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Kinetic Theory and Boundary Conditions for Flows of Highly Inelastic Spheres.

Quarterly Progress Report

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Introduction

In the last quarter, we focused on steady, fully developed, gravity-driven flows of identical, smooth spheres down bumpy inclines, with flow depths greater than one particle diameter, solid fraction profiles everywhere less than .65, and dimensionless granular temperature $T(y=\beta)$ at the top between 0 and 20. For prescribed boundary bumpiness ($r=1$ and $\Delta=0$), restitution coefficients of the boundary ($e_w=.5$) and the flow particles ($e=.5$), and angles of inclination ($\phi=21°$, 22.5°, and 24°), we determined the *complete* range of $T(y=\beta)$ (within $0<T(y=\beta)\leq 20$) for which such flows can be maintained. For these angles, and all others in the range $14°<\phi<26°$ for which at least one steady, fully developed flow could be maintained, we found that there was a nonzero minimum value and a finite maximum value of $\dot{m}$ between which steady, fully developed flows can be maintained.

In this quarter, we continue the same parameter study by choosing other sets of values of r, Δ, e, e_w, and ϕ for which we know that at least one solution of the type described above may be maintained, and determine the *complete* range of $T(y=\beta)$ (within $0<T(y=\beta)\leq 10$) for which such flows can be maintained. To each value of $T(y=\beta)$ in this range, there correspond values of mass hold-up (m_t), mass flow rate ($\dot{m}$), and depth β. In addition, for each value of $T(y=\beta)$ in this range, we can calculate the total fluctuation energy per unit area (E), the slip velocity (v) at the base, the velocity (U_{top}) at the top, and the depth-averaged values of solid fraction, velocity, and granular temperature. In this manner, we thoroughly characterize *all* the steady, fully developed, gravity driven flows that are possible for prescribed sets of r, Δ, e, e_w, and ϕ.

In this manner, we find that there are boundaries, flow particles, and inclinations for which the kinetic theory predicts that steady, fully developed flows can be maintained at *all flow rates* above a minimum value. These are qualitatively different from the results presented last quarter, in which maximum flow rates (above which stedy flows could not be maintained) were determined for each case considered. The fact that it is possible to find circumstances under which there are *no maximum flow rates* that limits the occurrence of steady flows may be useful when in practice it is necessary to steadily transport extremely high volumes of granular materials.

Overview

In Figure 1, we show as a darkened area in the ϕ-Δ plane the values of ϕ and Δ for which at least one value of $T(y=\beta)$ within the range $0\leq T(y=\beta)\leq 10$ yielded a solution to the boundary value problem for steady, fully developed, inclined flows (within the bounds $\beta\geq 1$ and $v\leq .65$.) for $0<\Delta<.732$, when $r=1$ and $e=e_w=.5$. This figure was included in our last quarterly report, but is included here for completeness. It demonstrates that when $\Delta=.414$, for example, steady,

full developed flows are possible when ϕ is approximately between 21° and 35°. In what follows, we consider three intermediate angles ϕ for these two values of Δ, and for each angle determine the *full range* of $T(y=\beta)$ that yields solutions to the boundary value problem.

In addition, we characterize the solutions by calculating the corresponding values of mass hold-up (m_t), mass flow rate ($\dot{m}$), depth β, total fluctuation energy per unit area (E), slip velocity (v) at the base, the velocity ($u(y=\beta)$) at the top, and the depth-averaged values of solid fraction $\bar{\nu}$, velocity $\bar{u}$, and granular temperature $\bar{T}$. Here, the dimensionless depth β and distance y from the base are nondimensionalized by particle diameter σ, the dimensionless velocities u, v, and $u(y=\beta)$ are nondimensionalized by $(\sigma g)^{1/2}$, the dimensionless granular temperature T is nondimensionalized by σg, the depth-average of any quantity q(y) is defined by,

$$\bar{q} \equiv \frac{1}{\beta} \int_0^\beta q(y)dy \quad , \tag{1}$$

and the depth-totaled quantities m_t, $\dot{m}$, and E are equal to $\beta\bar{\nu}$, $\beta\bar{\nu}\bar{u}$, and $\beta\bar{\nu}\bar{T}$, respectively.

