Unfolding the fission prompt gamma-ray energy and multiplicity distribution measured by DANCE

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Unfolding the fission prompt γ-ray energy and multiplicity distribution measured by DANCE

A. Chyzh and C.Y. Wu

Lawrence Livermore National Laboratory, Livermore, CA 94551

T. Bredeweg, A. Couture, M. Jandel, J. Ullmann, and A. Laptev

Los Alamos National Laboratory, Los Alamos, NM 87545

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Abstract

The nearly energy independence of the γ-ray efficiency and multiplicity response for the DANCE array, the unusual characteristic elucidated in our early technical report (LLNL-TR-452298), gives one a unique opportunity to derive the true prompt γ-ray energy and multiplicity distribution in fission from the measurement. This unfolding procedure for the experimental data will be described in details and examples will be given to demonstrate the feasibility of reconstruction of the true distribution.

INTRODUCTION

DANCE (Detector for Advanced Neutron Capture Experiments [1]) is a γ-ray calorimeter and designed to study the neutron-capture reactions on small quantities of radioactive and rare stable nuclei. These reactions are important for the radiochemistry applications and modeling the element production in stars. The recognition of the capture event is made by the summed γ-ray energy $E_\sum$ which is equivalent of the reaction Q-value and unique for a given capture reaction. For a selective group of actinide nuclei, where the fission reaction competes favorably with the capture reaction, additional signature is needed to distinguish between fission and capture γ-rays for the DANCE measurement. This can be accomplished by introducing a detector system to detect fission fragments, which establishes a unique signature for the fission event. Once this system is implemented, one has the opportunity to study not only the capture but also fission reaction.

For the fission reaction, one has a unique opportunity to study the prompt γ-ray energy and multiplicity distribution by taking advantage of the unusual characteristic of DANCE, which was reported in our earlier technical writing (LLNL-TR-452298). Here we describe an unfolding procedure to reconstruct the true γ-ray energy and multiplicity distribution in fission from the DANCE measurement. The validity of this unfolding procedure is established by examining the cases for three radioactive sources, ^{60}Co, ^{88}Y, and ^{22}Na. The plan to measure these quantities for a selective group of actinide nuclei will be discussed.

γ-RAY MULTIPLICITY

In the DANCE data analysis we utilize a concept of cluster multiplicity M_{cl}, where a cluster is a group of adjacent crystals that fire above the 150-keV energy threshold each within a given time coincidence window, typically $T_{coin} = 5 - 20$ ns. Because of the finite efficiency, the number of γ rays detected by DANCE is not always equal to the number of emitted γ-rays, the true γ-ray multiplicity M_γ.

