EFFECT OF NON-UNIFORM HEAT GENERATION ON THERMIONIC REACTORS

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The penalty resulting from non-uniform heat generation in a thermionic reactor is examined. Operation at sub-optimum cesium pressure is shown to reduce this penalty, but at the risk of a condition analogous to burnout. For high pressure diodes, a simple empirical correlation between current, voltage and heat flux is developed and used to analyze the performance penalty associated with two different heat flux profiles, for series-and parallel-connected converters. The results demonstrate that series-connected converters require much finer power flattening than parallel converters. For example, a ±10% variation in heat generation across a series array can result in a 25 to 50% power penalty.

INTRODUCTION

Since thermionic converter performance is a very sensitive function of emitter temperature, it is generally well understood that thermionic reactors will require a greater degree of power flattening than conventional nuclear reactors. In spite of the best efforts at power flattening, however, the heat generation rate in a reactor can never be completely uniform, because of the following factors:

1. Inaccuracies in calculational methods and physical data (cross-sections).
2. Impracticability of continuous variation of the fuel element composition and/or geometry.
3. Need to avoid impractically tight manufacturing tolerances.
4. Changes in power profile as a result of control element movement.
5. Changes in power profile as a result of non-uniform fuel burnup.

The present study examines the effect of heat generation non-uniformity on the power output of an array of series or parallel connected thermionic converters. The quantitative results derived should aid in answering the following questions:

1. What is the penalty for a given degree of non-uniformity?
2. How much effort (e.g., in tight manufacturing tolerances, critical mockup tests, etc.) is warranted to improve the power flattening?
3. How do the performance penalties for a given non-uniformity among series-connected converters compare with those for parallel-connected diodes?
4. Do the results suggest any lessons in how to design a thermionic reactor and how to interconnect its converters?

In addition, the analytical approach presented in this paper may be applied by the reader to any other assumed heat flux distribution.

ANALYSIS

Before proceeding, we require a mathematical relation between output current, voltage, and emitter heat flux, which can in turn be derived from a suitable correlation of current, voltage, and emitter temperature. Note, however, that this correlation must be based on data measured at a single cesium reservoir temperature since, for the present analysis, that is the condition we assume to exist in the reservoir. (Also, that is the condition when a series of stacked diodes is connected to a single cesium reservoir.)

Fortunately, Reference 1* presents some careful measurements of the required nature. Figure 1 illustrates the data for a relatively high cesium pressure, while Figure 2 illustrates the case of a somewhat lower reservoir temperature. From the viewpoint of single diode performance, both of these reservoir temperatures are within the range of practical interest. In general, raising the temperature results in cesium desorption and hence a higher emitter work function. Eventually, the current decrease due to the higher potential barrier exceeds the current increase resulting from the higher emitter temperature, leading to a net decrease in current density. At high cesium pressures (Figure 1) this cross-over effect only occurs at very high emitter temperatures, whereas at lower cesium pressures (Figure 2) the effect occurs well within the normal range of operating temperatures.

For every point on the shown current-voltage characteristics, we now compute a corresponding emitter heat flux, consisting of electron cooling and thermal radiation. To compute the electron cooling, we assume that the effect of emitter temperature T on current density J is given by

$$J = AT^2 \exp(-eV_B/kT), \tag{1}$$

where A is the Richardson-Dushman constant, V_B is the potential barrier the electrons must surmount, e is the electronic charge, and k is the Boltzmann's constant. The electron cooling rate q_e per unit emitter area is then given by

$$q_e = (JkT/e)\left[\log (J/AT^2) + 2\right], \tag{2}$$

where the factor 2 represents the mean kinetic energy of the electrons crossing the energy barrier. The radiation term of the emitter heat flux was computed by assuming an effective emissivity (including mutual reflections) of 0.25, based on some laboratory measurements on open-circuited diodes.

Once the emitter heat flux has been computed, we can crossplot current versus voltage for fixed emitter heat flux, as illustrated by the solid curves in Figures 3 and 4 for the previous two cases.

