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Relating Confidence to Measured Information Uncertainty in Qualitative Reasoning

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***Abstract*—Qualitative reasoning makes use of qualitative assessments provided by subject matter experts to model factors such as security risk. Confidence in a result is important and useful when comparing competing results. Quantifying the confidence in an evidential reasoning result must be consistent and based on the available information. A novel method is proposed to relate confidence to the available information uncertainty in the result using fuzzy sets. Information uncertainty can be quantified through measures of non-specificity and conflict. Fuzzy values for confidence are established from information uncertainty values that lie between the measured minimum and maximum information uncertainty values.**

I. Introduction

Logic gate trees are used to enumerate an exhaustive set of possible scenarios for a system of interest. The identified scenarios each share common states of interest; although, each scenario has a distinct combination of values for these states of interest. In many applications the available values for the states are provided by subject matter experts (SME) and are qualitative. Approximate reasoning (AR) has been used on many engineering and control applications involving qualitative or imprecise data [17], [1], [12]. AR models emulate expert judgments [14] and it has been used in conjunction with logic gate models, using a series connected inferences, to draw conclusions about a particular criterion of interest common among all the identified scenarios. Bott et al. extended AR to logic gate trees [2] which has been used to model security [7] and risk [6], [3]. AR can be used to draw conclusions from vague or imprecise representations of the states of interest involved in the scenario. Similar to AR, evidential reasoning (ER) is used with logic gate trees as an alternative approach to draw conclusions about a certain aspect of a system; however, ER is used when it is uncertain which qualitative value represents the state of interest. One major difference between the AR and ER-logic gate model approaches is in the uncertainty quantified.

The imprecision associated with describing a specific state of interest x_i qualitatively can be captured with the degree of membership of x_i in the fuzzy set while the uncertainty associated with assigning x_i to one qualitative value over another can be captured through the expert's degree of belief, or basic evidence assignment that x_i is a particular set [9]. AR-logic gate models use the degree of membership in fuzzy sets in the inferences while ER-logic gate models use the the expert's degree of belief in the inferences. AR-logic gate models produce simple results consisting of a vector of qualitative set values and the degree of membership of x_i in each of the qualitative set values see Chavez et al. [4]. A simple of example of a vector consisting of qualitative values for economic consequences and their respective degree of membership are as follows:

$$\vec{R} =$$

$$[verylow(0), low(0), medium(0), high(0.25), veryhigh(0.75)]$$

ER-logic gate models also produce results consisting of a vector of qualitative set values; however, instead of the degree of membership, the degree of belief of x_i in each qualitative set values is used. The uncertainty associated with imprecise boundaries of the qualitative sets is not addressed in ER and is discussed in Section II. Fuzzy ER has been proposed [20]; however, it does not quantify both assignment and linguistic uncertainties present. This study refers collectively to both AR and ER-logic gate models as qualitative reasoning models and the reader is referred to Chavez et al. [4] for a broader discussion on these qualitative models. Separate underpinings are provided for the confidence obtained using the AR and ER models results.

An issue of concern for competing scenarios and their vector results for a particular criterion of interest is the confidence level associated with each vector result produced. The confidence level identified here quantifies how believable the result is based on the available data. It is similar to the Bayesian statistical interpretation of confidence level [11], in that it answers how believable the result is in containing the true, based on the the available information. In this sense, the confidence level is distinct from the frequentist statistical interpretation of the confidence level which is associated with the percentage of confidence intervals containing the true value [18] and is based on a potentially infinite number of trials. The desired confidence level associated with each vector result should convey how believable is the result and should not have a greater precision than the available data used to determine the vector result; thus a qualitative value for confidence is proposed in this study.