Fig. 1 shows the simulated M_{cl} response of DANCE to the emitted (true) M_γ. From now on the words emitted and true are interchangable. Each plot contains three spectra and each spectrum was simulated with 10^5 mono energetic γ-rays: **0.5 MeV in black**, 1 MeV in red, 2 MeV in blue. For the event with the true γ-ray multiplicity M_γ=1, the total efficiency
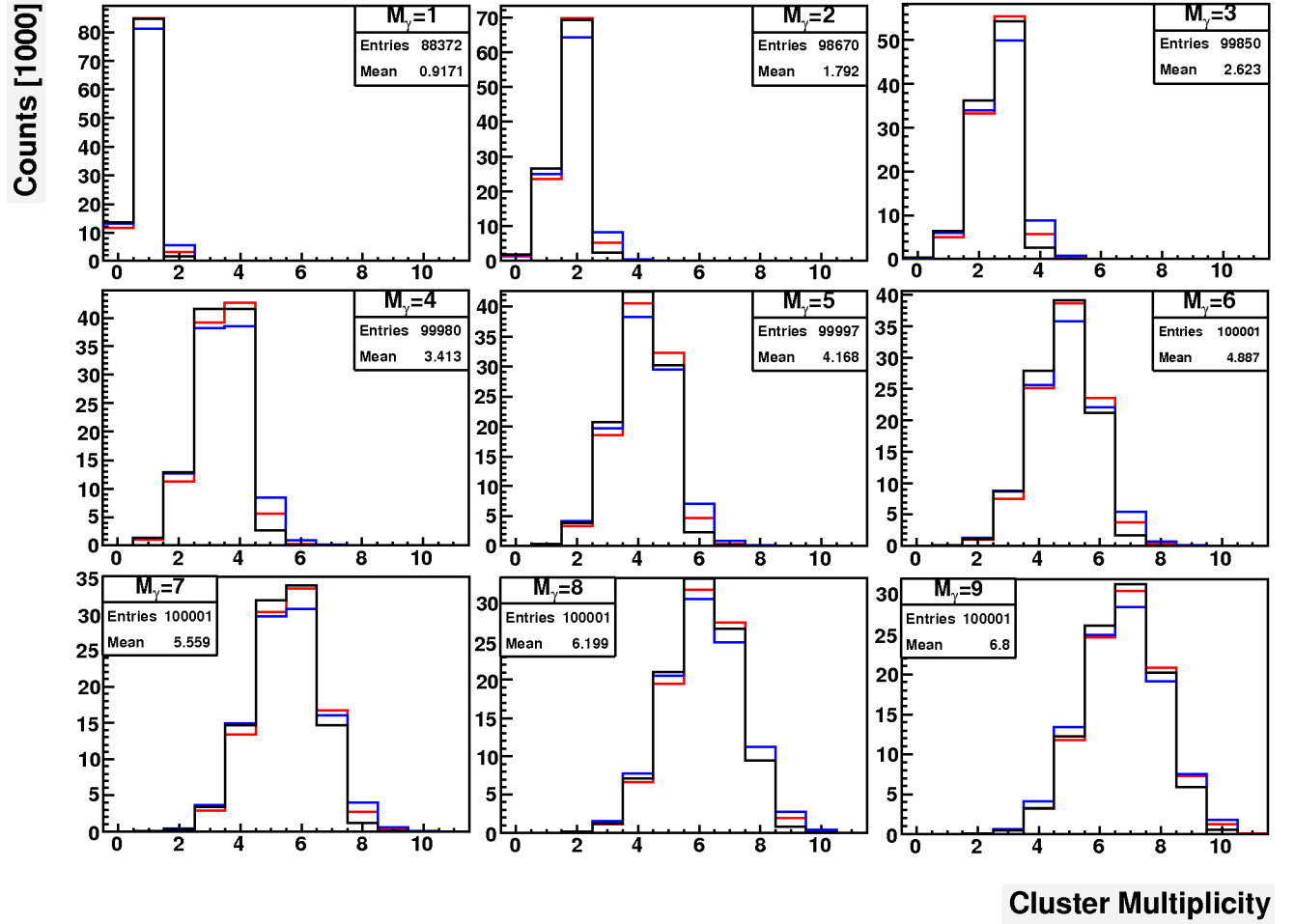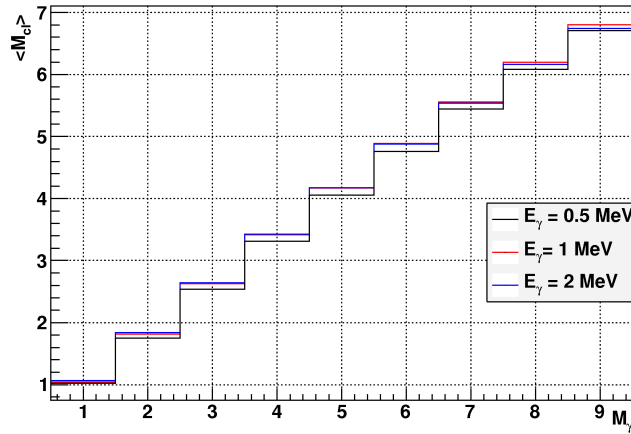
FIG. 1: The M_{cl} spectra for different E_γ. $<M>$ mean value is a function of M_{cl}. The spectrum integrals (*Entries*) do not include the M_{cl}=0 counts.

$\epsilon_{tot}(0.5 \text{ MeV})$ is 88.3%, from which $\epsilon(M = 1)$=85%, $\epsilon(M = 2)$=2%, and $\epsilon(M = 0)$=13%. Thus, the measured mean $<M_{cl}>$=0.92 corresponds to the real M_γ=1. The dependece of the mean value $<M_{cl}>$ from M_γ for 3 different γ-ray energies is shown in Tab I and Fig. 2.

The DANCE response to the M_γ=2 events spans over the cluster multiplicity between 1 and 4; the M_γ=6 events are detected anywhere between $M_{cl} = 2$ and 8 with corresponding probabilities shown in Fig. 1. Thus, what we measure with DANCE is not the true γ-ray multiplicity M_γ, but rather the M_{cl} distribution folded through the DANCE response.

TABLE I: Mean values $<M_{cl}>$ of measured M_{cl} as a function of true M_γ for 3 different energies E_γ. Each row/column corresponds to a certain M_{cl}/E_γ.

