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Title:	A Comparative Study of Staggered and Cell-centered Lagrangian Formulation for Multimaterial Hydrodynamics
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Los Alamos National Laboratory, Los Alamos, NM 87544, USA

Abstract

We compare a staggered Lagrangian formulation with a cell-centered Lagrangian formulation for a two-material compressible flow. In both formulation, we assume a single velocity field and rely on pressure relaxation techniques to close the system of equations. We employ Tipton's mixture model for both formulation. However, for the cell-centered formulation, employing Tipton's model for the mixture cell results in loss of conservation of total energy. We propose a numerical algorithm to correct this energy discrepancy. We test both algorithms on the two-materials Sod shock tube test problem and compare the results with the analytical solution.

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Lagrangian Hydrodynamics Equations

$$\rho\frac{D}{Dt}\left(\frac{1}{\rho}\right)-\frac{\partial u}{\partial x}=0$$

$$\rho\frac{Du}{Dt}+\frac{\partial p}{\partial x}=0$$

$$\rho\frac{DE}{Dt}+\frac{\partial}{\partial x}(pu)=0$$

+ Perfect gas equation of state

$$p=\rho e(\gamma-1)$$

Density ρ

Velocity u

Pressure p

Total energy $E=e+\frac{1}{2}u^2$

Specific internal energy e

Staggered Lagrangian Formulation with Tipton's Closure Model

- Velocity at nodes and pressure at cell-centers
- Standard predictor-corrector time integration scheme (Shashkov, 2008)
  - Half-time step: Node position, cell volume, densities, pressure (adiabatic approximation)
  - Final update: velocity, node position, cell volume, densities, specific internal energies, pressure
  - Pressure augmented by artificial viscosity (von Neumann and Richtmyer)
- Each material has its own mass, material interface may not coincide with mesh faces (mixed cells)
- Tipton's model for pressure relaxation after half-time step

$$\delta V^{n+1/2}=\sum_k \delta V_k^{n+1/2} \qquad p_i^*-\left(\rho_i^* c_i^{*2}\right)\left(1+\frac{L^*}{c_i^* \delta t}\right)\left(\frac{\delta V_i^{n+1/2}}{V_i^*}\right)=p^{n+1/2}$$

Speed of sound c
Length (cell-length) L
Volume V
Material indice k

- Consistent internal energies with total energy conservation

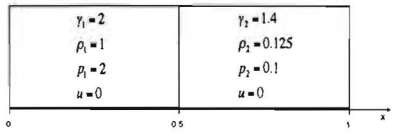
Cell-centered Lagrangian Formulation with Tipton's Closure Model

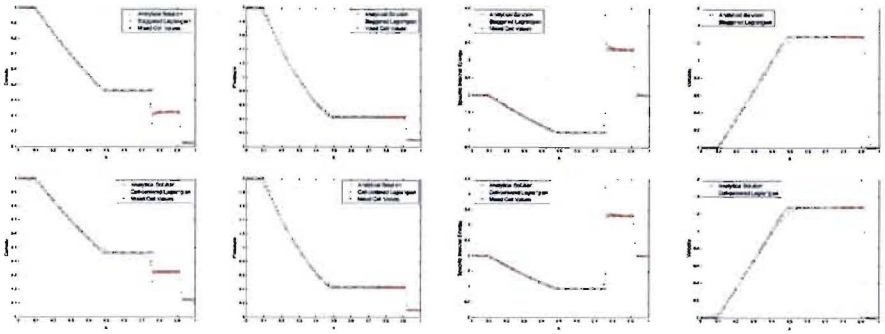
- Velocity and pressure at cell-centers
- 2nd order Godunov method with acoustic Riemann solver
- Predictor-corrector time integration scheme (Maire and Shashkov, 2008)
  - Half-time step: mass, momemtum, energy update with Riemann states u* and p* from time n data
  - Final update: mass, momemtum, energy update with Riemann states u* and p* from time n+1/2 data
- Each material has its own mass, material interface may not coincide with mesh faces (mixed cells)
- Tipton's model for pressure relaxation after predictor and corrector steps $p=\sum_k f_k p_k$
- Conservation of total energy by redistributing discrepancy in total internal energies to either material internal energies based on the sign of the discrepancy