Results and Discussion

Here we consider the case in which r=1, $e=e_w=.5$, $\Delta=.414$, and choose four angles ϕ=25.5°, 27.505°, 28°, and 34° in the range $21°<\phi<35°$. In Figure 2, we show for each angle the variations of β, m_t, $\dot{m}$, $\bar{T}$, $\bar{\nu}$, E, $\bar{u}$, v, and $u(y=\beta)$ with $T(y=\beta)$. At the maximum value of $T(y=\beta)=10$, the flows are quite deep and quite dilute. In each case, as $T(y=\beta)$ decreases from its maximum value, the flows become more massive and more shallow. However, as $T(y=\beta)$ nears its minimum value, the flow depths reach their minimum values and then begin to increase. For ϕ=25.5° and 27.505°, the increase in depth is not sufficient to mitigate the increase in mass hold-up, and the assemblies become too dense to flow as $T(y=\beta)$ decreases near its minimum value. For these two lower inclinations, the theory predicts that for values of $T(y=\beta)$ below the minima, the solid fraction somewhere within the flows exceeds .65. For these inclinations, when $T(y=\beta)$ is exactly equal to its minimum value, the flow rate assumes a corresponding *finite maximum* value. However, for the two upper inclinations ϕ=28° and 34°, the increase in depth as $T(y=\beta)$ decreases near its minimum value is sufficient to compensate for the increase in mass hold-up. For these two higher inclinations, the assemblies do not become too dense to flow, and the mass flow rates increase without bound as

$T(y=\beta)$ approaches its minimum value. **Consequently, we find here that for $r=1$, $e=e_w=.5$, $\Delta=.414$** (unlike for the case considered last quarter) **there are relatively high inlinations at which the flow rates are unbounded.** For the same values of r, e, e_w, and Δ, there are lower inclinations at which the flow rate is bounded by a finite maximum. In the next quarter we will focus on the transition from inclinations at which the flow rates are bounded to those at which the flow rates are unbounded.

To make the variations with $T(y=\beta)$ near its minimum value more clear, in Figure 3 we replot the variations of $\dot{m}$, and $\bar{v}$ with $T(y=\beta)$ on log-log scales. The variations of $\dot{m}$ with $T(y=\beta)$ shown in both Figures 2 and 3 also suggest that corresponding to each inclination is a *nonzero minimum* value of $\dot{m}$ above which steady, fully developed flows can be maintained.

According to Figure 2, as $T(y=\beta)$ decreases from 20 over most of its range, the flows become less thermalized (as might be expected) and slower. However as $T(y=\beta)$ continues to decrease near its minimum value, these trends are reversed. Furthermore, the quantities m_t, $\dot{m}$, $\bar{v}$, and E are extremely sensitive to changes in $T(y=\beta)$ near its minimum value but relatively insensitive to changes in $T(y=\beta)$ away from its minimum value. These observations indicate that there is no simple relationship between $T(y=\beta)$, which can not be controlled experimentally, and such parameters as $\dot{m}$ and E, which may be controllable. For this reason, when presenting the results, it is probably better to parameterize the solutions by either $\dot{m}$ or E.

In Figure 4, for example, we eliminate $T(y=\beta)$, and plot the variations of E and m_t with $\dot{m}$ when $r=1$, $e=e_w=.5$, and $\Delta=.414$, for $\phi=25.5°$, $27.505°$, $28°$ and $34°$. The left-hand panel of Figure 4 indicates how much thermal energy should be imparted to the particles at the inlet to to ensure that their initial states are near to the steady, fully developed states predicted by the theory. For each inclination, the endpoints of these curves at the lower flow rates correspond to very deep, very dilute flows of nonzero mass hold-ups. The endpoints for the two lower inclinations at the upper flow rates correspond to relatively dense flows in which the solid fraction is somewhere equal to .65. The two upper inclinations have no corresponding (high-flow-rate) endpoints.