Lui and Lui [13] have proposed measures for the credibility on fuzzy set values and Peng et al. [16] applied credibility on fuzzy variables. They define credibility on fuzzy variables as the expected value of a membership function of a fuzzy set and is thus not relevant here. These approaches are not applicable here, as the output of the AR-logic gate model is focused on a specific resulting state represented qualitatively using the degree of membership in each fuzzy set which does not involve or include all the states included in the entire fuzzy set. Therefore an approach is proposed in here to obtain a measure of confidence in the result from the available uncertainty in the model. Chavez et al. [4] have quantified the information uncertainty associated with either an AR or ER-logic model results. The available uncertainty in a result is related to confidence [5] and Chavez et al. suggested that the greater the quantity of information uncertainty the lower the confidence. However, Chavez et al. do not extend the quantification of information uncertainty to a measure of confidence. Their work is further developed here by relating a qualitative measure of confidence to the measured quantity of information uncertainty present in the vector result.

Before proceeding to the presentation of the proposed approach, a brief overview of both AR and ER-logic gate models is provided in section II. In section III the methods used to quantify information uncertainty in AR and ER-logic gate results are presented followed by section IV which introduces the proposed method used to determine the confidence level from the measured information uncertainty in a vector result. The proposed approach provides a novel means to quantify confidence in a AR or ER-logic gate vector result which is necessary for segregating and ranking competing scenarios. The significance and findings of the proposed method are discussed further in section V.

II. Qualitative Methods

AR or ER methods are used in conjunction with logic gates models to draw conclusions about a particular system whose components, or contributing variables, states have various possible qualitative values. A detailed discussion on the AR and ER processes involved in obtaining the results is beyond the scope of this paper and the reader is referred to [4], [7], [3] for a thorough discussion. Only a brief overview of the qualitative reasoning approaches is provided here. Qualitative methods consist of two parts: (1) a logic gate model and (2) an inference model. The function of a logic gate model (see [2]) is to enumerate all the possible scenarios for the system under investigation. While an inferential model (see [7]) is created to draw conclusions about an outcome or criteria of interest, z, such as risk, where each identified scenario has a specific outcome state z_i. Each scenario consists of several connected states of interest, x and y, and the qualitative values for each of the specific states of interest x_i and y_i involved in the scenario contributes to the value of z_i. A simple inferential model is used to draw conclusions about the value of z_i from the available qualitative values of x_i and y_i involved in a particular scenario.

A. *Approximate Reasoning*

An AR model is a type of inferential model which uses rules combined into a series of rule bases, developed from SMEs, to draw conclusions from the available information. The AR approach is primarily intended for systems consisting of qualitative values, imprecisely or vaguely defined linguistic set values, with the uncertainty referred to as fuzzy or linguistic uncertainty. A fuzzy set is denoted as $\tilde{A}$ and the boundary of set is imprecise or fuzzy. The uncertainty associated with describing x_i imprecisely with $\tilde{A}$ is captured using the degree of membership of x_i in $\tilde{A}$, $\mu_{\tilde{A}}(x_i)$. AR approaches are simple in that it is not necessary to define the fuzzy sets through the entire membership function for each fuzzy set. The method is simplified by only requiring the degree of membership for the specific state of interest in each fuzzy set, which can be elicited from the SME. If a specific state x_i is a member of the $\tilde{A}$, then this mapping is given by Equation 1.

$$\mu_{\tilde{A}}(x_i) \in [0,1] \quad (1)$$

The complement of $\tilde{A}$ is defined in 2:

$$\mu_{\bar{\tilde{A}}} = 1 - \mu_{\tilde{A}}(x_i) \quad (2)$$

A simplified AR-logic gate tree model result is provided here. For example, conclusions are drawn about security risk for each scenario from the resulting imprecise values for success likelihood and economic consequences. Each scenario produces a specific outcome state for security risk which is assessed using AR. The identified linguistic values for security risk consist of "very low", "low", "medium", "high", and "very high" while the vector result for security risk for three competing scenarios was determined to be:

Scenario A: Security Risk [0, 0, 0.57, 0, 0]
Sceanrio B: Security Risk [0, 0, 0.75, 0.2, 0.1]
Scenario C: Security Risk [0, 0.15, 0.85, 0, 0]

The three scenarios produce a medium security risk result and there is a different level of confidence associated with each result.

