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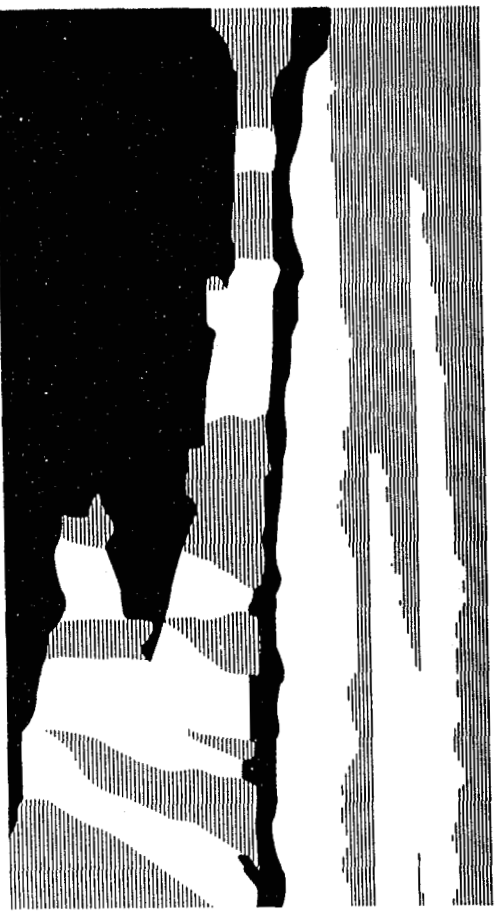
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Title: IDENTIFICATION OF DYNAMIC PROPERTIES FROM AMBIENT VIBRATION MEASUREMENTS

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IDENTIFICATION OF DYNAMIC PROPERTIES FROM AMBIENT VIBRATION MEASUREMENTS

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SUMMARY

The cross-correlation function between two response measurements made on an ambiently excited structure is shown to have the same form as the system's impulse response function. Therefore, standard time-domain curve-fitting procedures such as the complex exponential method, which are typically applied to impulse response functions, can now be applied to the cross-correlation functions to estimate the resonant frequencies and modal damping of the structure. A direct comparison of resonant frequencies identified by curve-fitting the cross-correlation functions, using traffic excitation as the ambient vibration source, and modal properties identified by standard forced vibration testing of a highway bridge, after traffic was removed, showed a maximum discrepancy of 3.63 percent. Similar comparisons for the average modal damping values identified by the two methods showed a 9.82 percent difference. This experimental verification implies that the proposed method of analyzing ambient vibration data has the potential to accurately assess the dynamic properties of large structures subjected to seismic excitations and small structures that are tested on a shake table.

INTRODUCTION

To better understand the dynamic behavior of structures under normal dynamic loads as well as extreme loads such as those caused by seismic events or high winds, it is desirable to measure the dynamic properties (resonant frequencies, mode shapes and modal damping) of these structures. The use of measured input-measured response frequency response functions (FRFs) to identify a structure's dynamic properties is well documented in the technical literature [4]. However, when large structures are subjected to seismic excitation, it is difficult, if not impossible, to measure the input to the structure. The extension of system identification methods to ambient vibration cases, where an input can not be measured, has received considerably less attention in the technical literature. However, the size of most large structures makes ambient vibration testing the only practical experimental method available for studying their dynamic response. This study will focus on the development of an ambient system identification method and its application to a 425 ft (130 m) section of a highway bridge. Results are compared to dynamic properties identified from conventional forced vibration tests conducted after traffic was removed from the bridge.

The technical literature is replete with ambient vibration studies of bridges. One of the earliest attempts to fully characterize the dynamic parameters of bridges undergoing ambient vibrations was reported by McLamore, Hart, and Stubbs [7] using an extension of a spectral technique developed by Crawford and Ward [3]. In this work, the recorded motion of the bridge was measured with a series of accelerometers. Frequencies associated with peaks in the power spectral density function (PSD) of each recorded motion provided estimates of resonant frequencies. The half-power bandwidth method (HPBW) was used to estimate the modal damping associated with these peaks. Amplitude and phase information contained in cross-power spectra (CPS) between a designated reference measurement and the other measurements provided estimates of mode shapes. This method of system identification from ambient response measurements has been summarized more recently in Bendat and Piersol [1]. Following this earlier work, numerous ambient vibration tests, most of which analyze the response measurements in a similar manner have been reported. Drawbacks of the system identification methods used in these studies are the need for very high frequency resolution (the necessary resolution has been quantified by Bendat and Piersol [1] around the resonance to adequately define the half-power points and the difficulties in identifying closely spaced modes because of spectral overlap.

