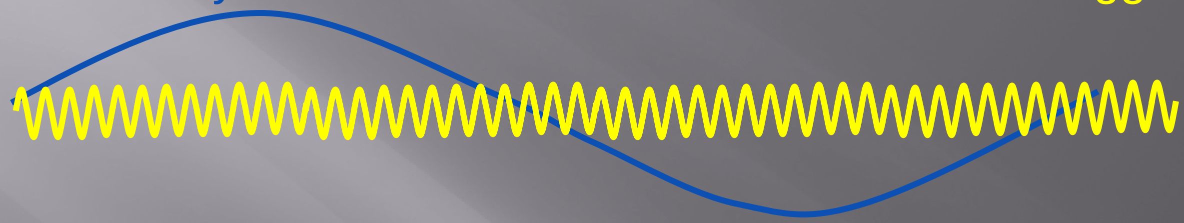
Divide both sides by c^2 to convert to 2^{nd} derivative with respect to z

$$y'' = -\left(\frac{k_w a_w}{\gamma}\right)^2 y$$

Betatron Motion

Betatron motion in y

Wiggler motion in x



$$y'' + k_\beta^2 y = 0$$

$$x'' + k_w x = 0$$

$$k_\beta = \frac{k_w a_w}{\gamma}$$

Betatron motion is slow, large amplitude motion in the y direction over many wiggler periods due to gradient of B_y along the y direction. The field amplitude is proportional to the square of deviation from the center.

$$B_y \approx B_0 \left[1 + \left(k_w y \right)^2 \right] \cos\left(k_w z \right)$$

Single-Plane Focusing

- Vertical envelope equation with emittance

$$\frac{d^2R_y}{dz^2} + k_\beta^2 R_y = \left(\frac{\varepsilon_{ny}}{\gamma}\right)^2 \frac{1}{R_y^3}$$

- Find matched beam envelope radius by setting $\frac{d^2R_y}{dz^2}$ to zero

$$R_{y0}^4 = \left(\frac{\varepsilon_{ny}}{\gamma k_\beta}\right)^2 \qquad k_\beta = \frac{k_w a_w}{\gamma}$$

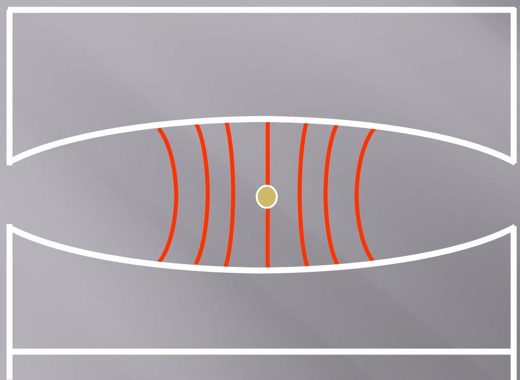
Matched beam rms radius

$$R_{y0} = \sqrt{\frac{\varepsilon_{ny}}{k_w a_w}}$$

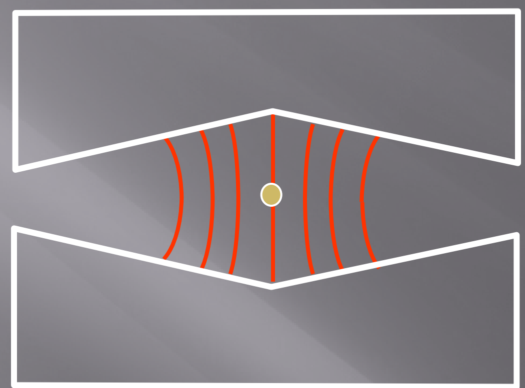
With single-plane focusing (planar) wiggler, the electron beam is focused in y at the entrance and in x in the wiggler of the wiggler. The beam is elliptical at the entrance and exit, and round in the middle.

Two-Plane Weak Focusing

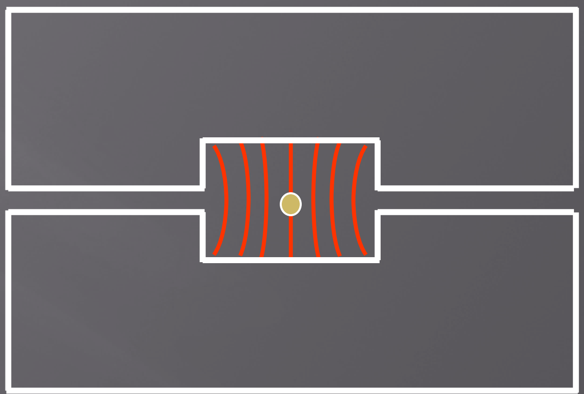
Parabolic Pole Face



Dual Canted Pole Face

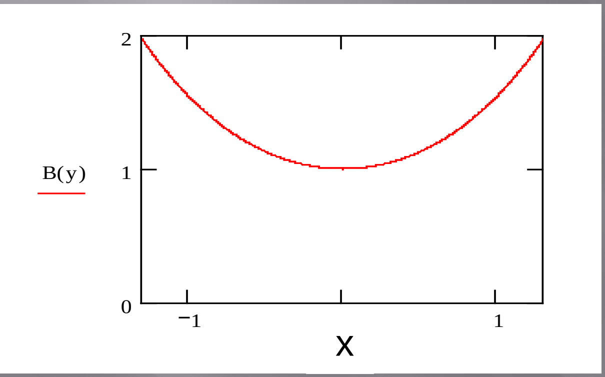


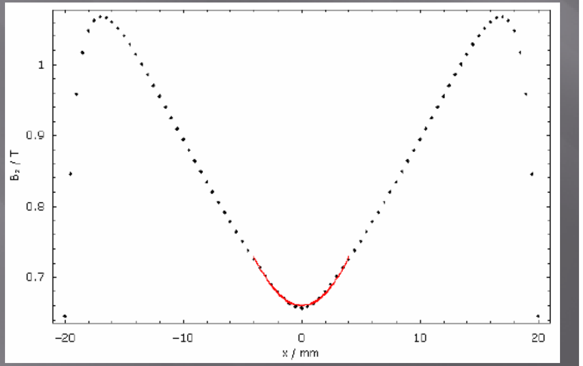
Slotted Pole Face

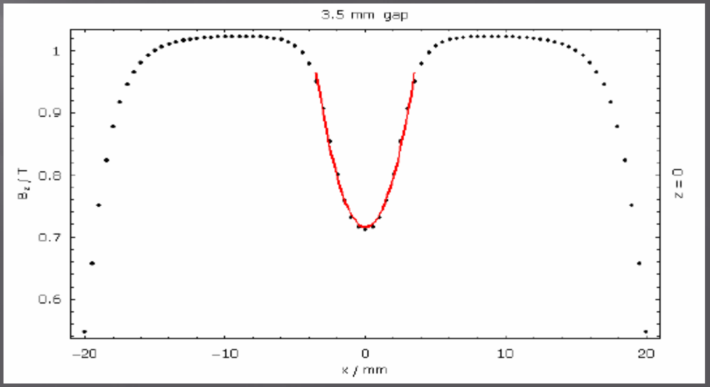


Quadratic x^2 dependence

Approximate x^2 dependence of B_y at the center







$$B_y = B_0\left[1+\frac{(k_w x)^2}{2}\right]$$

Magnetic field along the x axis in a two-plane focusing wiggler. Note the parabolic dependence similar to the field of a sextupole magnet.

Matched Beam for Two-plane (Weak) Focusing Wiggler

Equations for x and y envelope radii in two-plane focusing wigglers

$$\frac{d^2R_x}{dz^2} + k_x^2 R_x = \left(\frac{\varepsilon_{nx}}{\gamma}\right)^2 \frac{1}{R_x^3}$$

$$\frac{d^2R_y}{dz^2} + k_y^2 R_y = \left(\frac{\varepsilon_{ny}}{\gamma}\right)^2 \frac{1}{R_y^3}$$

where

$$k_x^2 + k_y^2 = k_\beta^2$$

Equal two-plane focusing

$$k_x = k_y = \frac{k_\beta}{\sqrt{2}}$$

Matched beam rms radii in x and y

$$R_{x0} = \sqrt{\frac{\sqrt{2}\varepsilon_{nx}}{k_w a_w}}$$

$$R_{y0} = \sqrt{\frac{\sqrt{2}\varepsilon_{ny}}{k_w a_w}}$$

In a two-plane weak focusing wiggler, the focusing strength and matched beam radii are independent of beam energy.

Summary of Part 4

Most wigglers are based on permanent magnet Halbach or hybrid designs. **Halbach design** is used for gap-to-period ratio >0.3 and **hybrid design** is used for ratio <0.3 (small gap, large period).

If not properly matched, the electron beam undergoes **betatron oscillations** in the y direction for plane-polarized wigglers and in both directions for two-plane weak focusing wigglers.

Two-plane **weak focusing** can be accomplished by shaping the wiggler magnets.

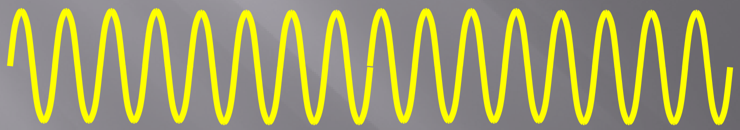
Part 5
Spontaneous Emission, FEL Gain & Efficiency

Spontaneous Emission Spectrum

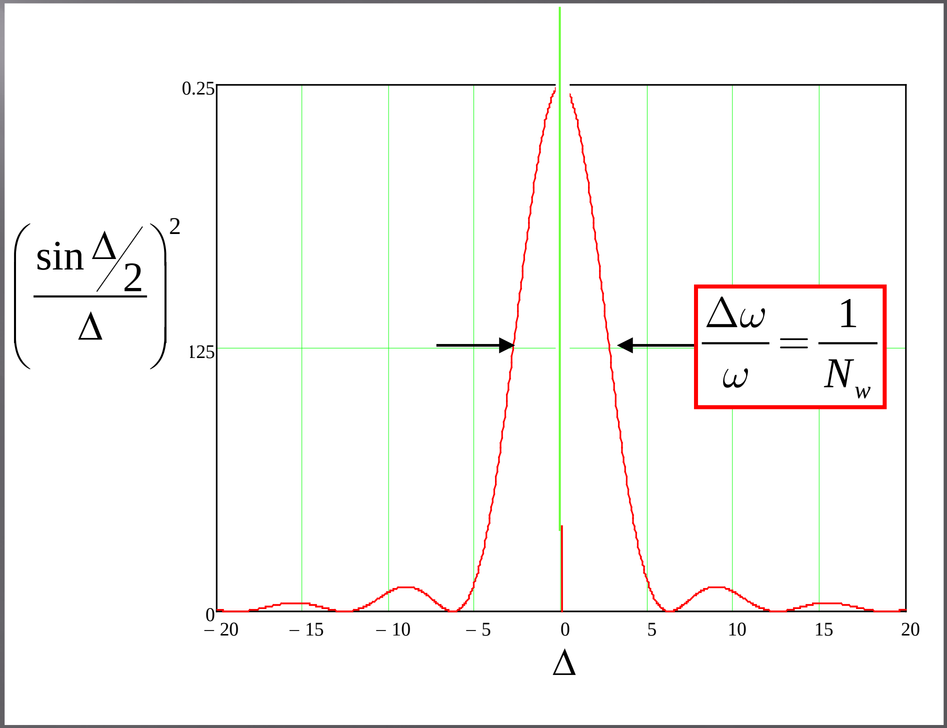
$$\frac{dW}{d\omega d\Omega} = \frac{e^2\gamma^2 N_w^2 N_e}{2\pi\varepsilon_0 c}\left[JJ(a_w)\right]^2\left(\frac{a_w}{1+a_w^2}\right)^2\left(\frac{\sin \Delta/2}{\Delta}\right)^2$$

$$\Delta = 4\pi N_w\left(\frac{\Delta\gamma}{\gamma_R}\right) = 2\pi N_w\left(\frac{\Delta\omega}{\omega_R}\right)$$

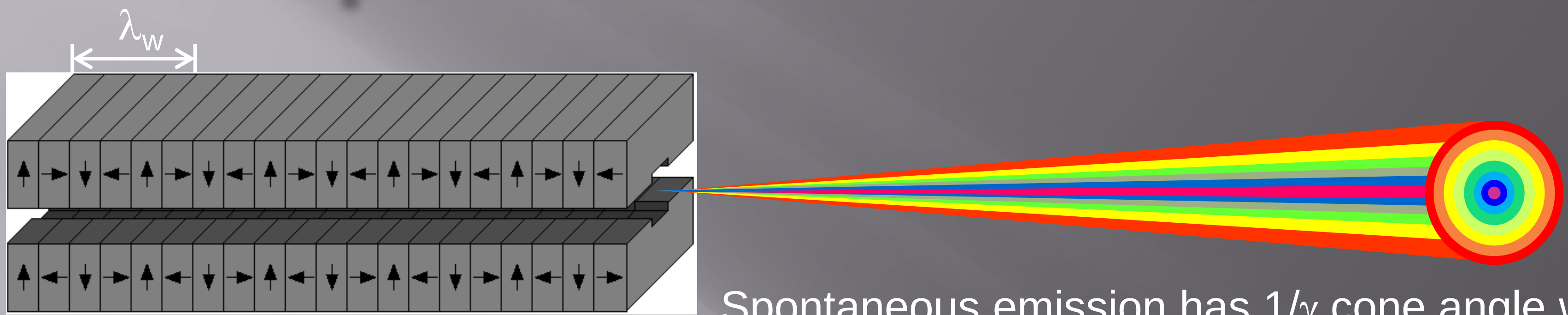
A wave train with N_w wavelengths has relative bandwidth equal to the inverse of N_w



Spontaneous spectrum is the Fourier transform of a rectangular train of N_w waves



Number of Coherent Photons in Spontaneous Emission



Spontaneous emission has 1/γ cone angle with high energy at center, low energy on the edges

