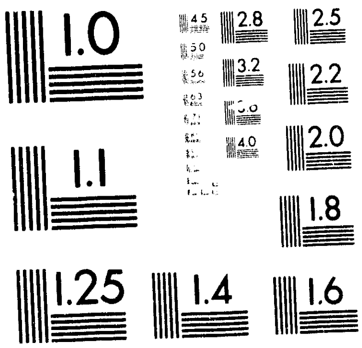

1.0
2.8
2.5
3.2
2.2
2.0
1.1
1.8
1.25
1.4
1.6

Conf-930312b--10

UCRL-JC-114331
PREPRINT

FREEZE-OUT AND THE FAILURE OF RICHTMYER'S PRESCRIPTION

Karnig O. Mikaelian
University of California
Lawrence Livermore National Laboratory
Livermore, California 94550

This paper was prepared for the proceedings of the
4th International Workshop
on the Physics of Compressible Turbulent Mixing
held in Cambridge, England, March 29 - April 1, 1993

April 23, 1993

Lawrence Livermore National Laboratory

FREEZE-OUT AND THE FAILURE OF RICHTMYER'S PRESCRIPTION

Karnig O. Mikaelian
University of California
Lawrence Livermore National Laboratory
Livermore, California 94550

In the standard Richtmyer-Meshkov[1,2] (RM) instability perturbations at a shocked interface grow after the passage of a shock. Freeze-out refers to the phenomenon whereby the perturbations do **not** grow, i.e., freeze-out, after the passage of a shock. This is fairly straightforward, at least theoretically (no experiments have been done so far) in a doubly shocked system: The first shock induces a growth which can be completely neutralized by a second shock, provided that the direction and the strength and timing of the second shock are properly chosen (see ref. 3). This type of double-shock freeze-out occurs in compressible as well as incompressible fluids, and is easy to understand.

Somewhat more subtle is single-shock freeze-out; in our pursuit of this phenomenon we found that in certain cases Richtmyer's prescription fails to give the correct growth rate.

Fig. 1 shows the system and our notation: An incident shock moves at speed W_i from fluid A into fluid B having densities ρ_A and ρ_B and specific heat ratios γ_A and γ_B respectively. The interface between A and B has a sinusoidal perturbation of wavelength λ and initial amplitude $\eta_0 \equiv \eta_{before}$. After the shock strikes the interface the perturbation $\eta(t)$ grows linearly in time at a rate given by

$$\frac{\dot{\eta}}{\eta_0 \Delta v k} = \text{NGR} = A_{effective} \qquad (1)$$

where Δv is the jump velocity of the interface ($\Delta v = u_1 = u_2$ in Fig. 1), $k = 2\pi/\lambda$, and NGR stands for Normalized Growth rate, a dimensionless quantity. Eq. (1) is limited to the linear regime, i.e., $\eta k << 1$, the regime considered by Richtmyer.

The question is: What is $A_{effective}$? For incompressible fluids

$$A_{effective}=A_{before}=\frac{\rho_B-\rho_A}{\rho_B+\rho_A}\ . \tag{2}$$

Of course $A_{before}=A_{after}$ and $\eta_{before}=\eta_{after}$ for incompressible fluids, i.e., the Atwood number as well as the amplitude immediately after the shock are the same as before the shock: There is no compression. In this case freeze-out becomes completely trivial: the NGR in Eq. (1) vanishes if and only if $\rho_B=\rho_A$, i.e., for identical fluids. The same is true for the Rayleigh-Taylor instability also.

The situation is not so trivial for compressible fluids: One can have $\rho_A=\rho_B$ with $\gamma_A\neq\gamma_B$. In fact Richtmyer's prescription for $A_{effective}$ is:

$$A_{effective}=A_{after}\times \text{Compression Factor}$$
$$=\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)\left(1-\frac{\Delta v}{W_i}\right) \tag{3}$$

which he gave on the basis of three numerical examples.[1] His prescription was to use $A_{after}\eta_{after}$ instead of $A_{before}\eta_{before}$ in the classical incompressible result $\dot{\eta}=\Delta v k A \eta_0$ which he had obtained first by treating the shock as an instantaneous acceleration. In Eq. (3) $A_{after}=(\rho_1-\rho_2)/(\rho_1+\rho_2)$ and the compression factor is $\eta_{after}/\eta_0=1-\Delta v/W_i$.