This paper presents a ambient vibration system identification method, referred to as the Natural Excitation Technique (NExT) [6] that circumvents the drawbacks of the methods previously discussed. The NExT method essentially involves applying time domain curve-fitting algorithms to cross-correlation measurements made

between various response measurements on an ambiently excited structure to estimate the resonant frequencies and modal damping. To justify such a system identification procedure, it must be shown that for an input, which is not measured but assumed to be white noise, the cross-correlation function between two response measurements is the sum of decaying sinusoids and these decaying sinusoids have the same damped resonant frequencies and damping ratios as the modes of the system. This result implies the cross-correlation functions will have the same form as the system's impulse response function. Therefore, multi-degree-of-freedom time domain curve-fitting algorithms such as the polyreference method [12] or complex exponential method [2], which were developed to analyze impulse response functions, can be applied to the cross-correlation functions to obtain the resonant frequencies and modal damping exhibited by the structure. These methods attempt to simultaneously identify all modes within a given frequency range and to compensate for the influence of out-of-band modes. Therefore, these curve-fitting methods account for the spectral overlap that has caused problems for the methods used in previous studies. The method proposed has the ability to identify closely spaced modes and, in general, provides a more accurate estimate of damping than the HPBW method. Mode shapes are again determined from magnitudes and phases in the CPS at the identified resonant frequencies.

BASIS OF THE ANALYSIS METHOD

For an n degree of freedom system, the equations of motion can be represented in matrix form as

$$[m]\{\ddot{x}(t)\} + [c]\{\dot{x}(t)\} + [k]\{x(t)\} = \{f(t)\},\tag{1}$$

where $[m]$ = nxn mass matrix,
$\quad\quad [c]$ = nxn damping matrix,
$\quad\quad [k]$ = nxn stiffness matrix,
$\quad\quad \{\ddot{x}(t)\}$ = nx1 acceleration vector,
$\quad\quad \{\dot{x}(t)\}$ = nx1 velocity vector,
$\quad\quad \{x(t)\}$ = nx1 displacement vector, and
$\quad\quad \{f(t)\}$ = nx1 applied force vector.

When proportional damping is assumed and Eq. 1 is transformed into modal coordinates, a set of uncoupled scalar equations of the following form results

$$\ddot{q}^r + 2\zeta^r\omega_n^r\dot{q}^r + \left(\omega_n^r\right)^2 q^r = \frac{1}{m^r}\left\{\phi^r\right\}^T\{f(t)\},\tag{2}$$

where the superscript r denotes values associated with the rth mode, $q, \dot{q},$ and $\ddot{q}$ are the displacement, velocity and acceleration in modal coordinates, ϕ is the mode shape vector, ω_n is the natural frequency, and m is the modal mass. These equations may be solved by the convolution integral, assuming a general forcing function and zero initial condition, and back-transformed into the original coordinates yielding

$$\{x\} = \sum_{r=1}^{n}\left\{\phi^r\right\}^T \int_{-\infty}^{t}\left\{\phi^r\right\}^T\{f(\tau)\}g^r(t-\tau)d\tau,\tag{3}$$

where $g^r(t) = \dfrac{1}{m^r\omega_d^r}e^{-\zeta^r\omega_n^r t}\sin(\omega_d^r t)$ is the impulse response function associated with mode r, ω_d^r is the damped natural frequency associated with mode r, and n is the number of modes.

The response at location i caused by an input at location k, x_{ik} and $f_k(t)$, respectively, can be expressed as

$$x_{ik} = \sum_{r=1}^{n}\phi_i^r\phi_k^r \int_{-\infty}^{t} f_k(\tau)\,g^r(t-\tau)d\tau,\tag{4}$$

where ϕ_i^r is the ith component of the mode shape vector.

If $f(\tau)$ is a Dirac delta function at $\tau=0$, then the response at location i resulting from the impulse at location k is

correlation functions to estimate the system's resonant frequencies and modal damping values. The ambient vibration system identification method is now applied to ambient vibration data from traffic excitation obtained on a highway bridge.

TEST STRUCTURE

The existing I-40 bridge over the Rio Grande consists of twin spans made up of a concrete deck supported by two welded-steel plate girders and three steel stringers. Loads from the stringers are transferred to the plate girders by floor beams located at approximately 20 ft (6.1 m) intervals. Cross-bracing is provided between the floor beams. Each span carries three lanes of traffic under normal operating conditions. Figure 1 shows an elevation view of the portion of the bridge that was tested. Each bridge is made up of three identical sections. Except for the common pier located at the end of each section, the sections are independent. A section has three spans; the end spans are of equal length, approximately 131 ft (39.9 m), and the center span is approximately 163 ft (49.7 m) long. All subsequent discussions of the bridge will refer to the bridge carrying east bound traffic, particularly the three eastern spans, which were the only ones tested. A detailed description of the test structure, the experimental procedures described below, and all results obtained can be found in [5].