B. *Evidential reasoning*

Alternatively, ER is focused on assignment uncertainty or the uncertainty associated with assigning a x_i to particular but well defined linguistic sets, A. The SME's degree of belief that x_i is a particular qualitative value captures the uncertainty in assigning x_i to a particular value and is referred to as assignment uncertainty. The SME assigns x_i to the linguistic sets of the power set $P(X)$, i.e. the set of all subsets of X, and associates a degree of belief with each assignment. The SME's degree of belief that x_i is a particular A_j is called the basic probability assignment or basic evidence assignment *bea*. The uncertainty associated with imprecisely describing x_i linguistically is not quantified with the *bea*. The *bea*, (m), must satisfy the following boundary conditions:

$$m(\emptyset) = 0 \quad (3)$$

$$\sum_{A_j \in P(X)}^{j=1,2,3\cdots,n} m(A_j) = 1 \qquad (4)$$

Equation 3 indicates that the *bea* assigned to the null set is equal to 0 and Equation 4 indicates that the sum of all the *bea* must equal 1. The *bea* is distinct from probability in that it is not required to satisfy the excluded middle axioms and it is defined on the $P(X)$ rather than X [9]. The *bea* used here does not involve the *bea* assigned to the entire set but rather the *bea* that x_i is a particular A_j. For each identified scenario, an ER-logic gate model produces a vector result comprised of various A_j and their associated *bea* for the output state of interest z. As an example consider a specific outcome state for three different scenarios which is to be assigned a qualitative value for *effectiveness of physical inventory*. For each scenario, there are four identified linguistic values for *effectiveness of physical inventory* (from left to right) *not applicable*, *low*, *moderate*, and *excellent*. In each scenario, there is an associated *bea* with each linguistic value. For example, three scenarios are provided to illustrate the difference in the ER vector results.

Scenario 1:Effectiveness of phys. inventory[0, 0.1, 0.90, 0]
Scenario 2:Effectiveness of phys. inventory[0, 0.1, 0.85, 0.05]
Scenario 3:Effectiveness of phys. inventory[0, 0, 1.0, 0]

In which of the two scenarios can the decision makers have the most confidence.

III. Information Uncertainty

A logic gate tree combined with a qualitative reasoning model produce numerous scenarios, each with an associated vector result. A consistent means to segregate and compare competing results is critical in areas such as asset allocation for such areas as security risk assessment. Moreover, decision makers are interested in the level of confidence associated with each result. Chavez et al. [4] have proposed using information uncertainty to compare qualitative reasoning vector results. Information uncertainty is comprised of *conflict* and *non-specificity* [9], both of which can be quantified in the vector results [4].

Shannon first addressed the quantification of information uncertainty, or entropy, in 1948 [19]. His proposed measure quantifies conflict involved in random uncertainty involving probability. The importance of information uncertainty is demonstrated with the following example. Consider a normal die having six faces, all of which are equally likely to be thrown, and there exists a six sided trick die with one side being twice as likely to be thrown as the remaining five sides. The regular die has a greater quantity of conflict than the trick die because all sides are equally likely to occur in the regular die. The trick die is less uncertain because one side is twice as likely to be thrown as each of the remaining five; thus, one can have more confidence in the trick die.

Klir and Wierman [10] extended Shannon's measure of conflict to evidence theory and Chavez et al. [4] further extends the measure to vector results, i.e. x is A_j. Conflict in an ER vector result is calculated using Equation 5, where $\vec{R}$ is the resulting vector.

$$C(\vec{R}_{ER}) = -\sum_{j=1}^{n} m_{A_j}(x) log_2 m_{A_j}(x) \qquad (5)$$

Pal and Bezdek [15] provide an overview of the numerous measures available to measure the conflict due to the fuzzy uncertainty associated with a membership function of a fuzzy set. Previous applications involving the quantification of information uncertainty involved all the possible states, elements, described by a particular fuzzy set and Chavez et al. extended the quantification to situations involving one state described linguistically using various fuzzy sets. Conflict in AR vector result is calculated through Equations 6.