Therefore, we should expect freeze-out **not** when $A_{before}=0$ (incompressible result), but when $A_{after}=0$, according to Richtmyer's prescription. This is easy to achieve if fluid A is highly compressible (following Richtmyer we are considering cases where $\rho_A<\rho_B$): a highly compressible low density fluid can be shocked to a density equal to or greater than the density of a less compressible fluid even though the latter is initially more dense. Referring to Fig. 1, one can start with $\rho_A<\rho_B$ yet achieve $\rho_1=\rho_2$ (freeze-out) or even $\rho_2>\rho_1$ (Atwood number reversal) provided that fluid A is much more compressible than B.

An example with γ_A=1.1 and γ_B=5/3 is shown in Fig. 2 where we plot A_{after} and the compression factor as functions of ε for various initial Atwood numbers: A_{before}=0.0, 0.25, 0.5, 0.75, and 0.95. Here ε is a measure of shock strength,

$$\varepsilon = 1 - \frac{p_0}{p_3},$$

related to the Mach number of the shock via

$$M_s = \sqrt{1 + \frac{\gamma_A + 1}{2\gamma_A}\left(\frac{\varepsilon}{1-\varepsilon}\right)} \; .$$

Clearly, for $\varepsilon \to 0$ the compression factor $\to 1$ and $A_{after} \to A_{before}$. As ε increases A_{after} decreases because fluid A compresses more than fluid B and A_{after}, in some cases, passes through zero and is indeed negative at higher ε. The compression factor is always positive, as seen in Fig. 2b. If we concentrate on the case A_{before}=0.5, we see from Fig. 2a that $A_{after} \leq 0$ for $\varepsilon \geq 0.87$. Therefore according to Richtmyer's prescription, Eq. (3), we should have freeze-out ($\dot{\eta}=0$) at ε=0.87, and phase reversal ($\dot{\eta}<0$) at higher ε.

Direct numerical simulations, however, do **not** confirm this behaviour. In Figs. 3 and 4 we show the cases ε=0.87 and ε=0.95 respectively. Clearly, there is no freeze-out in Fig.3 nor phase reversal in Fig. 4. The time scale, microseconds, in these figures follow from setting $\rho_0 = \rho_{atmosphere}$ and $p_0 = p_{atmosphere}$, while the dimensions, centimeters, follow those of the CalTech shock-tube.[4] We should point out, however, that such strong shocks cannot be generated at the CalTech shock-tube.

These and other examples given in ref. 5 show that Richtmyer's prescription fails in certain cases. The cause of this failure is not clear, and we emphasize that it is **not** the strength of the shock. In fact the examples which Richtmyer considered were strong shock problems, and they all agree with his prescription, which we also verified. An example of a weak shock, ε=0.4, which does not agree with Richtmyer's prescription is given in ref. 5. For weak shocks his prescription can be written explicitly as

$$A_{effective}=A_{before} + \varepsilon F/\gamma_A + \text{terms of order } \varepsilon^2 + \ldots . \quad (4)$$

where

$$F=2\left[1-R+2\frac{R(R-y^2)}{y(R+1)}\right](R+1)^{-1}(y+1)^{-1} , \quad (5)$$

with the definitions $R\equiv\rho_B/\rho_A$ and $y\equiv(R\gamma_B/\gamma_A)^{1/2}$.

In contrast, we found that an analysis of the RM problem given by Fraley[6] agreed with our simulations. Richtmyer's prescription and Fraley's analysis both give the correct limit, $A_{effective}\rightarrow A_{before}$ as $\varepsilon\rightarrow 0$, i.e., they both reproduce the expected leading term A_{before} in the expansion given in Eq. (4). The next term, however, is different; Fraley's expression for F, after correcting a misprint, is[6]

$$F=\left[(y-1)^2+4\frac{R^2+y^2}{y(R+1)} -2R-2y\right](R+1)^{-1}(y+1)^{-1} \quad (6)$$

to be compared with Eq. (5).