References
Maire P.-H., Shashkov M., A one dimensional free Lagrange cell-centered- scheme, Los Alamos technical report. LA-UR-06-6867, 2006
Shashkov M., Closure models for multimaterial cells in arbitrary Lagrangian-Eulerian hydrocodes, International Journal for Numerical Methods in Fluids, 56:1497-1504, 2008

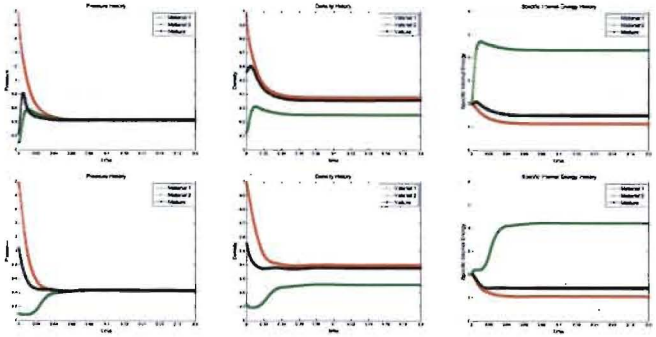
Two-material Sod Shock Tube

- perfect gas with different gammas
- one-dimensional, 100 cells
- mixed cell volume fractions f_1=0.5, f_2=0.5
- results at final time t=0.2





Time-history plots in the mixed cell with Tipton's mixture model



Staggered Lagrangian Formulation

Cell-centered Lagrangian Formulation

Observations and Future Work

- Differences are noticeable in the specific internal energy plots and in the time history plots.
- The transient mixed pressure is always within the two material pressures for the cell-centered Lagrangian formulation, which is not the case for the staggered Lagrangian formulation.
- Other correction algorithms to ensure conservation of total energy could be designed and give other time-history behavior.

Acknowledgments
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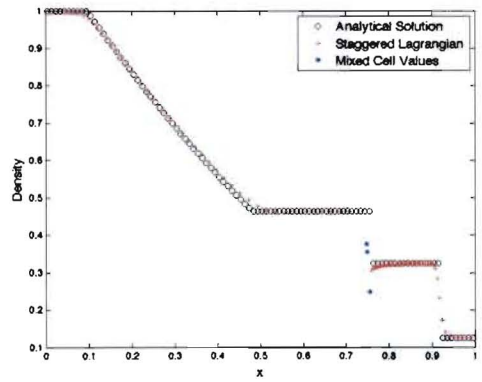
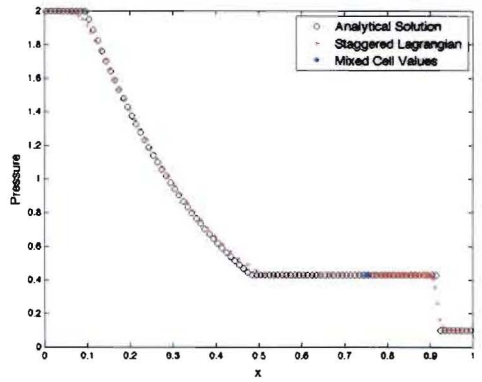
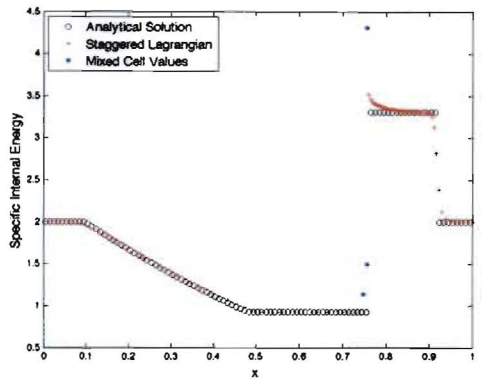
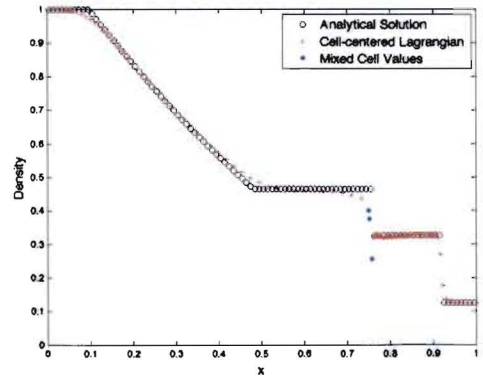
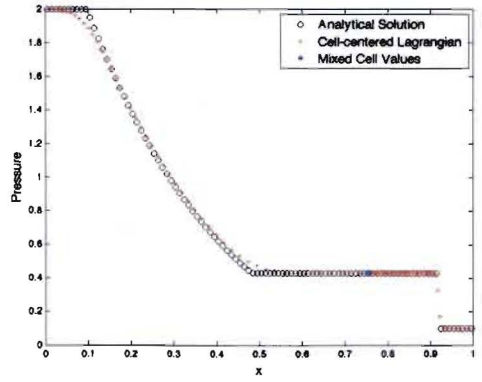
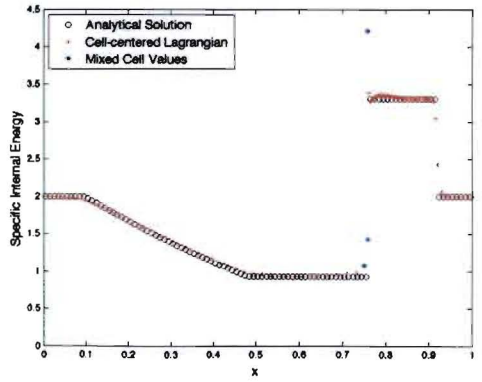