Operation at Lower Cesium Pressures

Since we are interested in minimizing the effect of non-uniform heat generation on thermionic performance, one may be tempted to take advantage of the cross-over condition at lower cesium pressures. As shown by the illustrative load line in Figure 2, under these conditions relatively large changes in emitter temperature only produce a small change in electrical output. However, the danger in this mode of operation is shown by the same load line in Figure 4. As can be seen, if the heat generation rate should increase (even if only locally) from 70w/cm^2 to 75 w/cm^2, with the load resistance or voltage remaining the same, almost all of the cesium would desorb from the affected part of the emitter. Such a condition, which is equivalent to an open circuit with almost complete loss of electron cooling, is closely analogous to burn-out in boiling heat transfer.

To further illustrate the potential instability of this operating regime, consider the 75 w/cm^2 heat flux curve in Figure 4. The variation of emitter temperature is

*1. TE 7-66, S. Kitrilakis et al, "Final Report for the Thermionic Research Program, Task Iv, Contract 950761, "Vol. I, 8/2/65, Thermo Electron Engineering Corporation.

indicated along the solid portion of the curve, and the extension to very high emitter temperatures (where most of the cesium has left the emitter) is qualitatively represented by the dashed curve.

Starting from point 1 on the cesiated J-V curve, increasing the load resistance lowers the current density and the electron cooling rate, which raises the emitter temperature. Finally, when the load reaches line A the temperature rises so rapidly as to cause virtually complete cesium desorption, and the operating point makes a sudden jump from 3 to 5. Upon further increase of the load resistance, the operating point moves along the uncesiated curve from 5 to 6. When the process is reversed, decreasing the load resistance raises the current and the electron cooling rate and lowers the emitter temperature, until point 4 is reached, where cesium is reabsorbed and the operating point shifts to point 2. Thus, if the heat input is maintained constant and the load resistance is cycled slowly enough, the output should exhibit a hysteresis effect.

In view of the previously described danger of "burn-out" at lower cesium pressures it appears preferable to operate at high pressures, even though this leads to greater penalties for small heat flux variations.

Analytical Model for High Cesium Pressure Case

To analyze the high pressure case, we require an analytical fit to the constant heat flux curves shown in Figure 3. Fortunately, these curves are not only very close to straight lines but can in fact be represented by a family of straight lines passing through a common point ($V = V^* = 2.15$ volt, $J = J^* = -3.80$ amp/cm^2). Moreover, the slopes of these straight lines are approximately proportional to the emitter heat flux q, so that the J-V-q curves can be represented by

$$J = J^* + aq(V^* - V), \tag{3}$$

where $a = 0.16$ volt^{-2}. The adequacy of this curve fit can be judged by comparing the solid and dashed lines in Figure 3.

To assess the effect of heat flux non-uniformity, we must at all times satisfy the requirement that no diode (and no part of any one diode) must exceed some maximum permissible emitter temperature (e.g., 2000°K). Since our principal interest is in the vicinity of the maximum power point (J_o, V_o) for a given temperature, the corresponding J-V curve can be represented by the equation of the tangent at that point,

$$J = J_o \left[2 - (V/V_o) \right] \tag{4}$$

For T = 2000°K, for example, the values of J_o and V_o given by Figure 2 are 18 amp/cm^2 and 0.41 volt, respectively. Thus, the performance model used in the subsequent analysis is represented by the dashed lines in Figure 3.

Comparison of Parallel and Series Arrangements

Even a qualitative examination of Figure 3 reveals that series-connected diodes are much more sensitive to heat generation differences than are parallel-connected converters. Consider, for example, the case where the hottest diode operates at 2000°K and receives a heat flux of 70 w/cm^2. Cooler diodes in parallel with this must operate at the same output voltage (0.58 v). As seen from Figure 3, a diode with a 60 w/cm^2 heat flux would produce 12.2/14.8 = 82% of the maximum power, and even a 50 w/cm^2 diode would still produce 9.6/14.8 = 65% of the hot diode power. By contrast, diodes in series with the hot unit would have to produce the same current density (14.8 w/cm^2). As seen from Figure 3, a 60 w/cm^2 diode would only produce 0.20/0.48 = 42% of the maximum power, while a 50 w/cm^2 diode would operate far into the negative voltage quadrant, i.e., it would be a power consumer. In other words, some of the power produced by the hotter diodes would be used up in forcing the current through the cool (50 w/cm^2) diode. Clearly, series-connected diodes require a very uniform heat generation rate.