In the upper portion of Figure 5, we plot the variations of β, $\bar{v}$, and $\bar{u}$ with $\dot{m}$ corresponding to those of E and m_t shown in Figure 4. Of particular interest is the variation of $\bar{v}$ with $\dot{m}$ shown in the middle panel. At the two lower inclinations, Near the higher-flow-rate end points, the depth-averaged solid fraction increases rapidly with small changes in flow rate. This indicates that these endpoints occur because the assemblies become too dense to flow rapidly. At the two lower inclinations, the depth-averaged solid fractions are quite insensitive to large increases in the high flow rates, and appear to approach relatively dilute asymptotic values. This indicates that at these inclinations, the flows do not become too dense regardless of how large the flow rates become.

For completeness, we have included in the lower portion of Figure 5, the corresponding variations of β, $\bar{v}$, and $\bar{u}$ with E.

Figure Captions

Figure 1: The area in the ϕ-Δ plane in which steady, fully developed, gravity driven flows are possible when r=1, $e=e_w=.5$, and $0<\Delta<.732$.

Figure 2: The variations of β, m_t, $\dot{m}$, $\bar{T}$, $\bar{v}$, E, $\bar{u}$, v, and $u(y=\beta)$ with $T(y=\beta)$ for $\phi=25.5°$, 27.505°, 28° and 34° when r=1, $e=e_w=.5$, and $\Delta=.414$.

Figure 3: The variations of $\dot{m}$, and $\bar{v}$ with $T(y=\beta)$ for $\phi=25.5°$, 27.505°, 28° and 34° when r=1, $e=e_w=.5$, and $\Delta=.414$.

Figure 4: The variations of E and m_t with $\dot{m}$ for $\phi=25.5°$, 27.505°, 28° and 34° when r=1, $e=e_w=.5$, and $\Delta=.414$.

Figure 5: The variations of β, $\bar{v}$, and $\bar{u}$ with $\dot{m}$ and E for $\phi=25.5°$, 27.505°, 28° and 34° when r=1, $e=e_w=.5$, and $\Delta=.414$.