Analytical Solution
Staggered Lagrangian
Mixed Cell Values
Density
x

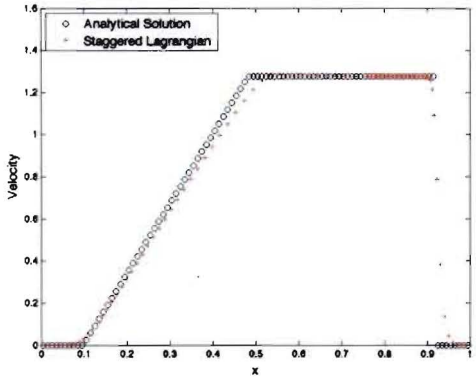

Analytical Solution
Staggered Lagrangian
Mixed Cell Values
Pressure
x

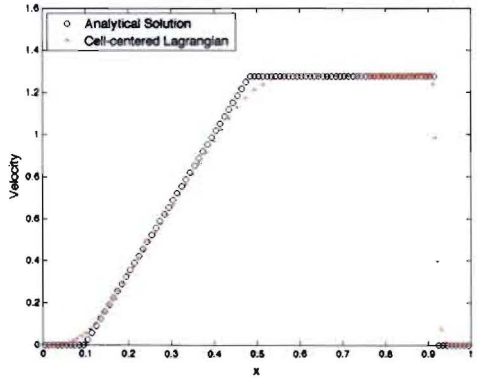

Analytical Solution
Staggered Lagrangian
Mixed Cell Values
Specific Internal Energy
x


Analytical Solution
Cell-centered Lagrangian
Mixed Cell Values
Density
x


Analytical Solution
Cell-centered Lagrangian
Mixed Cell Values
Pressure
x


Analytical Solution
Cell-centered Lagrangian
Mixed Cell Values
Specific Internal Energy
x