EFFECT OF NON-UNIFORM DISTRIBUTION

To carry out the quantitative study, we let R denote the maximum to minimum heat flux ratio, and examine three specific heat flux distributions: the chopped cosine distribution

$$q(x) = q_m \cos [(\text{arcsec } R) x/X], \tag{5a}$$

the linear distribution

$$q(x) = q_m [1 - (1 - R^{-1}) x/X], \tag{5b}$$

and the secant distribution

$$q(x) = q_m R^{-1} \sec [(\text{arcsec } R) x/X] ; \tag{5c}$$

where x denotes position extending from 0 to X, and q_m designates the maximum heat flux. The subsequent approach can easily be applied to any other heat flux distribution.

PARALLEL ARRANGEMENT

Let us first consider the case of parallel diodes, all operating at some voltage V. (Note that this section also applies to a single long diode, with non-uniform heat generation.) The average power density in this case is given by

$$P = \frac{V}{X} \int_0^X J dx. \tag{6}$$

Inserting equation 3 for J, we obtain

$$P = \frac{V}{X} \int_0^X [J^* + a(V^* - V)q] dx, \tag{7}$$

where q is given by equations 5a, b, c, depending on the assumed heat flux distribution. Inserting the appropriate expression and integrating from 0 to X, we find that the average power density in all three cases can be expressed as

$$P = V [J^* + a(V^* - V) q_m F] , \tag{8}$$

where F is a measure of heat flux uniformity, respectively defined by

$$F = (1 - R^{-2})^{\frac{1}{2}}/\text{arcsec } R, \tag{9a}$$

$$F = \tfrac{1}{2} (1 + R^{-1}), \tag{9b}$$

$$F = \{ \log [R + (R^2 - 1)^{\frac{1}{2}}] \} /R \text{ arcsec } R. \tag{9c}$$

Consider now the maximum current density J_m in the system, i.e., the current density of the hottest diode, where $q = q_m$. This current density must satisfy equation 3,

$$J_m = J^* - a(V^* - V)q_m . \tag{10}$$

Moreover, since the hottest diode cannot exceed the specified emitter temperature T, J_m must also satisfy equation 4:

$$J_m = J_o [2 - (V/V_o)] . \tag{11}$$

Combining equations 8, 10 and 11 we obtain

$$P = V[J^* (1 - F) + J_o F(2 - V/V_o)] . \tag{12}$$

From equation 12 it is easily shown that the voltage V_m which maximizes the output power is given by

$$V_m = V_o [1 + \tfrac{1}{2} (F^{-1} - 1)J^*/J_o] . \tag{13}$$

Finally, inserting this in equation 12 and recalling that the maximum possible power density (at a uniform temperature) is $P_o = J_o V_o$, we obtain

$$P_m / P_o = [\tfrac{1}{2} (J^*/J_o)(1 - F) + F]^2 / F. \tag{14}$$

SERIES ARRANGEMENT

In the series-connected case, all diodes operate at a common current density J, and their average power density is given by

$$P = \frac{J}{X} \int_0^X V dx, \tag{15}$$

where the local voltage V is obtained from equation 3,

$$V = V^* - (J - J^*)/aq, \tag{16}$$

and q is again given by equations 5a, b, c, depending on the assumed heat flux distribution. Inserting the appropriate expressions into equation 16, 16 into 15, and integrating from 0 to X, we obtain

$$P = J [V^* - (J - J^*)F/aq_m], \tag{17}$$

where F is again a measure of heat flux uniformity, this time defined by

$$F = \{ \log[R + (R^2 - 1)^{\frac{1}{2}}] \} / \operatorname{arcsec} R, \tag{18a}$$

$$F = R \log R/(R - 1), \tag{18b}$$

$$F = (R^2 - 1)^{\frac{1}{2}} / \operatorname{arcsec} R, \tag{18c}$$

respectively, for the three assumed heat flux distributions.