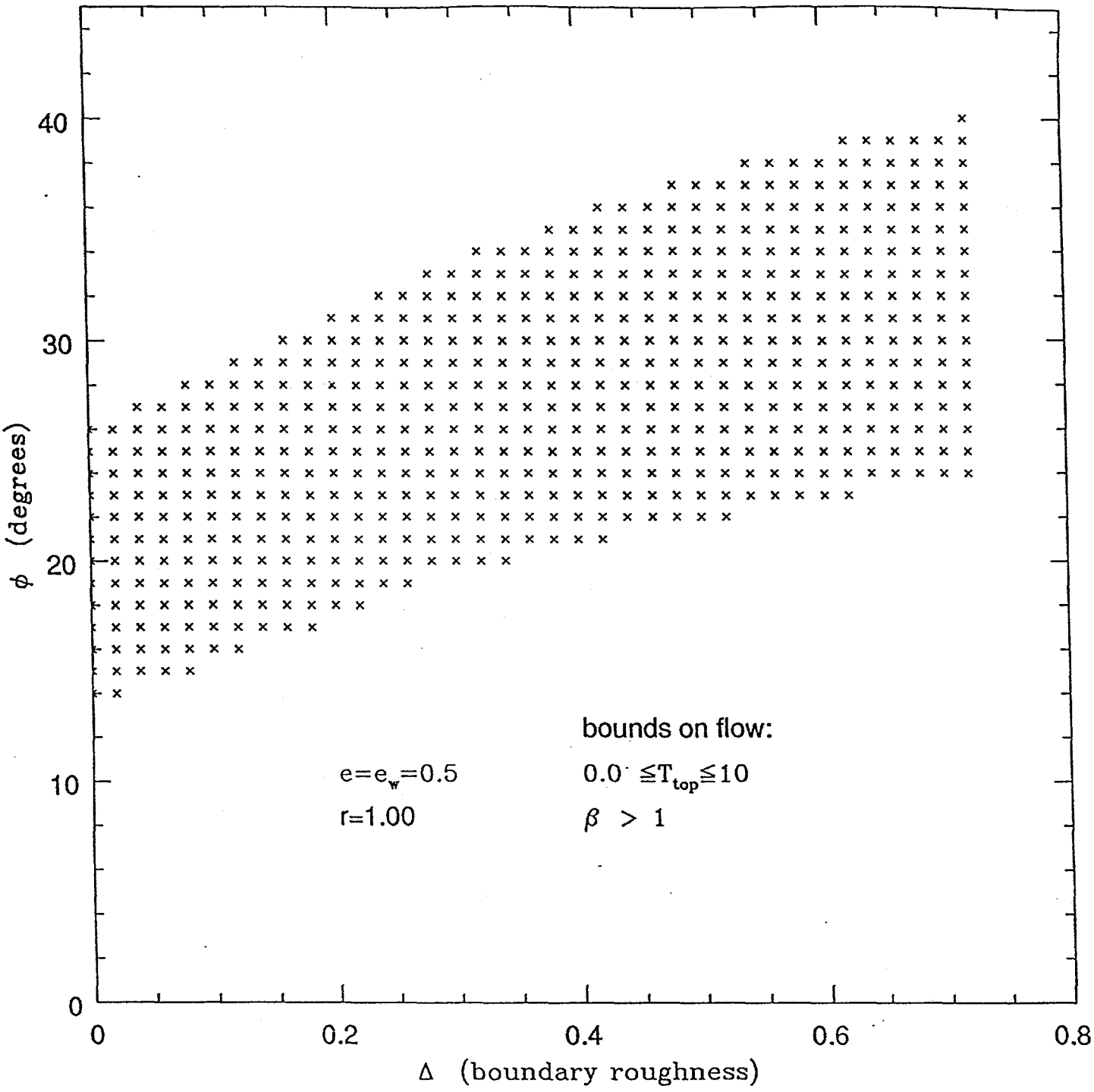


Figure 1: The area in the ϕ-Δ plane in which steady, fully developed, gravity driven flows are possible when r=1, $e=e_w=.5$, and $0<\Delta<.732$.