Two cases naturally come to mind in comparing Eq. (5) with Eq. (6): When $\gamma_A=\gamma_B$, Eq. (5) gives

$$F(\gamma_A=\gamma_B)=-2(\sqrt{R} - 1)/(R+1) \quad (7a)$$

while Eq. (6) gives

$$F(\gamma_A=\gamma_B)=-(\sqrt{R} - 1)/(R+1) , \quad (7b)$$

i.e., half of Eq. (7a). One must remember, however, that F is the coefficient of the ε/γ_A term in Eq. (4) for small ε and therefore can be easily masked if the leading term A_{before} is large.

The second case is $A_{before}=0$, i.e., R=1, for which Eq. (5) gives

$$F(\rho_A=\rho_B) = -1 + \frac{1}{y} \quad (8a)$$

while Eq. (6) gives

$$F(\rho_A=\rho_B) = (-1 + \frac{1}{y})(1 - \frac{y}{2}) . \quad (8b)$$

Note that for y=2 Eq. (8a) gives -0.5 while Eq. (8b) gives 0. Since the leading term $A_{before}=0$, we conclude that $A_{effective}$ is negative according to Richtmyer's prescription while it is zero according to Fraley's analysis. Examples of this type are reported in ref. 5 where we did find freeze-out ($\dot{\eta}=0$) for a case with $A_{before}=0$ but $A_{after}\neq 0$.

In summary, we found a case of freeze-out which, according to Richtmyer's prescription, should not have been there: $\dot{\eta}=0$ while $A_{after}\neq0$. Conversely, freeze-out cases expected from Richtmyer's prescription, i.e., $A_{after}=0$, did not exhibit freeze-out, an example of which is Fig. 3. Nor did the case of Atwood number reversal, Fig. 4, exhibit phase reversal. This lack of correlation between the sign of A_{after} and the sign of $\dot{\eta}$ is, we believe, definite proof that Richtmyer's prescription fails in certain cases. We should add that Fraley's analysis passed these tests, correctly showing where freeze-out should occur and where it should not. Equally if not more important is the fact that Richtmyer's prescription gives an accurate result in many cases, in particular for previous experiments, physical or numerical, starting of course with Richtmyer's own numerical experiments. There are 4 free variables in the RM problem: ρ_A/ρ_B, ε, γ_A and γ_B. Unfortunately we have no simple recipe for finding where in this 4-dimensional space Richtmyer's prescription is guaranteed to work. The combination of high Atwood numbers, weak shocks and high γ's appears to be a safe bet.

ACKNOWLEWDGMENT

This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48.

REFERENCES

[1]R. D. Richtmyer, "Taylor instability in shock acceleration of compressible fluids," Commun. Pure Appl. Math **13**, 297 (1960).

[2]E. E. Meshkov, "Instability of the interface of two gases accelerated by a shock wave," Fluid Dynamics **4**, 101 (1969) (trans. from Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza **5**, 151 (1969)).

[3]K. O. Mikaelian, "Richtmyer-Meshkov instabilities in stratified fluids," Phys. Rev. A **31**, 410 (1985).

[4]M. Brouillette, "On the interaction of shock waves with contact surfaces between liquids of different densities," Ph. D. Thesis, California Institute of Technology, 1989; R. Bonazza, "X-ray measurements of shock-induced mixing at an Air/Xenon interface," Ph. D. Thesis, California Institute of Technology, 1992.

[5]K. O. Mikaelian, "Freeze-out and the effect of compressibility in the Richtmyer-Meshkov instability," LLNL report UCRL-JC-112495 (January 1993).

[6]G. Fraley, "Rayleigh-Taylor stability for a normal shock wave-density discontinuity interaction," Phys. Fluids **29**, 376 (1986).