We now argue that the maximum diode voltage V_m must coincide with the maximum heat flux q_m. Hence, from equation 3,

$$J = J^* + aq_m (V^* - V_m). \tag{19}$$

Moreover, since this must occur at the maximum allowable emitter temperature, V_m must also satisfy equation 4:

$$J = J_o [2 - (V_m / V_o)]. \tag{20}$$

Combining equations 17, 19 and 20, we obtain

$$P = J \{ V^* (1 - F) - V_o [2 - (J/J_o)] F \}. \tag{21}$$

To maximize P, we set J equal to

$$J_m = J_o [1 + \tfrac{1}{2} (F^{-1} - 1) V^*/V_o] \tag{22}$$

Inserting this in equation 21 and dividing by $P_o = J_o V_o$, the maximum power density with a uniform heat flux, we obtain

$$P_m / P_o = [1 + \tfrac{1}{2} (F^{-1} - 1) V^*/V_o]^2 F. \tag{23}$$

Figure 5 illustrates the effect of the maximum to minimum heat flux ratio R on the optimized power output ratio, for the six cases analyzed above. The figure demonstrates the high sensitivity of thermionic reactors to non-uniform heat generation rates. Even small variations in emitter heat flux lead to considerable penalties in power output, particularly in the case of series-connected converters. The results suggest the desirability of arranging many diodes in parallel groups, before connecting these groups in series.