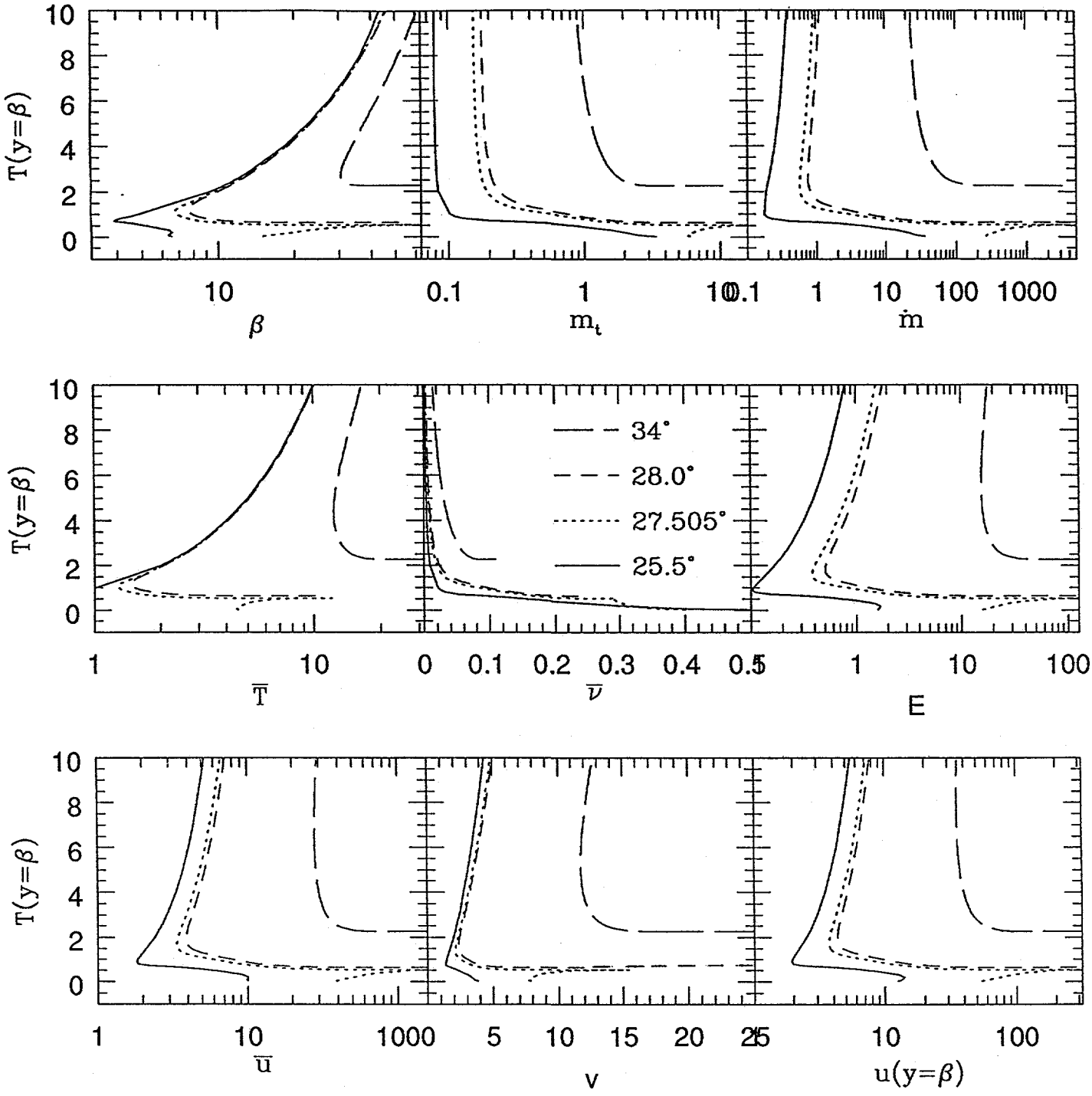
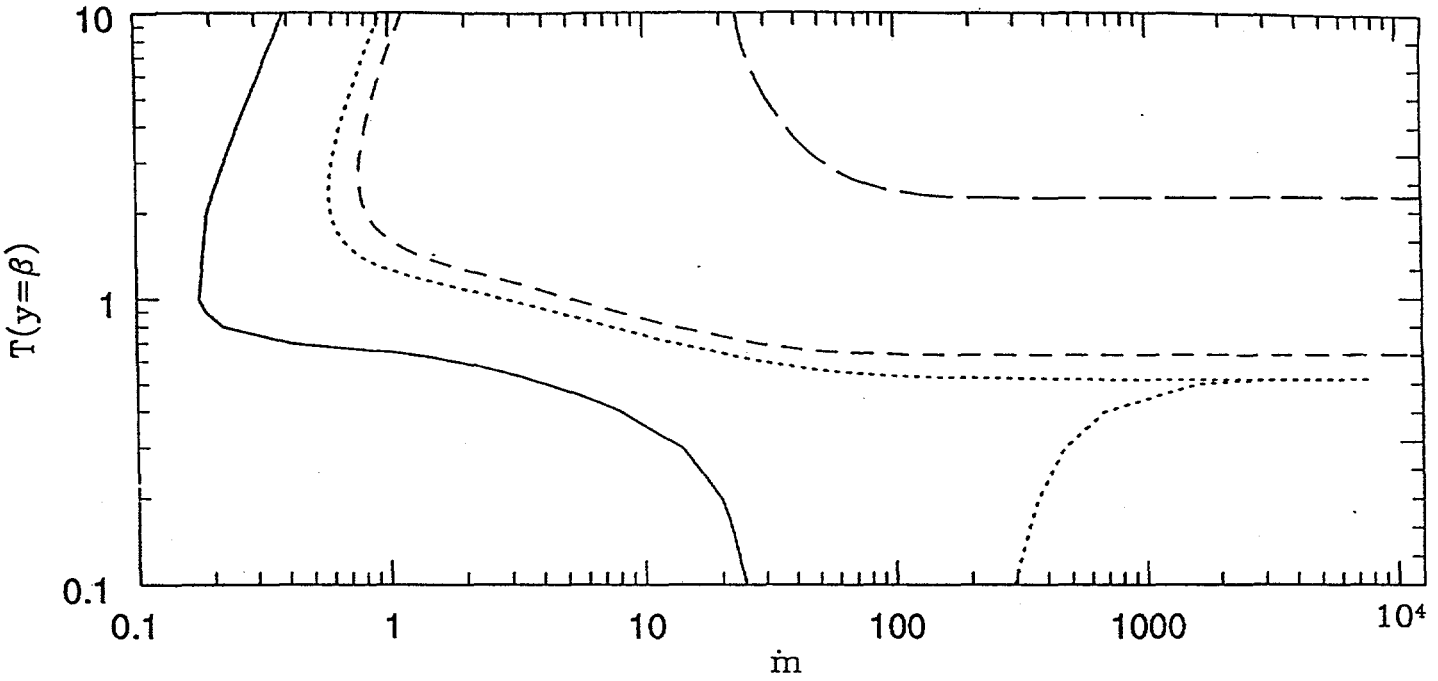