AMBIENT VIBRATION TEST PROCEDURE

Twenty-six accelerometers were mounted in the global Y direction shown in Fig. 2. The data acquisition system used in the modal tests consisted of a computer workstation which controlled 29 input modules and a signal processing module. The workstation was also the platform for a commercial data acquisition/signal analysis/modal analysis software package. The input modules provide power to the accelerometers and performed analog to digital conversion of the accelerometer voltage-time histories. The signal processing module performed the needed fast Fourier transform calculations. A 3500 watt generator was used to power this system in the field.

During the tests, traffic had been directed onto the two northern lanes. Significantly different traffic flow could be observed at various times when data was being acquired. During morning and afternoon rush hours the traffic would slow down considerably thus producing lower level excitations in the bridge. At midday the trucks crossing the bridge at high speeds would cause high level excitations that would often over range some of the data acquisition channels. A final ambient vibration test was conducted just prior to the subsequent forced vibration tests when all traffic had been removed from the eastbound bridge. For this test the ambient vibration source was provided by the traffic on the adjacent new eastbound bridge and the existing westbound bridge that was transmitted through the ground to the piers and abutment. During all ambient tests, no attempt was made to characterize the input to the bridge.

The data acquisition system that was used did not estimate cross-correlation functions directly. Instead, CPS were determined from measured acceleration response data and these functions were inverse Fourier transformed to obtain the needed cross-correlation functions. The CPS were calculated with an additional accelerometer located at either S-2 or S-6 in Fig. 2 specified as the reference channel. Sampling parameters were specified that calculated the CPS from 64-s or 32-s time windows discretized with 1024 samples. A Hanning window was applied to the time signals to minimize leakage and AC coupling was specified to minimize DC offsets. Data acquisition occurred over time periods ranging up to almost three hours. Table 1 summarizes the different ambient vibration tests that were conducted.

Table 1				
Ambient vibration test summary				
Test Designation	Frequency Range (Hz)	No. of Averages	Reference Channel[1]	Time
t1tr	0 - 6.25	100	S-2	9:27 AM - 12:17 PM
t10tr	0 - 6.25	35	S-6	2:42 - 3:30 PM
t15tr[2]	0 - 12.5	15	S-2	4:45 - 5:00 PM

[1]See Fig. 3.

[2] This test was performed immediately before forced vibration tests when traffic had been routed to new spans. Ambient excitation was caused by traffic on the adjacent spans.

$$x_{ik} = \sum_{r=1}^{n} \frac{\phi_i^r \phi_k^r}{m^r \omega_d^r} e^{-\zeta^r \omega_n^r t} \sin(\omega_d^r t).$$

(5)

The cross-correlation function $R_{ijk}(t)$ relating two measure responses at locations i and j caused by a white noise random input at k is given by Bendat and Piersol [1] as

$$R_{ijk}(T) = E\{x_{ik}(t+T)x_{jk}(t)\},$$

(6)

where E { } indicates the expectation operator.

Substituting Eq. 4 into Eq. 6 and noting that $f_k(t)$ is the only random variable yields

$$R_{ijk}(T) = \sum_{r=1}^{n} \sum_{s=1}^{n} \phi_i^r \phi_k^r \phi_j^s \phi_k^s \int_{-\infty}^{t} \int_{-\infty}^{t+T} g^r(t+T-\sigma) \ g^s(t-\tau) \ E\{f_k(\sigma)f_k(\tau)\} \ d\sigma d\tau.$$

(7)

Based on the assumption that f(t) is a white noise function and using the definition of the autocorrelation function given in [1], the following relationship can be established for the autocorrelation function of f

$$E\{f_k(\sigma)f_k(\tau)\} = \alpha_k \delta(\tau - \sigma)$$

(8)

where α_k is a constant and $\delta(t)$ is the Dirac delta function.

Substituting Eq. 8 into Eq. 7 and changing the variable of integration to $\lambda = t - \tau$ yields

$$R_{ijk}(T) = \sum_{r=1}^{n} \sum_{s=1}^{n} \alpha_k \phi_i^r \phi_k^r \phi_j^s \phi_k^s \int_{0}^{\infty} g^r(\lambda + T) \ g^s(\lambda) \ d\lambda.$$

(9)

From the previous definition of g^r and the trigonometric identity for the sine of a sum, $g^r(\lambda +T)$ can be expressed with terms involving T separated from those involving λ resulting in

$$g^r(\lambda + T) = \left[e^{-\zeta^r \omega_n^r T} \cos(\omega_d^r T)\right] \frac{e^{-\zeta^r \omega_n^r \lambda} \sin(\omega_d^r \lambda)}{m^r \omega_d^r} + \left[e^{-\zeta^r \omega_n^r T} \sin(\omega_d^r T)\right] \frac{e^{-\zeta^r \omega_n^r \lambda} \cos(\omega_d^r \lambda)}{m^r \omega_d^r}$$

(10)