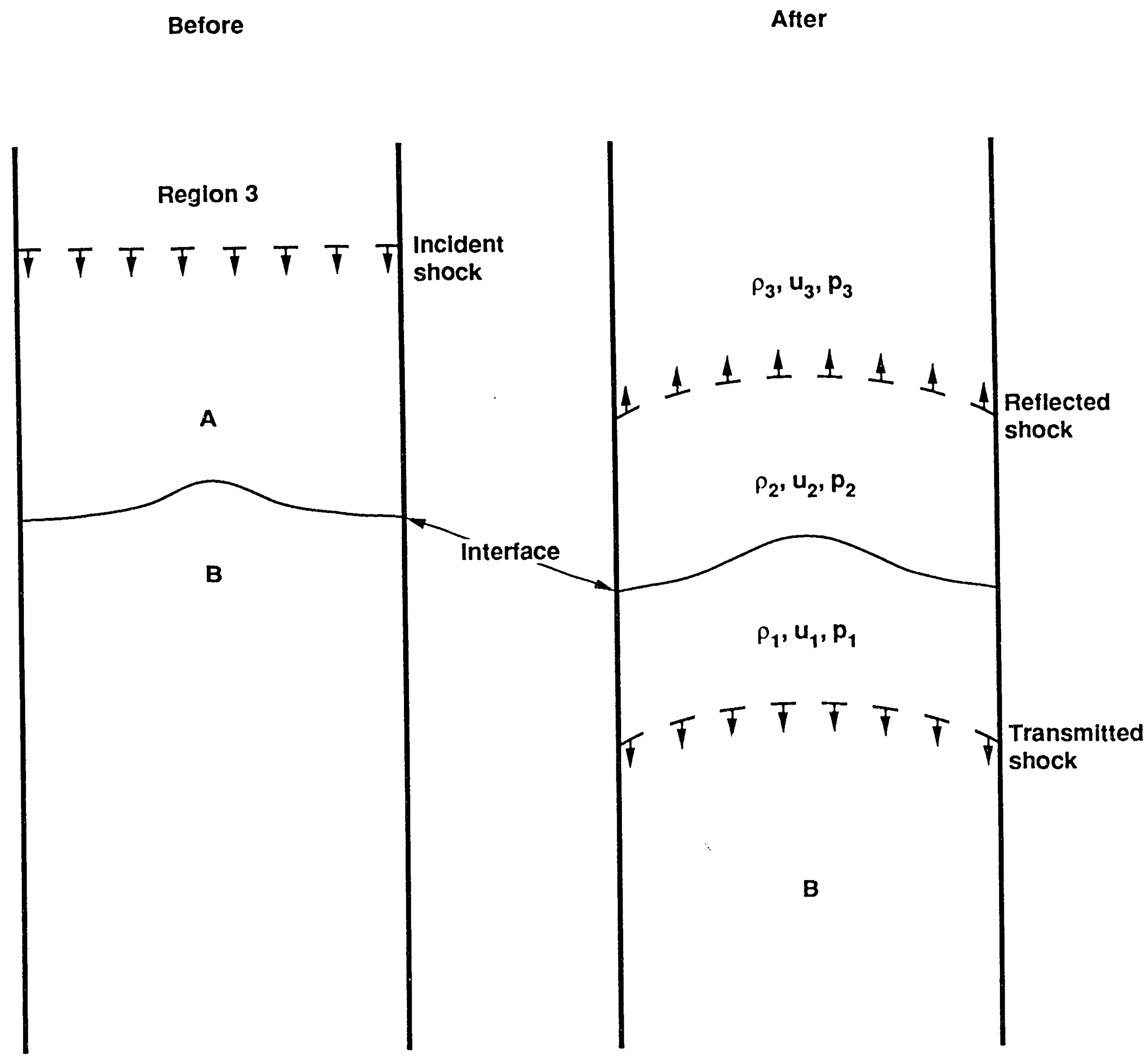

Before
After
Region 3
Incident
shock
ρ_3, u_3, p_3
A
Reflected
shock
ρ_2, u_2, p_2
Interface
B
ρ_1, u_1, p_1
Transmitted
shock
B