Coherent spectral bandwidth

$$\frac{\Delta\omega}{\omega} = \frac{1}{N_w}$$

Coherent angle

$$\theta = \sqrt{\frac{\lambda}{L_w}}$$

Solid angle

$$\pi\theta^2 = \frac{\pi\lambda}{N_w\lambda_w}$$

Number of coherent spontaneous photons per electron

where α = fine structure constant

$$\alpha \approx \frac{1}{137}$$

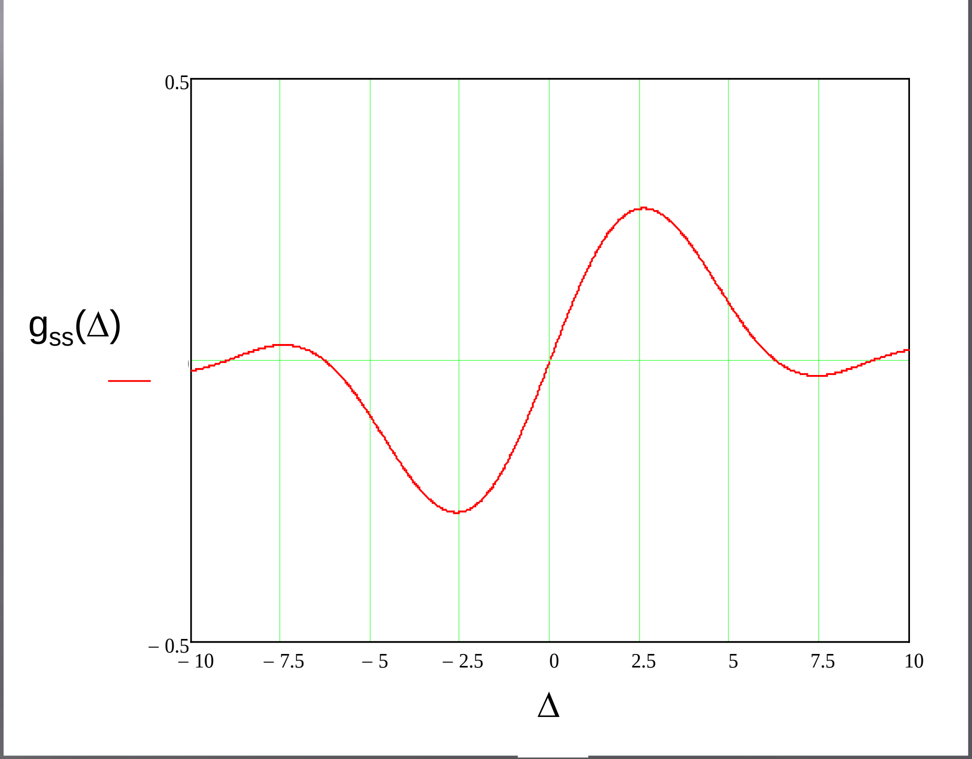
$$\frac{N_{coh.photon}}{N_e} = \pi\alpha\left[JJ(a_w)\right]^2\left(\frac{a_w}{1+a_w^2}\right)^2$$

Madey Theorem

Madey theorem: The FEL gain spectrum (gain versus energy detuning) is the derivative of the spontaneous emission spectrum.

$$g_{ss}(\Delta) = \frac{4(4\pi\rho N_w)^3}{\Delta^3}\left(1 - \cos\Delta - \frac{\Delta}{2}\sin\Delta\right)$$

Maximum gain occurs at **positive detuning = 2.6**, zero on resonance and negative (absorption) at negative detuning.



Definition of energy detuning

$$\Delta = 2\pi N_w \left(\frac{\Delta\lambda}{\lambda}\right)$$

Madey theorem predicts equal gain and absorption off resonance.

Phase-space Illustration of Madey Theorem

Electrons' energy > Resonance energy

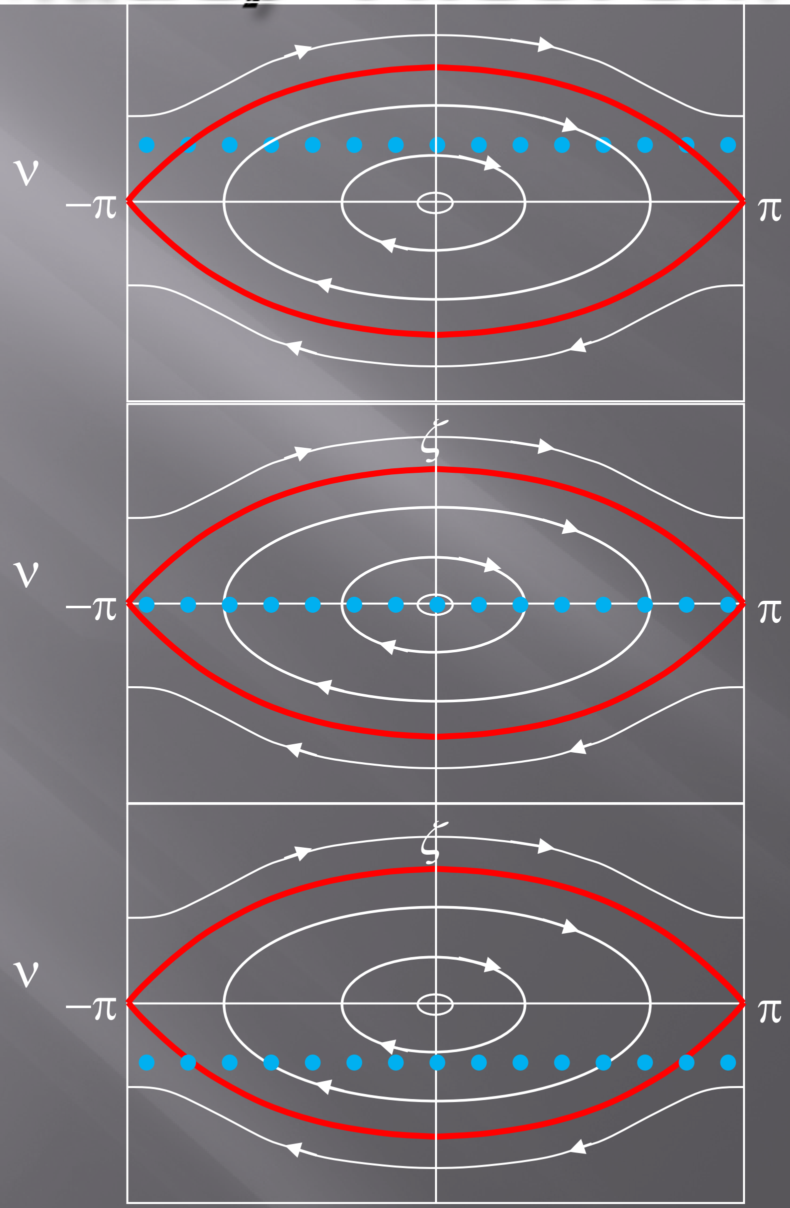
FEL gain > 0

Electrons' energy = Resonance energy

FEL gain = 0

Electrons' energy < Resonance energy

FEL gain < 0



Low-Gain, Small-Signal (Low-Field) Gain Spectrum

Small-signal gain at peak

$$g_{ss} = \left(\frac{2\pi N_w}{\gamma}\right)^3 \left(\frac{[JJ]a_w}{\sigma k_w}\right)^2 \left(\frac{I}{I_A}\right)$$

Gain scales linearly with peak current (not average)

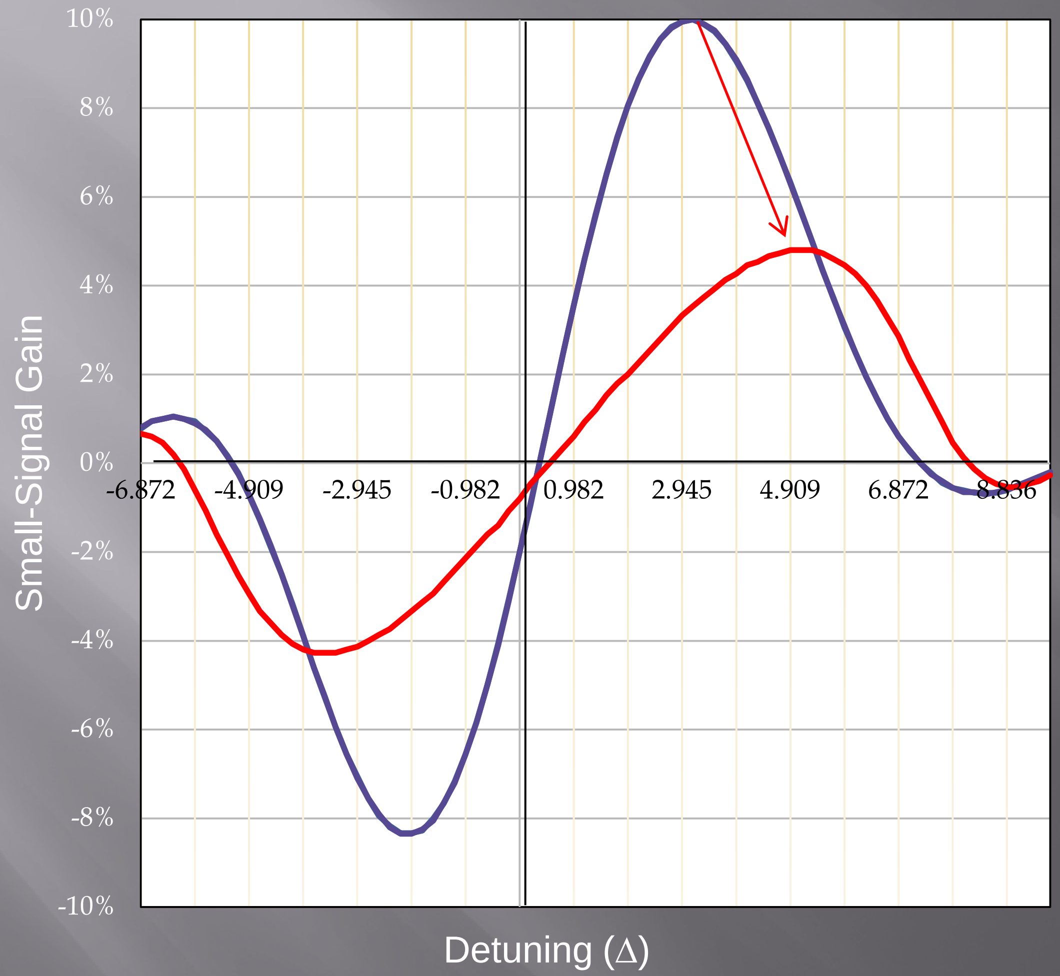
Amplification

$$P_{out} = (1 + g_{ss})P_{in}$$

Peak of gain curve shifts toward larger detuning (~3 instead of 2.6) due to optical diffraction.

Diffraction introduces loss at resonance wavelength.

Low-Gain, Large-Signal (High-Field) Gain Spectrum



Gain curve shifts to longer wavelength (larger Δ)

Peak gain is reduced

Gain spectrum is broadened

Gain Saturation

Gain decreases at high input power

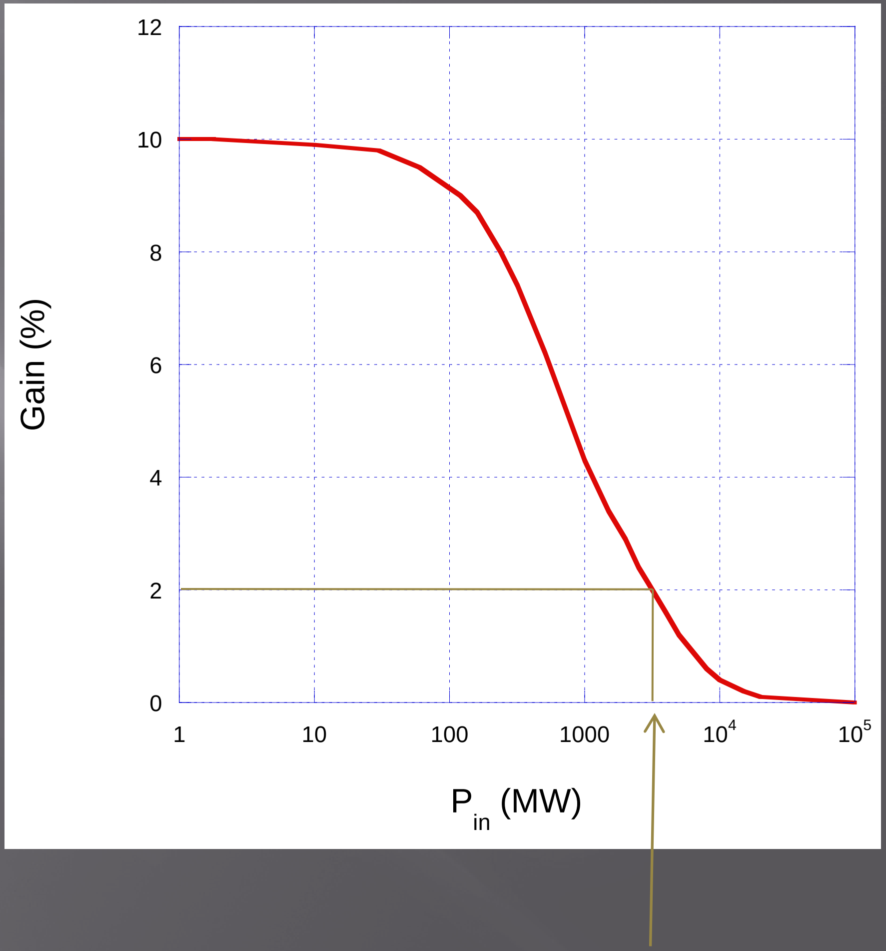
Power saturates when gain = loss

For oscillator FEL, this is the intracavity power, not output FEL power

Optimum outcoupling ~ ¼ of g_{ss}

For g_{ss} = 10%, optimal outcoupling is 2% and the intra-cavity power saturates at 3 GW. FEL external power is 60 MW.

Note these numbers denote peak power; to get FEL pulse energy, multiply peak power by the electron pulsewidth (FWHM for a gaussian pulse).



FEL Gain (Pierce) Parameter

Dimensionless Pierce parameter

$$\rho = \frac{1}{2\gamma}\left(\frac{[JJ]a_w}{\sigma k_w}\right)^{\frac{2}{3}}\left(\frac{I}{I_A}\right)^{\frac{1}{3}}$$

Small-signal gain relationship to ρ

$$g_{ss} = (4\pi N_w \rho)^3$$

The small-signal gain is proportional to the cube of the interaction length in the low-gain regime. As the number of wiggler periods increases, FEL power grows exponentially with z instead of z^3. With exponential power growth, log of FEL power increases linearly with z.