Analytical Solution
Staggered Lagrangian
Velocity
x


Analytical Solution
Cell-centered Lagrangian
Velocity
x

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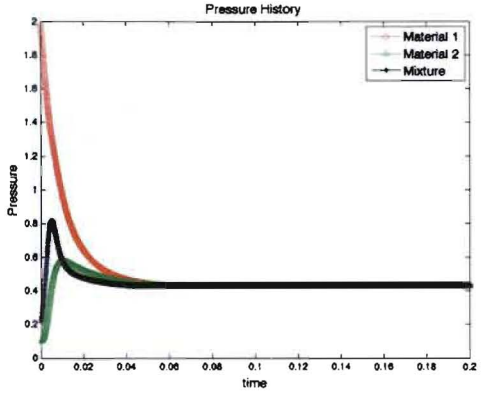
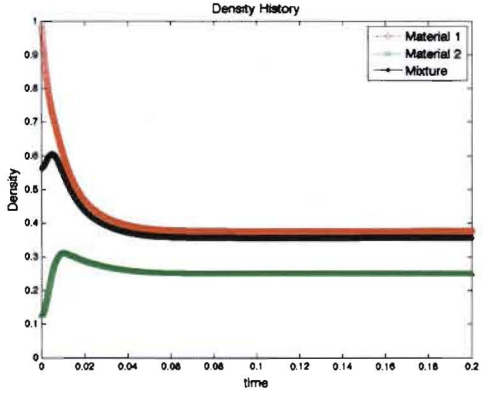
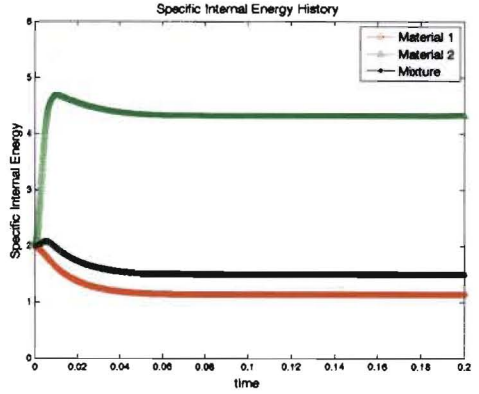
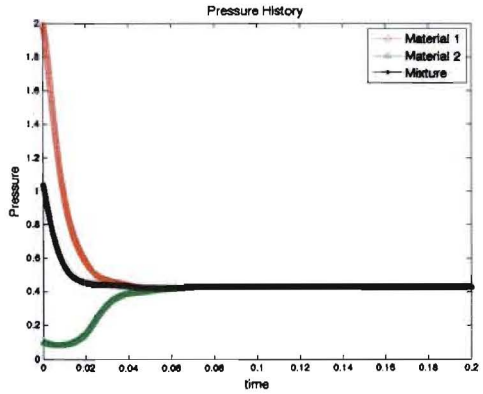
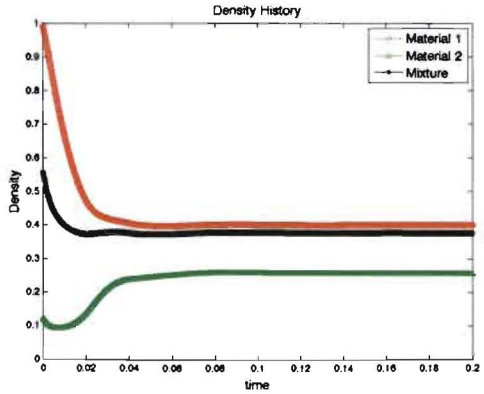

Pressure History
Material 1
Material 2
Mixture
Pressure
time


Density History
Material 1
Material 2
Mixture
Density
time


Specific Internal Energy History
Material 1
Material 2
Mixture
Specific Internal Energy
time


Pressure History
Material 1
Material 2
Mixture
Pressure
time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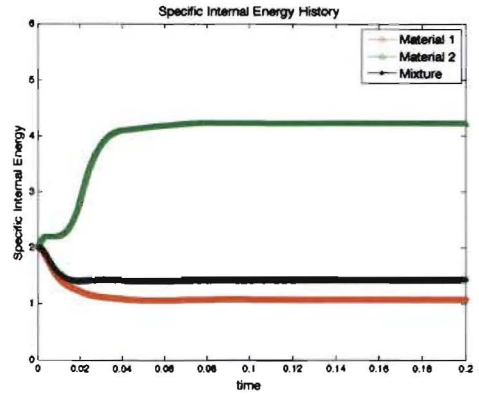
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