Fig. 1

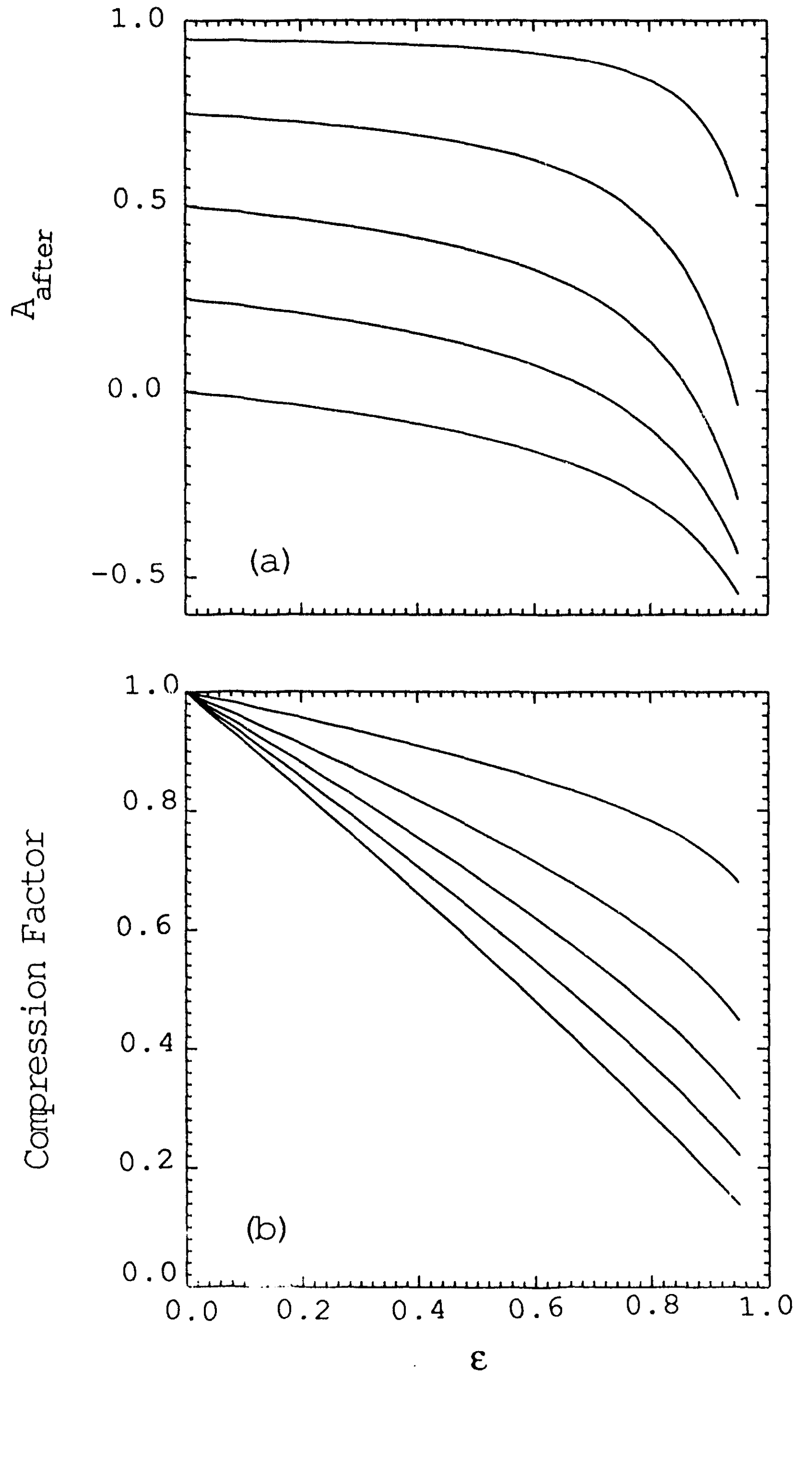