High-Gain FEL

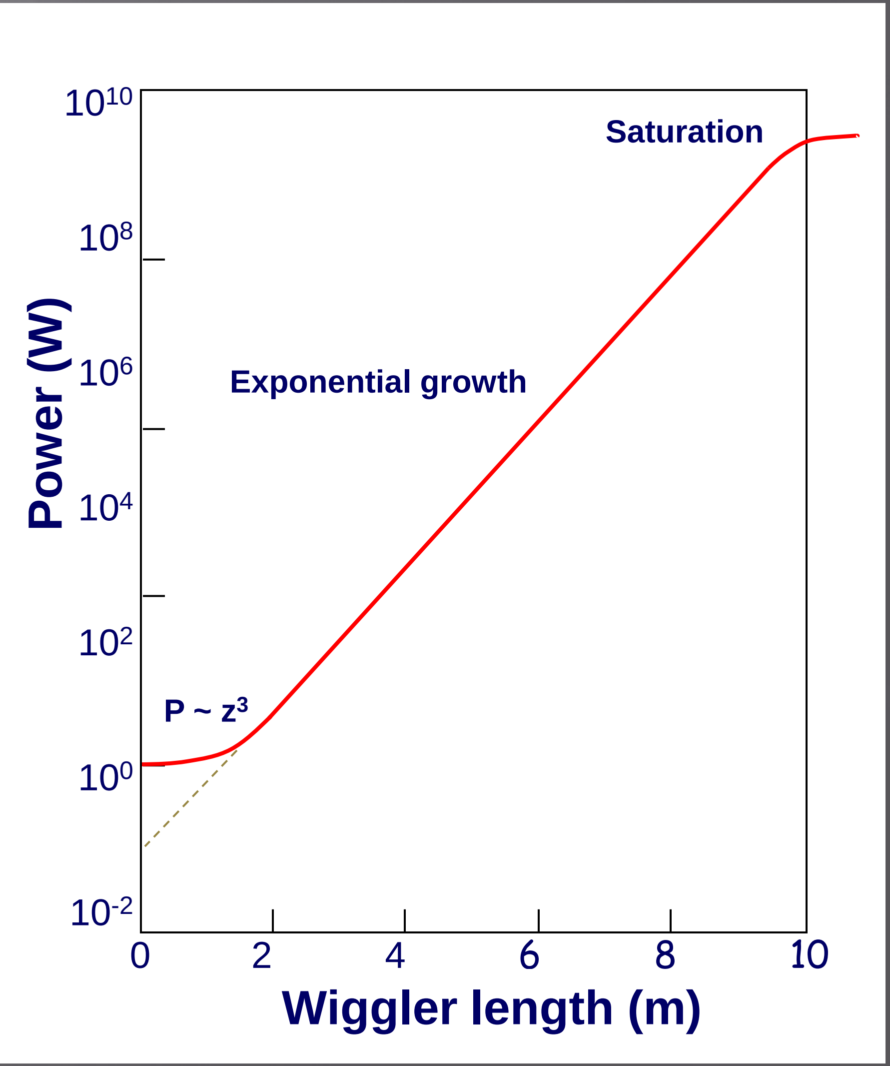
Gain length is wiggler length needed for power to grow by 2.7

$$L_G = \frac{\lambda_w}{4\pi\sqrt{3}\rho}$$

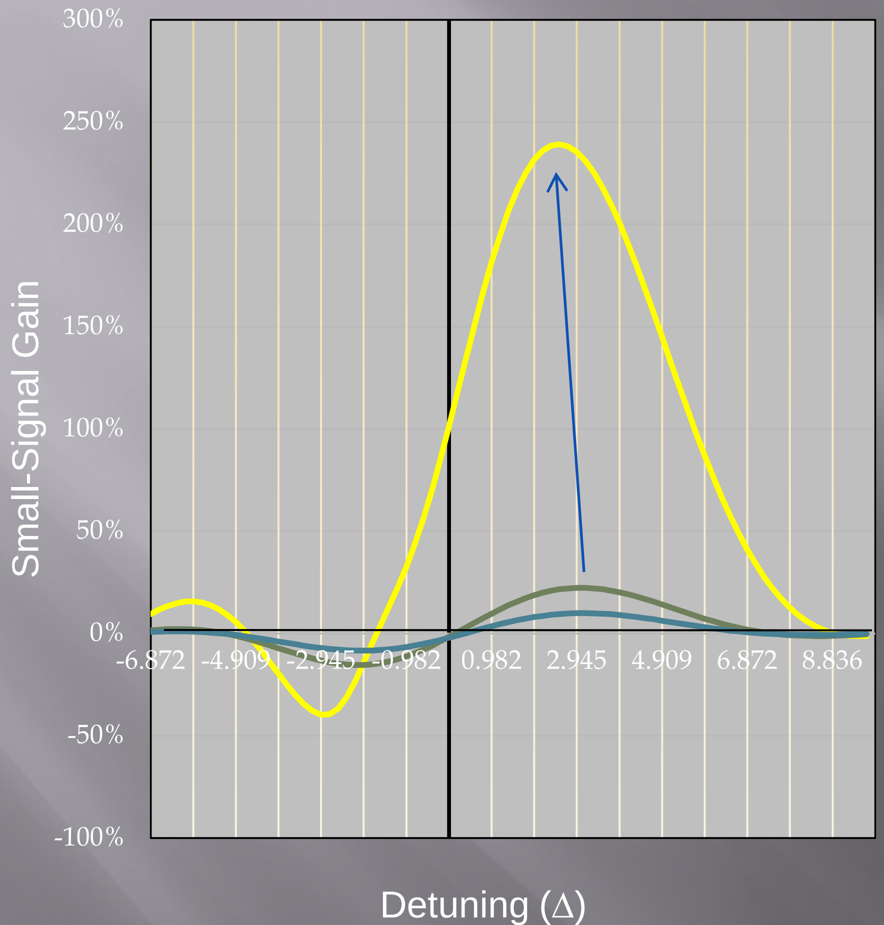
Seed laser power is reduced by 9 due to mode competition (growing, decaying and oscillatory modes)

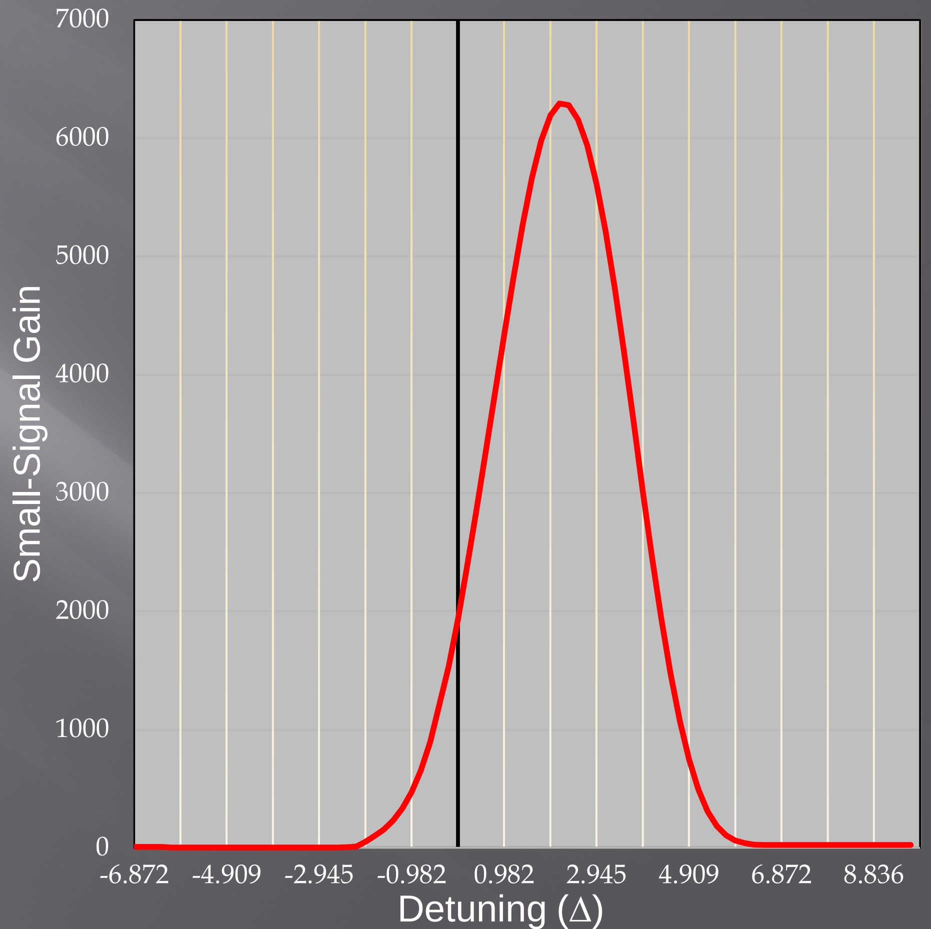
In a long wiggler, power grows exponentially with wiggler length

$$P = \frac{P_0}{9}\exp\left(\frac{z}{L_G}\right)$$



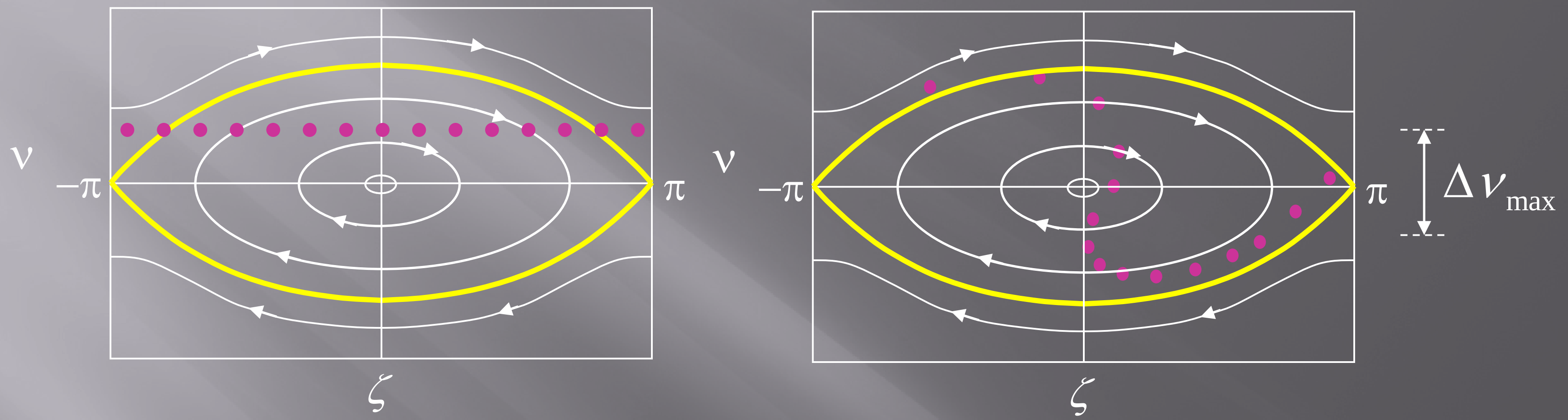
High Gain Spectrum





Gain curve shifts to shorter wavelength (smaller Δ) and higher peak gain.
Unlike the low-gain case, there is significant gain at the resonance wavelength.

FEL Extraction Efficiency



Low-gain oscillator

$$\Delta \nu_{\max} = \frac{1}{2N_w}$$

High-gain amplifier

$$\Delta \nu_{\max} = \rho$$

Example:

JLab FEL oscillator

$N_w = 30$
$\Delta\nu_{max} = 1.6\%$

Tapering Wiggler Period/Field

As the electron beam loses energy along the wiggler, the resonance condition is shifted toward lower beam energy. To maintain resonance, the wiggler period or a_w must be reduced. It is easier to increase the wiggler gap to reduce the wiggler parameter a_w than to reduce the wiggler period.

Resonance condition at z_0

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1 + a_w^2\right)$$

Resonance condition at $z_0 + \Delta z$

$$\lambda = \frac{\lambda_w}{2\left(\gamma - \Delta\gamma\right)^2}\left[1 + \left(a_w - \Delta a_w\right)^2\right]$$

Rate of resonance energy change with respect to z

$$\frac{d}{dz}\gamma_R^2 = -k_w a_w a_s \sin\phi_R$$

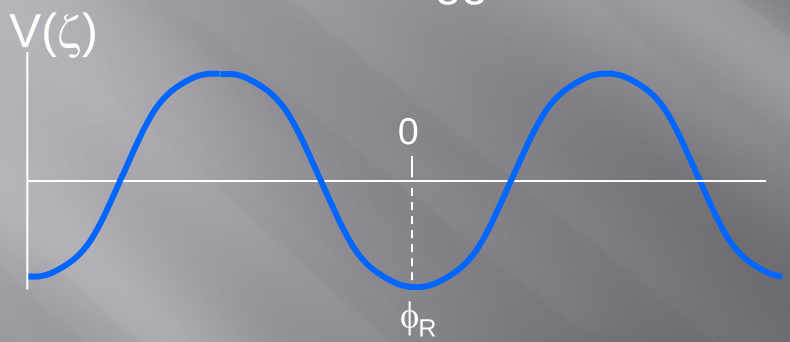
Energy taper vs. taper in a_w

$$\frac{d}{dz}\left(\frac{\Delta\gamma}{\gamma}\right) \approx \frac{a_w^2}{1 + a_w^2}\frac{d}{dz}\left(\frac{\Delta a_w}{a_w}\right)$$