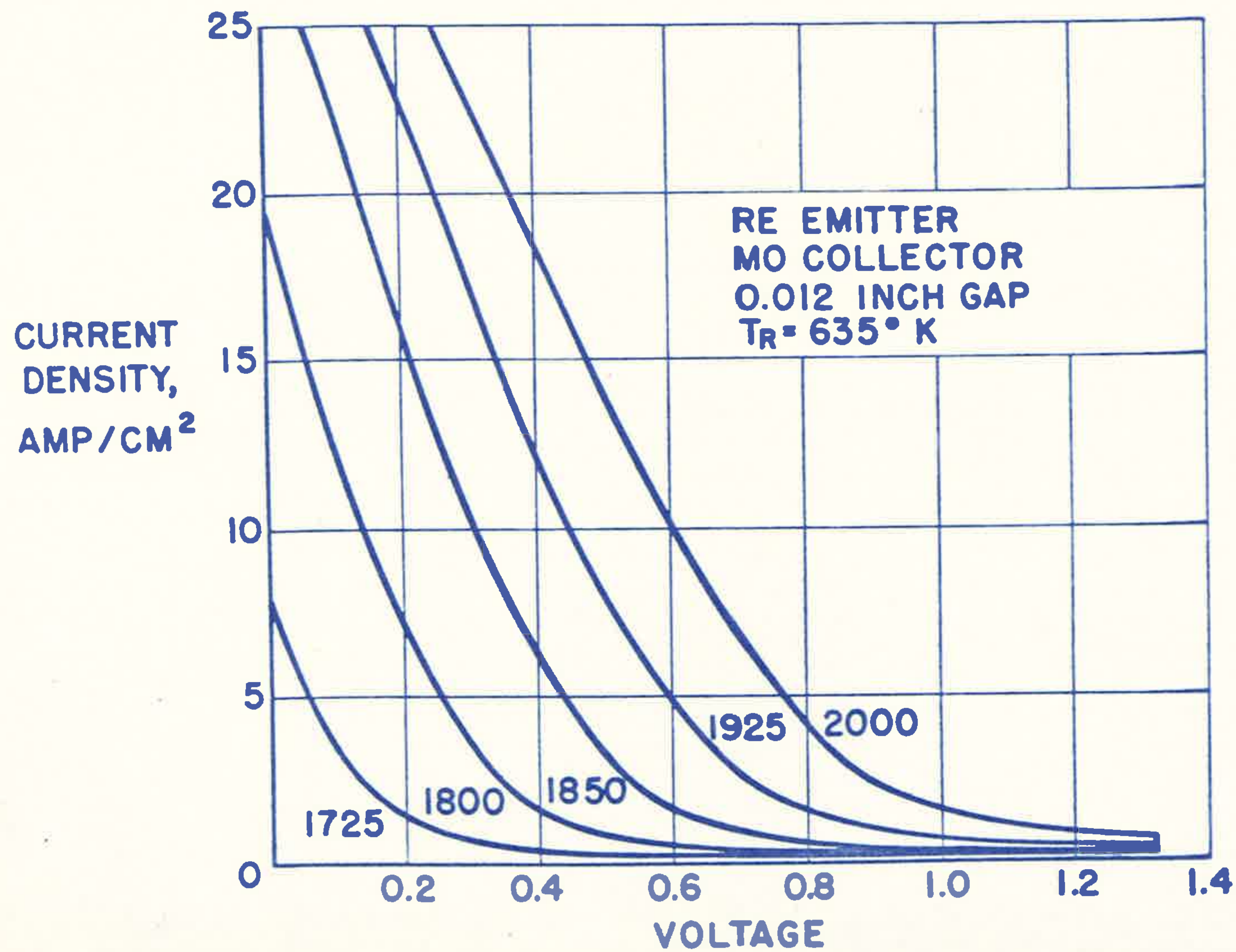

Figure 1: Typical J-V-T curves for high cesium pressure

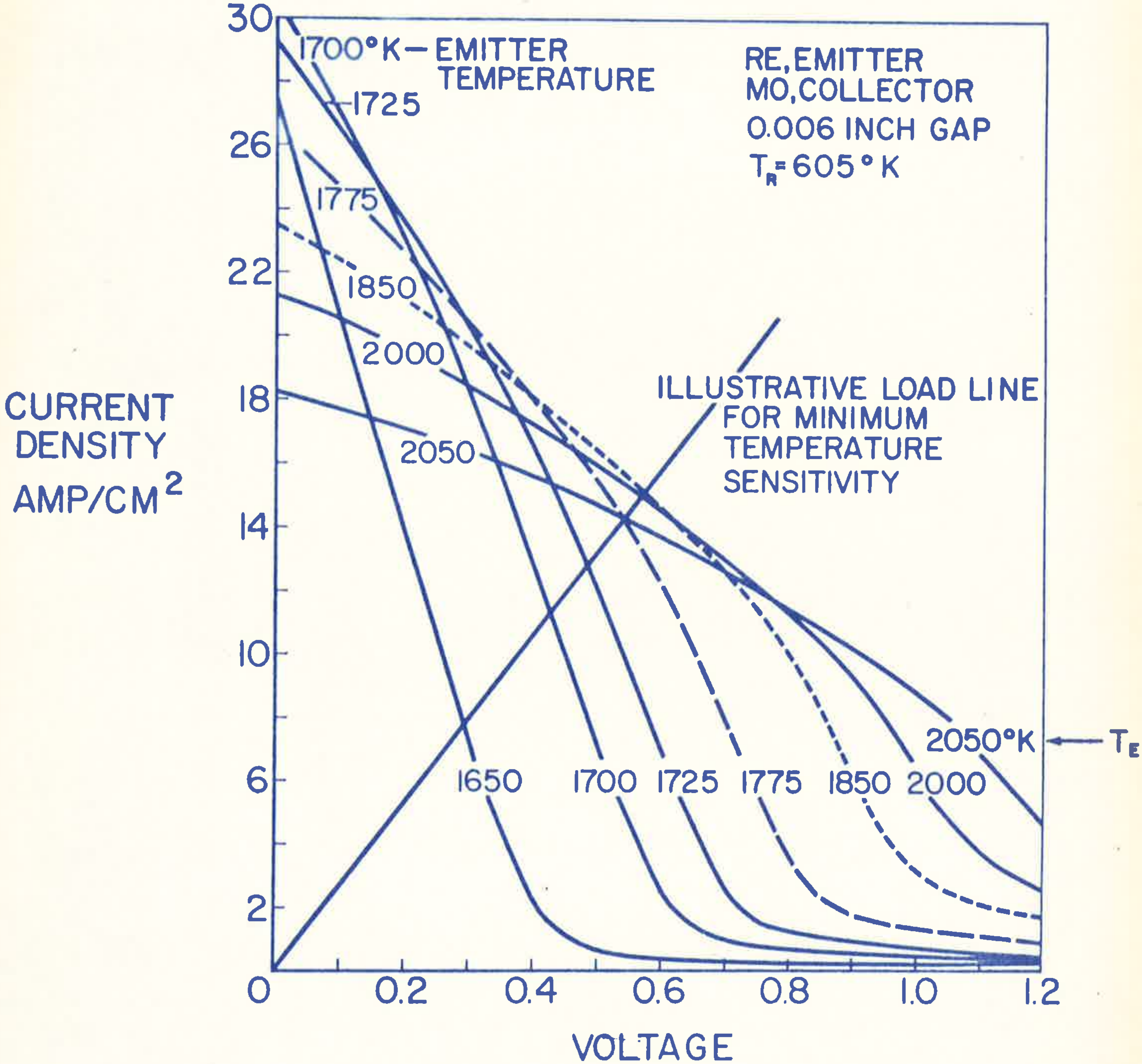

Figure 2: Typical J-V-T curves