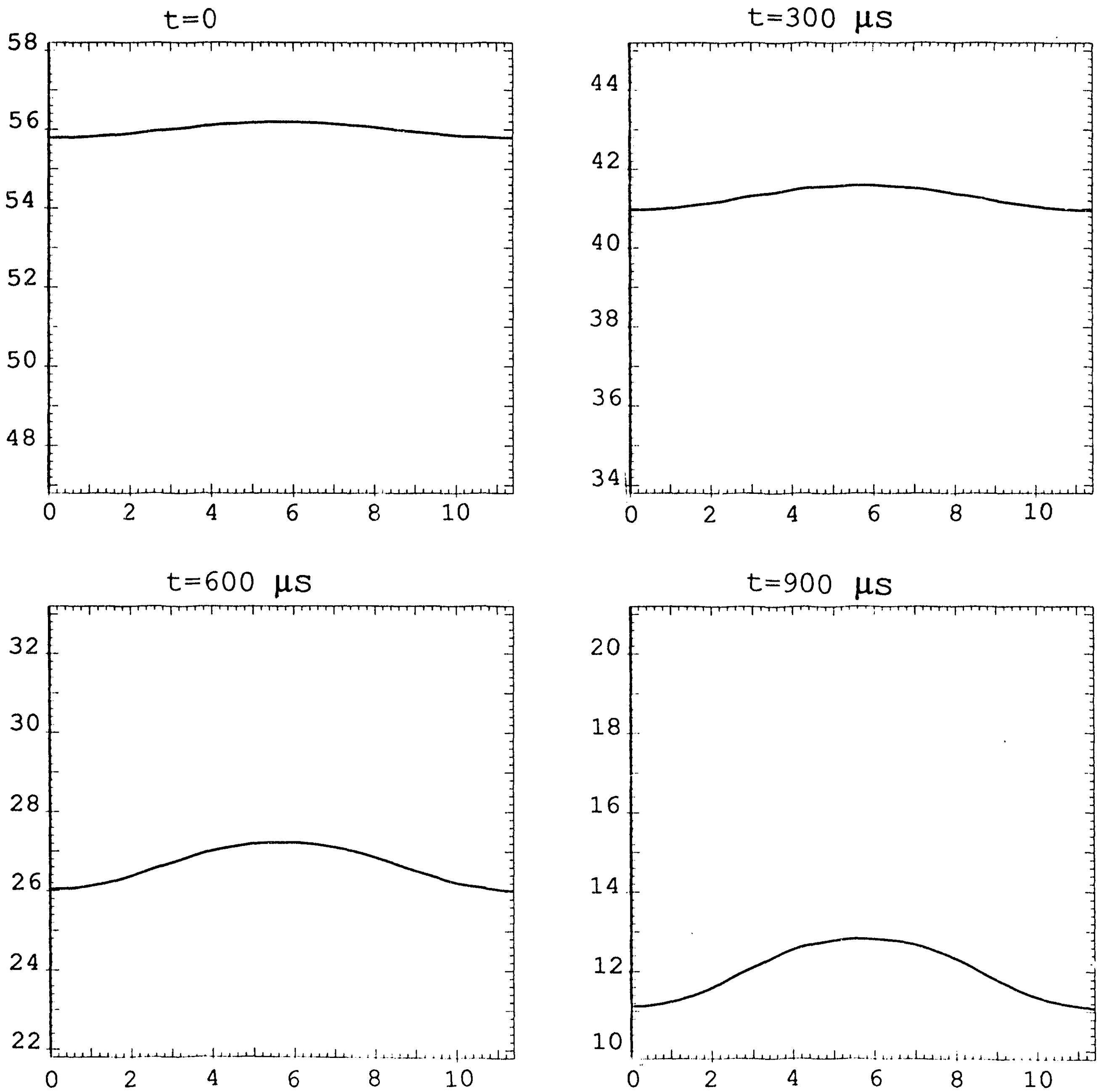