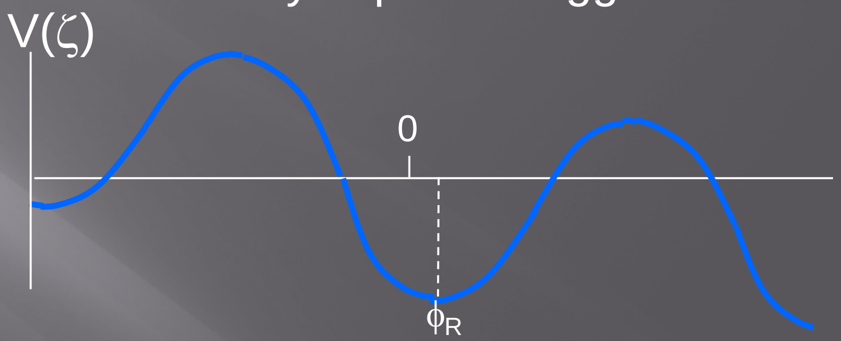
Ponderomotive Potential

$$H = \frac{v^2}{2} + V(\zeta)$$

Uniform Wiggler



Linearly Tapered Wiggler



$$V(\zeta) = -|a|\cos\zeta$$

$$V(\zeta) = -|a|\cos(\zeta + \phi_R) + \zeta\sin\phi_R$$

$$\dot{v} = -\frac{\partial H}{\partial \zeta} = -|a|\sin\zeta$$

$$\dot{v} = -\frac{\partial H}{\partial \zeta} = -|a|\sin(\zeta + \phi_R) + \sin\phi_R$$

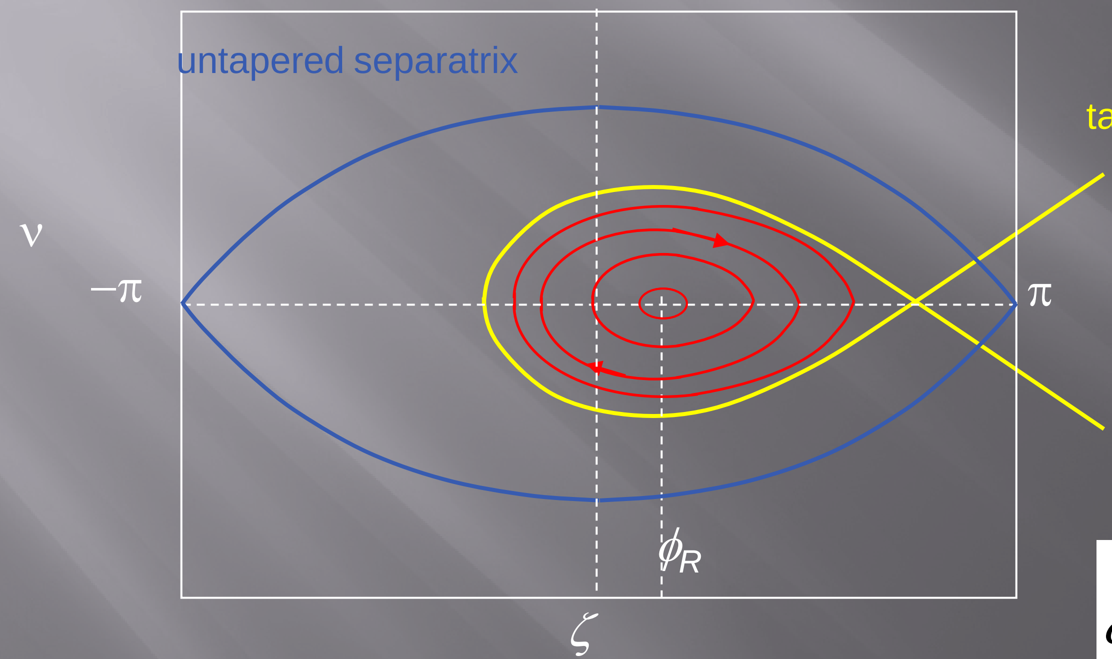
Tapered Wiggler Phase Space

Tapered wiggler pendulum equation

$$\dot{\nu} = |a|\sin(\zeta + \phi_R) + \delta$$

Energy exchange amplitude

$$|a| = \frac{k a_s a_w}{\gamma^2}$$

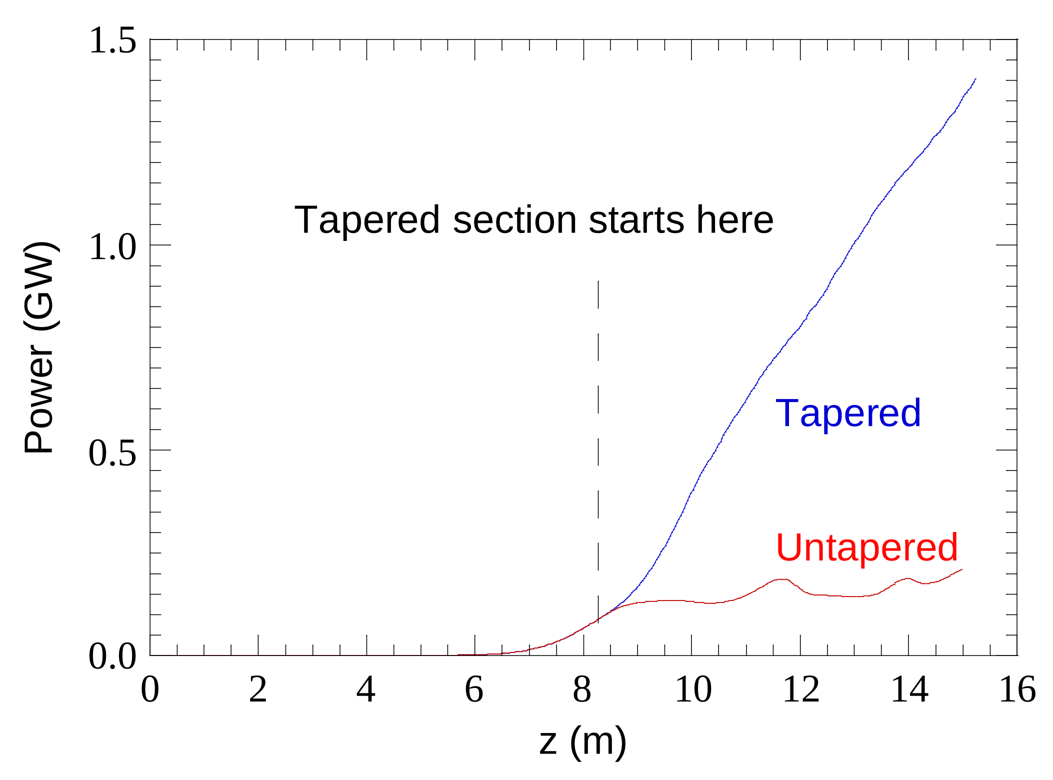


New term in pendulum eq.
= phase acceleration

Phase acceleration

$$\delta = \sin\phi_R = \frac{1}{k_w a_s a_w}\frac{d}{dz}\left(\frac{\Delta\gamma}{\gamma_R}\right)$$

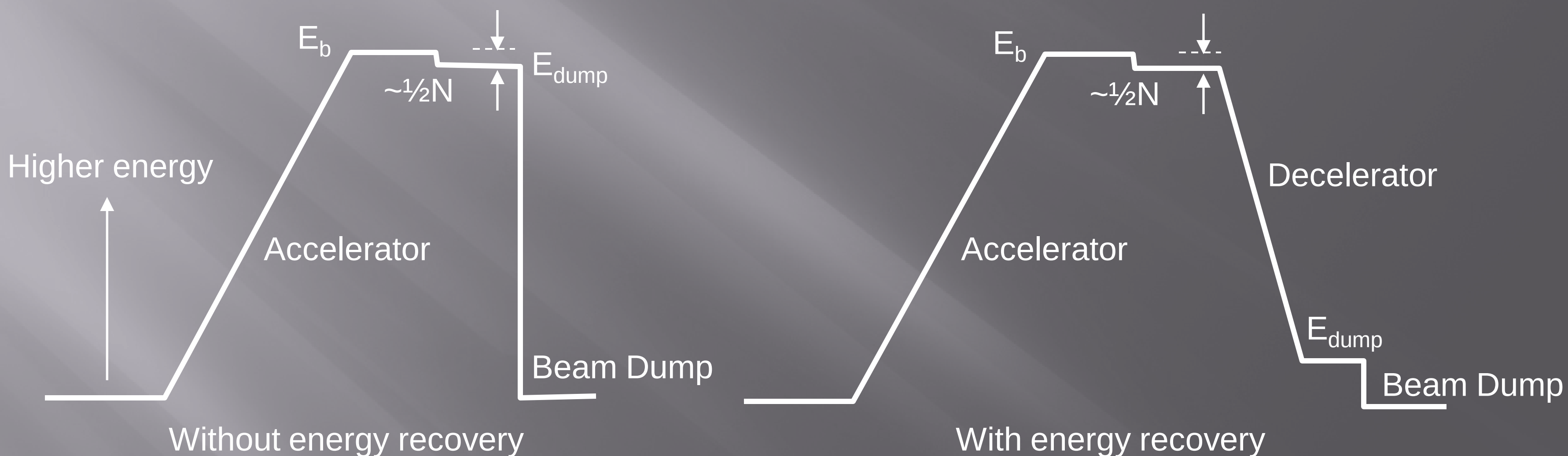
Efficiency Enhancement with a Linearly Tapered Wiggler



Depending on the trapping fraction and taper length, tapering can increase the power by 2 – 5. The electron trapping fraction decreases with increasing resonant phase and becomes zero if resonant phase = π.

Efficiency Enhancement with Energy Recovery

Oscillator FEL efficiency is typically 1%. The spent electron beams still have ~99% of the initial energy. Dumping the high-power electron beam is wasteful and creates radiation hazards (E_{dump} is beam energy before the beam dump).



With energy recovery, the efficiency of electron-to-FEL conversion is enhanced by the ratio of beam energy at the wiggler to beam dump energy.

$$\eta_{electron-FEL} = \frac{E_b}{2N_w E_{dump}}$$

Summary of Part 5

Spontaneous emission (same as undulator radiation) is partially coherent temporally and incoherent spatially.

Madey Theorem: the small-signal gain spectrum is the derivative of the spontaneous spectrum with respect to energy detuning.

The **small-signal gain** peaks at wavelengths slightly longer the resonant wavelengths or for a fixed wavelength, at higher beam energy.

For **high gain FEL**, the peak of the gain curve moves closer to the resonant wavelength.

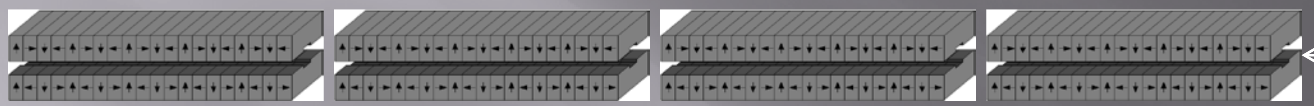
FEL **extraction efficiency** is $\sim 1/2N_w$ for oscillator and $\sim\rho$ for amplifier.

FEL efficiency can be increased by **tapering**, **detuning** (to higher energy or longer wavelength) or **energy recovery**.

Part 6
Optical Architectures

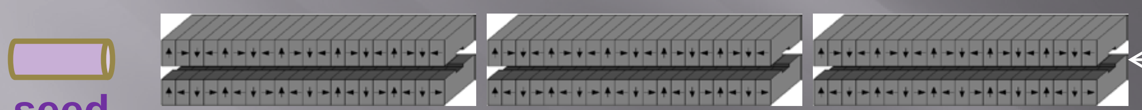
Gain & optical feedback determine optical architecture

Self-Amplified Spontaneous Emission (SASE)



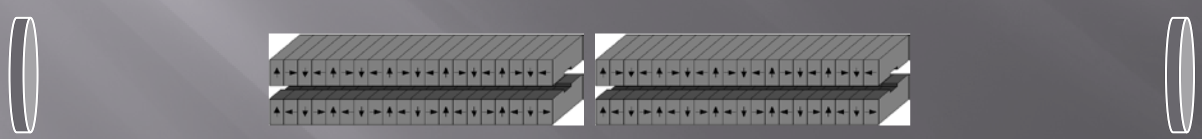
Very high gain
No optical feedback

Seeded Amplifier (or Pre-bunched Amplifier)



Regenerative Amplifier

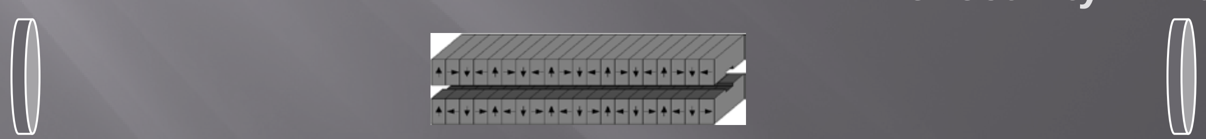
Reflectivity < 1%



High gain
Small optical feedback

Oscillator

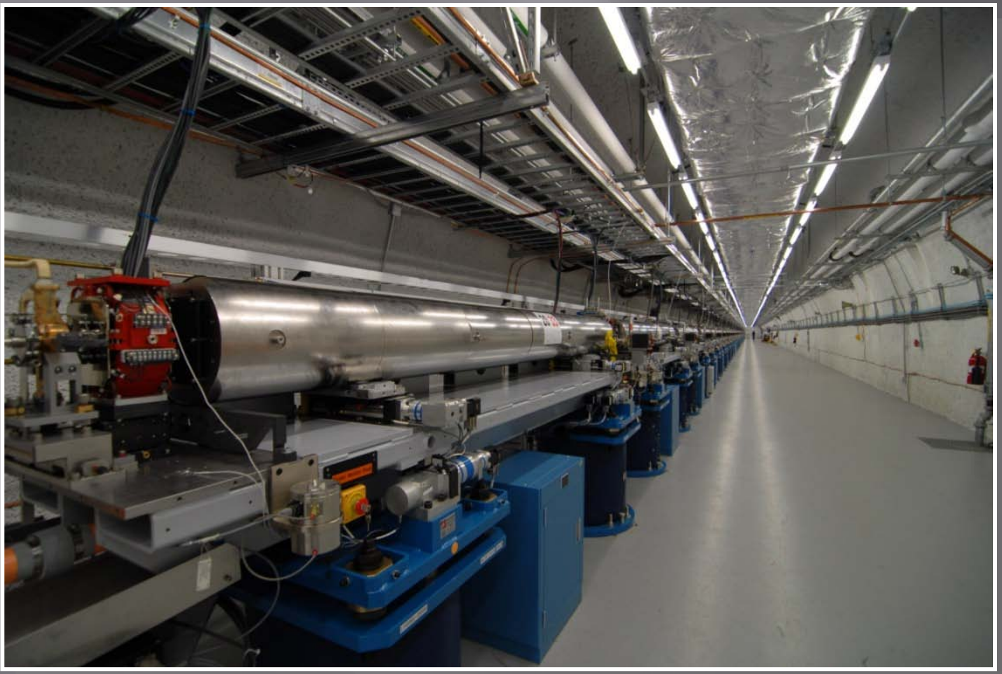
Reflectivity > 1%



Low gain
Large optical feedback

Self-Amplified Spontaneous Emission (SASE)