for sub-optimum cesium pressure

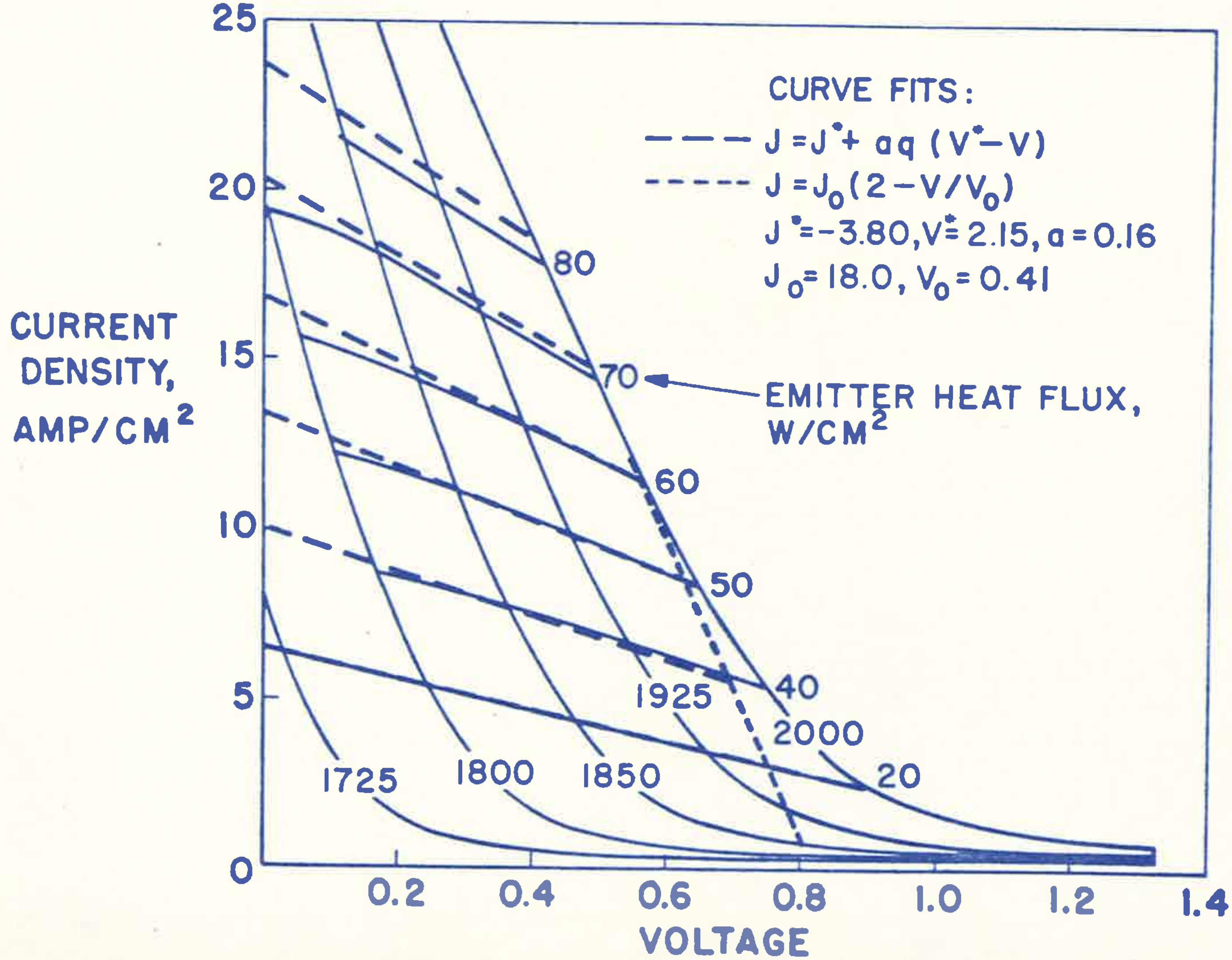

Figure 3: Typical J-V-q curves for high cesium pressure, showing closeness of linearized curve fit.