$$C(\vec{R}_{AR}) = -\sum_{i=1}^{n} \mu_{\tilde{A}_i}(x) log_2 \mu_{\tilde{A}_i}(x) + \mu_{\bar{\tilde{A}}_i}(x) log_2 \mu_{\bar{\tilde{A}}_i}(x) \qquad (6)$$

Another type of information uncertainty, identified by Hartley [8], is associated with the ambiguity in specifying the exact solution and is referred to as the non-specificity [9]. This lack of specificity is simply related to the number of available alternatives. Chavez et al. proposed a measure for non-specificity in an AR or and ER vector result is thus quantified using Equation 7 related to the number of alternatives.

$$N(\vec{R}) = log_2|R| \qquad (7)$$

Where R is the number of linguistic sets in the resulting AR or ER vector having a non-zero degree of membership or non-zero *bea*, respectively.

A. Information Uncertainty in AR vector results

Consider the following simplified inference model involving the expected economic consequences for a terrorist attack. A series of connected inferences are used to determine aggregate consequences from qualitative values for the identified states in the scenario; likewise, the likelihood of successful attack is determined from the identified states in the scenario. The resulting *economic consequences* and the resulting *likelihood of a successful attack* for each scenario are ultimately used to draw conclusions about the risk. Figure 1 provides the degree of membership values for the resulting states of antecedents, likelihood of successful attack and economic consequences, which are used to draw conclusions about the consequent, expected risk. Equations 6 and 7 are used to quantify the conflict and non-specificity associated with the AR vector result. Note the linguistic values identified in Figure 1 are not complements of one another.

$$\begin{aligned} C(\vec{R}_{AR}) = -(0.57 * log_2 0.57 + 0.43 * log_2 0.43 + \\ 1 * log_2 1 + 1 * log_2 1 + \\ 1 * log_2 1 + 1 * log_2 1 = \\ 0.9868 \end{aligned} \qquad (8)$$

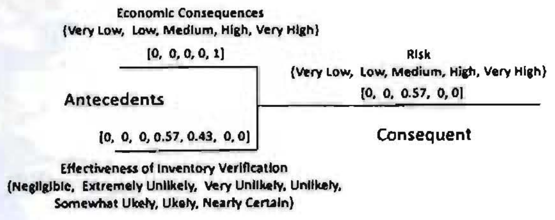

Fig. 1. Simplified Inferential AR Model

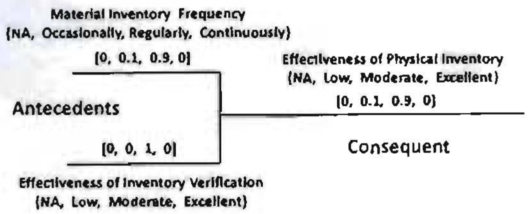

Fig. 2. Simplified Inferential ER Model

$$N(\vec{R}) = log_2|1| = 0$$

B. Information Uncertainty in ER vector results

Consider the following ER inference involving the consequence *effectiveness of physical inventory* for a facility which will ultimately be used to determine the facility vulnerability. The *material inventory frequency* and *effectiveness of inventory verification* for each scenario are antecedents used to determine the *effectiveness of phyisical inventory*. Figure 2 provides the degree of belief values for the states involved in the antecedents which are used to determine the the degree of belief for the consequent qualitative values describing the resulting state. Equations 5 and 7 are used to quantify the conflict and non-specificity associated with the ER vector result.

$$C(\vec{R}_{AR}) = -(0.1 * log_2 0.1 + 0.9 * log_2 0.9) = 0.4689$$

$$N(\vec{R}) = log_2|2| = 1.0$$

IV. Fuzzy Confidence

The correlation between a qualitative value of confidence and the quantity of information uncertainty is developed in this study. A consistent method is proposed which relates confidence to the quantity of information uncertainty. Measured values of conflict and non-specifity are correlated to qualitative confidence values using maximum and minimum potential values of information uncertainty. The conflict and non-specificity values each characterize a unique aspect of information uncertainty; however, in the case of a uniform distribution, i.e. the *bea* values are each equivalent to 1/n, conflict and non-specificity are equivalent [9]. Similarly, when only one alternative is possible, i.e. *bea* is equal to 1 for the single alternative, the conflict and non-specificity are again equivalent. This characteristic has led some researchers to consider non-specificity as a special case of conflict; however, this consideration is ill conceived as non-specificity quantifies a different aspect than does conflict [9]. An aggregate uncertainty quantification has been previously proposed [10], but it is the intent of this study to segregate the two information uncertainty metrics to maintain their individual significance.