- Start from spontaneous emission (noise)
- Use very long wiggler (undulator)
- Rely on very high brightness electron beams
- Saturate in a single pass



LCLS Undulator

Courtesy of P. Emma

SASE Power vs. Length

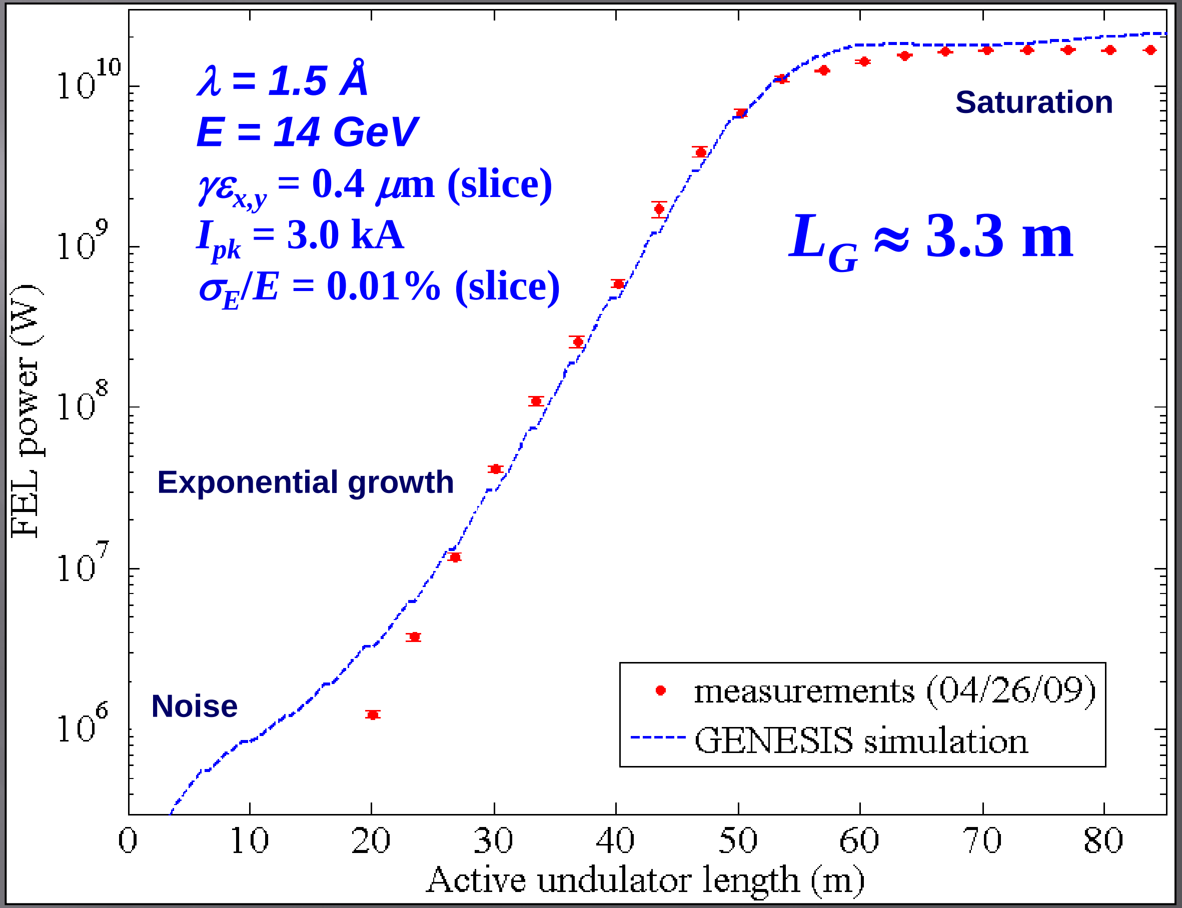
About 20 gain lengths are needed to reach saturation

Saturation Power

$$P_{sat} = \frac{\rho I_p E_b}{e}$$

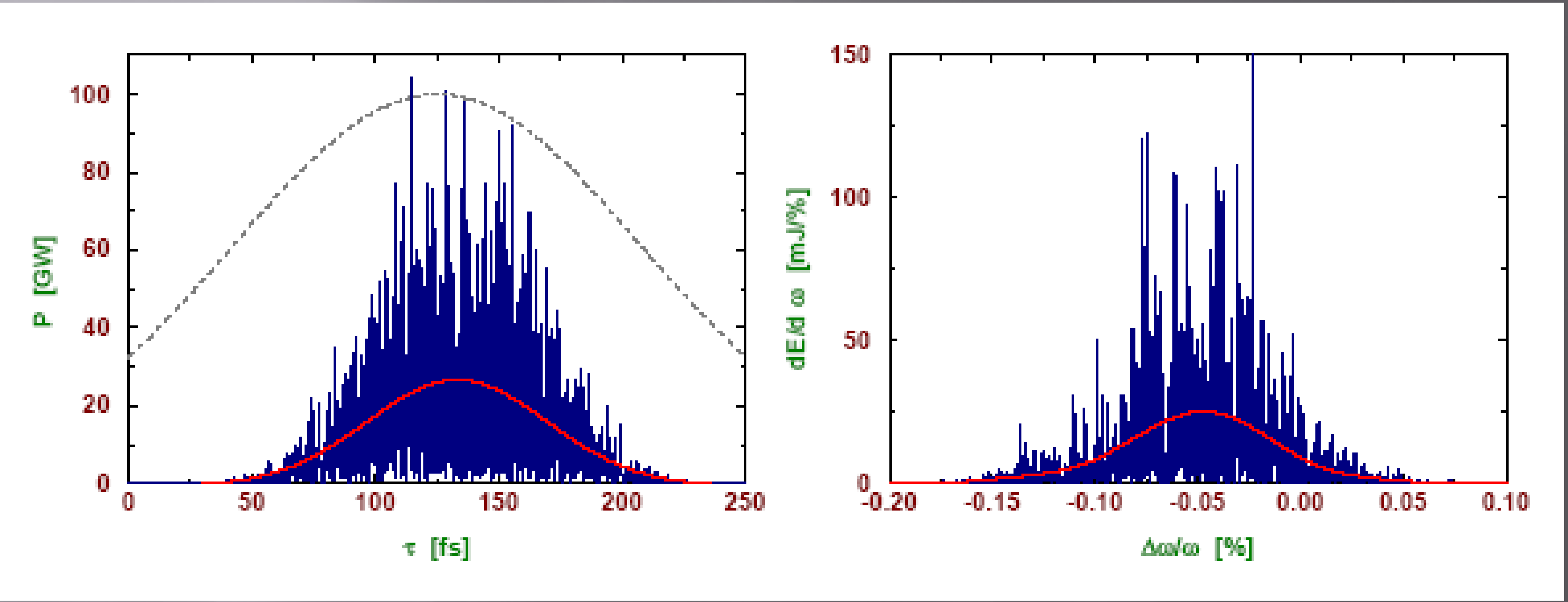
Rho parameter

$$\rho = \frac{1}{2\gamma}\left(\frac{[JJ]a_w}{\sigma k_w}\right)^{\frac{2}{3}}\left(\frac{I}{I_A}\right)^{\frac{1}{3}}$$



LCLS FEL Power versus z

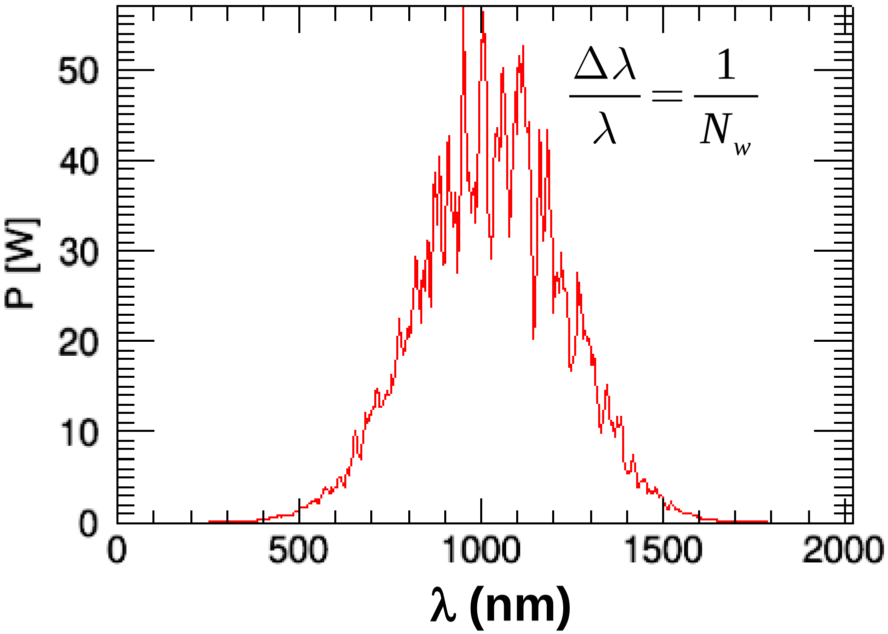
SASE output is chaotic temporally and spectrally



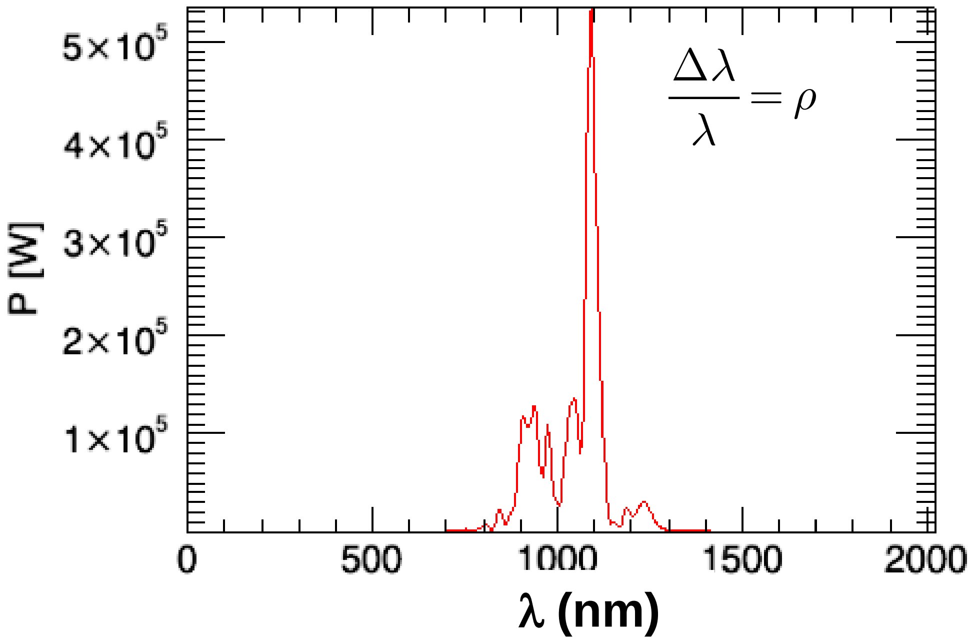
Temporal profile **Spectral profile**

SASE temporal profile consists of several spikes. The corresponding spectral profile FWHM is inversely proportional to the spikes' temporal width. The narrow spectral line is the inverse of the full temporal pulse.

SASE spectrum is slightly narrower than spontaneous emission



Incoherent Spontaneous Emission



SASE near saturation

Shot-to-shot fluctuations depend on the number of spikes in the radiation pulse.

$$\frac{\Delta I}{I} = \frac{1}{\sqrt{M}}$$

Seeded Amplifier

- Start with coherent signals from a seed laser or pre-bunched electron beam
- Reduce wiggler length needed to reach saturation
- Reduce spectral width and fluctuations, increase coherence

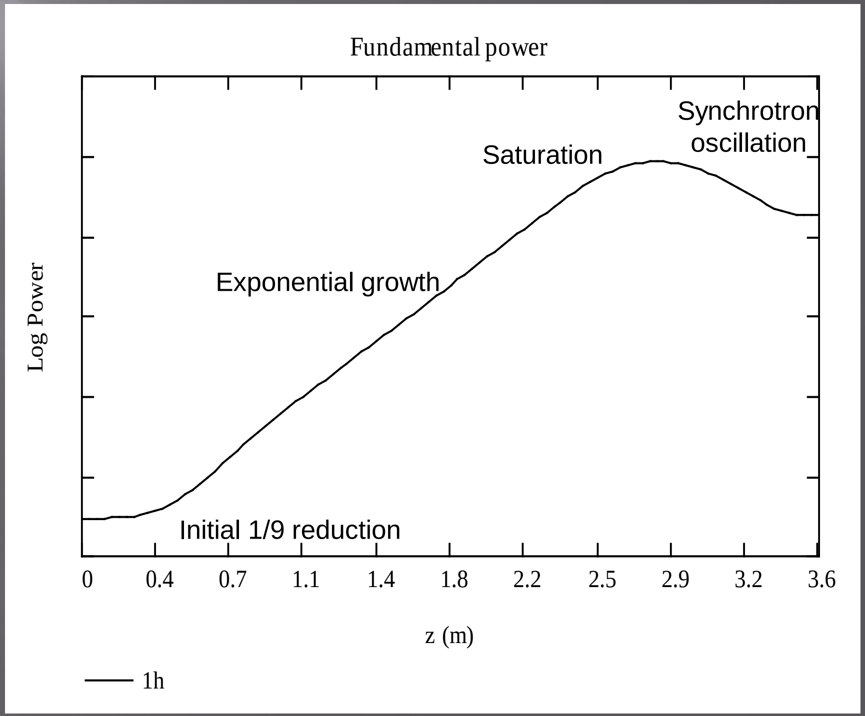
$$P(z) = \frac{1}{9} P_0 \, exp\left(\frac{z}{L_G} \right)$$

Power grows exponentially with distance by one e-folding every power gain length.