	E_γ=0.5 MeV	E_γ=1 MeV	E_γ=2 MeV
M_γ=1	1.02	1.04	1.06
M_γ=2	1.75	1.81	1.84
M_γ=3	2.53	2.62	2.64
M_γ=4	3.31	3.41	3.42
M_γ=5	4.05	4.16	4.17
M_γ=6	4.75	4.89	4.88
M_γ=7	5.44	5.55	5.54
M_γ=8	6.09	6.20	6.16
M_γ=9	6.71	6.80	6.73

FIG. 2: Mean values of M_{cl} depend on true M_γ and almost don not depend on E_γ.

M-response matrix of DANCE

The transition from M_γ to M_{cl} is determined by the DANCE response and can be arranged into the M-response matrix which explicitly describes dependencies between these two quantities.

Fig. 3 shows how the M-response matrix is structured. The X axis corresponds to the true multiplicity M_γ, and the Y axis corresponds to the measured multiplicity M_{cl}. For instance, 100,000 events of M_γ=1 event are measured as follows: 65,100 events are M_{cl}=1, 3,300 events are M_{cl}=2, and 11 events are M_{cl}=3. Another example is how the M_γ=2 events are measured: 23.5% are measured as M_{cl}=1, 69.8% are M_{cl}=2, 5.2% are M_{cl}=3, and so forth (Tab. II). The M_{cl}=0 values represent events that are not detected by DANCE.
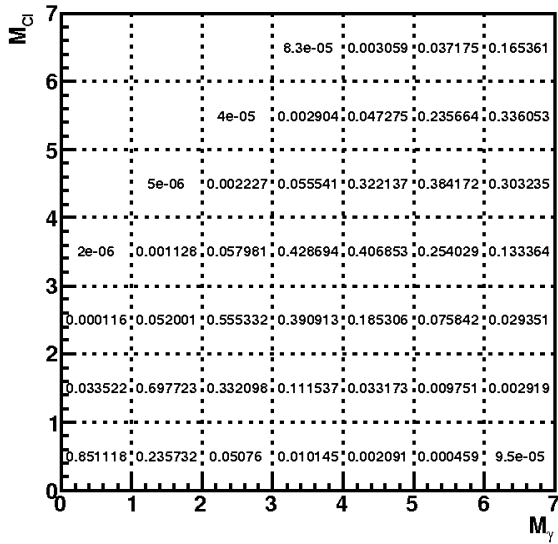
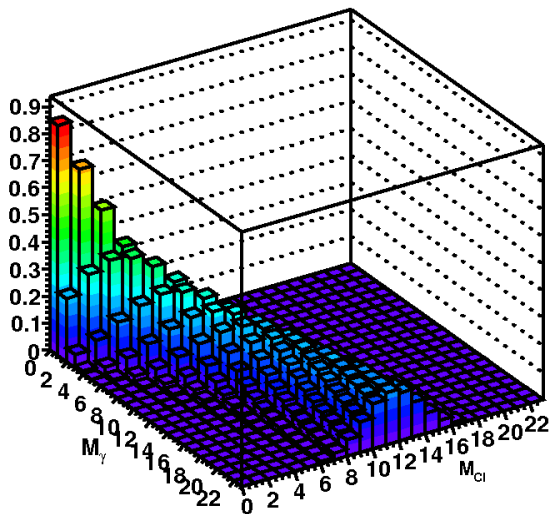**FIG. 3:** M-response matrix of DANCE. Each columns corresponds to a M_γ value and contains M_{cl} values.
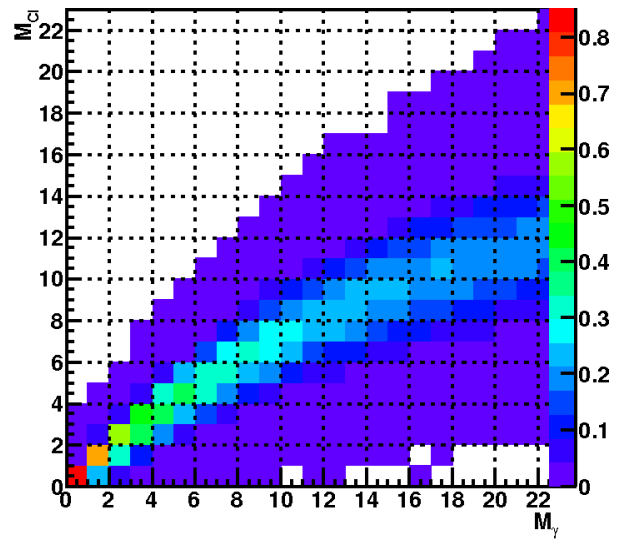

FIG. 5: M-response matrix of DANCE. The multiplicity range is M = 1 − 23.