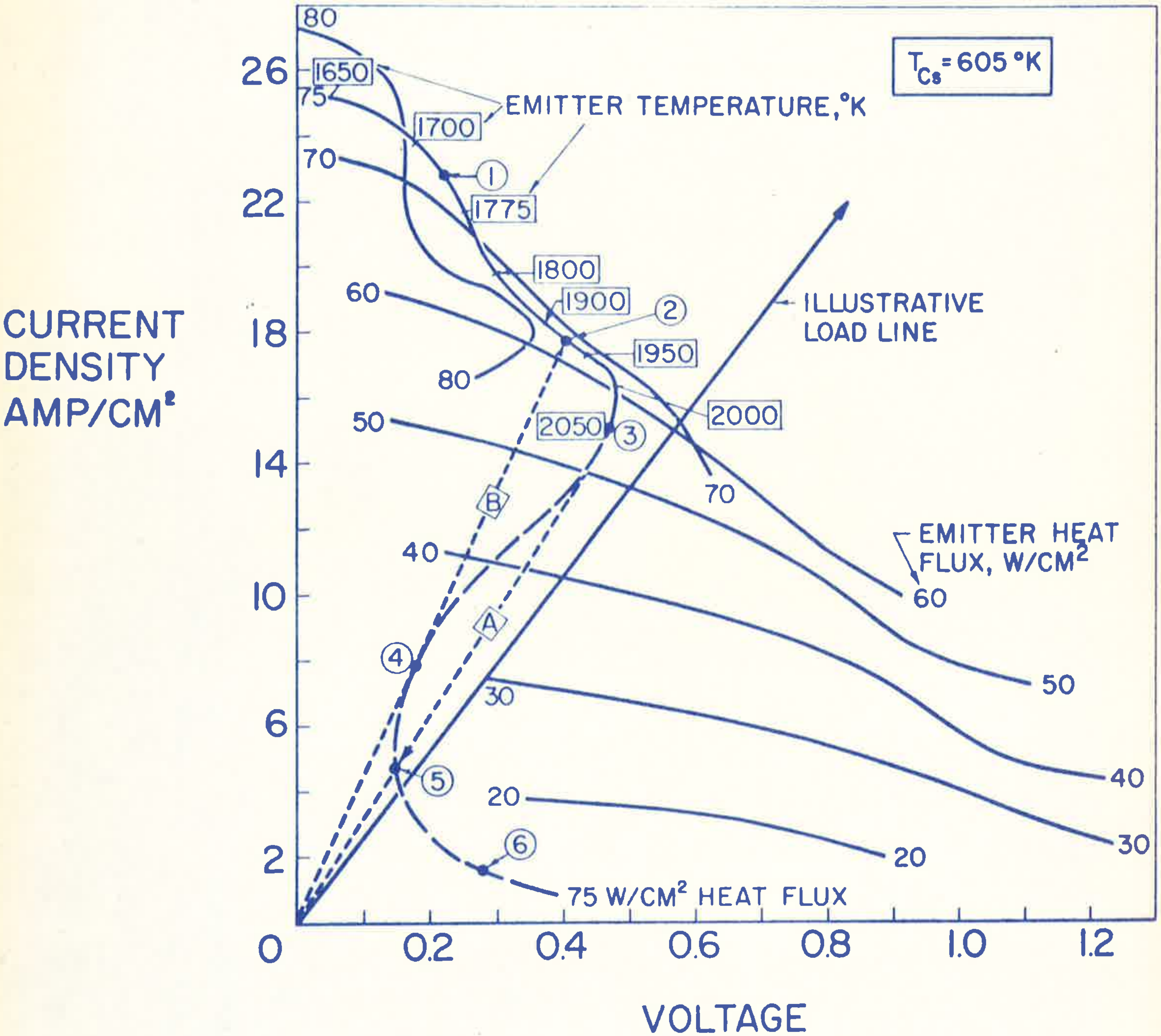

Figure 4: Typical j-V-q curves for sub-optimum cesium pressure, illustrating danger of "burn-out".