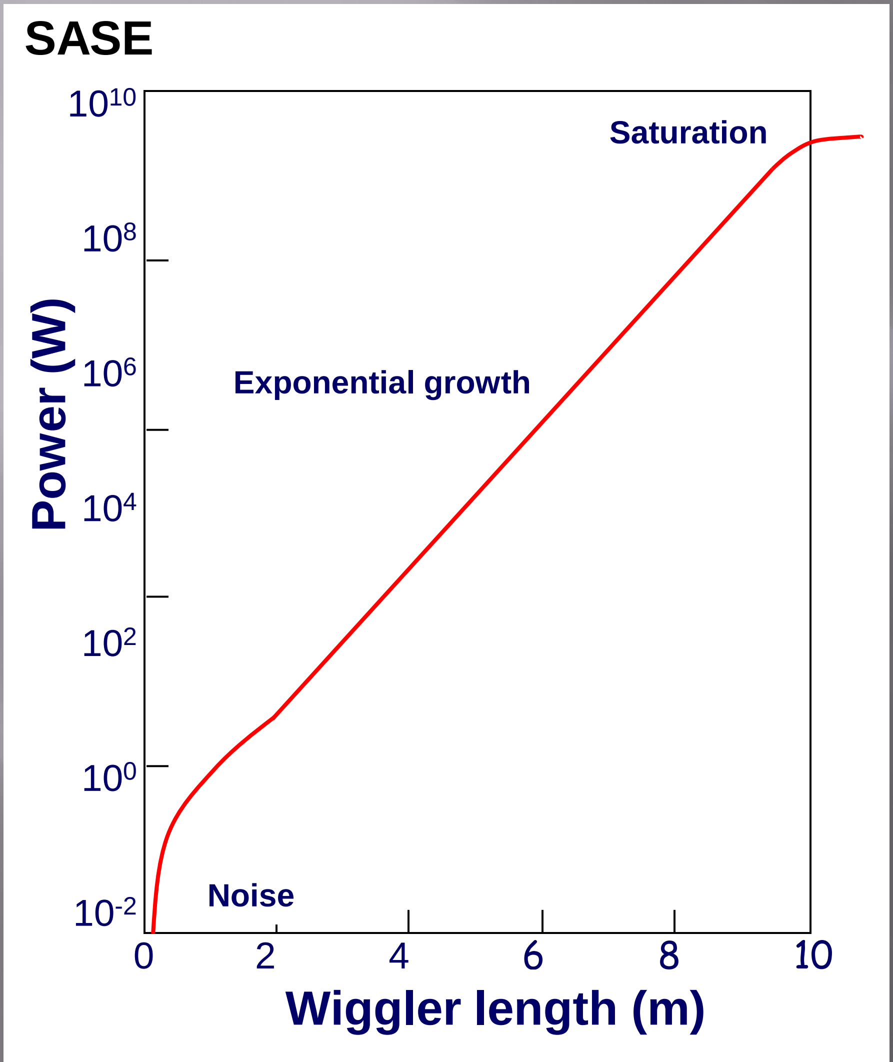
Power gain length

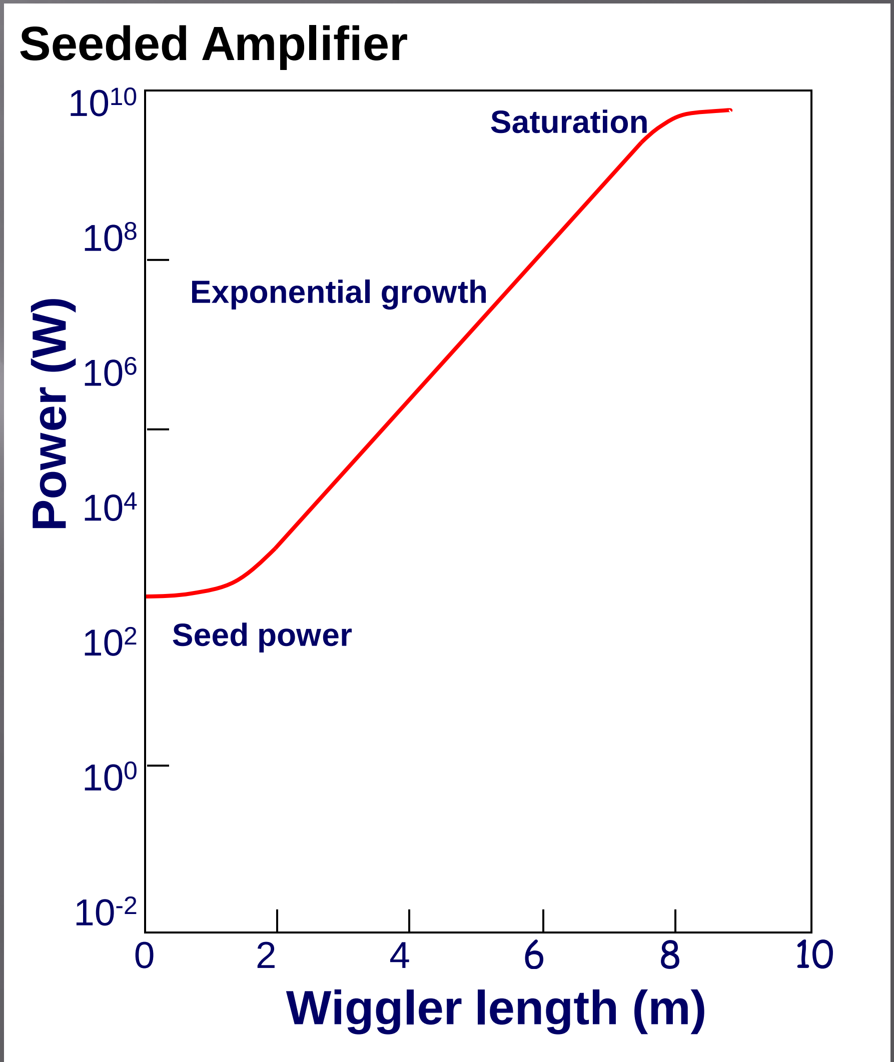
$$L_G = \frac{\lambda_w}{4\pi\sqrt{3}\rho}$$

Power versus z position in the wiggler

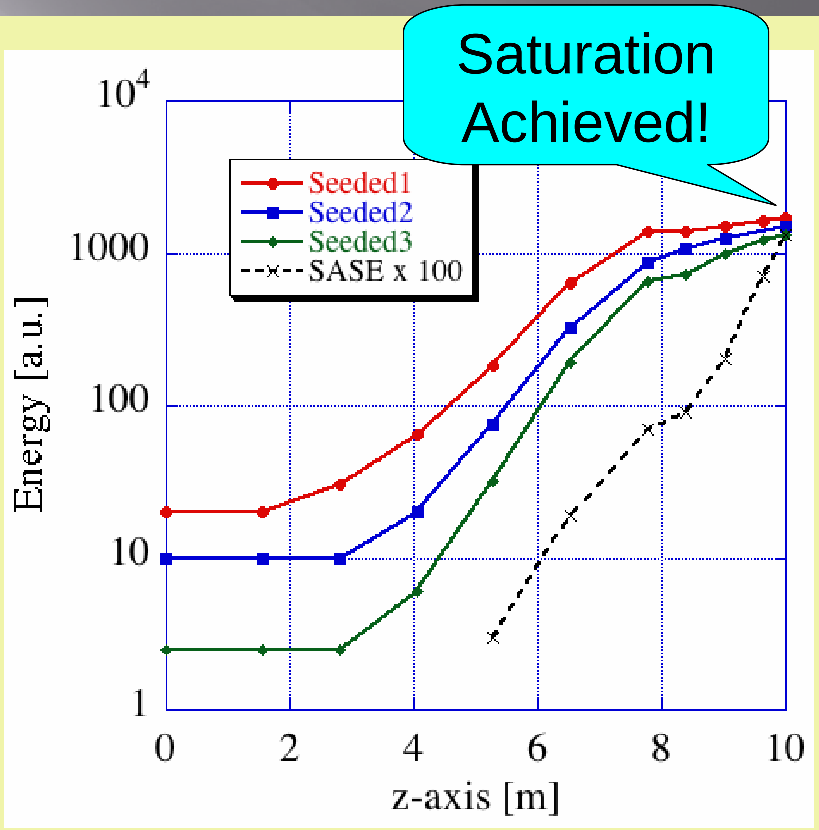


Seeded amplifier reduces wiggler length needed to saturate





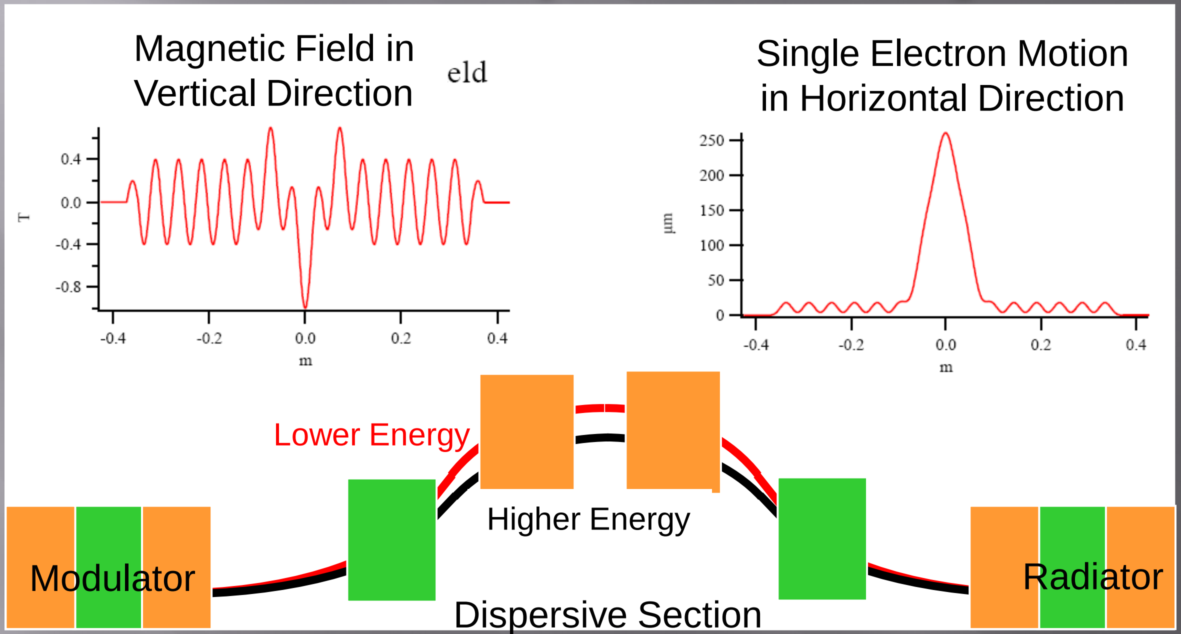
Seeded Amplifier Experiments with Different Seed Power

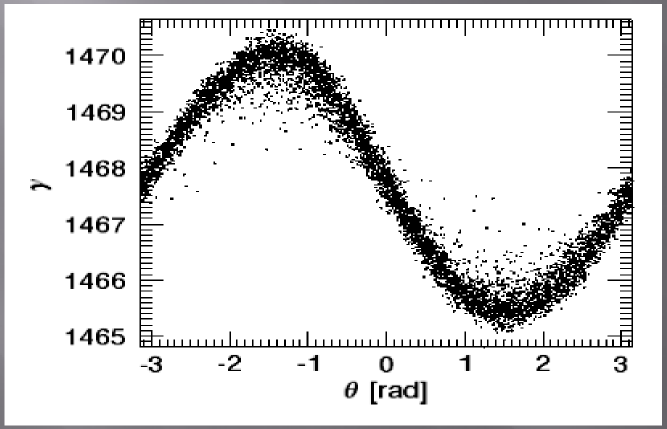


Three orders of magnitude FEL gain was observed.

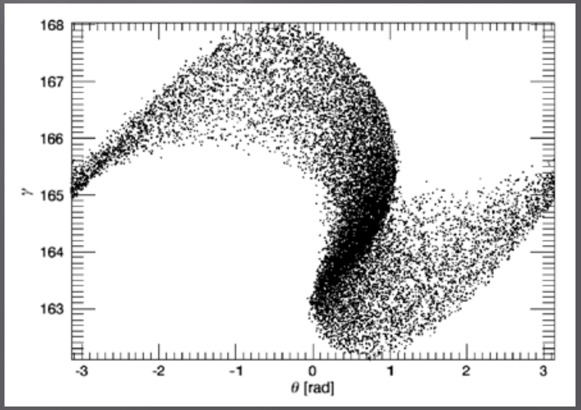
Courtesy of Jim Murphy

Pre-bunching Electron Beams with an Optical Klystron





Energy modulation



Density modulation

Prebunched Amplifier

- Inject electron beams that have density modulations into wiggler
- Reduce wiggler length needed to reach saturation
- Increase optical guiding and thereby relaxing beam's emittance requirement