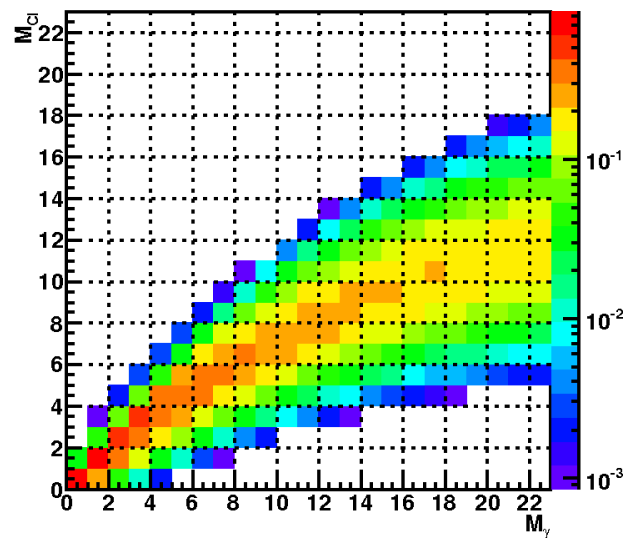**FIG. 4:** M-response matrix of DANCE. 3D view.

FIG. 6: M-response matrix of DANCE. Z-axis log view.

TABLE II: DANCE response to true M_γ is distributed over M_{cl} values.

	M_γ=1	M_γ=2	M_γ=3
M_{cl}=0	11.7%	1.3%	0.1%
M_{cl}=1	85.0%	23.5%	5.0%
M_{cl}=2	3.3%	69.8%	33.2%
M_{cl}=3	0.0%	5.2%	55.5%
M_{cl}=4	0.0%	0.01%	5.7%
M_{cl}=5	0.0%	0.0%	0.2%

The colored representations of the M-response matrix (Fig. 5 and 6) show that the multiplicity range spans up to M = 23. The number of BaF_2 crystals in DANCE is 160, but the multiplicity cannot go that high for two reasons: a cluster may contain several crystals, and clusters may overlap thus reducing M_{cl} value. That's why the matrix is truncated to the size of 23 for the practical purpose. The columns of the matrix become increasingly linearly dependent on each other, i.e., as M_γ increases, the dependence of M_{cl} on M_γ decreases. The more linearly dependent matrix is, the more difficult is to solve it. That is, solving the matrix becomes nearly impossible once its determinant closes to zero within the limits of computer accuracy. Fig. 4 is a 3D view of the M-response matrix.

It is necessary to mention that the present matrix is built for the 1-MeV mono energetic photons. The same matrix for the 2-MeV photons would look slightly different because the DANCE detection efficiency weakly depends on the γ-ray energy E_γ (see Ref. [3]), where the probabilities for detecting the M=1 events with the 1 MeV and 2 MeV energies are 87% and 85% correspondingly. For the higher multiplicity case such as M_γ=5, the largest M_{cl} values varies less than 5%, but the total area of the M_{cl} distribution, which is the total efficiency, changes only less than 2%. The majority of γ-rays produced in the neutron capture and neutron induced fission reactions on many nuclei lay within 100 keV to several MeV. Thus, for the first approximation, this M-response matrix can be used to obtain true M_γ distributions without knowing the exact γ-ray energy if we assume the M-response matrix for 1 MeV is the same for other energies.

Multiplicity unfolding

The M-response matrix can be used to derive the true γ-ray multiplicity M_γ from the measured cluster multiplicity M_{cl}, the so-called **unfolding** procedure. Also the same matrix can be used for **folding** the true M_γ into the measured M_{cl}.

The folding procedure can be demonstrated in Fig 7. The Brunson distribution [4] for the ^{252}Cf fission prompt γ-ray output (red) represents a true M_γ distribution. The M_{cl} distribution (**black**) is the simulated DANCE response to the Brunson distribution, i.e., the Brunson distribution is fed into the GEANT4 model of DANCE as an input and the M_{cl} distribution is the output. The folded M_{fold} distribution (blue) is the matrix product. The

folding procedure is the matrix product:

$$V(M_{cl}) = R(M) \cdot V(M_\gamma) \tag{1}$$

where $V(M_{cl})$ is the vector of the M_{cl} values, $R(M)$ is the M-response matrix, and $V(M_\gamma)$ is the vector of the M_γ values. Before taking the matrix product of $R(M)$ and $V(M_\gamma)$ both of them need to be truncated to the same dimensions. The folding procedure should generate M_{fold} distribution equal to the M_{cl} distribution. The agreement between these two is very good but not perfect. For M = 16 and higher the folded M distribution remains constant instead of decreasing as it should. The computer accuracy is limited by 64 bits (type *double* in C++ and *Double_t* in ROOT).
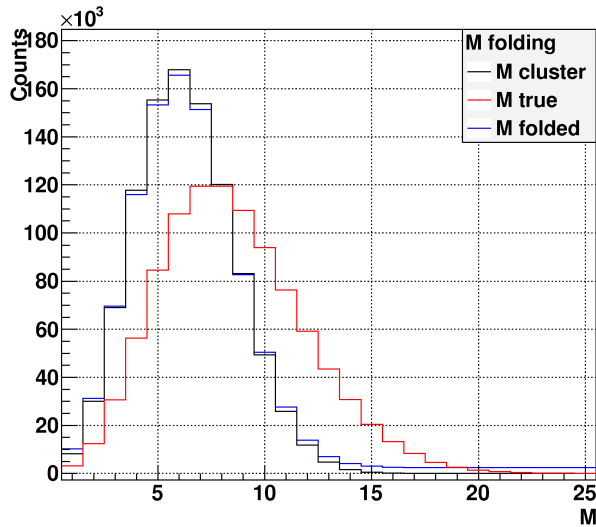

FIG. 7: Folding Brunson distribution of M_γ into cluster multiplicity M_{cl}.
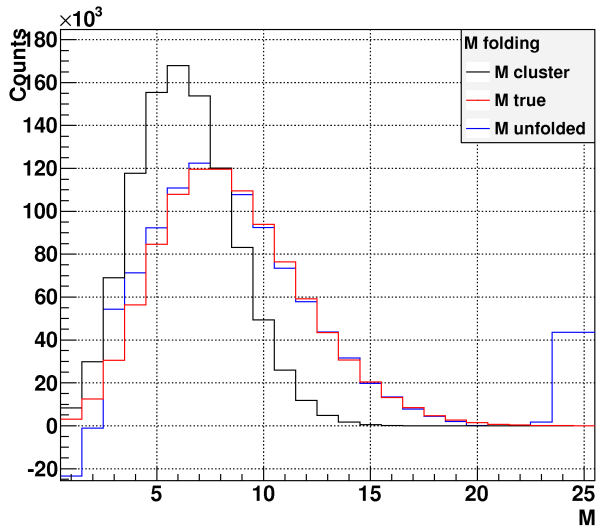
FIG. 8: M-response matrix of DANCE. The multiplicity range is M = 1 − 23

The unfolding procedure is more complicated than folding. The most straight forward way to unfold the measured M_{cl} into the true M_γ is to invert the matrix $R(M)$ and take the matrix product:

$$V(M_\gamma) = R^{-1}(M) \cdot V(M_{cl}) \tag{2}$$

where $R^{-1}(M)$ is the inversion of the M-response matrix $R(M)$. The inversion of the 23×23 matrix precisely is difficult if possible at all because of the increasing linear dependence of matrix elements. If the inversion is precise, product $R(M) \times R^{-1}(M)$ will produce a unit matrix I with diagonal elements equal to 1 and off-diagonal equal to 0. But in this case the

I matrix is not completely unit, some off-diagonal elements fluctuate in the range of 10^{-16} – 10^{-3}. Solving $R(M)$ by taking its inversion produces unrealistic fluctuation in the $V(M_\gamma)$ values.

The other way to solve Eq. 1 relative to the $V(M_{cl})$ vector is to decompose the $R(M)$ matrix using Singular Value Decomposition (SVD) method [5]. The SVD method is a factorization of a rectangular or square real matrix. if R is an $n \times n$ matrix with real nonnegative numbers, then there exists a factorization of the form:

$$R = UZV^*$$ (3)

where U is an $n \times n$ unit matrix, Z is an $n \times n$ diagonal matrix with nonnegative real numbers on the diagonal, and V^* is the conjugate transpose of V. The SVD method orders the diagonal entries of Z to be in descending order, then Z is uniquely determined by R, although U and V may not be unique. The diagonal values of Z are singular values of R.

Using the SVD method Eq. 1 can be solved relative to $V(M_{cl})$ with a unique solution stored in the main diagonal of the Z matrix. Fig 8 shows the unfolding procedure for the Brunson M_γ distribution. The M true is in red and the M unfolded is in blue. The solution has some deviations in the range M $= 1 - 5$, other than that the agreement is very good. No manual or automatic fitting was performed here yet, although it can be done to remove the deviations by iterations to achieve a better agreement. This figure shows that unfolding of the true γ-ray multiplicity M_γ from the measured cluster multiplicity M_{cl} is indeed possible, thus, DANCE can deliver a true M_γ distribution without knowing the true γ-ray energy.