1.0
0.5
0.0
-0.5
A_after
(a)
1.0
0.8
0.6
0.4
0.2
0.0
Compression Factor
(b)
0.0
0.2
0.4
0.6
0.8
1.0
ε

Fig. 2



t=0
58
56
54
52
50
48
0
2
4
6
8
10
t=300 µs
44
42
40
38
36
34
0
2
4
6
8
10
t=600 µs
32
30
28
26
24
22
0
2
4
6
8
10
t=900 µs
20
18
16
14
12
10
0
2
4
6
8
10

Fig. 3

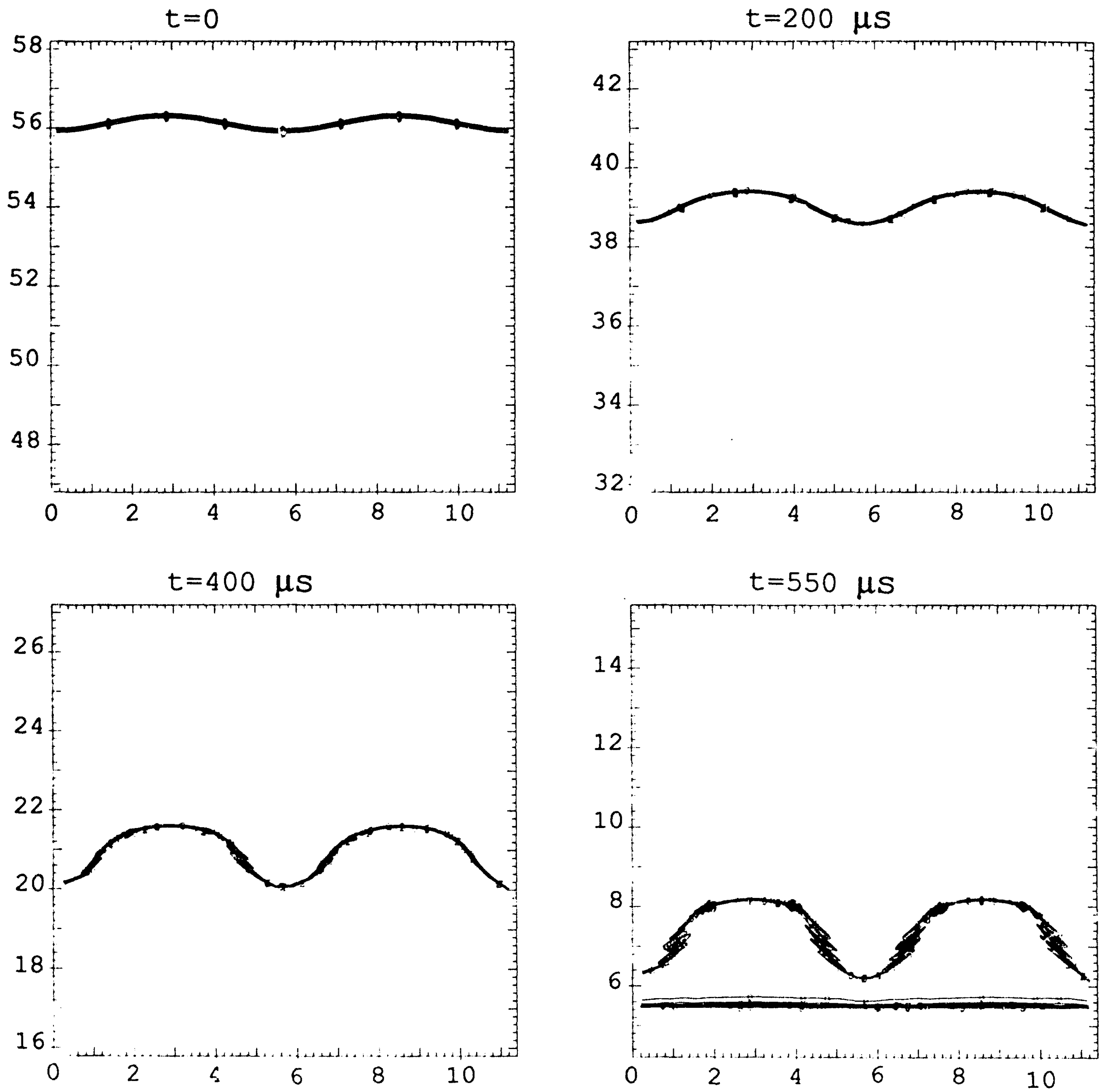

t=0
58
56
54
52
50
48
0
2
4
6
8
10
t=200 μs
42
40
38
36
34
32
0
2
4
6
8
10
t=400 μs
26
24
22
20
18
16
0
2
4
6
8
10
t=550 μs
14
12
10
8
6
0
2
4
6
8
10

Fig. 4

DATE
FILMED
11 / 26 / 93
END