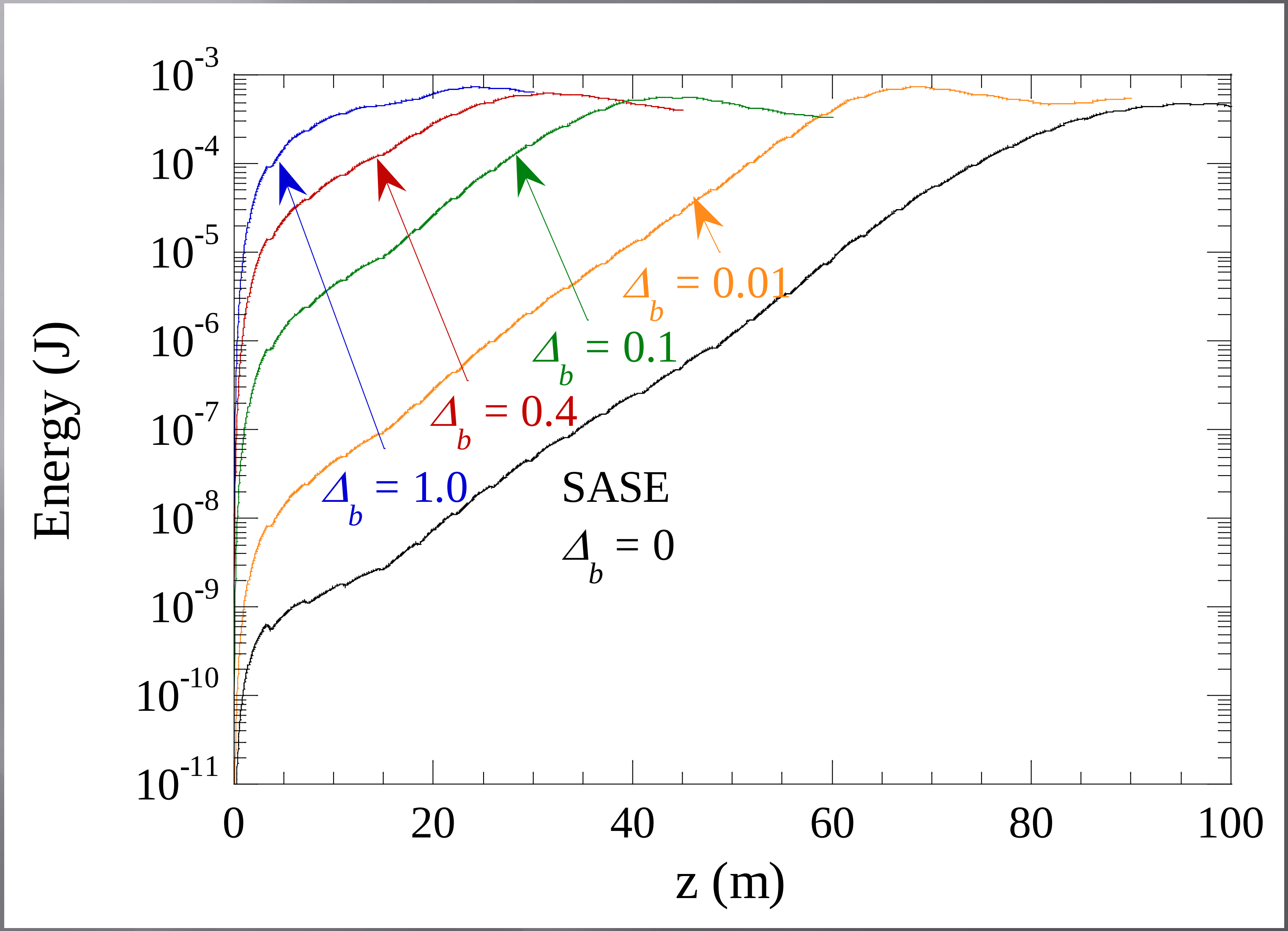
Start-up power without a seed laser depends on the number of correlated electrons, characterized by the initial bunching coefficient, and the resulting equivalent start-up power.

$$b_1 = \frac{1}{N_e} \sum_{k=1}^{N_e} e^{i\omega t_k(0)}$$

$$P_{prebunch} = 0.22 |b_1|^2 \rho \left(\frac{IE_b}{e} \right)$$

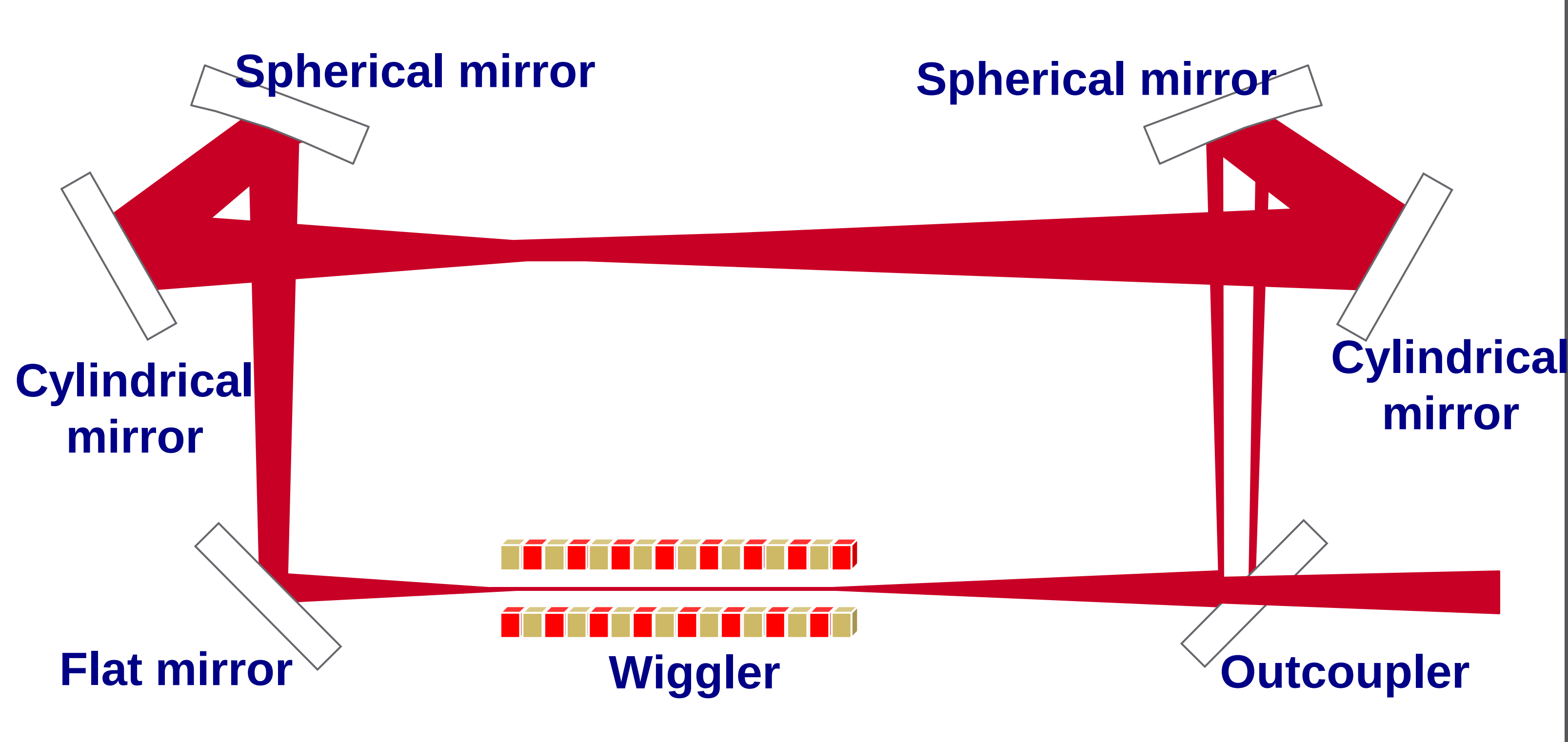
Since the saturated power is $P_{sat} = \frac{\rho I_p E_b}{e}$ the FEL amplifier needs to provide a single-pass gain of $G = \frac{1}{0.22 |b_1|^2}$ to reach saturation

Wiggler lengths needed to saturate vs prebunching factor

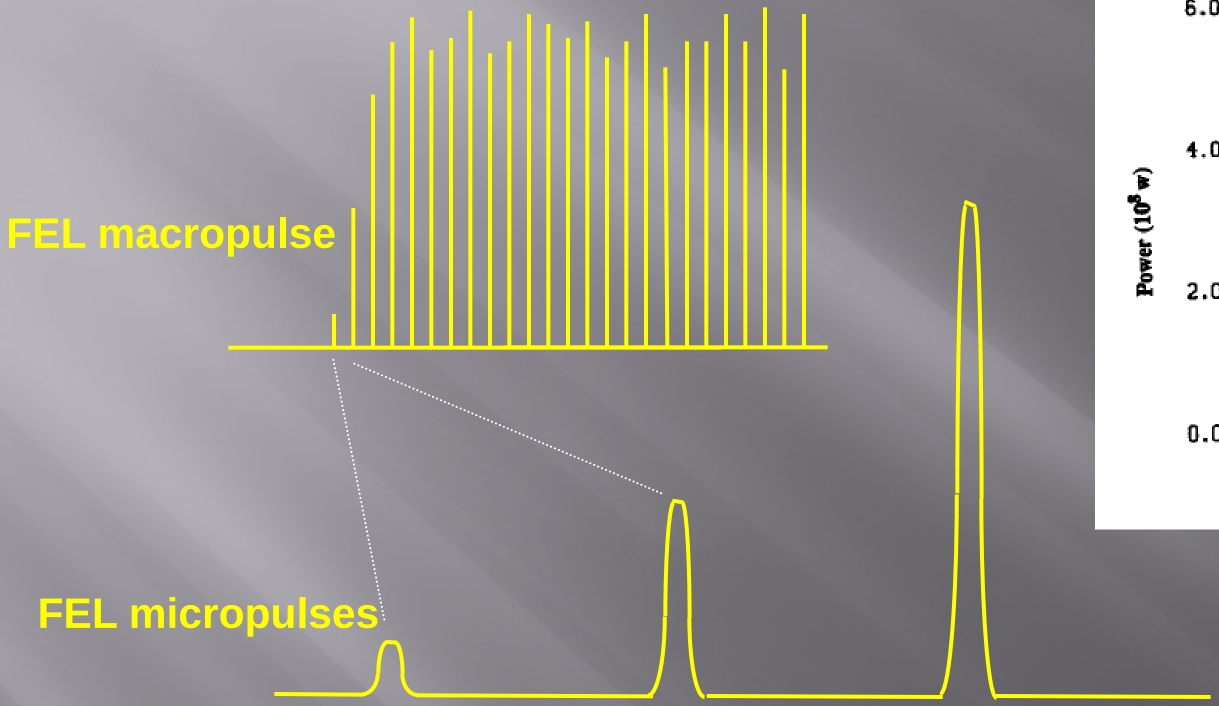


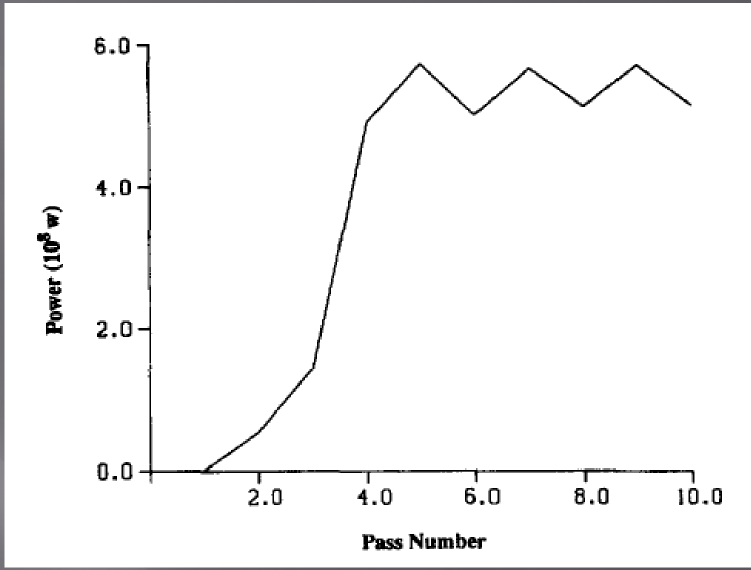
Regenerative Amplifier

- Use mirrors to feedback a small fraction (<1%) of FEL beam
- Similar to Seeded Amplifier except it does not need a seed laser
- Cavity length has to be multiple of electron bunch arrival time



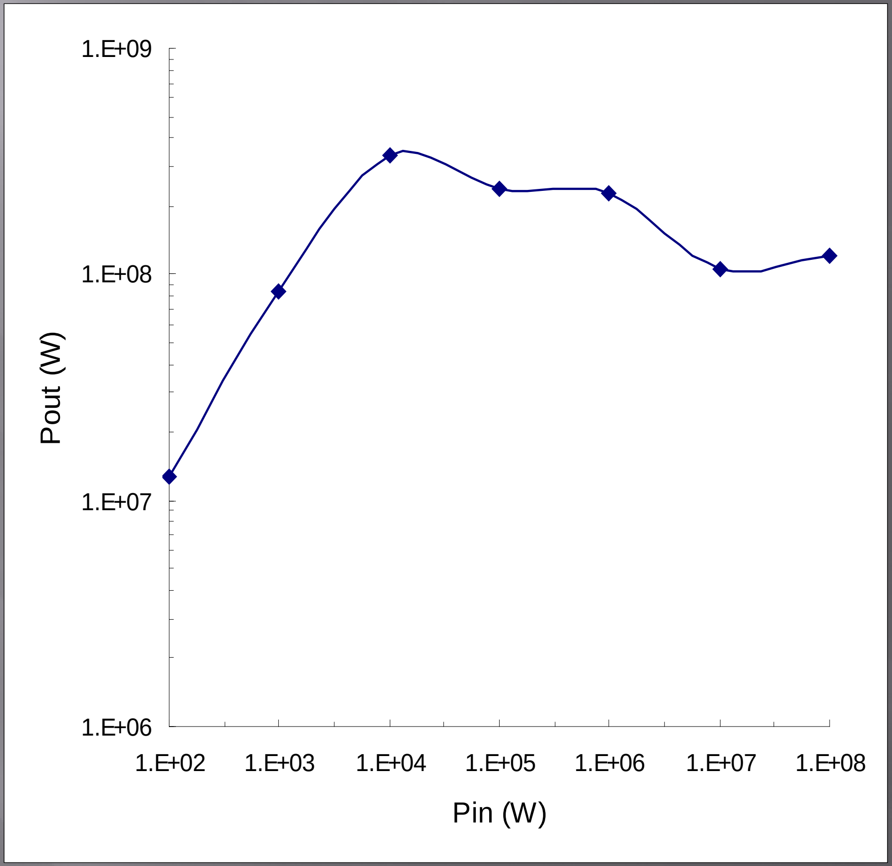
RAFEL saturates with a few electron bunches through the wiggler





RAFEL micropulse power fluctuates between high and low values, caused by feedback fraction exceeding the optimum value.