When Eq. 10 is substituted into Eq. 9 along with the corresponding term for $g^s(l)$, the terms involving T can be factored out of the integral and the summation on s yielding the following form for the cross-correlation function

$$R_{ijk}(T) = \sum_{r=1}^{n} G_{ijk}^r \left[e^{-\zeta^r \omega_n^r T} \cos(\omega_d^r T)\right] + H_{ijk}^r \left[e^{-\zeta^r \omega_n^r T} \sin(\omega_d^r T)\right],$$

(11)

where

$$\begin{Bmatrix} G_{ijk}^r \\ H_{ijk}^r \end{Bmatrix} = \sum_{s=1}^{n} \frac{\alpha_k \phi_i^r \phi_k^r \phi_j^s \phi_k^s}{m^r \omega_d^r m^s \omega_d^s} \int_{0}^{\infty} e^{(-\zeta^r \omega_n^r - \zeta^s \omega_n^s)\lambda} \sin(\omega_d^r \lambda) \begin{Bmatrix} \sin(\omega_d^r \lambda) \\ \cos(\omega_d^r \lambda) \end{Bmatrix} d\lambda.$$

(12)

From Eqs 11 and 12 it is evident that the cross-correlation function between two response measurements that result from an unknown white noise excitation have the form of decaying sinusoids, and these decaying sinusoids have the same characteristics as the system's impulse response function. Therefore time-domain system identification techniques, which are typically applied to impulse response functions, can be applied to these cross-

FORCED VIBRATION TEST PROCEDURE

When traffic was removed from the bridge and the final ambient tests had been complete, forced vibration tests were performed. Eastbound traffic had been transferred to a new bridge just south of the one being tested. The westbound traffic continued on the original westbound bridge. These forced vibration tests were conducted so that results from a conventional experimental modal analysis of the bridge could be compared with the ambient vibration test results. In this context experimental modal analysis refers to the procedure whereby a measured force excitation is applied to a structure and the structure's acceleration response is measured at discrete locations that are representative of the structure's motion. Both the excitation and the response time histories are transformed into the frequency domain so that modal parameters (resonant frequencies, mode shapes, modal damping) can be determined by curve fitting a Laplace domain representation of the equations of motion to the measured frequency domain data [4]. The data acquisition system, mounting blocks, cabling, accelerometers, and generator used for the forced vibration tests were identical to those used for the ambient vibration tests. An additional input module was used to monitor the force input.

A hydraulic shaker was used to generate a measured force input. The shaker consisted of a 21,700 lb (96.5 kN) reaction mass supported by three air springs resting on top of drums filled with sand. A 2200 lb (9.79 kN) hydraulic actuator bolted under the center of the mass and anchored to the top of the bridge deck provided the input force to the bridge. The shaker was located over the south plate girder directly above point S-3 as shown in Fig. 2. A detailed description of the shaker can be found in [8]. Sampling parameters were specified so that responses with frequency content in the range of 0 - 12.5 Hz could be measured. All computed frequency response functions were based on 30 averages with no overlap. A Hanning window was applied to all time samples used in these calculations.

RESULTS

A typical cross-power spectrum between two measurements (reference location S-2 and response location N-7) measured during the ambient vibration test designated t1tr is shown in Fig. 3. The inverse Fourier transform of this measurement yields the cross-correlation function, Fig. 4. In practice, a particular peak in the cross-power spectrum was isolated, as shown in Fig. 3, by zero-padding the spectrum on either side of the peak. The inverse transform of this modified spectrum yields the cross-correlation function, Fig. 5, that was curve-fit to obtain the resonant frequencies and modal damping values. Each peak in the cross-power spectrum was analyzed in this manner. Figures 4 and 5 show the circular nature of the cross-correlation function as discussed by Bendat and Piersol [1]. In practice, only the decaying half of the function shown in Fig. 9 was curve-fit. When closely spaced modes are present, the curve-fitting procedure attempts to fit multiple modes to the portion of the spectrum being analyzed. The cross-correlation functions are based on averaged spectra generated from windowed time-histories. Table 2 summarizes the resonant frequencies and modal damping values determined when the ambient vibration parameter identification method was applied to the cross-correlation functions. Both parameters were calculated in a global manner using a complex exponential curve-fitting method, that is, each measured CPS was inverse transformed and the resulting cross-correlation functions were used to estimate the modal parameters. The mean value of these parameters, obtained from individual analysis of the 26 measurements, was then calculated. These mean values appear in Table 2. A detailed discussion of the curve-fitting procedure is given in [5]. Also shown in Table 2 are the resonant frequencies and modal damping values determined by conventional modal analysis using a measured input. A rational-fraction polynomial global curve-fitting algorithm in a commercial modal analysis software package [9] was used to fit the analytical models to the FRF data and extract resonant frequencies, mode shapes and modal damping values.