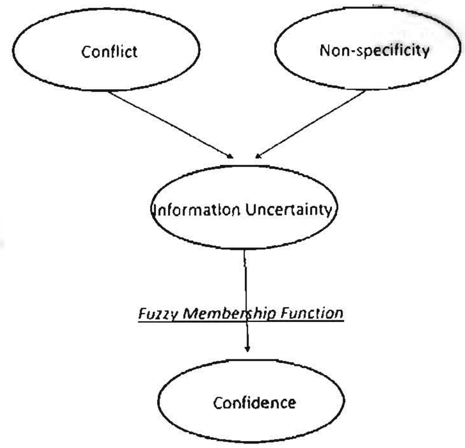

Fig. 3. Relationship for Determining Confidence

Each competing scenario in a family of sceanrios share the same possible qualitative values, i.e. the linguistic values *NA*, *Low*, *Moderate*, and *Excellent* for Effectiveness of Physical Inventory in Figure 2; thus, the maximum and minimum values for conflict and non-specificity for the family of scenarios can be determined. Similarly, the conflict and non-specificity for each individual scenario vector result can also be calculated. As illustrated in Figure 3 and described, here the measured values of conflict and non-specificity can then be related to qualitative values for confidence.

Maximum conflict and non-specificity occur in an ER vector result that contains equivalent *bea* values for each n qualitative values in the vector. Minimum conflict and non-specificity occur in an ER vector result with one possible qualitative value having its *bea* equal to 1. Similarly, maximum conflict and non-specificity in an AR result, with a resulting vector consisting of non-complementary qualitative values, occurs when each qualitative set in the vector has a degree of membership equivalent to 0.5. The domain of AR vector result may not necessarily include the linguistic set and its complement and therefore it may not sum to 1. Whereas the domain of an ER vector result is considered the power set and should sum to 1. Equation 6 accounts for the degree of membership of the state of interest in a particular linguistic set and in its complement with maximum conflict produced when there is 0.5 membership in both the linguistic set value and its complement for all linguistic sets in the idenitifed domain. Equation 9 is proposed as a means to relate the quantified

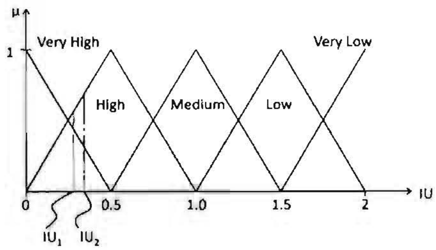

Fig. 4. Membership function for confidence

conflict and non-specificity to information uncertainty (IU). In Equation 9 the measured values for conflict and non-specificity for each scenario are each normalized by their respective maximum value for the family of scenarios which are then summed into a combined measure.

$$IU(\vec{R}) = \frac{C(\vec{R})}{C(\vec{R}_{max})} + \frac{N(\vec{R})}{N(\vec{R}_{max})} \quad (9)$$

An IU value of 0 is produced when there is no conflict or non-specificity in the result which is considered the minimum IU value while the maximum IU value of 2 occurs when both conflict and non-specificity have maximum value. These values are used as the upper and lower bound values for the confidence, see Figure 4, and they confine the confidence linguistic values *Very High, High, Medium, Low, Very Low.* Figure 4 provides the membership functions on each of the identified fuzzy qualitative value sets. The determined value of IU for each scenario is mapped to a specific value of confidence which permits comparison of each scenario on the level of confidence. The manner in which the resulting information uncertainty, IU_1 and IU_2, is mapped to confidence is illustrated in Figure 4.