Figure 2: The variations of β, m_t, ṁ, $\bar{T}$, $\bar{\nu}$, E, $\bar{u}$, v, and u(y=β) with T(y=β) for ϕ=25.5°, 27.505°, 28° and 34° when r=1, e=e_w=.5, and Δ=.414.



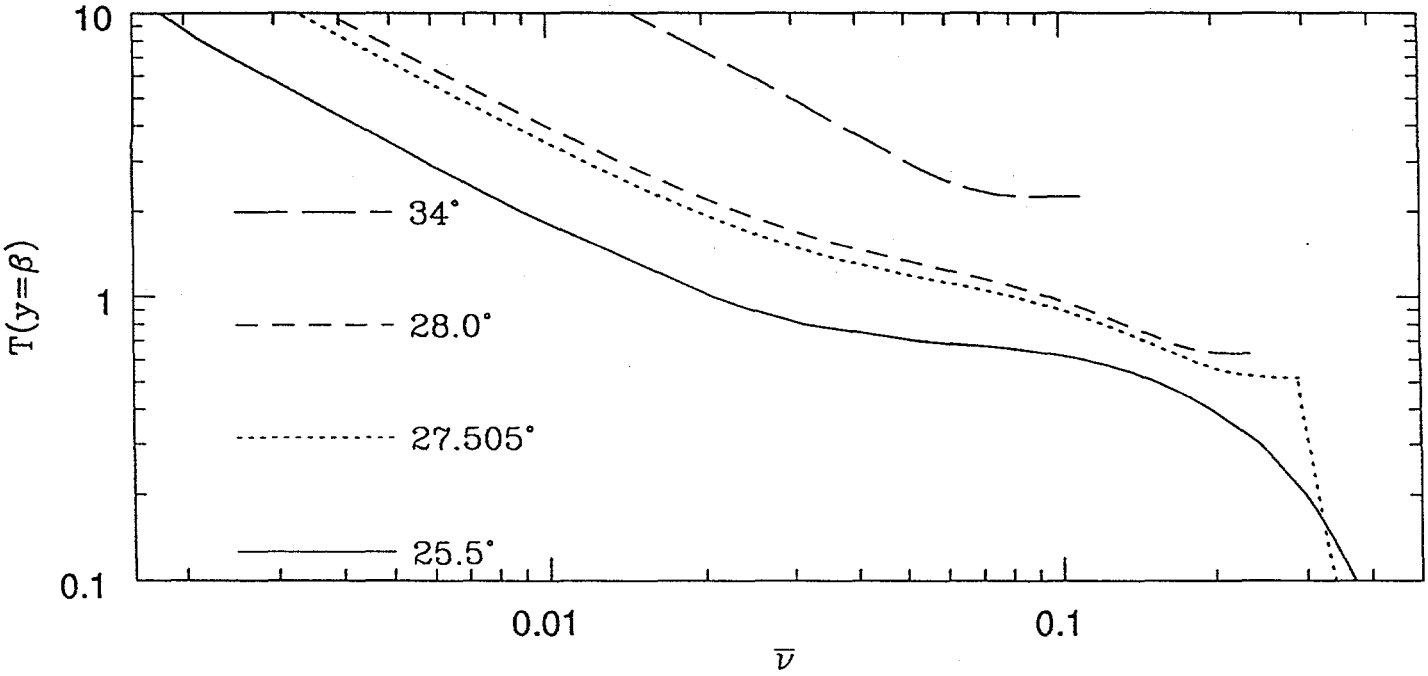


Figure 3: The variations of $\dot{m}$, and $\bar{\nu}$ with T(y=β) for ϕ=25.5°, 27.505°, 28° and 34° when r=1, e=e_w=.5, and Δ=.414.