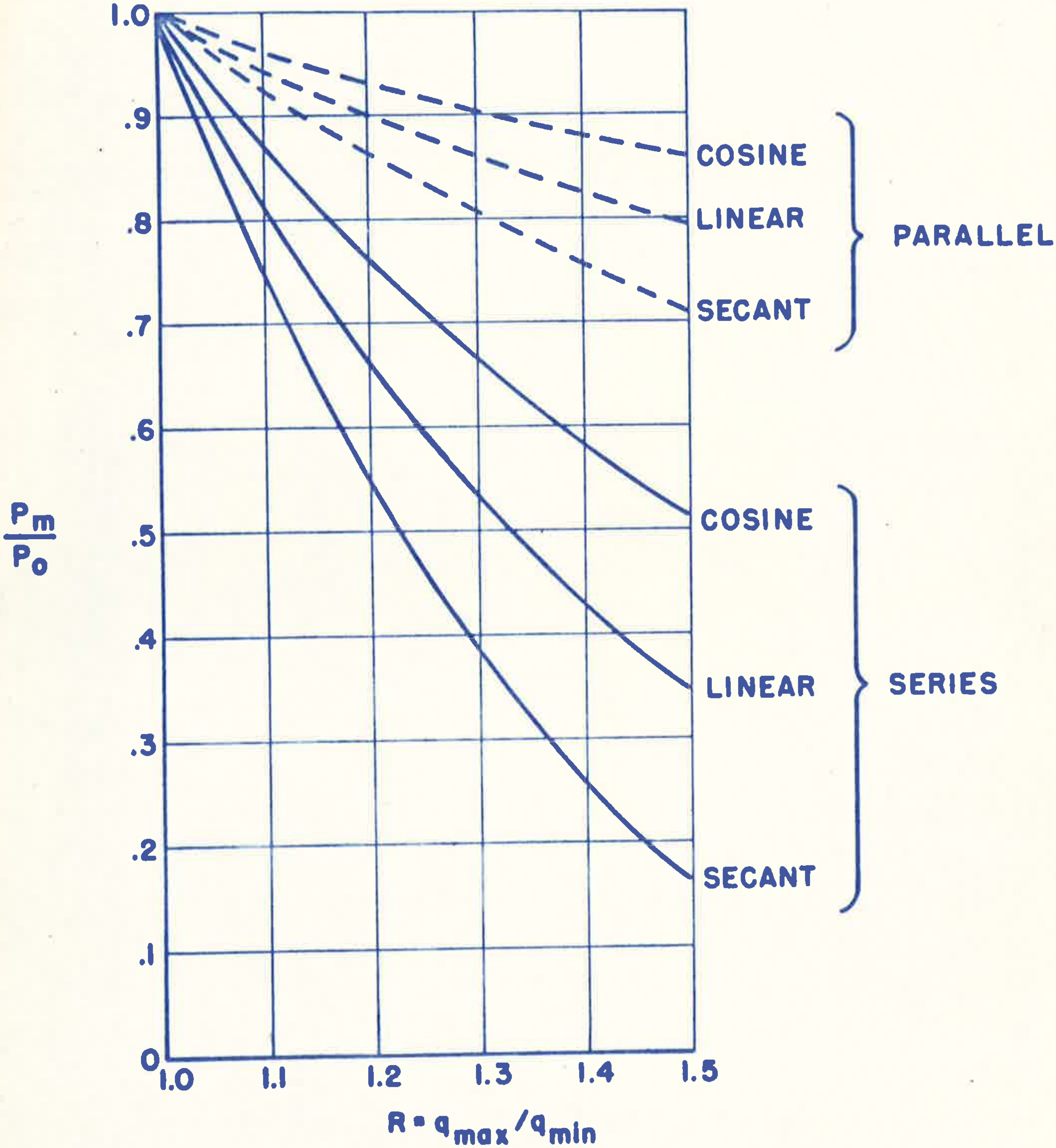

Figure 5: Effect of heat flux ratio on power output of parallel and series connected diodes, for various heat flux distribution.