A. Confidence in AR and ER results

The proposed approach is illustrated here using the AR and ER vector results provided in Section II. Equations 6 and 7 are used to calculate the conflict and non-specificity, respectively, involved in the AR result for Scenarios A, B and C. The maximum conflict and non-specificity present in the set of scenarios is determined to be 5 and 2.322, respectively. Note, the qualitative sets provided are not considered complementary of one another. The resulting values for conflict, non-specificity, and the associated IU, are provided in Table I while the confidence associated with the determined IU is provided in Table II. The resulting confidence vector is contained in the linguistic sets [Very High, High, Medium, Low, Very Low] and their respective degree of membership for the resulting confidence state, determined from the IU and the confidence membership functions, in each linguistic value is provided in Table II along with the resulting linguistic value.

Equations 5 and 7 are used to calculate the conflict and non-specificity, respectively, involved in the ER result for Scenarios 1, 2, and 3. The maximum conflict and non-specificity present in the set of scenarios is determined to be 2 and 2, respectively,

TABLE I
CONFLICT, NON-SPECIFICITY AND RESULTING INFORMATION UNCERTAINTY IN THE AR RESULTS.

Scenario	*Conflict*	*Non-Specificity*	*IU*
Scenario A	0.986	0	0.197
Scenario B	2.002	1.585	1.083
Scenario C	1.220	1.000	0.675

TABLE II
INFORMATION UNCERTAINTY AND CONFIDENCE IN AR RESULTS.

Scenario	*IU*	*Confidence Vector*	*Confidence*
Scenario A	0.197	[0.606,0.394,0,0,0]	Very High to High
Scenario B	1.083	[0,0,0.834,0.166,0]	Medium
Scenario C	0.675	[0,0.65,0.35,0,0]	High to Medium

which is produced when each qualitative value equivalent *bea* values. Table III provides the conflict, non-specificity, and the associated IU, while Table IV provides the resultant confidence for Scenario 1, 2, and 3.

The results demonstrate the utility of relating qualitative confidence levels to information uncertainty to compare competing AR scenario results or ER scenario results. In the ER example illustrated, (see Table III) the three scenarios result in a *mostly moderate* qualitative value for the effectiveness of physical inventory; however, there is an observable difference in each resulting vector. A realistic comparison is not possible without the use of information uncertainty which is related to qualitative value of confidence for a more meaningful comparison. In the case of the AR results, Table I, qualitative values of conflict provided a useful means to compare the competing scenarios.

V. CONCLUSION

This study provides a means to determine a qualitative level of confidence for both AR or ER-logic gates model vector results. The determined confidence values are used to compare competing scenarios and understand the influence on the desired consequence or metric of interest, such as risk. Moreover, the determined confidence values can also be used to compare the results for possible resource allocations and used to reduce risk. Maximal and minimal potential information uncertainty is easily identified in various AR and

TABLE III
CONFLICT, NON-SPECIFICITY AND RESULTING INFORMATION UNCERTAINTY FOR ER RESULTS

Scenario	*Conflict*	*Non-Specificity*	*IU*
Scenario 1	0.469	1.000	0.734
Scenario 2	0.748	1.585	1.166
Scenario 3	0	0	0

TABLE IV
INFORMATION UNCERTAINTY AND CONFIDENCE IN ER RESULTS.

Scenario	*IU*	*Confidence Vector*	Confidence
Scenario 1	0.734	[0,0.531,0.469,0,0]	High to Medium
Scenario 2	1.166	[0,0,0.667,0.333,0]	Medium
Scenario 3	0	[1,0,0,0,0]	Very High

ER vector results and is related to minimal and maximal confidence levels, respectively. A simple algorithm is presented to correlate the information uncertainty quantified in a result to a fuzzy confidence value. A confidence value is an extremely useful metric when comparing different scenarios and it is easily understood by policy and decision makers who require understandable yet defensible metrics. Due to the absence of quantitative information, direct validation was not pursued, future work will involve comparisons of the results obtained using the proposed confidence metrics to rank the results to those obtained from a SME ranking of the results. The methods used to quantify information uncertainty do not discern the difference between a vector result with a bimodal distribution and one that is not bimodal. Vector results with a bimodal distribution have signaled an error in the inference.

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