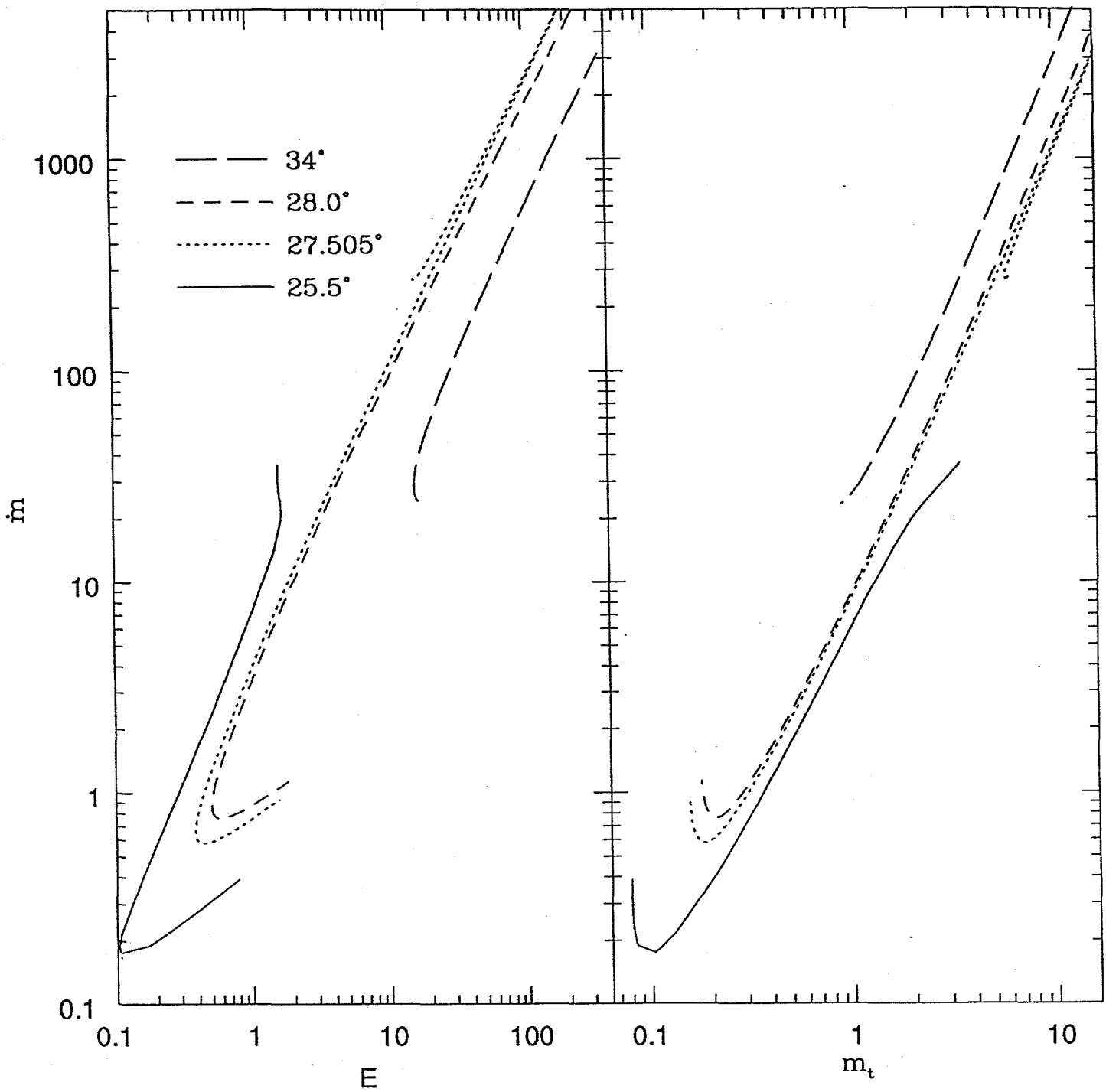


Figure 4: The variations of E and m_t with $\dot{m}$ for ϕ=25.5°, 27.505°, 28° and 34° when r=1, e=e_w=.5, and Δ=.414.