γ-RAY ENERGY

DANCE is a 4π γ-ray calorimeter and it can measure the total energy of an event E_Σ, as well as individual γ-ray energies E_γ. The energy of one cluster E_{cl} corresponds to the γ-ray energy deposited in the crystals adjacent to each other. The relation between E_{cl} and E_γ is shown in Fig. 9. Each measured (simulated) E_{cl} spectrum is the DANCE response to the single true E_γ energy. The studied range is 0.30 – 10 MeV, these spectra were simulated using the GEANT4 model of DANCE (see Ref. [2]) and it was validated in Ref. [3].
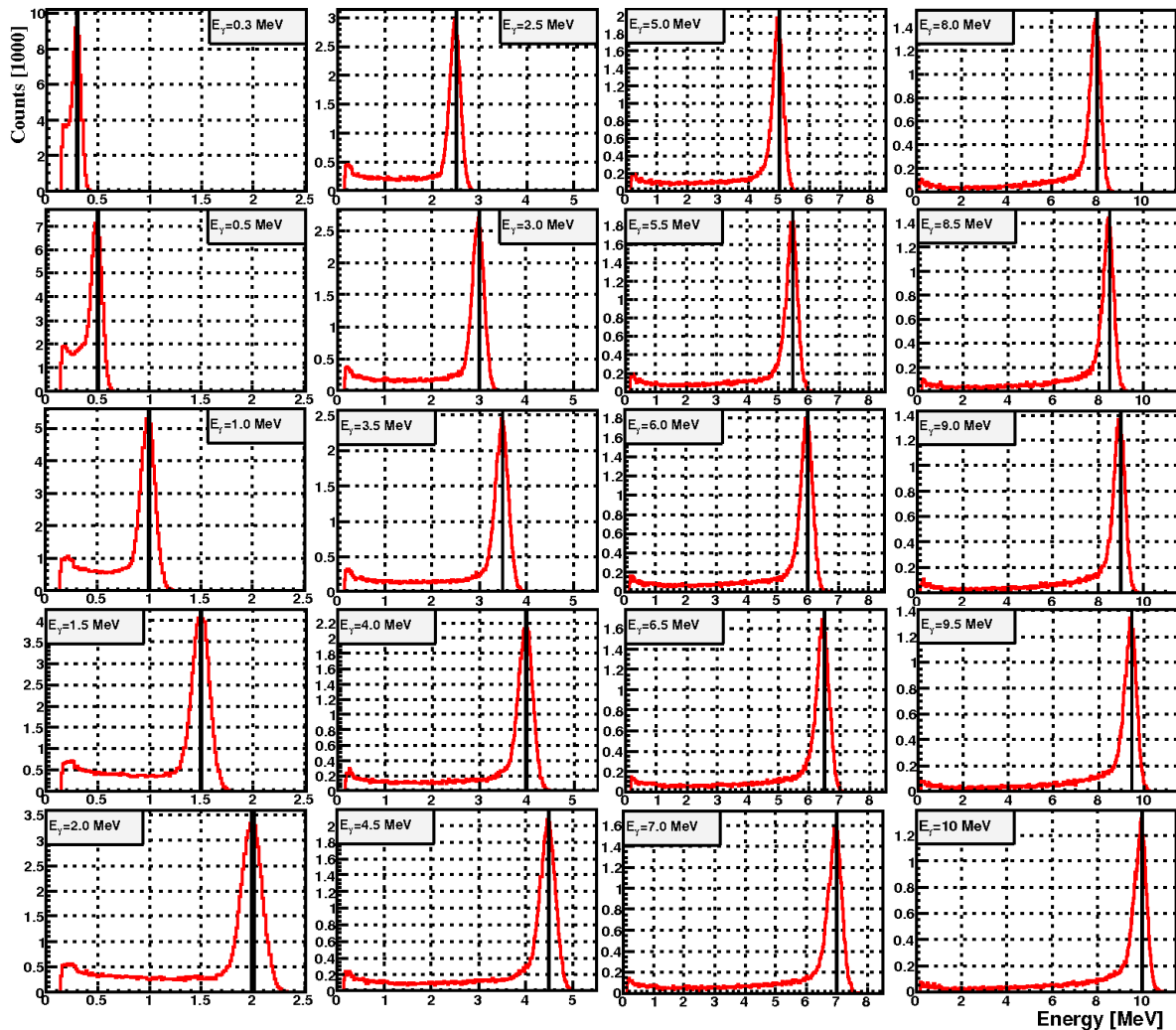
FIG. 9: Each E_{cl} energy corresponds to the true E_γ.