The mode shapes for the first three modes identified from amplitude and phase information contained in the CPS measured during the ambient vibration test designated t1tr are shown in Fig 6. Figure 6 also shows the corresponding modes identified from the conventional measured input modal analysis. A modal assurance criterion (MAC), sometimes referred to as a modal correlation coefficient [4] was calculated to quantify the correlation between mode shapes measured during different tests . The matrix listed below in Table 3 show the MACs that compares modes identified from data measured during the ambient test designated t1tr with modes identified during the forced vibration test. This matrix shows that the six modes identified from the ambient vibration data are, in fact, closely correlated with the modes measured during the forced vibration test. Similar correlation was obtained for modes identified from the other ambient vibration tests and modes identified during the forced vibration test. Modes 4 and 5, which are closely spaced in frequency and which were difficult to identify during several ambient vibration tests did not always show good correlation with the modes determined during the forced vibration tests.

Table 2

Resonant frequencies and modal damping values identified from ambient vibration response compared with similar properties identified from conventional forced vibration tests

Test	Mode 1 Freq. (Hz)/ Damp. (%)	Mode 2 Freq. (Hz)/ Damp. (%)	Mode 3 Freq. (Hz)/ Damp. (%)	Mode 4 Freq. (Hz)/ Damp. (%)	Mode 5 Freq. (Hz)/ Damp. (%)	Mode 6 Freq. (Hz)/ Damp. (%)
t1tr	2.39/ 1.28	2.92/ 1.18	3.42/ 1.00	3.96/ 0.94	4.10/ 1.58	4.56/ 1.56
t10tr	2.42/ 1.15	2.93/ 1.18	3.46/ 0.85	3.99/ 0.70	4.12/ 0.59	4.61/ 0.97
t15tr	2.52/ 1.28	3.04/ 0.38	3.53/ 0.89	4.10/ 1.08	4.17/ 0.92	4.71/ 0.60
Forced Vibration Test	2.48/ 1.06	2.96/ 1.29	3.50/ 1.52	4.08/ 1.10	4.17/ 0.86	4.63/ 0.92

Table 3
Modal assurance criteria:
mode shapes identified from ambient vibration Test t1tr compared with mode shapes identified from forced vibration test

Mode/test	1/Forced	2/Forced	3/Forced	4/Forced	5/Forced	6/Forced
1/t1tr	0.989	0.008	0.000	0.004	0.002	0.001
2/t1tr	0.004	0.985	0.000	0.001	0.001	0.004
3/t1tr	0.002	0.003	0.984	0.000	0.009	0.001
4/t1tr	0.005	0.002	0.001	0.901	0.102	0.009
5/t1tr	0.000	0.001	0.005	0.066	0.917	0.005
6/t1tr	0.001	0.003	0.002	0.004	0.004	0.984

SUMMARY AND CONCLUSIONS

In this study the cross-correlation function between two response measurements made on an ambiently excited structure was shown to have the form of decaying sinusoids, similar to the system's impulse response function. The significance of this derivation is that standard time-domain curve-fitting procedures such as the complex exponential method, which are typically applied to impulse response functions, can now be applied to the cross-correlation functions to estimate the resonant frequencies and modal damping of the structure. The advantage of this system identification method over standard procedures that identify resonant frequencies from peaks in the power spectrum and damping from the width of the power spectrum is the ability to identify closely spaced modes and their associated damping. The ambient vibration system identification method was applied to an in-service highway bridge where traffic provided the vibration source. Although the assumption that the traffic on a bridge produces a white noise input was not verified; the random weights of vehicles, as discussed by Turner and Pretlove [11]; their random arrival times; the random nature of the vehicles' suspension systems; and the randomly distributed road surface irregularities suggest that this assumption is valid. Subsequently, after traffic had been re-routed, the same bridge was tested with conventional measured-input force vibration procedures. Results from these tests allow the following conclusions to be made:

1. Ambient vibration from traffic provides an adequate source of input for identifying the dynamic properties of the bridge. The results obtained in this study were repeatable (resonant frequency values measured with traffic on the bridge did not vary more than 2%) and were independent of the selected reference measurement.

2. The method presented in this paper allowed closely spaced modes (0.07 Hz or approximately 2.24 Δf apart) such as modes 4 and 5 to be identified, and this method identified the associated modal damping values. However, both the ambient system identification method and the conventional measured-input system identification method were able to identify the dynamic properties associated with these modes.

3. All measured modes were lightly damped with modal damping values ranging from 0.38 to 1.58%. These modes can be accurately approximated as real modes. Phase angles of the CPS associated with the resonant

frequency were typically close to either 0 or 180 degrees. An average modal damping value of 1.01 percent was obtained from the three ambient tests. The forced vibration test yielded an average modal damping value o 1.12 percent, a 9.82 percent difference from the ambient results. The identified values are consistent with those obtained by other investigators for similar bridges as summarized by Tilly [10].