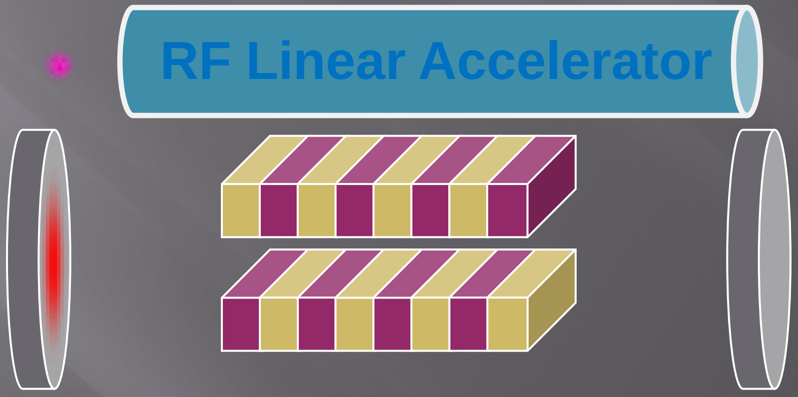
RAFEL output with high gains peaks at low feedback power



Optimum feedback fraction is only 10^{-4} which makes RAFEL possible with low-reflectivity mirrors (e.g., those at x-ray wavelengths).

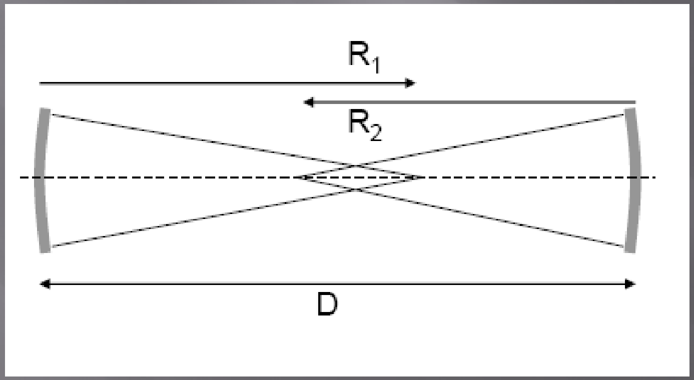
Oscillator FEL

- Use mirrors to feedback a large fraction (>50%) of FEL beam
- If cavity loss is low, the remaining power exits optical cavity
- Optical cavity determines the optical mode to a good approximation
- Cavity length has to be exactly multiple of electron bunch arrival time
- Oscillator has the highest average power than any other architectures



Optical Resonator

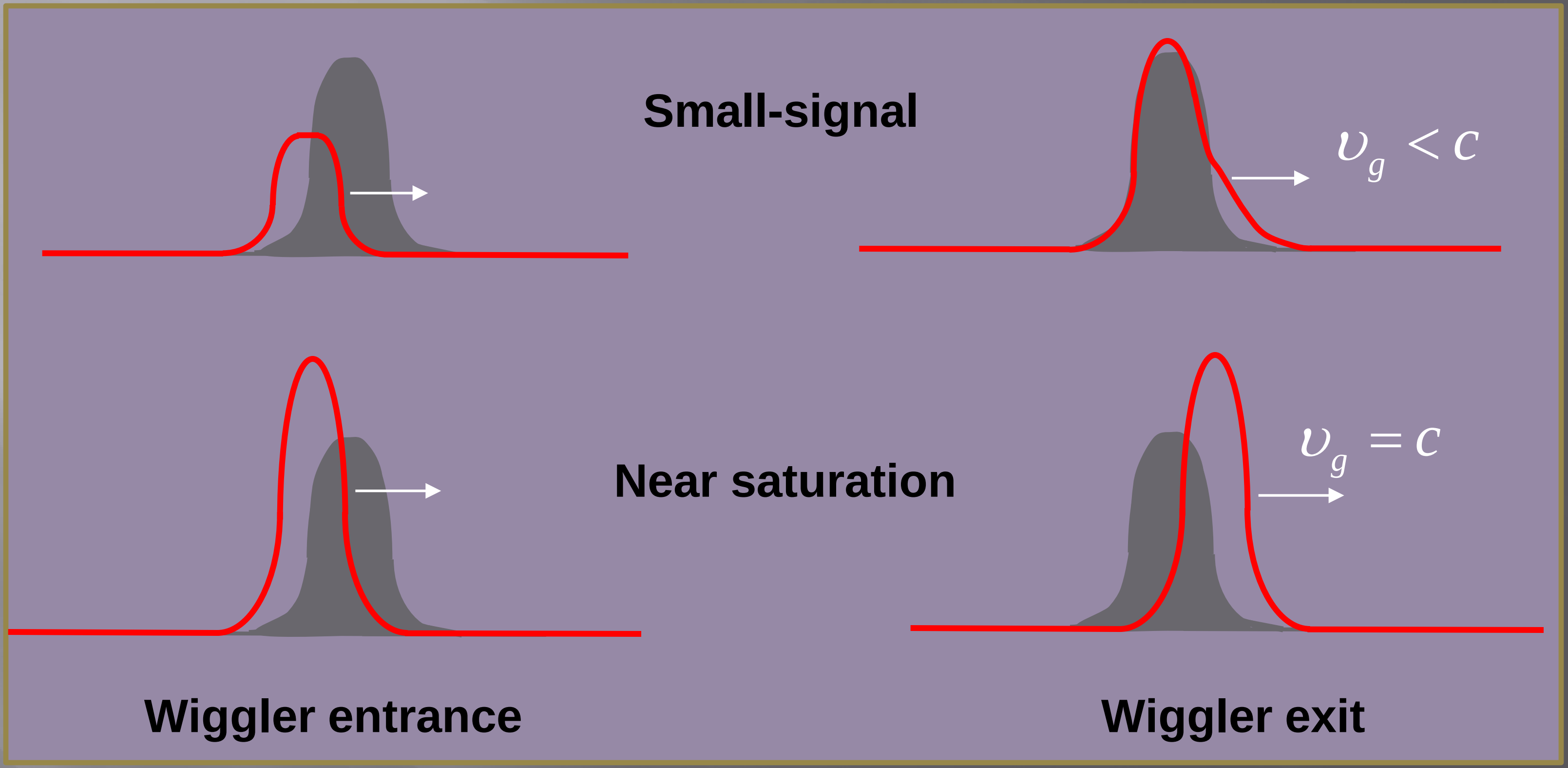
- The optical resonator consists of two concave mirrors with radii of curvature R_1 and R_2. The mirrors are separated by a distance D.



- A measure of stability is the overlap between two foci of the mirrors.
- The Rayleigh length is a measure of the FEL beam’s divergence. The shorter the Rayleigh length, the larger the divergence.

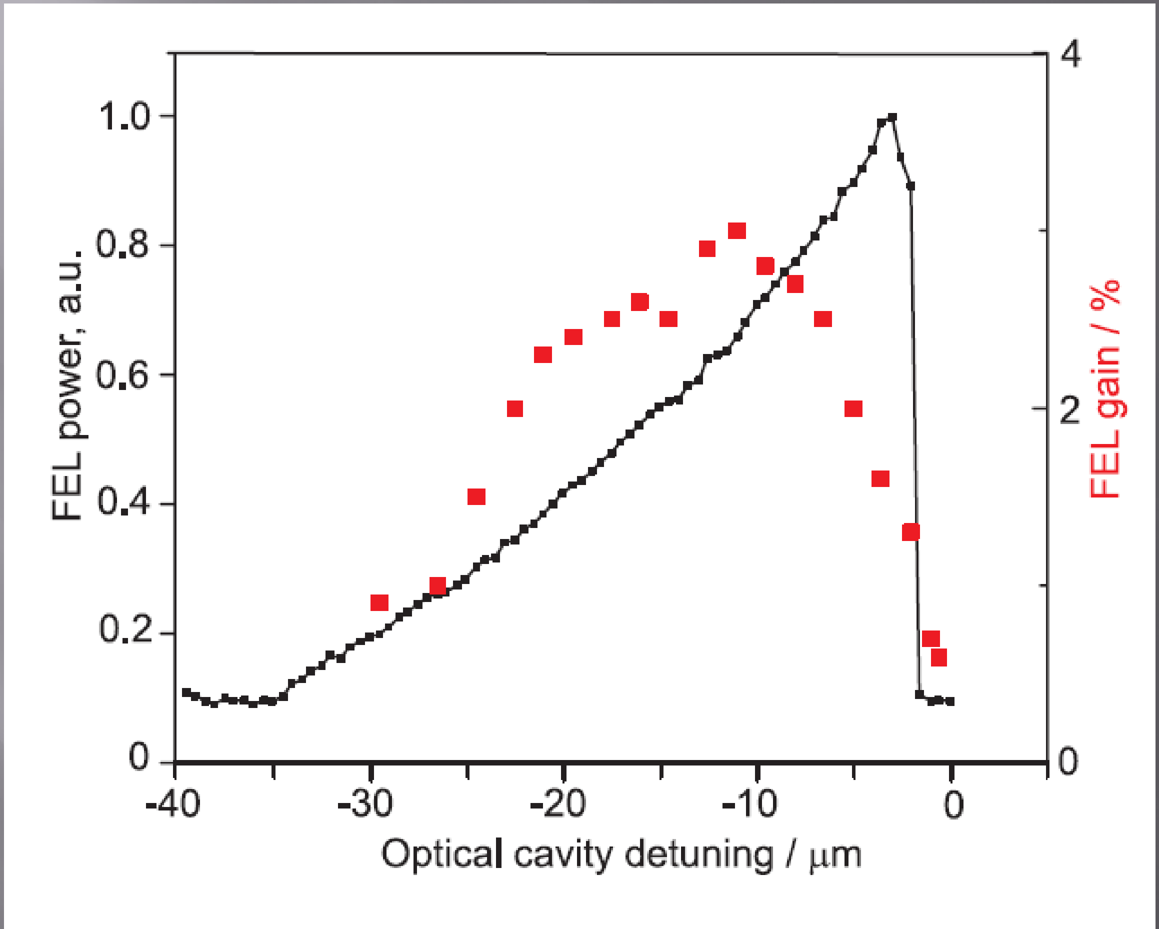
Slippage and Lethargy in FEL

Resonance condition dictates that the FEL pulse slips ahead of electron pulse one wavelength every wiggler period.



In the small-signal regime, FEL pulse is pulled back by slower electrons. Near saturation, FEL pulse travels at speed *c* as gain is reduced.

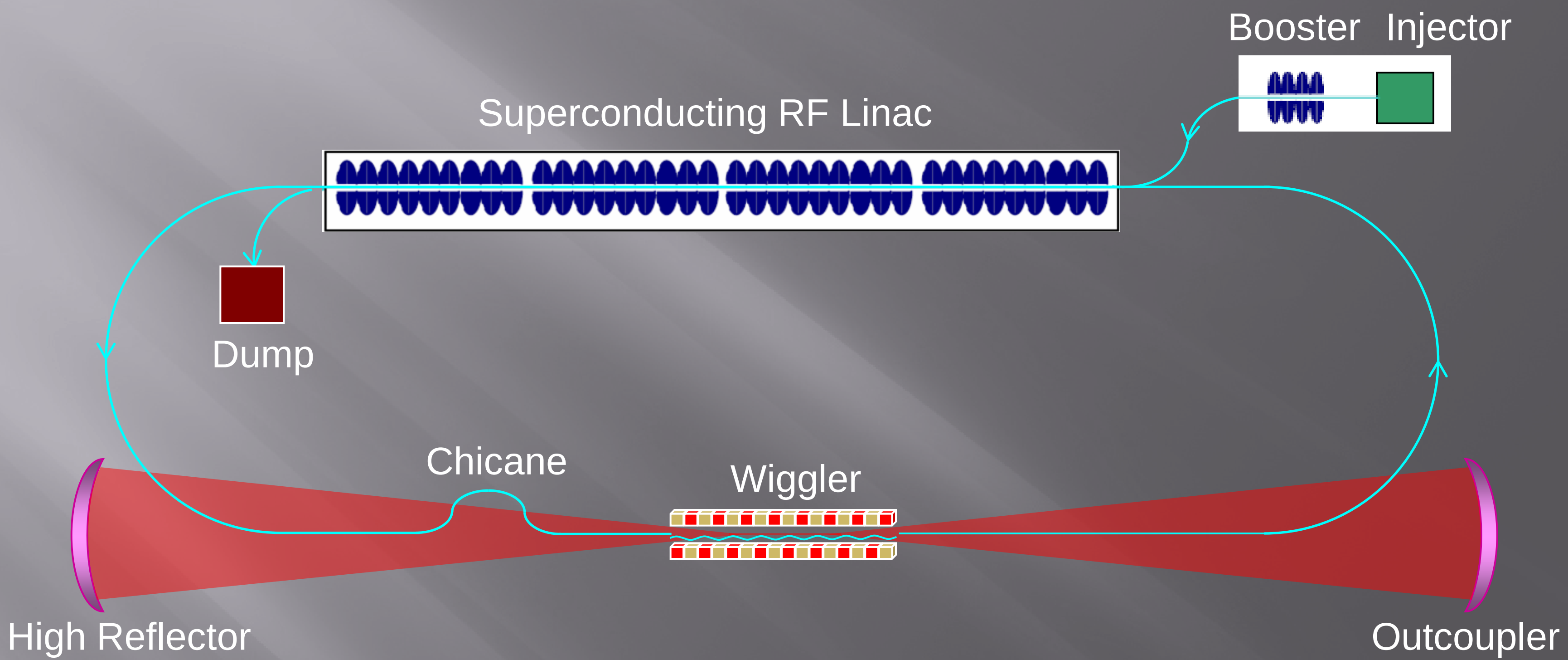
Cavity Length Detuning



Cavity length must be within 4-5 wavelengths of the correct length.

Cavity length for max gain is slightly shorter than that for max power.

Jefferson Laboratory FEL



Jefferson Laboratory FEL has produced 14kW, the world record of FEL average power, thanks to energy recovery linac design.

Summary of Part 6

SASE (Self-Amplified Spontaneous Emission) has the highest single-pass gain and starts with zero input optical signal (starts from noise).

Seeded Amplifier FEL uses an external laser or prebunching the electron beams

Regenerative Amplifier FEL relies on high single-pass gain and very small optical feedback.

Oscillator FEL has the lowest single-pass gain and relies on high reflectivity mirrors to provide sufficient optical feedback.

Most optical cavities for oscillator FEL are near-concentric resonator.

The cavity detuning length for oscillator FEL is only a few wavelengths.