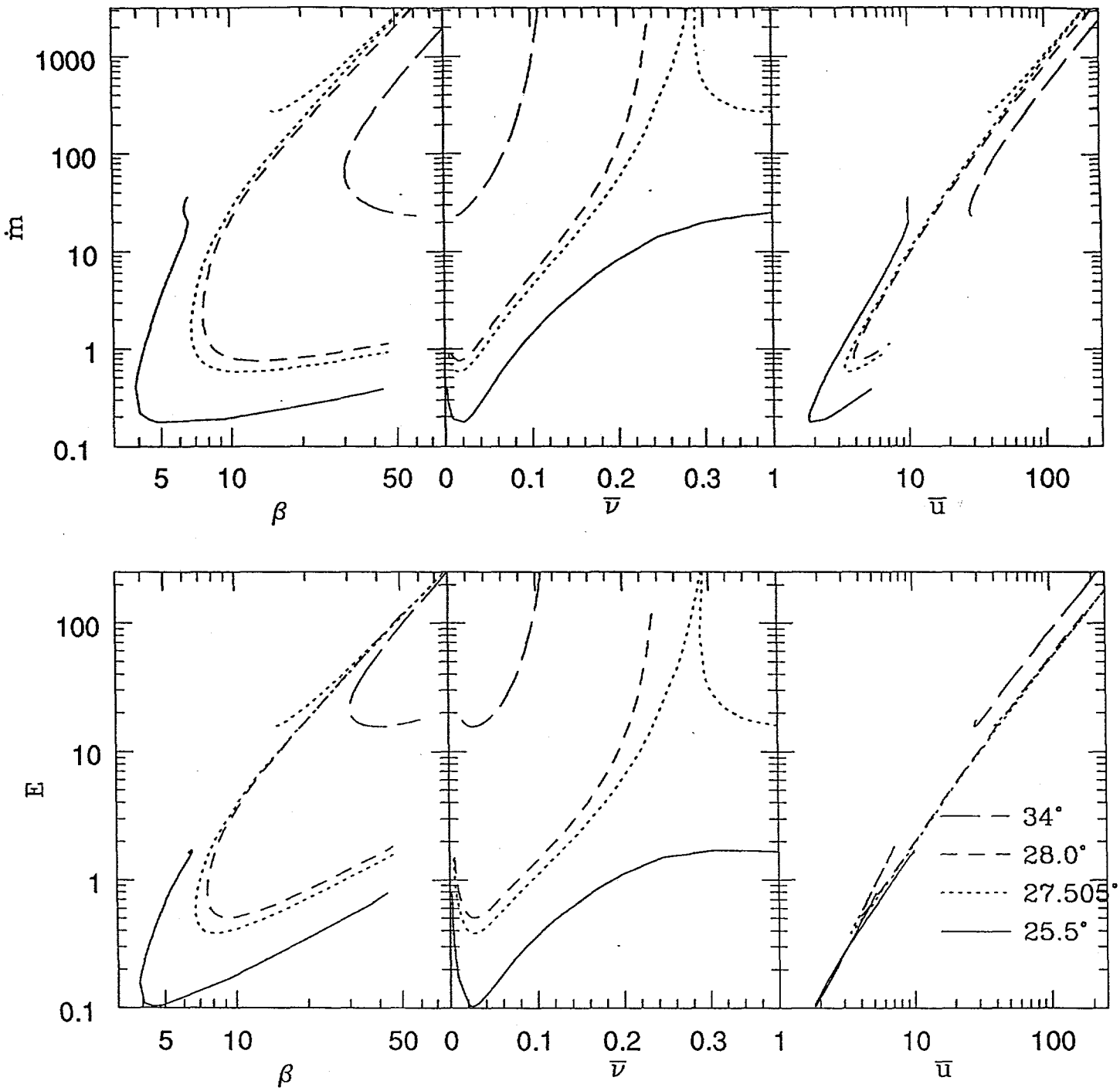


Figure 5: The variations of β, $\bar{v}$, and $\bar{u}$ with $\dot{m}$ and E for ϕ=25.5°, 27.505°, 28° and 34° when r=1, $e=e_w$=.5, and Δ=.414.