4. During test t15tr, when traffic was not on the bridge, and during the forced vibration test, generally higher frequencies were measured for each mode as compared to the results from tests when traffic was on the bridge. These higher frequencies are attributed to the reduced mass when traffic was removed from the bridge.

5. Background sources of ambient vibration from traffic on the adjacent bridges were of sufficient magnitude that the dynamic properties of the structure could be determined by measuring the response to this excitation source as was done in test t15tr.

This experimental verification of the accuracy of the ambient vibration system identification method implies that the proposed method can be used to accurately assess the dynamic properties of bridges and other large structures in a non-intrusive manner. Further studies are necessary to assess the effects of the non-white-noise nature of seismic inputs on the accuracy of this method.

ACKNOWLEDGMENTS

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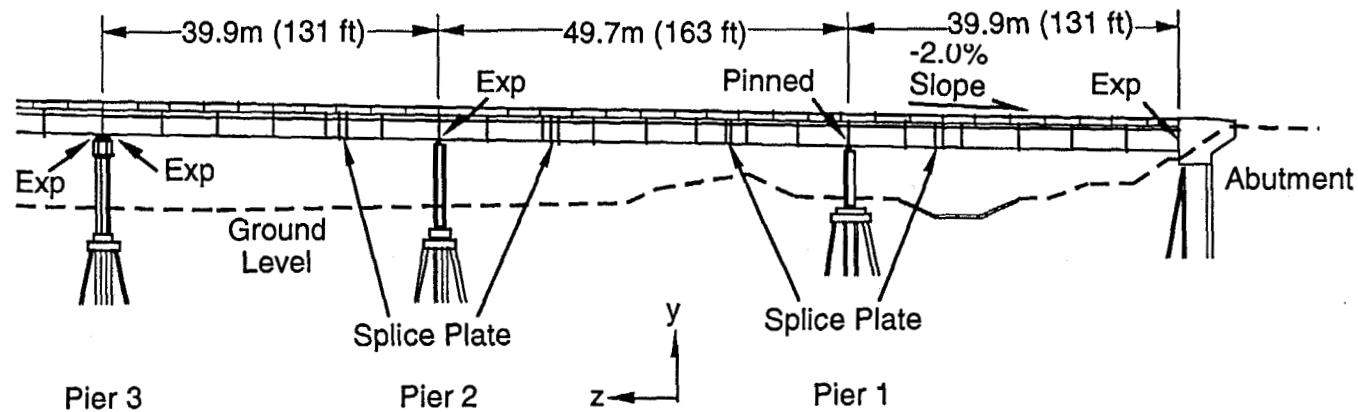
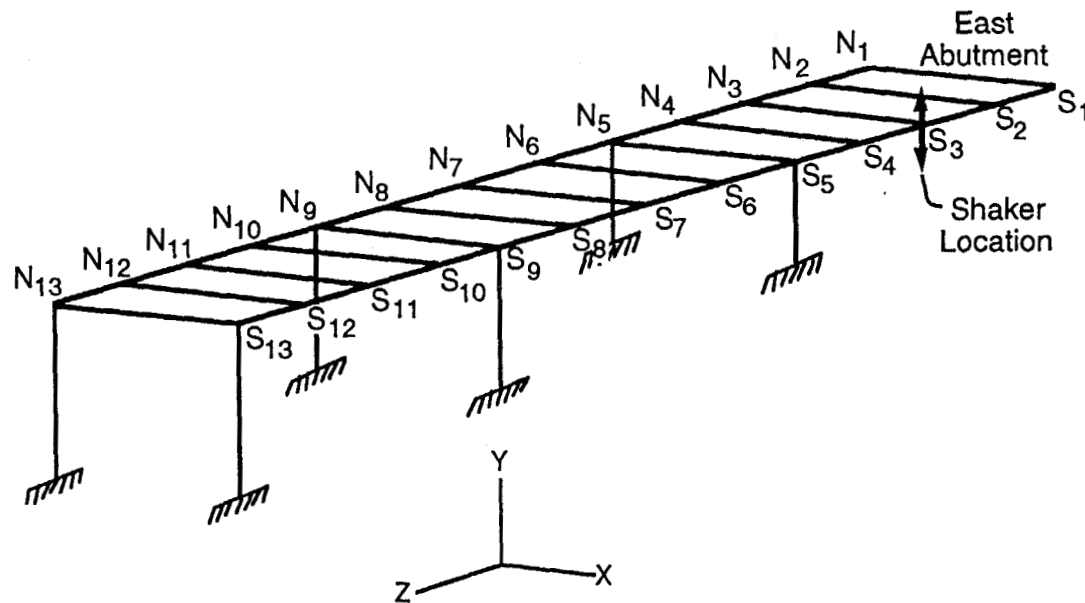Figure 1. Elevation view of the portion of the eastbound bridge that was tested.