E-response matrix of DANCE

The DANCE energy response to 1 MeV γ-ray is not just a photopeak at 1 MeV, but also a compton distribution from the 1-MeV peak down to the 150-keV threshold (see Fig. 9). Obviously, the measured energy E_{cl} is a function of the true γ-ray energy E_γ, and for each E_γ value there has a unique E_{cl} response. Thus, the E-response matrix can be built for DANCE (see Fig. 10). For a given energy of E_γ, the E_{cl} spectrum (DANCE response) is produced through simulations and normalized to the true number of γ-rays. Then it is integrated with the 50-keV increment in the range 150 keV – 5 MeV.

The E-matrix is filled for E_γ = 150 keV to 5 MeV, total of 97 bins, each bin is 50 keV wide. The lower limit of 150 keV is the DANCE energy threshold and the upper limit of 5 MeV is
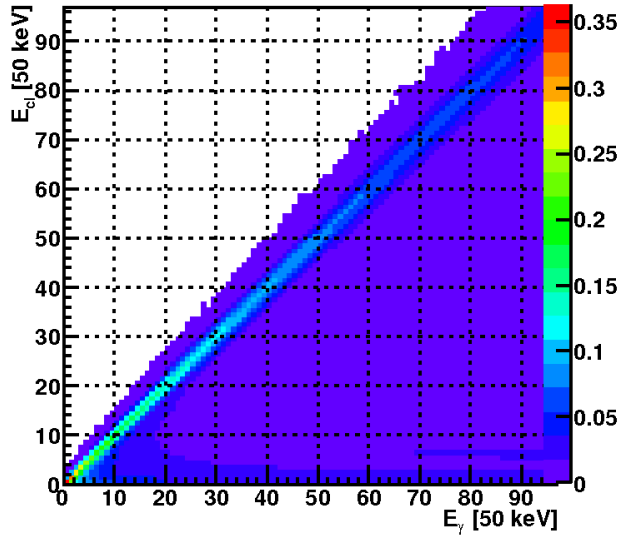
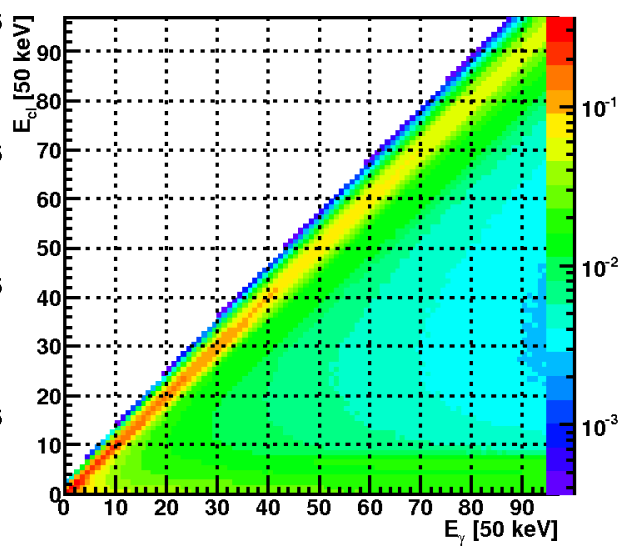FIG. 10: E-response matrix of DANCE. X-axis labels are bin numbers. Each bin is 50 keV wide.

FIG. 11: E-response matrix of DANCE. Z-axis log view.

to make sure the entire range of interest covered. In many nuclei the neutron capture and neutron induced fission reactions produce γ-rays within $0.15 - 5$ MeV. The GEANT4 model of DANCE was validated up to 3 MeV using the calibration sources and the extrapolation is employed for the higher energy. The E-matrix can be used to reconstruct the true E_γ energy from the measured E_{cl} spectrum.

Energy unfolding

The DANCE energy response folds the true energy spectrum as follows:

$$V(E_{cl}) = R(E) \cdot V(E_\gamma) \tag{4}$$

where $V(E_{cl})$ is the measured energy spectrum, $V(E_\gamma)$ is the true energy spectrum, and $R(E)$ is the E-response matrix. The energy unfolding is done the same way as the multiplicity unfolding. Eq. 4 is solved relative to the vector $V(E_\gamma)$ by factorizing the matrix $R(E)$ using the Singular Value Decomposition (SVD) method [5].