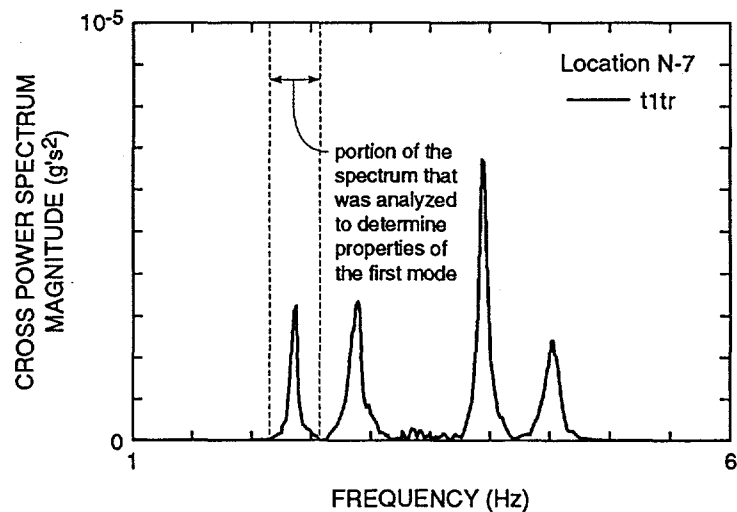Figure 2. Accelerometer locations.

Figure 3.   Cross-power spectrum between ambient vibration measurements made at locations S-2 and N-7.
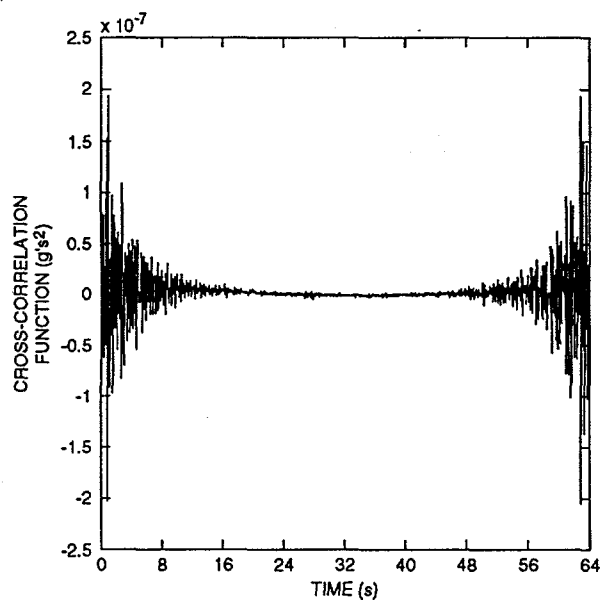

Figure 4.   The cross-correlation function obtained from the inverse Fourier transform of the CPS shown in Fig. 3.
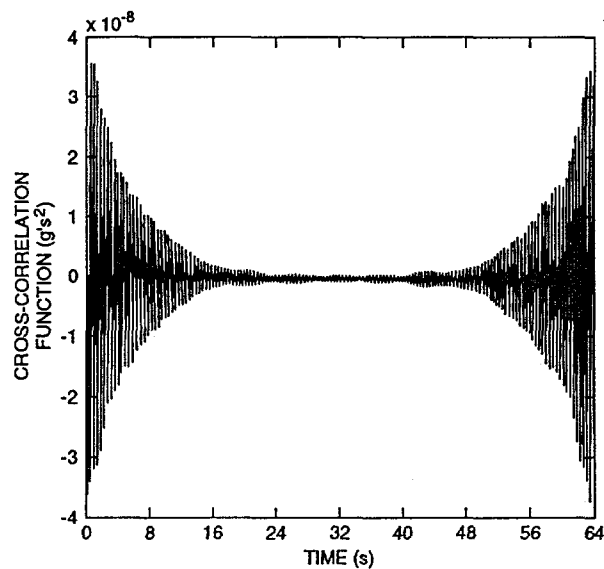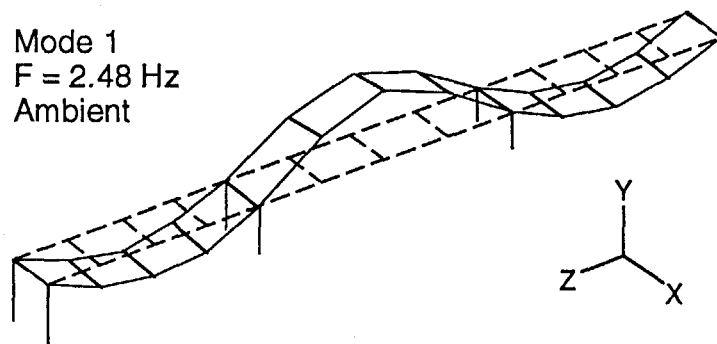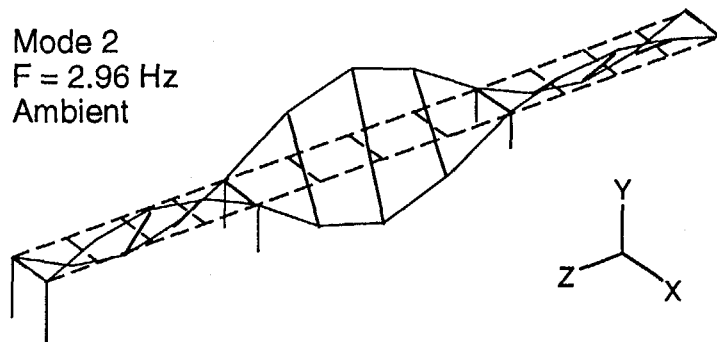

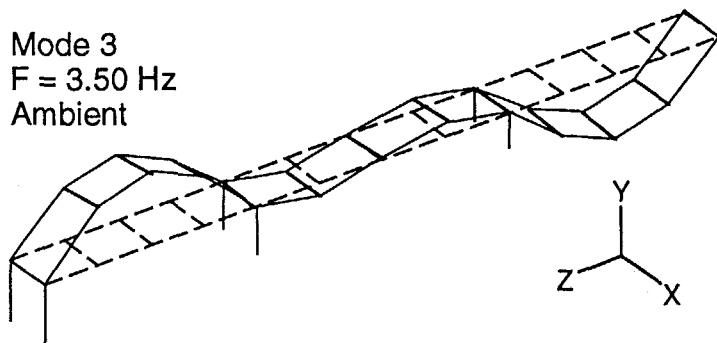Figure 5.   The cross-correlation function corresponding to the isolated portion of the spectrum shown in Fig. 3.
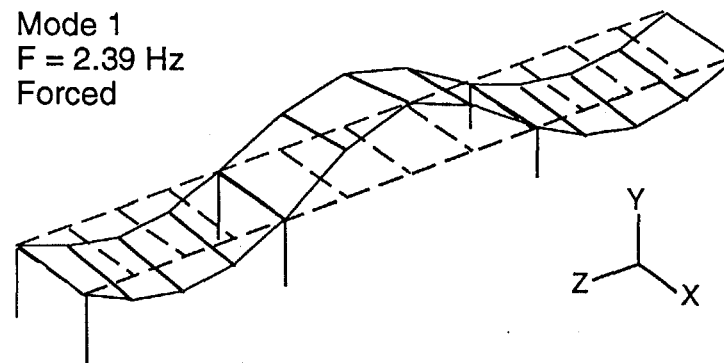
Mode 1
F = 2.48 Hz
Ambient

Mode 1
F = 2.39 Hz
Forced

Mode 2
F = 2.96 Hz
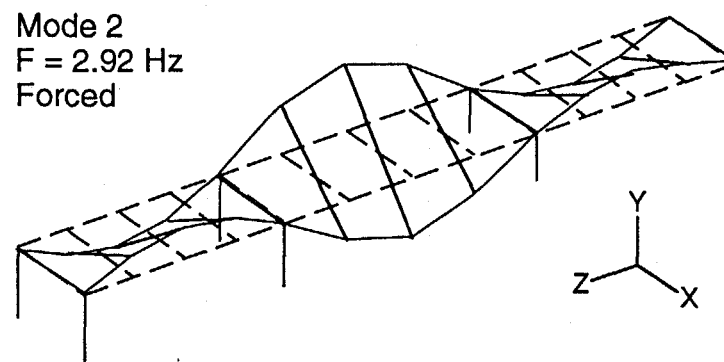Ambient

Mode 2
F = 2.92 Hz
Forced

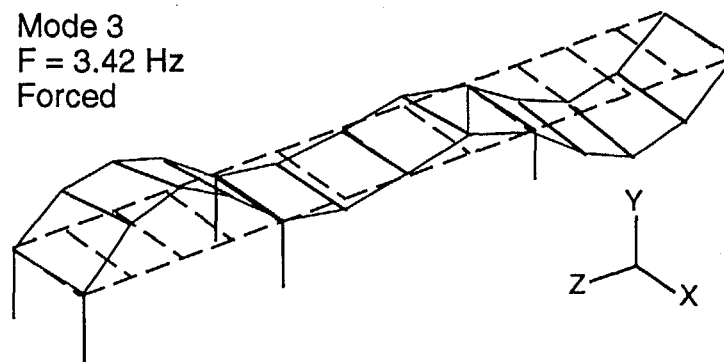Mode 3
F = 3.50 Hz
Ambient

Mode 3
F = 3.42 Hz
Forced

Figure 6.   The first three modes identified during ambient vibration tests compared to the first three modes identified during the conventional measured forced input vibration test.