The energy unfolding has been made for the three calibration sources: ^{60}Co (Fig. 12), ^{88}Y (Fig. 13), and ^{22}Na (Fig. 14). Each figure contain two spectra: the measured one in **black** and the unfolded one in red.
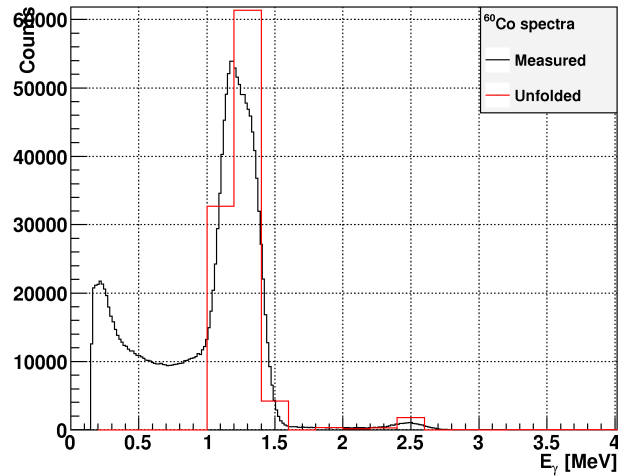
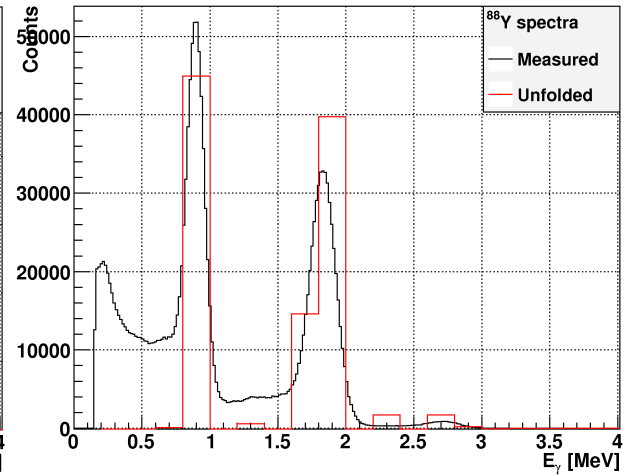**FIG. 12:** Unfolding the true E_γ from the measured E_{cl} for ^{60}Co.

FIG. 13: Unfolding the true E_γ from the measured E_{cl} for ^{88}Y.
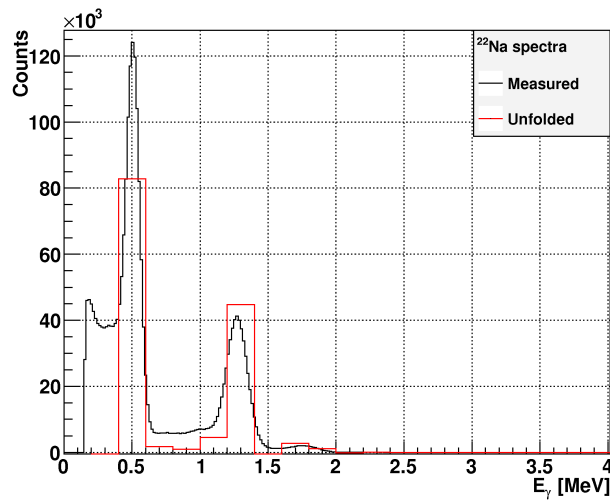

FIG. 14: Unfolding the true E_γ from the measured E_{cl} for ^{22}Na.

The binning of the measured spectra is 16 keV and 200 keV for the unfolded spectra. The energy unfolding improves not only the energy resolution, but also the signal-to-noise ratio, that is, one can almost entirely eliminate compton background and get a clean peak of E_γ. More importantly, the relative intensity ratio of the unfolded E_γ peaks is conserved for each spectrum. Note that the unfolding for the 100 keV bin γ-ray spectrum also works well.

There is no doubt that the unfolding of the γ-ray energy and multiplicity distribution from the DANCE measurement is imperfect since approximations mentioned above are made for the detector response to measure these quantities. However, one still can obtain a

reasonable solution for the true distribution under certain favorable conditions for unfolding. For instance, the modeling of the DANCE response to the γ-ray efficiency and multiplicity can be made reasonably well using the known γ-ray calibration sources. The nearly γ-ray energy independence of the detector response, discovered during the modeling stage, is the most important ingredient for the success of this proposed unfolding process. We are going to continue to explore various approaches to refine the unfolding method and achieve a better solution for the prompt γ-ray energy and multiplicity distribution in fission.

DISCUSSION

For CY2010 we have two scheduled experiments to measure the neutron-induced reactions on ^{239}Pu and ^{241}Pu using the DANCE array in conjunction with a new fission PPAC, designed and fabricated at LLNL. These provide an excellent opportunity to study the fission prompt γ-ray energy and multiplicity distribution in the Pu isotopes. In addition, we are going to install a ^{252}Cf fission PPAC to study the prompt γ-ray energy and multiplicity distribution in spontaneous fission, which complements to our study for the neutron-induced fission. It certaintly will enhance our understanding of the γ-ray production in fission and benefit not only the modeling effort but also the nuclear power industry.

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